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THE CHOICE OF METHOD TO THE EVALUATION OF MEASUREMENT UNCERTAINTY IN METROLOGY

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Abstract – This paper discusses the selection of appropriate uncertainty framework in metrology related to the class of problem to be solved.

Keywords: solution method, measurement uncertainty, metrology

1. INTRODUCTION

This paper is concerned with the need to ensure that an appropriate uncertainty framework is selected when determining the measurement uncertainty of a particular problem in metrology. The GUM uncertainty framework [1] is indubitably the most widely used method and thus its adequacy will always be tested against more elaborate methods. It will be attempted to establish some simple guidance rules based on the selection parameters.

Although the theoretical grounds for the application of the mainstream GUM are well defined, they are often overlooked and will result in inadequate applications. On the other hand, situations exist where it is known that GUM provides accurate results despite the fact that not all requirements for its application are met. It would therefore be useful to have some knowledge on the factors that influence mostly the outcome and adequacy of GUM applications. This methodology requires proper validation tools and a Monte Carlo method (MCM) will generally be used for that purpose.

Another important approach to the evaluation of measurement uncertainty is based on Bayesian methods. Its fundamentals will concisely be explained and the merits of its application will be discussed.

The differences between approaches will be explored and a comparison between GUM, MCM and Bayesian methods will be drawn, based on examples of different classes of typical metrology problems. The objective of generic method selection guidelines will be attempted.

2. APPROACH

Different approaches can be used to provide a best estimate of the measurand and the associated measurement uncertainty, and a coverage interval for the measurand for a prescribed coverage probability.

This is the whole set of information that GUM uncertainty framework can provide. It operates with summary information derived from the probability density functions (PDFs) for the input quantities, such as best estimates of the input quantities and the associated standard uncertainties, and through a Taylor expansion of a functional relationship will provide those measurand parameters.

A Monte Carlo method (MCM) implements the propagation of distributions [2] by sampling from the PDFs for the input quantities to provide a posterior PDF for the measurand. From this PDF the statistics parameters associated with the measurand can readily be obtained. This latter distinction in comparison with the mainstream GUM, represents an important advantage as it will be shown later.

Bayesian methods, on the other hand, can also incorporate a prior PDF for the measurand in its probabilistic formulation, accounting for previous knowledge, e.g., physical knowledge on the output quantity, which can be relevant when, for example, physical limitations to the outcome result are known. As will be illustrated with examples, this feature of the method can determine its selection as the best suitable approach for some classes of problem.

Considering that all methods have a process based on two stages, called formulation stage and calculation stage, and that they share similar requirements on the information needed for the formulation stage (the mathematical model and the PDFs of the input variables), the main differences that can define its suitability to each metrological problem are necessarily connected with the calculation stage requirements.

In this way, a main task is to identify the relevant characteristics of metrological problems and the constraints of the evaluation methods, taking this information as a basis to aggregate these metrological problems under similar conditions to allow a classification suitable to act as guidance to the metrologist.

3. DISCUSSION

The selection of an appropriate methodology for the evaluation of measurement uncertainties is, in certain circumstances, preponderant for the correctness of that evaluation with respect to the physical reality it intends to represent [3].

The mathematical models used as the support of that representation may differ in the number of variables and its combinations, some of which are particularly common in metrology, such as ratio, power and exponential expressions, by themselves or in some sort of combination. They will all introduce some degree of non linearity or asymmetry in the output quantity whose influence needs to be studied.

However, the particular mathematical model will not define alone the best suited approach to its evaluation. Rather, the order of magnitude between uncertainties and the PDF associated with each of those input quantities will also have a very important role to play.

Generically, it can be stated that the analytical approach is appropriate to validate other methods, and should applied whenever possible. Its main shortcoming lies in the scope of its applicability which is limited to simple models. Therefore, its application in real life experiments is almost never considered.

The GUM uncertainty framework, on the other hand, is particularly suited to differentiable linear models, or with mild non linearity, symmetric input PDFs, and Central Limit Theorem conditions, or the level of approximation provided will be difficult to estimate.

Finally, the methods based on numerical simulations have a broader application, even to strongly non linear models, provide more information due to access to the outcome PDF and can converge rapidly to near exact solutions.

