

Wave Overtopping of a Porous Structure: Numerical and Physical Modeling

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ABSTRACT

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This paper illustrates the application of the new version of a nonlinear shallow water numerical model, AMAZON, to study the mean wave overtopping discharge at a porous breakwater that protects the Portuguese harbor of Póvoa de Varzim. The results are compared with two-dimensional physical model data collected at the National Civil Engineering Laboratory, Portugal. The implications of using different porous flow parameters in the Forchheimer equation for stationary turbulent flow within the porous layer of the breakwater are discussed. The maximum velocity that the water can reach during the exchange between the free-flow and porous layers has been included as an input to AMAZON and its impact on the overtopping results is analyzed. A suitable choice of the values of the porous flow parameters and of the maximum velocity leads to a good agreement between the AMAZON results and the data. The specified maximum velocity was found to be the parameter which mostly affects the obtained results.

ADDITIONAL INDEX WORDS: Breakwaters, Mean overtopping discharge, AMAZON nonlinear shallow water model, physical model data

INTRODUCTION

In recent years, due to the continuous increase in computer power, numerical models of wave overtopping have been developed further and their use is becoming increasingly attractive. However, for realistic simulations of wave overtopping, numerical models must be able to simulate all the important hydrodynamic processes involved and should be capable of running sufficient random waves to give reasonably consistent results.

At present, there are still no numerical models capable of being, simultaneously, both accurate and computationally efficient. Nevertheless, there are different kinds of models, each capable of meeting some of the required criteria. The use in practical engineering applications of the more comprehensive models, based on the fuller Navier-Stokes equations (e.g. DALRYMPLE and ROGERS, 2006; LOSADA *et al.*, 2008), still has limitations, mainly because these models are computationally very demanding, *i.e.*, their results cannot be obtained within manageable computation times, especially within the time limits usually available to consultants. The nonlinear shallow water (NLSW) equation models (e.g. VAN GENT, 1994; DODD, 1998; HU, 2000; CLARKE *et al.*, 2004,) in spite of their restrictions (mainly relating to the shallow water assumptions and to the fact that waves entering the computational domain will already have broken or will begin to break), allow realistic, but simplified, fast simulations. These models are already being used for the purposes of design and flood forecasting, since they enable trains of several thousand random waves to be rapidly simulated.

The existing NLSW models have mainly been validated for impermeable structures (e.g. DODD, 1998; HU *et al.*, 2000) and for

permeable beaches (e.g. VAN GENT, 1996; CLARKE *et al.*, 2004). They have not been systematically validated to study the wave overtopping of porous structures. Furthermore, these models have assumed that the pressure gradient at the interface between the free-flow and the porous layers of the model is not greater than unity. This assumption means that the maximum velocity that the water can reach during the exchange between the two layers is constant for given values of the layer porosity, representative particle diameter and porous flow parameters, and it is not an input to the models. Consequently, these models do not consider the impact of the maximum velocity on the overtopping results.

In the present work, the NLSW model AMAZON (HU, 2000) is applied, together with physical model tests, to study the overtopping of a porous breakwater protecting a Portuguese harbor. The maximum velocity that the flow can attain during the exchange of water between the porous and the free-flow layers has been included as an input to AMAZON. The physical model results are used to check AMAZON's applicability to porous structures. The end result of this study is the development of a user-friendly numerical overtopping model incorporating a porous layer in the structure and that provides a good compromise between computational effort and accuracy in terms of overtopping results.

Following this introduction, the paper begins with a brief description of AMAZON. Next, the case study is presented, together with the physical model tests carried out at the National Civil Engineering Laboratory (LNEC), Portugal. Then, the AMAZON results are shown, compared with the physical model data and discussed. Finally, conclusions are drawn and suggestions on future developments of AMAZON are made.

THE NUMERICAL MODEL AMAZON

The AMAZON model was originally developed, at Manchester Metropolitan University (HU, 2000) and it comes as both a one-dimensional model and as a two-dimensional plan model. The one-dimensional version is used and described here.

