Numerical modelling for interpretation of stress measurements by overcoring

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ABSTRACT

The overcoring method for measuring in situ stresses is frequently used in rock engineering projects. Most models for its interpretation assume that the rock is homogeneous, linear elastic and isotropic. The elastic constants are determined by various methods, namely by biaxial tests on the recovered core.

A three-dimensional finite element model was developed, which allows to compute the stress state from the strain measurements obtained during overcoring, assuming a transversely-isotropic behaviour of the rock mass. Different testing methodologies for elastic constants determination of anisotropic rocks are introduced and discussed. Several numerical tests were performed to analyse the influence of the weakness planes that sometimes occur in the rock core owing to the tensile strains that are generated during the biaxial test. To avoid the tensile strains in the biaxial test, a new triaxial testing procedure of the overcored rock sample with the cell was developed. The effect of the degree of rock anisotropy on in situ stress determination and the error involved in neglecting rock anisotropy are analysed. Differences on the principal stresses and principal directions calculated considering the usual isotropic model and the developed numerical model for a real case of anisotropic rock are presented.

1 INTRODUCTION

The knowledge of the in situ stress state is indispensable for design and construction of underground excavations in rock masses. However, determination of the state of stress in a rock formation is still a challenging problem in rock engineering.

Stress measurements by overcoring methods are widely used for determination of the state of stress in rock masses. In this indirect technique, borehole deformations induced by stress relief are measured, and they are then converted in stresses using the elastic constants of the medium.

The strain tensor tube (STT) is a 2 mm thick hollow cylinder with a 35 mm external diameter, with 10 electrical resistance strain gauges embedded midway from the inside to the outside surface, developed at LNEC for this purpose. Stress measurements using this technique consist of cementing into a 37 mm diameter borehole the strain cell and releasing the stresses by overcoring with a larger diameter, usually around 120 mm. The state of stress in the rock mass is computed from the strains obtained in the 10 gauges in the overcoring final stage and the elastic constants are determined from a biaxial test on the recovered large diameter core, with the cell. Comparing with other tests reviewed by Amadei (1996), the biaxial test presents the advantage of being carried out on the overcored rock. However, it has been verified by the authors that the axial tensile strains that result from the loading condition, often significantly affect the values calculated for the elastic constants.

The model currently used for interpretation of the overcoring test assumes linear elastic and isotropic behaviour of the rock, both for calculating the state of stress from the strain measurements and for calculating the elastic constants from the biaxial tests. However, in many types of rock this assumption is not realistic, and its consideration may significantly affect the calculated values, as demonstrated by Amadei (1983) and other authors.

Despite its relevance, anisotropy is often disregarded in engineering practice. This is partly because more parameters have to be determined, and partly because of their spatial variation, which puts problems to their determination when more than one specimen is used.

The paper presents the steps that are being taken, aiming at a better interpretation of the STT test results. A numerical model is presented, which allows computing the stress state from the strain measurements obtained during overcoring and the elastic constants of a transversely-isotropic rock. This model is now also being used as a back-analysis tool for estimating the transversely-isotropic elastic constants from the biaxial test, using an original optimisation methodology based on artificial intelligence techniques. Calculations performed to assess the importance of the tensile strains introduced in the specimen during the biaxial test are presented. A new triaxial testing method, which avoids the tensile strains verified in the biaxial test, was developed and is now being used. Finally, the paper presents an example that shows how in situ stress determinations in anisotropic rock are affected by the degree of rock anisotropy.

