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Metrological quality of the excitation force in forced vibration test of concrete dams

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Metrological quality of the excitation force in forced vibration test of concrete dams

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Abstract. This paper describes the study of the metrological quality of the excitation force in the context of force vibration test of concrete dams. For this purpose, a measurement uncertainty evaluation was performed, based on available probabilistic information about the input quantities – rotation frequency, mass, radial position, dimension, diameter and density of the generator's rod – which support the determination of the excitation force in an eccentric masses vibration generator, used by LNEC in concrete dam's field observation. The uncertainty propagation from the input quantities to the output quantity was performed by a Monte Carlo method, considering the mathematical model used for the determination of the excitation force. Two experimental cases were studied: (A) the use of five weights in the generator in the frequency range of 1 Hz up to 6 Hz; and (B) the use of a single weight in the generator in the frequency interval comprised between 5 Hz and 15 Hz. In the first case, the excitation force estimates and expanded measurement uncertainties (considering a 95 % confidence interval) varied between 3.55 kN ± 0.14 kN and 127.68 kN ± 0.91 kN, being rotation frequency the major contribution for the obtained dispersion of force values. In the second case, the excitation force estimates and expanded measurement uncertainties varied between 16.71 kN ± 0.25 kN and 150.4 kN ± 1.8 kN, being the generator's rod diameter the main contribution for the output measurement uncertainty. The obtained knowledge is essential to assure confidence and rigorous knowledge about the applied excitation force, namely, in extreme situations near dynamical structural safety limits of the observed concrete dam and of the testing equipment.

1. Introduction

Forced vibration tests have always been considered a reliable method for dynamic characterization of concrete dams. In this field, LNEC – the Portuguese National Laboratory for Civil Engineering – has an extensive experience, having conducted a large number of tests on concrete dams [1]. Test methodologies have continuously evolved with substantial improvements in the control of the dynamic actions applied to the dam, the reliability of the structural behavior records, and the processing techniques to identify the structural dynamical parameters.

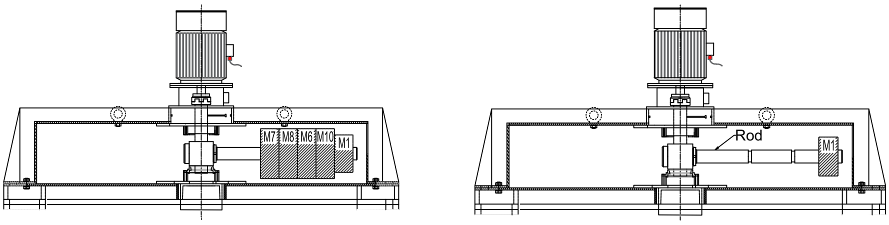
In this context, LNEC uses an eccentric masses vibration generator, which was designed and produced in-house [2] and has been used for several decades in the concrete dam's field observation, as shown in figure 1.

Figure 1. LNEC's eccentric masses vibration generator.

This equipment generates a controlled vibration using a set of weights eccentrically assembled in a rod connected to a vertical rotation shaft. This mechanical component is sup-ported in two bearings inserted a surrounding steel frame, which is fixed to the dam with eight M30 bolts. The applied excitation force is controlled by imposing the rotation frequency of the electrical engine and by the number and radial position of the weights mounted in the rod.

The main objective of this work was the determination of the measurement uncertainty of the excitation force applied by the mentioned dynamic testing equipment. Being a key issue for metrological quality evaluation, the obtained information is essential to assure confidence and rigorous knowledge about the applied excitation force, namely, in extreme situations near the dynamical structural safety limits of the observed concrete dam and of the used equipment.

In this case, since the excitation force is indirectly measured using a mathematical model (section 2), the measurement uncertainty evaluation (section 3) consisted, in a first stage, in defining the probabilistic formulation of the input quantities (rotation frequency, mass, radial position, dimension, diameter and density of the generator's rod), as described in sub-section 3.1. Experimental work was performed to provide traceability [3] to the International System of Units (SI) in the case of the generator's masses and rotation frequencies (see sub-section 3.2), while information related to the remaining input quantities was obtained from design and production requirements. In a second stage, the dispersion of values related to the input quantities was propagated through the mathematical model, using a Monte Carlo method [4], allowing the quantification of the excitation force measurement uncertainty and the identification of the main uncertainty contributions by performing a sensitivity analysis (sub-section 3.3). Two experimental cases were studied (see figure 2): (A) the use of five weights in the generator in the frequency range of 1 Hz up to 6 Hz; and (B), the use of a single weight in the generator in the frequency interval comprised between 5 Hz and 15 Hz.