AMAZON is based on solving the NLSW equations and is numerically very stable and robust. The pressure is assumed to be hydrostatic and the equations describe the water motions in terms of the instantaneous total water depth and of the depth-averaged velocity. The equations are solved using a high-resolution finite volume method that is second-order in time and space. The employed MUSCL-Hancock scheme (VAN LEER, 1979; VAN ALBADA *et al.*, 1982) is a Godunov-type method that uses a monotonic reconstruction of the conserved variables to obtain values at cell interfaces that prevent spurious oscillations in the solution. Solutions to local Riemann problems that are required for the corrector stage are calculated using the HLL (Harten, Lax and Van Leer) approximate Riemann solver, capable of capturing bore waves and of simulating supercritical flows (HARTEN *et al.*, 1983). It uses a “zero-equation” turbulence model. A full description of the computation scheme can be found in HU (2000).

AMAZON is capable of generating flexible computational meshes, i.e. grid cells with any shape (such as rectangular, triangular, hexagonal, etc., in two dimensional mode), allowing the definition of complex shaped grids, a finer grid where a precise calculation is needed and a coarser grid elsewhere in the calculation domain.

AMAZON simulates random waves which travel as bores. Across the bores, mass is conserved but energy is dissipated, as would be expected in natural breaking waves. It uses a non-reflective wave inlet boundary condition, which is able to remove at the seaward boundary more than 98% of the energy of any waves reflected from the modeled structures. As a consequence, the seaward boundary can be set close to the structure to avoid deep water conditions, where AMAZON has limitations. According to HU and MEYER (2005), AMAZON produces good results when its seaward boundary is located at a distance from the structure toe of approximately one wavelength, L_s , where L_s is the shallow water wavelength in depth d_s at the structure toe, calculated using the peak period of the incident waves, T_p ($L_s = T_p(gd_s)^{0.5}$ in which g is the acceleration due to gravity).

AMAZON can model sloping structures, with or without berms, and with or without a crown wall. Since it is a depth-averaged model, it does not model curved wave return walls. Vertical and nearly-vertical structures can be approximated by a steep slope and the results have been satisfactory (HU, 2000). At the crest of a structure, AMAZON is able to continue computing as the flows, either side of the crest, separate, overtop or return. The model also includes a bottom friction coefficient, f , to account for dissipation of wave energy across the structure and the foreshore due, for example, to slope roughness (HU, 2000).

The original version of AMAZON did not explicitly account for porous flow. The development of the porous flow model, briefly reported in this paper, includes the addition of one porous layer to the original model design and the porosity is taken as constant for the whole porous element. For a structure with more than one permeable layer with different characteristics and/or with a core, the choice has to be made whether the structure will be modeled as a homogeneous permeable structure or as a structure with an impermeable core.

To govern the water exchange between the porous cells, both the Darcy equation (valid for laminar stationary flows) and the Forchheimer equation (valid for turbulent stationary flows) are implemented in AMAZON:

$$\text{Darcy equation: } I = u / K \quad (1)$$

$$\text{Forchheimer equation: } I = au + bu|u| \quad (2)$$

where I is the pressure gradient, u is the depth-averaged velocity in the porous layer, K is the hydraulic conductivity and a and b are coefficients taken as constant in time and space. Expressions for the laminar coefficient, a , and the turbulent coefficient, b , are prescribed by many authors; see, for instance, GARCÍA (2007). In AMAZON, the following are used:

$$a = \alpha \frac{(1-n)^2}{n^3} \frac{v}{gD^2} \quad (3)$$

$$b = \beta \frac{1-n}{n^3} \frac{1}{gD}$$

where D is a representative particle diameter, v is the kinematic viscosity of water, n is the layer porosity and α and β are dimensionless coefficients which depend on the particle gradation, aspect ratio and shape, the Reynolds number and the Keulegan-Carpenter number. In the numerical model, constant values of α and β are used. The flows into and out of the porous layer (infiltration and exfiltration, respectively) are limited by the maximum velocity that the water can reach during the exchange between the free-flow (surface) and the porous layers of the model, hereafter called the interface permeability, IP . In the present version of AMAZON, the parameters K , α , β and IP should be calibrated.

The two model layers use the same computational mesh, i.e. each grid cell in the surface layer has a corresponding grid cell in the porous layer. Flow exchange between the two layers is updated at each time step:

- If a porous cell is not full and the corresponding surface cell is wet, water moves downward to the porous cell, i.e. infiltrates, limited by IP (Figure 1a).
- If a porous cell is full and water pressure from neighboring cells is higher than pressure from the surface cell, water exfiltrates, limited by IP (Figure 1b).