2 NUMERICAL MODEL

The STT triaxial cell, developed at the LNEC (Rocha and Silvério, 1969), has 10 strain gages located in the mid wall thickness of the hollow cylinder, which are oriented along the normal to the faces of an icosahedron, in order to give an isotropic sampling of the strains in the 3D space (Pinto, 1983). Figure 1 shows the STT cell and its geometry and orientation with respect to x, y, z co-ordinate system, where z is the axial direction. The position of each of the ten strain gauges (i = 1, 10) can be defined by two angles: α between the xy plane and the strain gauge direction; and θ between the x axis and the normal to the strain gauge (for i = 1, θ = 0º). Using cylindrical co-ordinates, the strain ε_i in gauge i is given by:

$$\varepsilon_i = \varepsilon_{\theta i} \cos^2\alpha_i + \varepsilon_{zi} \sin^2\alpha_i + \gamma_{\theta zi} \sin\alpha_i \cos\alpha_i \quad (1)$$

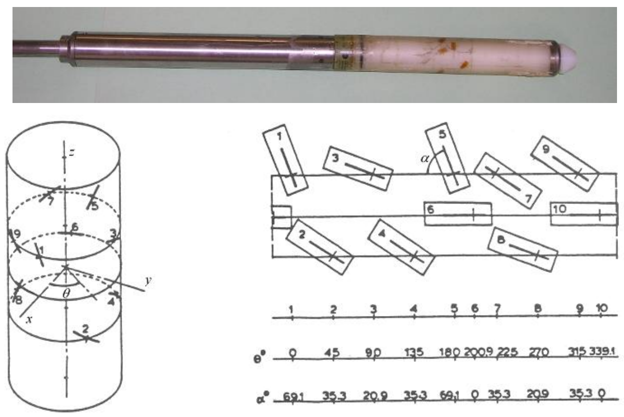

Figure 1: Stress Tensor Tube

The currently used semi-analytical model for interpretation of the STT tests (Pedro, 1972) assumes that the rock is homogeneous, linear elastic and isotropic, the relation between the length and the diameter of the plastic cylinder is high, and the stiffness of the plastic tube is significantly lower than the rock stiffness.

A three-dimensional finite element model was recently developed, which considers a transversely isotropic rock, characterised by 5 elastic constants and 2 angles that define the direction of the symmetry axis. The model simulates the plastic hollow cylinder, with its actual stiffness, and with the strain gauges in their exact location. The 3D-FEM mesh (Figure 2) is a 310 mm side cube and has 192, 20-nodes elements. The 2 mm thick plastic cylinder is simulated using 2 elements in the radial direction. One element is used for the 1 mm thick ring of glue between the plastic cylinder and the rock mass. The strain gauges are located at mid-height of the cylinder.

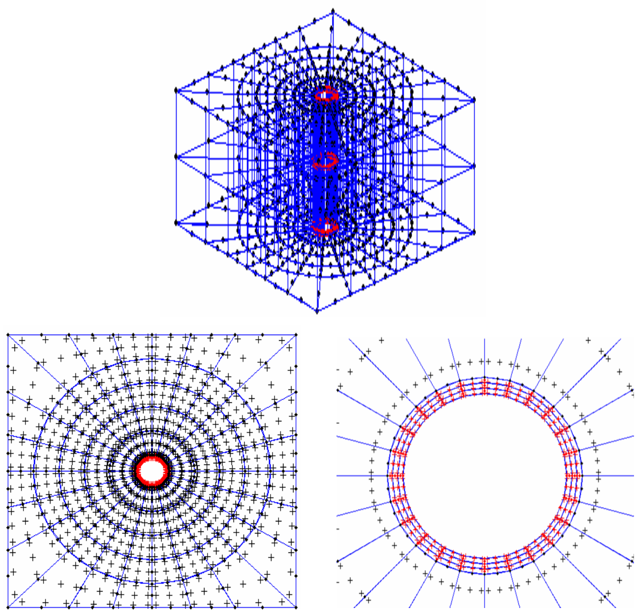

Figure 2: Three dimensional finite element model

Since linear elastic behaviour is assumed, the principle of superposition of effects can be applied and, at the end of the overcoring, the strain ε_i in gauge i is given by:

$$\varepsilon_i = a_{i1}\,\sigma_{xx} + a_{i2}\,\sigma_{yy} + a_{i3}\,\sigma_{zz} + a_{i4}\,\sigma_{xy} + a_{i5}\,\sigma_{yz} + a_{i6}\,\sigma_{xz} \quad (2)$$

where σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{yz}, σ_{xz} are the initial stress tensor components, and a_{ij} are coefficients that correspond to the strain in gauge i for a unit stress component j.