Case A Case B

Figure 2. Vertical section of the LNEC's eccentric masses vibration generator with the masses configuration for the study cases A and B.

The results obtained for each case are presented in section 4, namely, the estimates and measurement uncertainties of the excitation force, the output probability density function and the contributions of each input quantity to the output dispersion of values. The main conclusions obtained from this study are mentioned in section 5.

2. Excitation force mathematical model

In an eccentric masses vibration generator, the excitation force, F, is measured indirectly, knowing the mass, m_i, and the corresponding radial position, r_i, of each weight applied in the generator's rod, the rod's length, L, and the rotation frequency, f, i.e.

$$F = \left(\frac{1}{2} \cdot q \cdot L^2 + \sum_i m_i \cdot r_i\right) \cdot (2 \cdot \pi \cdot f)^2 \tag{1}$$

where q is the rod's linear mass given by

$$q = \rho \cdot \pi \cdot \left(\frac{D}{2}\right)^2 \tag{2}$$

being ρ and D the rod's density and diameter, respectively.

Figure 3 shows the corresponding functional diagram, where the input, intermediate and output quantities of the uncertainty propagation can be observed.

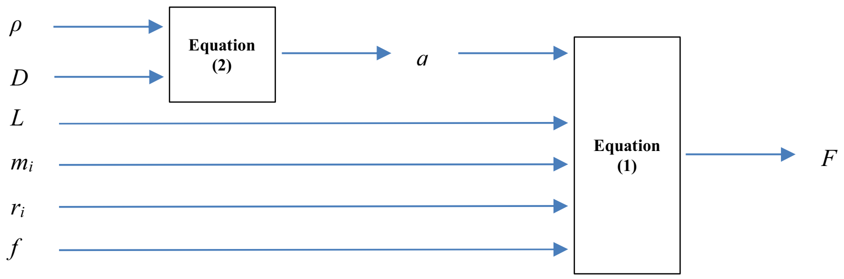


Figure 3. Uncertainty propagation functional diagram.

3. Measurement uncertainty evaluation

3.1. Probabilistic formulation of the input quantities

Two experimental cases – A and B – were considered in the performed uncertainty evaluation. In case A, a set of five weights (M1, M7, M8, M6 and M10) are mounted in the vibration generator and the rotation frequency varies between 1 Hz and 6 Hz (figure 2). Case B considers the use of a single weight (M1) while the rotation frequency is comprised between 5 Hz and 15 Hz (figure 2). Table 1 shows the adopted estimates for the mass and radial position of each weight considered in both studied cases (A and B).

Table 1. Studied configurations in the eccentric masses vibration generator.

Weight identification	Mass /kg	Radial position /mm	Case A (1 Hz – 6 Hz)	Case B (5 Hz – 15 Hz)
M1	15.573	686	Yes	Yes
M7	41.118	597	Yes	No
M8	41.172	486	Yes	No
M6	41.210	397	Yes	No
M10	41.324	286	Yes	No

Table 2 shows the adopted probabilistic formulation for the input quantities.

Table 2. Probabilistic formulation of the input quantities.

Input quantity	Quantity symbol	Measurement estimate	Probability distribution	Measurement standard uncertainty
Mass	m_i	See table 1	Gaussian	1.0 g
Radial position	r_i	See table 1	Uniform	1 mm / √3 = 0.58 mm
Rod length	L	749 mm	Uniform	1 mm / √3 = 0.58 mm
Rod diameter	D	59 mm	Uniform	1 mm / √3 = 0.58 mm
Rod density	ρ	7830 kg·m^{-3}	Uniform	10 kg·m^{-3} / √3 = 1.7 kg·m^{-3}
Rotation frequency	f	1 Hz – 6 Hz (case A) 5 Hz – 15 Hz (case B)	Gaussian	0.010 Hz

With the exception of the mass and rotation frequency quantities, a uniform probability distribution was adopted for the remaining input quantities, based on known design and production requirements of the vibration generator. The Gaussian probability distribution and the measurement standard uncertainty of the mass quantity were obtained from the calibration certificate of the set of weights related to the vibration generator. In the case of the rotation frequency, the same information was retrieved from an experimental study described in sub-section 3.2.