In AMAZON, IP is assumed to be constant and it is an input to the model. At present, reports on values of IP are rare. Some authors (e.g. VAN GENT, 1994; CLARKE *et al.*, 2004) have assumed that the pressure gradient, I , at the interface between the two layers is not greater than one, which leads to $IP \leq K$ for the Darcy equation and to $IP \leq [-a+(a^2+4b)^{0.5}]/(2b)$ for the Forchheimer equation.

AMAZON is written in C++ and has a friendly, effective and easy-to-use interface. Its input includes:

- Cross-sections of the overtopped structure and foreshore.
- Characteristics of the porous layer: geometry, porosity, representative particle diameter and porous flow parameters.
- Bottom friction coefficient (which may vary across the structure and foreshore).
- Water level (which may vary by adopting a sinusoidal tide curve).

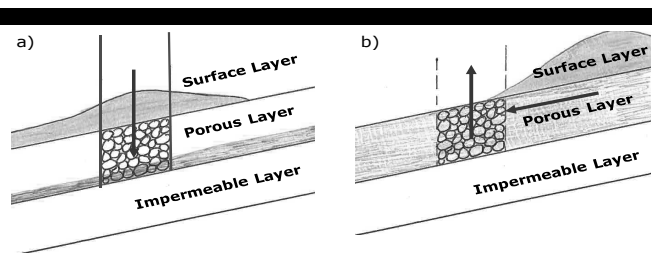


Figure 1. Water exchange between the free-flow (surface) and the porous layers.

- Incident waves: AMAZON allows the input of sinusoidal waves, random waves defined by an empirical spectrum (e.g. JONSWAP, Bretshneider-Moskowitz) or a user-defined spectrum, or a wave train obtained, for example, from physical modeling.

- Computation grid (uniform/non-uniform).

- Other parameters (e.g. minimum wet depth in each grid cell).

The output defines the free surface, depth-averaged velocities and, based on these values, discharge time-series, mean discharges and peak discharges.

AMAZON has been validated for a variety of representative test problems involving both steady and unsteady, inviscid and viscous, and subcritical and supercritical flows (HU, 2000). It has also been validated and extensively used to study the overtopping of impermeable dikes.

CASE STUDY: SOUTH BREAKWATER OF PÓVOA DE VARZIM HARBOR

The case study is of a proposed cross-section for the rehabilitation of the root of the South Breakwater of Póvoa de Varzim Harbor, located on the west coast of Portugal. The root of this breakwater directly protects the local Nautical Club building (Figure 2) and, therefore, it is the stretch for which overtopping should be minimal. Any overtopping that occurs on this stretch may cause unacceptable damage to the building and surrounding area and may disrupt local activities.

The proposed cross-section is basically a concrete vertical wall with a double-layer rock slope and berm in front of it (Figure 2). It was obtained by adding to the current cross-section a prism of 75 KN to 100 KN rocks. For the inner layer, the weight of the rock ranges from 10 KN to 50 KN.

To verify the efficiency of the proposed solution for the cross-section of the root of the South Breakwater, two-dimensional physical model tests of wave overtopping were performed at LNEC. Firstly, for each target test condition, different wave trains were produced, all conforming to the same target JONSWAP spectrum, for three different test durations. The number of random waves ranged from about 300 to 2400. Secondly, one of the main test conditions was again considered, but for twelve different test durations. In this case, the number of waves ranged from about 150 to 1900. For each test condition, the test repetitions enabled the analysis of the variability of the wave overtopping discharge, providing a range of measured values of the mean overtopping discharge (with a minimum and a maximum value) for each wave condition. The differences in overtopping are related to the different characteristics of the waves that approach the breakwater and their different breaking types.