The coefficients a_{ij} depend on the geometrical characteristics of the test (*e.g.* diameter of boreholes and gauge position) and on the elastic parameters of the rock. They can be determined from 6 numerical simulations of the overcoring test, applying in each of them a unit value of the stress tensor component. The following system of equations results for the 10 strain gauges, which can be solved by the least squares method in order to obtain the 6 stress components:

$$\varepsilon_i = a_{ij}\,\sigma_j \qquad \text{with } i = 1, 10 \quad \text{and } j = 1, 6 \quad (3)$$

3 DETERMINATION OF ELASTIC CONSTANTS

3.1 Advantages and disadvantages of biaxial tests

Biaxial tests allow determining the elastic constants of the rock and provide a check of the performance of the individual strain gauges. During testing, the rock core is subjected to a radial pressure, and the resulting strains are recorded by the strain gauges. The test sequence comprises both loading and unloading in order to study possible inelastic behaviour of the rock. The values of the Young's modulus, E, and of the Poisson's ratio, ν, are secant values, usually calculated from strain data obtained during unloading of the core specimen.

As mentioned earlier, the main advantage of biaxial tests is that the elastic parameters are obtained in the overcored specimen, and hence characterize the actual rock that was subjected to the stress relief. Therefore, it should be preferred to tests on other rock specimens.

However, in biaxial tests it is not possible to reproduce states of stress similar to those occurring in nature. While rock masses are subjected to 3D compressive stresses, in the biaxial test important tensile strains occur in the axial direction, which often significantly affect the values obtained for the elastic constants. According to the authors' experience, during the biaxial tests in some rocks, namely weak and schistous rocks, for which the assumption of isotropy may be reasonable in compression, test micro-fissures are created approximately normal to the core axis. These micro-fissures represent a damage, which can be simulated, in a very simple way, by decreasing the value of the Young's modulus normal to them, from E to E', thus resulting in a lower stiffness in the direction of the tensile strains.

In order to demonstrate the influence of this damage, numerical simulations of a biaxial test were performed considering a transversely isotropic model with the axis of symmetry in the core axial direction, characterized by Young's moduli E and E', Poisson's ratios ν and ν', and shear modulus G'. The range of admissible values for ν and ν' can be found in Amadei (1983). The relation between E (in the plane normal to the axis) and E' (along the axis) was varied between 1.0 and 2.0. A Poisson's ratio in the plane, ν, of 0.20 was assumed. Three values of ν' were considered (0.10, 0.20 and 0.25), and G' was calculated, as a function of the other parameters, using the Saint Venant's principle:

$$G' = \frac{E\,E'}{E + E' + 2\nu' E} \qquad (4)$$

A unit confining pressure was applied and the strains in the strain gauges were computed. With these strains, obtained for an anisotropic medium, ε_i^{ani}, the 3D-FEM model was used again, as a back-analysis tool to determine the equivalent Young's modulus and Poisson's ratio of an isotropic medium (E^{iso} and ν^{iso}). The values obtained in this way are presented in Figure 3. It results clear from the calculations, that neglecting the damage effect of the tensile axial strains in a rock core during the biaxial test can induce large errors, mainly in the Poisson's ratio. As E/E' increases, E^{iso} decreases and ν^{iso} increases. The high values of ν^{iso} for high ratios E/E' are in accordance with the values often obtained in biaxial tests.

In a second set of calculations it was assumed that the rock sample, for instance a schistous rock, has a preferential direction of weakness, represented by the normal to the schistosity planes. In the numerical model this direction is the axis of symmetry of the transversely isotropic material, which forms an angle β with the core axis. Two values of β were considered: 15º and 30º. The values of E^{iso} and ν^{iso} obtained are presented in Figures 4 and 5, where it can be clearly seen that the resulting values of E^{iso} become affected by a larger error as β increases, while the effect on ν^{iso} is less significant.