3.2. *Experimental work*

In the context of the performed study, experimental work was developed in order to establish traceability to the International System of Units (SI), for the mass and rotation frequency quantities, in order to have a rigorous knowledge of the corresponding estimates and measurement uncertainties.

The set of weights used in LNEC's vibration generator was subjected to calibration in a controlled environment of a metrology lab using, as a reference standard equipment, a SI traceable non-automatic weighing instrument (brand Sartorius, model F150S.D2-B, id. 81.04, with a range of 150 kg and a resolution of 1 g). The mass of each weight was estimated by the average value of a sample of five measurements. The 95 % expanded measurement uncertainty was equal to 2.0 g, considering a t-Student probability distribution with 97 effective degrees of freedom. This dispersion of mass values reflects the contributions of the weighing instrument calibration, drift, resolution and linearity, and also the measurement zero.

The calibration of the rotation frequency of LNEC's vibration generator was performed in situ, during a forced vibration test of the Daivões concrete arch dam, located in the North of Portugal. During the calibration operation, the air temperature varied between 23.6 ºC and 25.6 ºC, while the atmospheric pressure was comprised between 1001 mbar and 1009 mbar. An optical circuit with an infrared beam was established between an SI traceable digital tachometer (reference standard equipment) and a reflective patch glued in the vibration generator vertical rotation shaft. This allowed comparing the rotation frequency readings in the vibration generator indicator with the reference values obtained with the tachometer, for the studied configurations mentioned in table 1 for case A and B. Experimental samples of measurements were obtained for each rotation frequency testing step, being composed by 10 consecutive values obtained in time intervals of five seconds. The results are presented in table 3.

Several uncertainty sources were identified in the performed calibration, being characterized in a probabilistic perspective in table 4.

The application of the uncertainty components mentioned in table 4 into the Law of Propagation of Uncertainty [5], allows determining the rotation frequency standard measurement uncertainty (equal to 0.010 Hz), and the corresponding effective degrees of freedom (equal to 166).

Table 3. Results of the rotation frequency calibration.

Case A				Case B			
Nominal value /Hz	Reading /Hz	Reference value /Hz	Sample standard deviation / Hz	Nominal value /Hz	Reading /Hz	Reference value /Hz	Sample standard deviation / Hz
1	0.94	0.941 5	0.000 6	5	5.16	5.162	0.001
2	2.02	2.017	0.001	6	6.17	6.171	0.002
3	3.06	3.061	0.001	7	7.17	7.175	0.003
4	4.06	4.063	0.001	8	8.17	8.169	0.001
5	5.05	5.050	0.001	9	9.17	9.168	0.001
6	5.98	5.980	0.001	10	10.17	10.176	0.004
				11	11.12	11.118	0.002
				12	12.04	12.042	0.003
				13	13.03	13.028	0.002
				14	14.01	14.006	0.003
				15	14.95	14.948	0.002

Table 4. Measurement uncertainty budget for the rotation frequency calibration.

Uncertainty component	Uncertainty source	Probability distribution	Measurement standard uncertainty	Degrees of freedom
$u(f)_{\mathrm{cal}}$	Tachometer calibration	Gaussian	0.000 7 Hz	50
$u(f)_{\mathrm{dri}}$	Tachometer drift	Uniform	0.008 3 Hz / $\sqrt{3}$ =0.004 8 Hz	50
$u(f)_{\mathrm{res}}$	Measurement resolution	Uniform	0.008 3 Hz / $\sqrt{3}$ =0.004 8 Hz	50
$u(f)_{\mathrm{dev}}$	Systematic deviations	Uniform	0.006 Hz / $\sqrt{3}$ =0.003 5 Hz	50
$u(f)_{\mathrm{rep}}$	Repeatability	Gaussian	0.004 Hz / $\sqrt{10}$ =0.001 7 Hz	9

3.3. *Measurement uncertainty propagation*

The input measurement uncertainties mentioned in table 2, were propagated through the mathematical models given by expressions (1) and (2), using a Monte Carlo method and following the main guidelines of the GUM Supplement 1 [4]. A total of 10^6 runs were performed in each numerical simulation to assure a convergent solution for the dispersion of values related to the excitation force and a computational uncertainty lower than 0.01 kN. A dedicated calculation routine was developed in a MATLAB environment, being supported by a Mersenne-Twister pseudo-random number generator [6].

