XIX IMEKO World Congress
Fundamental and Applied Metrology
September 6–11, 2009, Lisbon, Portugal

UNCERTAINTY EVALUATION OF MULTI-SENSOR FLOW MEASUREMENT IN A SEWER SYSTEM USING MONTE CARLO METHOD

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Abstract – Technical and economical impacts of flow measurements in sewer systems are a major concern in today's system's management. Thus, the quality of the measurements is considered to be a critical issue. Considering the complex nature of the measurand and the metrological requirements of local installations, the best available level of accuracy in measurement results should be sought. Therefore, both the knowledge of the measurand estimates and measurement of uncertainties are required for achieving robust results.

Within this context, the quality of measurement results depends on the knowledge of the uncertainty contributions and on the selection of an appropriate method to evaluate the measurement uncertainty. The study of these aspects can be of major importance in providing information to management of the system, namely in upgrading and maintenance activities.

This paper discusses the advantages of using Monte Carlo method to perform the evaluation of the measurement uncertainty, considering the non-linear and multi-stage nature of the mathematical models adopted. The influence of geometric conditions and other relevant parameters are considered in the quality of measurements. The study was developed in the context of a specific sewer system, using a particular measurement system, from which measurement data was gathered.

Keywords: sewer systems, flow measurement, measurement uncertainty, Monte Carlo method.

1. INTRODUCTION

The measurement of flow in sewer systems is a complex task considering the dynamic behaviour of the measurand and the effects due to non-ideal conditions of operation [1]. When flow measurements are the basis for managing sewer systems, performance of the measurement system and the quality of measurement results becomes critical both to daily operation and to decision making processes within the utility.

Different solutions can be adopted in order to measure flow in free surface flow conditions in sewers [2]. One of the most common methods is the velocity-area, usually using multi-sensing flow meters composed by a combination of sensors for level and velocity measurement, often mounted in stainless steel rings or bands, to be fitted in the inner surface of sewer pipes. The flow can be calculated from measurement of different quantities, namely, level and velocity, applying the continuity equation. The slope-area method, using the Manning-Strickler formulae or similar formula, can also be used sometimes in conjunction with the velocity-area method to ensure redundancy. In both cases, calculation of the flow involves the use of non-linear mathematical models in a multi-stage system. Additionally, in general, these methods assume uniform flow conditions.

The actual probabilistic approach of Metrology defines the measurement result has a combination of the measurand estimate and its measurement uncertainty [3]. Given the nature of the mathematical models used the Monte Carlo method was pointed out as a suitable approach to perform the measurement uncertainty evaluation [4]. For the purpose of this paper, only the continuity equation is considered.

The development of the uncertainty budget requires the evaluation of contributions due to different uncertainty sources, which can be grouped in eight major factors: the measurand; the instrumentation metrological performance; the calibration; the sampling; the interface; the user; the environmental conditions; and the data reduction.

In the specific case under study, considering the technological development of instrumentation and data processing software, the non-ideal conditions of the measurand realization (i.e. non-uniform flow) appears as the main contribution.

The analysis of the instrumentation assembly and its installation *in situ* shows the relevance of a number of geometric requirements: probes placement, measuring angles and cross-sectional geometry. In addition, hydraulic conditions associated with the inner pipe characteristics (symmetry conditions, wall roughness, hydraulic jump, drops, curves and infrastructure irregularities) can generate different types of waves, energy losses and other disturbances towards non-uniform flow.

In order to study the sources of measurement uncertainties and their effects, a second aim of this paper is to obtain an assessment of the conditions that makes the contributions due to geometric variables dominant in the context of the uncertainty budget. An example of a field application is used in order to illustrate the proposed discussions and conclusions.

2. APPROACH

Flow is a quantity measured indirectly, usually obtained by the measurement of other quantities and applying mathematical models, the continuity equation being one of the most common.

The continuity equation, as given by (1), is a functional relation that yields the volumetric flow rate, Q, as a function of the mean velocity, U, and the cross sectional area of flow, A, according to the principle of conservation of mass.

$$Q = U \cdot A \qquad (1)$$

In practice, the input quantities of this mathematical model, obtained by indirect measurement of other measurands, creates a multi-stage metrological problem with several input and output quantities and functional relations between them to reach the final output measurand, Q.