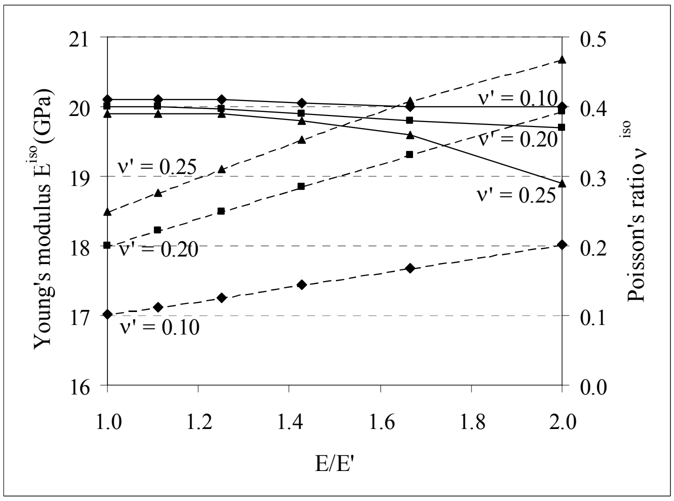

Figure 3: Variation of E^{iso} and ν^{iso} with the ratio E/E' (β = 0º)

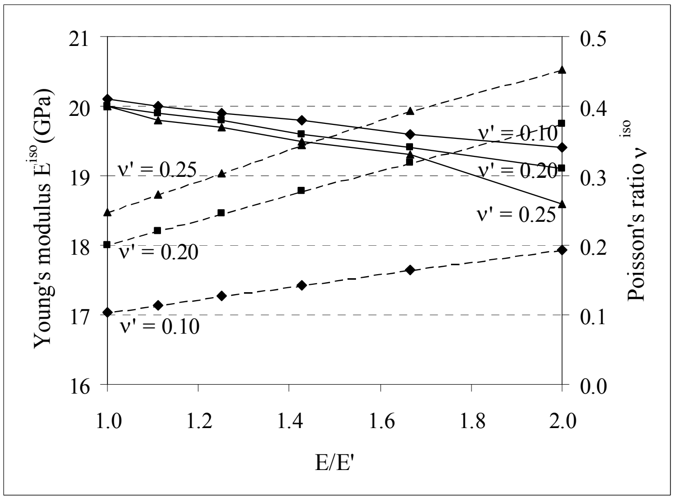

Figure 4: Variation of E^{iso} and ν^{iso} with the ratio E/E' (β = 15º)

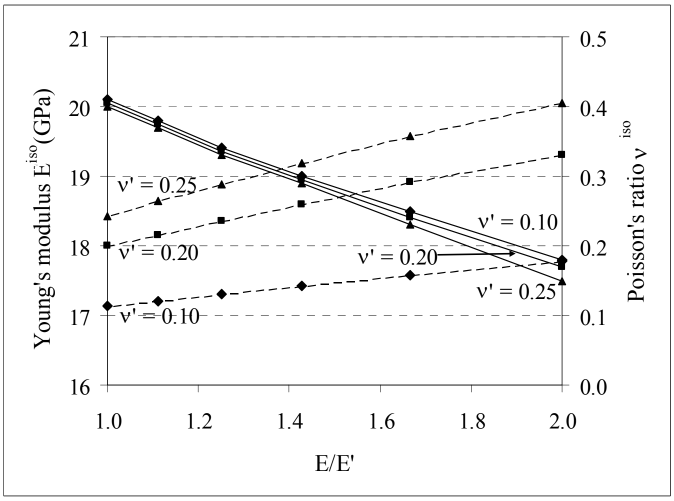

Figure 5: Variation of E^{iso} and ν^{iso} with the ratio E/E' (β = 30º)