A detailed description of the tests can be found at REIS *et al.* (2008b). This paper concentrates on the results of four tests, with duration, D , of 270 s. In Table 1, the mean overtopping discharges per meter length of structure, q_{PM} , are presented for these four tests, together with the variability of q_{PM} obtained for the whole set of tests (minimum and maximum values). In this table, the values for the wave conditions in front of the wave-maker (the

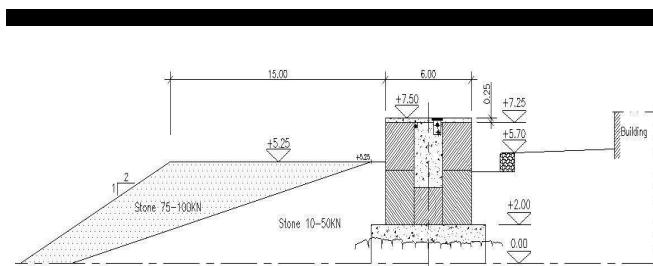


Figure 2. Proposed cross-section for the root of the South Breakwater (values shown are for the prototype, with levels relative to datum, ZH).

significant wave height, H_{os} , and the peak period, T_{op}) and in front of the structure (H_s , T_p) refer to incident values obtained using the MANSARD and FUNKE (1980) method applied to the data measured by three gauges located in front of the wave-maker and by three gauges positioned in front of the structure (at a distance from the structure toe of approximately one wavelength, L_s), respectively.

The tests are numbered in order of increasing discharge. As expected, the variation about the mean was greater for small magnitudes of overtopping discharges. The range of mean discharges presented for each test confirms that different wave trains (all conforming to the same JONSWAP spectrum) have a somewhat different impact on the total overtopping volume.

AMAZON RESULTS

To check the applicability of AMAZON to porous structures, use was made of the data collected at LNEC in overtopping physical model tests. AMAZON was applied (at model scale) for the four test conditions shown in Table 1.

AMAZON's landward boundary was a full absorption boundary set 0.16 m behind the crest of the wall. The location of the seaward boundary suggested by HU and MEYER (2005) was adopted in this study; that is, it was located at a distance from the structure toe of approximately one wavelength, L_s . Thus, the wave series input to AMAZON were the incident wave series obtained using the MANSARD and FUNKE (1980) method applied to the data measured by the three gauges positioned at a distance from the structure toe of approximately one wavelength, L_s . Consequently, they are likely to have been somewhat different from the incident wave series in the physical model, due to the inherent limitations of this method (LIN and HUANG, 2004). Furthermore, as mentioned above, the differences in the wave trains had a significant impact on the total overtopping volume. To account for these differences, AMAZON's performance was evaluated by comparing its results with data in which this variability was accounted for. Thus, the AMAZON result for each test was compared with the corresponding range of mean discharges obtained in the physical model (see Table 1), instead of comparing simply with the result of the corresponding physical model test (q_{PM}).

Table 1: Mean overtopping discharges per meter length of structure, q_{PM} , obtained for each one of the four physical model tests considered in this study and range of q_{PM} obtained for the whole set of tests.

Test	D (s)	H_{os} (m)	T_{op} (s)	H_s (m)	T_p (s)	q_{PM} ($m^3/s/m$)	Range of q_{PM} ($m^3/s/m$)
1	270	0.09	1.69	0.07	1.75	1.66E-5	0.90E-5 to 2.83E-5
2	270	0.08	2.24	0.07	2.24	2.65E-5	2.65E-5 to 8.44E-5
3	270	0.11	2.28	0.08	2.33	1.65E-4	1.15E-4 to 2.02E-4
4	270	0.14	2.21	0.09	2.93	4.44E-4	3.43E-4 to 4.69E-4

For tests 1 to 4, the values of the relative water depth, d/L_{op} , at AMAZON's seaward boundary ranged from 0.020 to 0.035, in which L_{op} is the deep water wavelength corresponding to the peak of the incident wave spectrum and calculated, according to linear wave theory, as $L_{op}=gT_{op}^2/2\pi$. Researchers have reported different maximum permissible values of d/L_{op} which were found to provide good results when used with the NLSW equations: they varied from 0.016 to 0.19 approximately (PULLEN and ALLSOP, 2003).

The total computational domain was 2.4 m long and the total number of cells was 555. A non-uniform computation grid was used: 1 cm for the foreshore at the deeper part of the computational domain and for the area behind the vertical wall; 4 mm for the foreshore at the toe of the structure; and 2 mm for the breakwater. The minimum water depth at each cell was set to 2×10^{-5} cm. Any cell with a water depth below this minimum value was treated as dry and was excluded from the computation. The minimum value was sufficiently small to represent a dry bed; a smaller value would have resulted in more computational effort for little gain.