As a first example we can look into a fairly simple problem of determining the measurement uncertainty associated with the estimate of a volumetric flow rate, where the measurand Q_v is given by

$$Q_v = \frac{a * b * h}{\Delta t} \tag{1}$$

Variables *a* and *b* are the width and length of the weighing tank, respectively, with assigned rectangular PDFs, and *h* is the liquid height in the weighing tank, having a Gaussian PDF. Lets assume that *a* and *b* have both the same limits [0,3495 – 0,3505] m, whereas *h* has a mean value of 0,08 m and an associated standard deviation of the mean of 0,0023 m. The time interval taken to fill the weighing tank is represented by Δt and this variable can be crucial to the shape of the output PDF and thus to the validity of the GUM approach.

If one considers first that Δt is well represented by a Gaussian PDF with mean = 6,0 s and σ = 0,0049 s the resulting output has a Gaussian shape as expected and the validity of the GUM is apparently unquestionable (see Figure 1).

However if one changes significantly the shape and magnitude of this variable, considering now, for example, that a rectangular PDF is instead assigned to it, with limits between [0,1 – 1,1] s the resulting output PDF does not resemble a Gaussian distribution whatsoever and the uncertainty evaluation associated with the corresponding volumetric flow rate, using GUM or a MCM (Figure 2) approach are likely to produce rather different results.

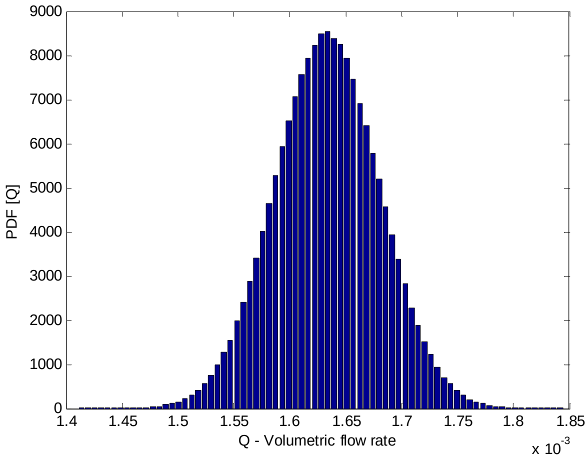

Figure 1 – Output PDF for input *t* with Gaussian PDF

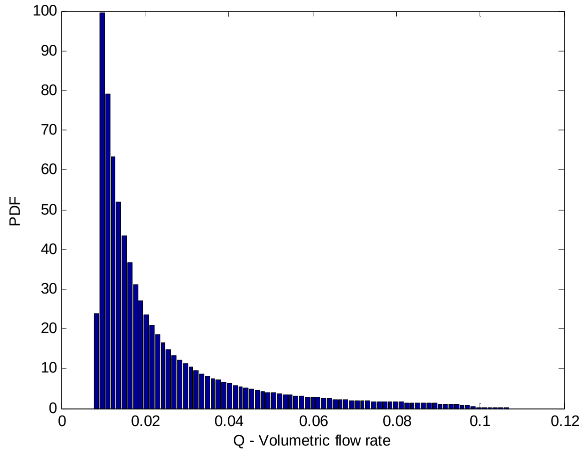

Figure 2 – Output PDF for input *t* with uniform PDF

In this context, and bearing in mind this example, one is confronted with the fact that a number of variables can influence the uncertainty evaluation, so that criteria to support the decision of method selection should be established.

Other examples shall be given, e.g., the exponential law of radioactive decay, in order to discuss not only the influence due to the nature of the mathematical models (namely, regarding linearity and symmetry) but also the relation between some types of boundaries and limits associated with values of the input variables and its expected output PDFs. The goal is the development of knowledge required to take decisions regarding the choice of methods to perform the evaluation of measurement uncertainty.

4. CONCLUSIONS

Examples have shown that, depending on the mathematical model, the value and assigned distributions for each input variable will influence the validity of the approach taken to the evaluation of the measurement uncertainty. This paper is set out to establish simple, general rules, to decide upon the correct choice of method to perform uncertainty calculations.

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