In order to analyse the errors between the values of the strains in an anisotropic medium, ε_i^{ani}, and the strains obtained in the strain gauges with the calculated isotropic elastic constants, ε_i^{iso}, the mean quadratic deviation was calculated. It is presented in Figure 6 for β equal to 0º, 15º and 30º, considering ν' = 0.2 and E/E' between 1.1

and 2.0. For $\beta = 0°$ the mean quadratic deviation is small, because the lower stiffness in the axial direction is reasonably simulated by a larger value of ν^{iso}. However, the mean quadratic deviation increases by one order of magnitude as E/E' increases from 1.1 to 2.0. For values of β different from 0º the errors are much higher, because it is no longer possible to calculate equivalent isotropic elastic constants that reasonably simulate this anisotropic effect. The mean quadratic deviation varies between 2×10^{-6} and 42×10^{-6}, and these are significant values when compared with the strains usually measured in tests.

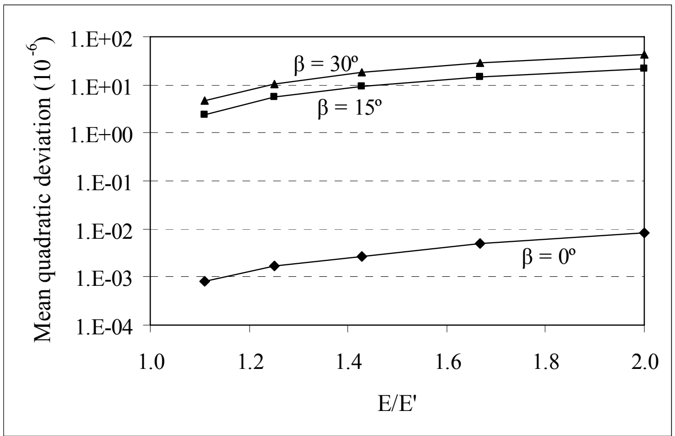

Figure 6: Mean quadratic deviation of the strain for ν' = 0.2

A triaxial testing procedure was implemented in order to avoid the axial tensile strains during the biaxial test. The same biaxial chamber is used for applying a confining pressure to the core with the cell, and the whole assembly is loaded in the axial direction using a servo-controlled testing machine (see Figure 7).

Figure 7: Triaxial test assembly of the core with the cell

3.2 Anisotropic constants

The importance of accounting for the mechanical anisotropy of rock in rock engineering has been thoroughly demonstrated by several authors. The results presented by Amadei (1983) indicate that an elastic modulus anisotropy ratio of 1.5 can introduce a 33% error in the calculated magnitudes of the principal stress and an anisotropy ratio of between 1.14 and 1.33 will have a definite effect on the interpreted in situ state of stress.

Different methodologies and testing techniques have been developed and implemented for determination of the mechanical properties of anisotropic rocks. Amadei (1996) provides an excellent review of many of the available laboratory and field techniques.

A long standing procedure for determination of the required parameters involves the testing in uniaxial compression of three different specimens, each cored at a specified and know orientation with respect to the axis of material symmetry.

A second approach used by Pinto (1979) and extended by Amadei (1996) and Chen *et al.* (1998), involves the diametral loading of Brazilian specimens. This methodology requires testing two specimens: one where the specimen and the anisotropy axes are coincident and the other where those axes are perpendicular.

Hollow cylinder tests have been reported by several authors, namely Talesnick *et al.* (1995). The main feature of this methodology is that a single hollow cylindrical specimen may be tested under different stress conditions, each enabling the determination of specific anisotropy parameters. This methodology applies to transverse isotropic materials and requires the axis of anisotropy to be coincident with the cylinder axis.