The flow through a given surface S is defined as the result of an integration of a velocity field over that target surface. Thus, U is the average of the field velocities over S. The pattern of the velocity field spatial distribution may vary significantly according to the type of flow (e.g. in completely filled pipes or free surface flow) and local conditions.

The best approximation to the average velocity U in a given flow should be obtained by measuring velocities in a large number of points distributed over the target surface, S.

In practice, the measurement of U is often carried out by transducers that capture the effect of the velocities along a straight line or, more realistically, along the conical dispersion of the beam [5,6], by assuming that certain flow distribution and symmetry conditions are well known and that yield feasible solutions. Then, the average surface velocity U is obtained from measured value (which can be either a beam average value or its maximum value) multiplied by an appropriate calibration factor.

The complexity of relations between quantities is presented in Fig. 1, and the set of quantities involved in Table 1.

The experimental performance of flow measurement in sewers implies that some influence quantities related to the method deviations, δQ_i, should be taken into account in the mathematical model, as included in (f_8). This modification of the mathematical model (1) is necessary in order to evaluate the measurement uncertainty. Both relations are in agreement if the average values of these quantities are null (as usually expected).

Table 1 – Set of quantities applied

Symbol	Description
$c_{\mathrm{us,w}}$	Ultrasound velocity in water (at ref. conditions)
f_S	Emitter frequency
β	Angle
Δf	Doppler frequency shift
$u_{\max}$	Peak flow velocity
C_u	Peak to average flow velocity factor
$\overline{U}$	Average flow velocity
$c_{\mathrm{us,air}}$	Ultrasound velocity in air (at ref. conditions)
$t_{\mathrm{tr,air},i}$	Wave time of transit
d_i, $\hat{d}$, $\overline{d}$	Displacement, estimate and average values
D	Diameter of conduit (at the cross-section area)
d_o	Displacement offset of the acoustic emitter
δh_{us}	Flow level variation in the measurement surface
h_{us}	Level of flow (measured with acoustic us instrument)
p_{w}, p_{atm}	Pressure of fluid (water) and atmospheric pressure
g	Gravity
ρ_{w}	Density of water (at ref. conditions)
h_p	Level of flow (meas. with pressure depth instrument)
r_{c}	Radius of conduit (at the cross-section area)
A	Cross-section area
δQ_i	Flow influence quantities related with the method
Q	Volumetric flow rate

In order to test the proposed approach for evaluating the flow measurement uncertainty, several measurement locations of a sewer system were studied. The information obtained was used in order to estimate the uncertainty contributions due to each input quantity and to discuss the relative influence of them, especially, those concerning the