The physical model geometrical characteristics of the foreshore and of the breakwater's envelope were reproduced within AMAZON. The foreshore and the vertical wall were represented as impermeable and frictionless, since any roughness was likely to be small, especially when compared to that of the rock armor. The permeability of the lower prism of rock was ignored. Only the top prism of rock was considered as a permeable layer, with a porosity of 0.54 and a mean rock diameter of 33.2 mm. The effect of its permeability was accounted for by applying, firstly, the Darcy equation and, secondly, the Forchheimer equation. The results obtained with AMAZON using the Darcy equation are shown in REIS *et al.* (2008a), so they are not presented here.

As explained before, in the present version of AMAZON, the use of the Forchheimer equation requires the calibration of three parameters: α , β , and IP. In this study, the calibration is made based on the values suggested in the literature (e.g. GARCÍA, 2007). Figure 3 shows the AMAZON results for tests 1 to 4 obtained by applying the Forchheimer equation using values of α and β in the ranges $1100 \leq \alpha \leq 1800$ and $0.55 \leq \beta \leq 1.1$. For these ranges of α and β , values of $IP \leq [-a+(a^2+4b)^{0.5}]/(2b)$ have been tested, that is $IP \leq 0.43$ m/s and $IP \leq 0.31$ m/s, approximately, for $\beta=0.55$ and $\beta=1.1$, respectively. Values of $0.05 \leq IP \leq 0.125$ m/s are shown here. The two dashed lines represent values of q/q_{PM} associated with the range of mean discharges obtained in the physical model for Tests 1 to 4. As the graphs suggest, if only these values of IP (0.43 m/s and 0.31 m/s) had been considered in running AMAZON (such as other numerical models do), the overtopping results would have been unsatisfactory when compared to the physical model data. This is mainly due to the fact that the simulated overtopping rates seem more sensitive to the value of IP than to the choice of the porous flow parameters, α and β . The impact of IP on the results depends on the test considered: this impact reduces for tests with greater overtopping discharges (tests 3 and 4). However, the impact is only weakly related to the quantity of overtopping water for the higher values of IP. The graphs also show that the best results have been obtained for values of IP of about 0.075 m/s, where the results are nearly all within the required ranges.

Figure 4 shows in more detail the impact of α and β on the results, for a value of $IP=0.075$ m/s. As expected, it is clear that the impact that β has on the results is greater than the impact of α .

The numerical tests were all run on an Intel(R) Core(TM)2 Duo CPU E6550 @2.33GHz with 2.00GB of RAM. The time AMAZON required for each run with the Forchheimer equation increased with increasing values of IP: it took between 4 and 5 hours, approximately, to complete each run, which corresponded