Nunes (2002) presents an analytical methodology to determine the parameters of orientation and deformability of transversely isotropic cylinders of rock using the CSIR triaxial cell (Leeman, 1971) under isotropic biaxial loading. This author uses theories developed by Lekhnitskii (1961) to derive an approximate solution for the stress at the inner surface of the cored sample, assuming isotropy of the rock. The relative error for this approximation decreases with increasing stiffness and decreasing anisotropy. Expressions for the tangential, axial, and shear strains on the inner surface of the core could then be obtained, and have a sinusoidal form. The direction of the anisotropy axis and the five elastic constants can be determined directly, because they are only a function of the cylinder geometry, the applied biaxial pressure and the measured strains during biaxial testing.

A crucial part of the methodology presented by Nunes (2002) is the applicability of the empirical Saint Venant's equation (4). Talenick and Ringel (1999) found that this equation underestimated the value of the shear modulus by up to 40% when compared to the results from torsion tests. They also proposed introducing a correction factor to the equation, based on the relative differences between E and E'. Barden (1963) showed analytically that, for plane stress conditions, the value of G' given by equation (4) is exact. These are the actual loading conditions of the biaxial tests of overcored samples, and therefore equation (4) can be used for estimation of the shear modulus with some confidence. However, the possible uncertainties in the determination of this parameter should be considered in the evaluation of the results.

A different approach is being developed at LNEC, using artificial intelligence techniques, for estimating the transversely-isotropic elastic constants from biaxial or triaxial tests on the overcored sample with the cell. The same 3D-FEM model is used as a back-analysis tool in the minimisation of an error function given by the mean quadratic deviation of the measured and the calculated strains in the strain gauges. This methodology is currently being tested with actual biaxial and triaxial test results.

4 STATE OF STRESS

In order to understand how in situ rock stress determinations are influenced by the degree of anisotropy, and to analyse the error involved in neglecting rock anisotropy by assuming isotropy, an example is presented, using the values of the elastic constants determined for the Bemposta dam site, in north Portugal. The migmatite rock was assumed to be a transversely isotropic material with the isotropic plane coincident with the schistosity.

The Young's modulus, in a direction forming an angle γ with the axis of transverse isotropy, $E(\gamma)$, is given by the following equation (Wittke 1990):

$$\frac{1}{E(\gamma)} = \frac{\sin^4\gamma}{E} + \frac{\cos^4\gamma}{E'} + \left(\frac{1}{G'} - \frac{2\nu'}{E'}\right)\sin^2\gamma\cos^2\gamma \qquad (5)$$

Using the results of 11 uniaxial compression tests on rock samples cut at different angles with respect to the planes of schistosity, the five elastic constants were determined by non-linear regression (Muralha, 2008): E=55 GPa, E'=19 GPa, ν=0.19, ν'=0.14, G'=16 GPa. Figure 8 presents the variation of $E(\gamma)$ with γ.

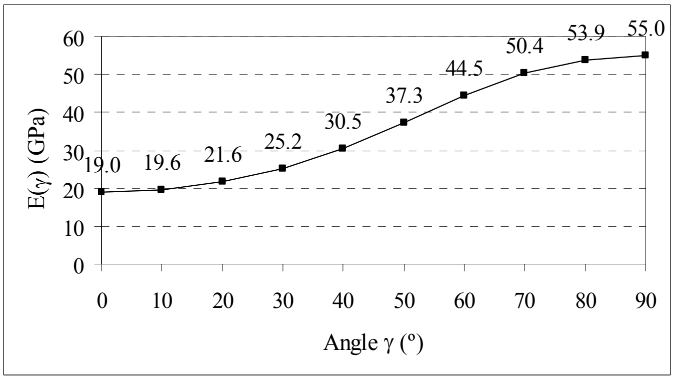

Figure 8: Variation of $E(\gamma)$ with the angle γ

Let us now consider a set of 7 simulations of biaxial tests performed with a unit confining pressure on a transversely isotropic rock core with those elastic constants. Each test corresponds to a different value of the angle β between the core axis and the axis of transverse isotropy (0º, 15º, 30º, 45º, 60º, 75º, 90º). Using the numerical model described in section 2, the strains in the location of the strain gauges were obtained. Taking these strains and applying the isotropic interpretation model of the biaxial tests, the values of E^{iso} and ν^{iso} were calculated for the 7 cases, and are presented in Figure 9. Confirming the results presented in the previous section, for $\beta = 0º$ E^{iso} is similar to E and ν^{iso} is high. As β increases, E^{iso} decreases until $\beta = 90º$, while ν^{iso} decreases rapidly until $\beta = 60º$.