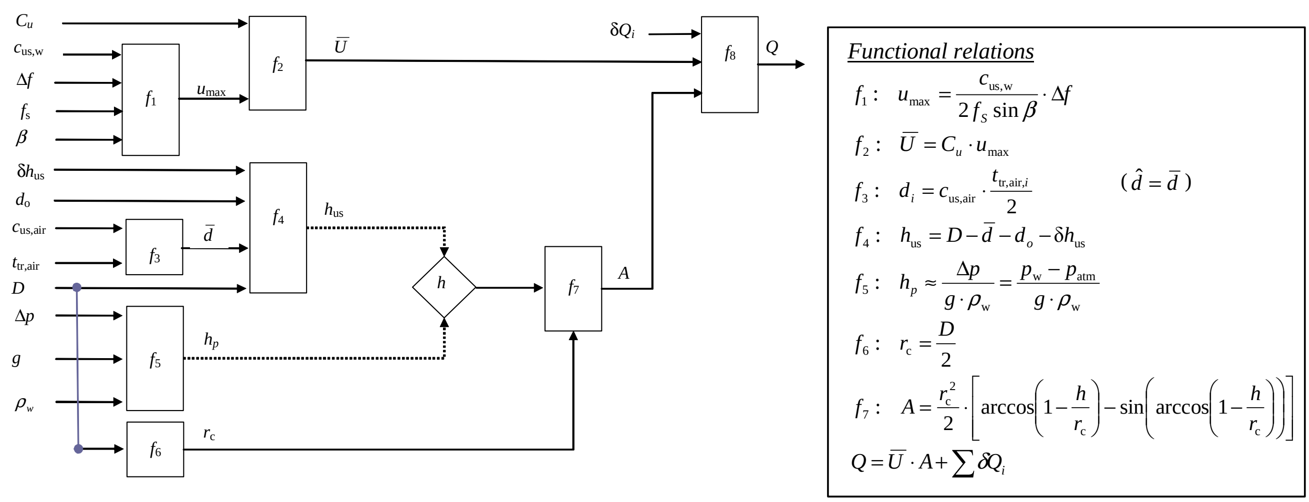

Figure 1 - Input quantities and functional relations to obtain volumetric flow rate

geometric conditions. The experimental data and the expanded uncertainty budget are presented as well as some remarks regarding the contributions of geometric conditions to the uncertainty budget.

Concerning the use of Monte Carlo method (MCM) to evaluate measurement uncertainties, it uses the relations (mathematical models) of the multi-stage system, by sampling from probability density functions (PDFs) of the input quantities considered, and gives a solution to the propagation of distributions in order to obtain the output quantities PDFs and to estimate their statistical parameters.

Among its merits is worth mentioning the absence of mainstream GUM requirements regarding the probability functions (symmetry or others), and the suitability to be applied to non-linear mathematical models.

The development of MCM numerical simulations is made by generating sequences up to 10^6 values for each quantity, depending on the required computational accuracy. The draws were based on the Mersenne Twister uniform random number generator [7] and the PDFs were obtained using validated methods like the Box-Muller transformation and the inverse cumulative distribution function (CDF) method [8]. Tests to verify the computational accuracy of the output PDFs were also made according with [9].

3. DISCUSSION AND CONCLUSIONS

Results to be presented include the volumetric flow rate probability functions, estimates of expanded measurement uncertainties and the discussion of the geometric conditions influence in the sewer system studied.

ACKNOWLEDGMENTS

The authors wish to thank the cooperation of SANEST, S.A. (Sistema Multimunicipal de Saneamento da Costa do Estoril) manager of the sewer system studied, especially for the providing of historical information and knowledge related with its performance.

REFERENCES

[1] Larrarte, F., “Velocity fields within sewers: An experimental study”, *Flow Measurement and Instrumentation*, n. 17, pp. 282-290, 2006.

[2] Bertrand-Krajewski, J-L, Laplace, D., Joannis, C., Chebbo, G., *Mesures en hydrologie urbaine et assainissement*. Techniques & Documentation, 2000.

[3] *Guide to the Expression of Uncertainty in Measurement*, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. Geneva, Switzerland, 1995.

[4] *Evaluation of measurement data − Supplement 1 to the “Guide to the expression of uncertainty in measurement”− Propagation of distributions using a Monte Carlo method*, BIPM, Sèvres, France, 2008.

[5] Edelhauser, M., “A Comparison of Continuous Wave Doppler vs. Pulsed Doppler Profiling Technology”, *MGD Technologies*, Inc., February, 1999.

[6] Jaafar, W., Fischer, S., Bekkour, K., “Velocity and turbulence measurements by ultrasound pulse Doppler velocimetry“, article in press - corrected proof, *Measurement*, 2008.

[7] M. Matsumoto and T. Nishimura, “Mersenne Twister: a 623-dimensionally equidistributed uniform pseudorandom number generator”, *ACM Transactions on Modelling and Computer Simulations*, vol. 8, nº1, pp. 3-30, January 1998.

[8] Gentle, J. E., *Random Number Generation and Monte Carlo Methods*, 2nd Ed. Springer-Verlag. New York, 2003.

[9] M. G. Cox, M. P. Dainton and P. M. Harris, *Software specifications for uncertainty calculation and associated statistical analysis*, National Physical Laboratory Report CMSC 10/01, NPL, Teddington, Mar. 2001.