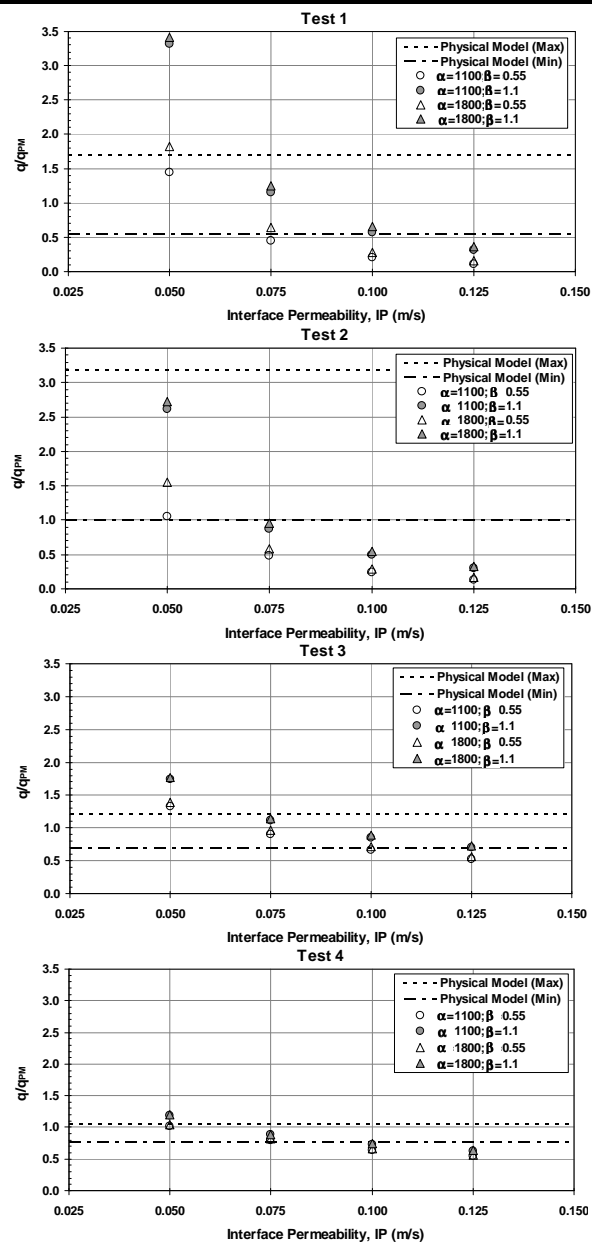


Figure 3. AMAZON results obtained using the Forchheimer equation with different combinations of α and β for Tests 1 to 4.

to a 270 s physical model test.

CONCLUSIONS

The paper illustrates the application of the new version of the NLSW numerical model, AMAZON, to study the mean wave overtopping discharge over a porous structure. The case study is the root of the South Breakwater of Póvoa de Varzim Harbor, Portugal. This cross-section of the breakwater is a combination of a concrete vertical wall with a double-layer rock slope in front of it. Physical model data of overtopping over this section were collected at the National Civil Engineering Laboratory, Portugal.

The physical model results are used to check AMAZON's applicability to porous structures. The experimental results confirm that the differences in the wave series (all conforming to

the same spectrum) have a significant impact on the measured overtopping volume. In order to account for this impact in the evaluation of AMAZON's performance, its results are compared with the ranges of mean discharges obtained in the physical model tests carried out specifically to take account of this variability.

A suitable choice of the values of the porous flow parameters in the Forchheimer equation, used to govern the water exchange between the porous cells, leads to a good agreement between the AMAZON results and the data: the results are nearly all within the ranges of mean discharges found in the physical model.

Unlike some other NLSW models, the maximum velocity that the flow can have during the exchange of water between the porous and the free-flow layers, IP, has been included as an input to AMAZON. The results show that the simulated overtopping rates are more sensitive to the value of the maximum velocity than to the choice of the porous flow parameters. The impact of the maximum velocity on the results reduces for tests with greater overtopping, but the impact is only weakly related to the quantity of overtopping water for the higher values of IP considered.

Further testing of the model is required, employing different structural configurations and different arrangements of rock or concrete armor blocks, in order to cover the most common types of sea defense structures. Due to the impact that the maximum velocity has on the results, research concerning the maximum infiltration and exfiltration velocities may also lead to further improvements of the model. Finally, it is important to note that AMAZON is computationally very efficient, especially when compared to more comprehensive overtopping numerical models.

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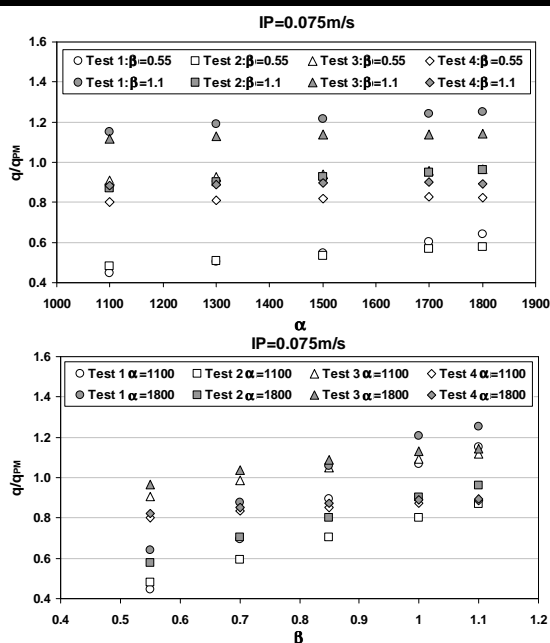


Figure 4. Impact of α and β on the AMAZON results obtained for $IP=0.075\text{ m/s}$ for Tests 1 to 4 using the Forchheimer equation.