Let us now assume that a transversely isotropic rock mass, with the same elastic constants, has a known state of stress σ^0. The strains that would be measured in the strain gauges of an STT cell during an overcoring test in this rock mass, ε^{oc}, can be computed using equation (2). Taking now ε^{oc} and the values of E^{iso} and ν^{iso} from Figure 9, different states of stress can be calculated for different values of β. Table 1 presents the components of σ^0 (obtained in an actual overcoring test, where z is vertical and along the core axis, x points to East and y to North) used for this example, and of the stress tensors calculated in this way. Figure 10 is a lower hemisphere projection of the principal directions, where the solid markers stand for σ^0. The principal stresses σ_I, σ_{II} and σ_{III} and the trace of the stress tensor, I_1, are presented in Figure 11.

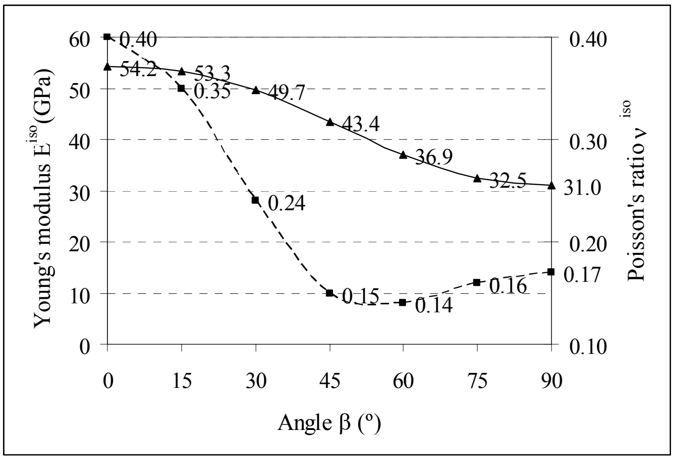

Figure 9: Variation of E^{iso} and ν^{iso} with the angle β

Table 1: States of stress obtained with an isotropic model (MPa)

	σ_{xx}	σ_{yy}	σ_{zz}	σ_{xy}	σ_{yz}	σ_{zx}
σ^0	2.90	4.60	7.50	1.10	1.80	1.50
σ (β=0º)	6.09	8.07	24.10	1.28	2.16	1.80
σ (β = 15º)	5.59	6.95	21.79	1.20	1.83	2.89
σ (β = 30º)	4.43	5.04	15.78	0.98	1.57	3.63
σ (β = 45º)	3.80	3.89	10.00	0.77	1.37	3.44
σ (β = 60º)	3.42	3.36	6.31	0.64	1.19	2.54
σ (β = 75º)	3.00	3.08	4.59	0.55	1.09	1.64
σ (β = 90º)	2.61	2.99	4.06	0.49	1.12	1.05

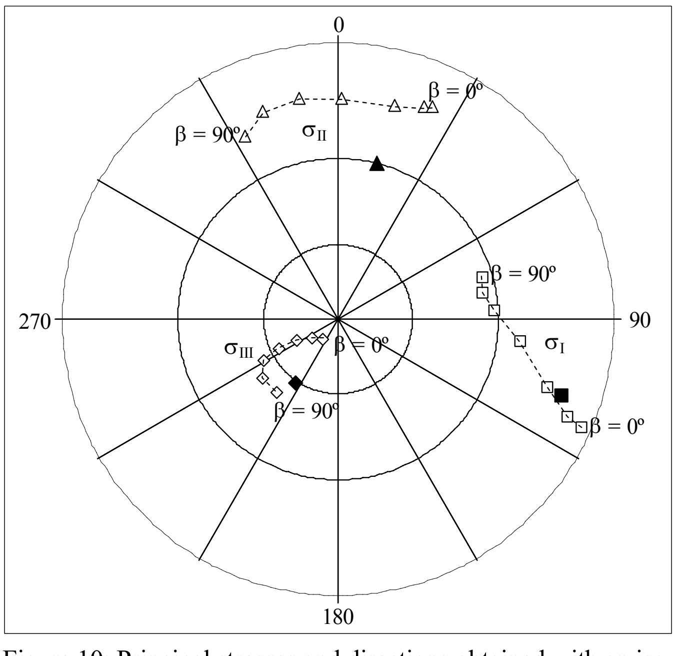

Figure 10: Principal stresses and directions obtained with an isotropic model for different values of the angle β

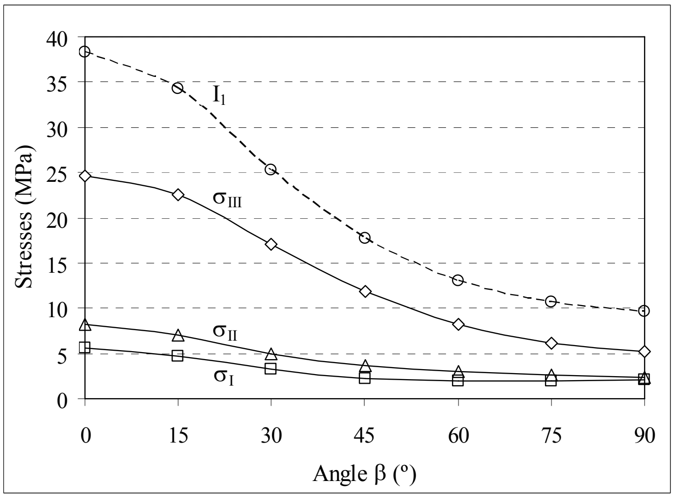

Figure 11: Variation of the principal stresses and of the trace of the stress tensor with the angle β

These 2 Figures show how the calculated stresses can vary so markedly, not only in value but also in direction. The values obtained are highly influenced by the loading conditions of the biaxial test and also by the angle β between the anisotropy and the core axes. The high values of E^{iso} obtained from the biaxial tests for low values of β result in higher calculated stresses. Of particular relevance are the very high values of the stress σ_{zz} along the core axis and of the maximum principal stress.

5 CONCLUSIONS

The paper showed the advantage of using the cores recovered from the overcoring tests to calculate the elastic constants of the rock. Biaxial tests used for this purpose are a good procedure, but in some types of rock, namely in some weak rocks and schistous rocks, these tests induce micro-fissures approximately normal to the core axis, owing to the tensile strains developed in this direction. Several numerical tests were performed to analyse their influence on the test results, and it was shown that neglecting the damage effect of the tensile axial strains in a rock core during the biaxial test can induce large errors, mainly in Poisson's ratio. In order to avoid this effect, a new triaxial testing procedure was developed.

A discussion on the existing methods for determination of the anisotropic parameters of rock was presented. Having in mind the advantage of using the actual overcore for this purpose and the difficulty in establishing a formulation for the direct interpretation of biaxial or triaxial tests on anisotropic overcores, an optimization algorithm, using artificial intelligence techniques, was developed in order to estimate the 5 elastic constants and the 2 angles that define a transversely isotropic material. This methodology is now being tested with actual test results.

The effect of the rock anisotropy on in situ stress determination and the errors involved in neglecting it were analysed, and it was demonstrated that assuming that the rock is isotropic may lead to large errors in the calculated in situ stresses. When the angle between the core and the anisotropy axes is small, a marked increase in the stresses was obtained, namely in the stress along the core axis.

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