The same routine was used for a sensitivity analysis aiming the identification of the main uncertainty contribution for the excitation force uncertainty. This was achieved considering a 25 % individual magnitude increase of each standard uncertainty mentioned in table 2 and normalizing the corresponding individual increase of the output measurement uncertainty.

4. Results

4.1. *Case A*

Numerical simulations by a Monte Carlo method were performed for the studied case A (set of five weights in the vibration generator), in a rotation frequency interval between 1 Hz and 6 Hz. The obtained estimates and the 95 % expanded measurement uncertainties (absolute and relative) are presented in table 5.

Table 5. Numerical simulation results for case A.

Frequency /Hz	Force estimate /kN	Absolute uncertainty /kN	Relative uncertainty /%	Computational uncertainty /kN
1	3.55	0.14	3.9	0.000 7
2	14.19	0.28	2.0	0.001 5
3	31.92	0.43	1.3	0.002 2
4	56.74	0.58	1.0	0.003 4
5	88.66	0.74	0.83	0.004 2
6	127.68	0.91	0.71	0.005 1

In the studied case A, the excitation force estimates and expanded (absolute) measurement uncertainties (considering a 95 % confidence interval) varied between 3.55 kN ± 0.14 kN and 127.68 kN ± 0.91 kN, which corresponds to an expanded relative measurement uncertainty between 3.9 % and 0.71 %. The output probability density function showed a Gaussian geometrical shape (see example in figure 4) in the studied rotation frequency interval.

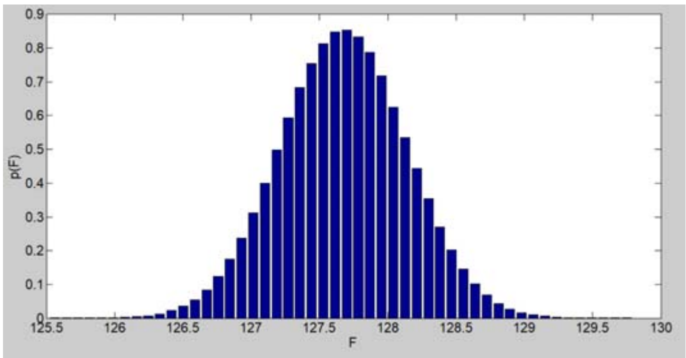


Figure 4. Output probability density function of the force – case A – rotation frequency 6 Hz.

The performed sensitivity analysis showed that, for case A, the rotation frequency measurement uncertainty is the major contribution (from, approximately, 80 % up to 100 %) for the excitation force dispersion of values. Small contributions arise from the rod diameter and the radial position of the weights, namely, when the rotation frequency increases up to 6 Hz, as shown in figure 5.

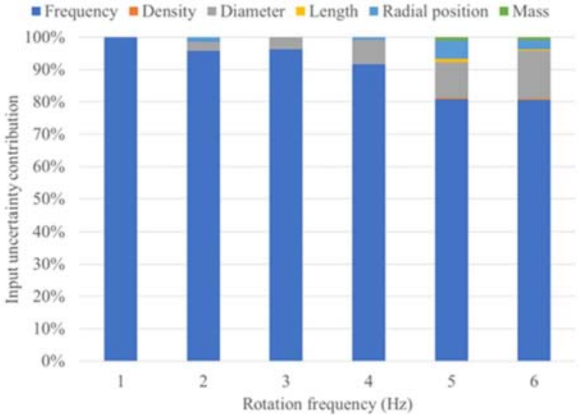


Figure 5. Measurement uncertainty contributions for case A.

4.2. Case B

Numerical simulations by a Monte Carlo method were performed for the studied case B (single weight in the vibration generator), in the rotation frequency interval between 5 Hz and 15 Hz. The obtained estimates and the 95 % expanded measurement uncertainties (absolute and relative) are presented in table 6.

Table 6. Numerical simulation results for case B.

Frequency /Hz	Force estimate /kN	Absolute uncertainty /kN	Relative uncertainty /%	Computational uncertainty /kN
5	16.71	0.25	1.5	0.001 1
7	32.76	0.44	1.3	0.001 6
9	54.15	0.71	1.3	0.002 3
11	80.9	1.0	1.2	0.002 9
13	113.0	1.4	1.2	0.004 0
15	150.4	1.8	1.2	0.004 6

In the studied case B, the excitation force estimates and expanded (absolute) measurement uncertainties (considering a 95 % confidence interval) varied between 16.71 kN ± 0.25 kN and 150.4 kN ± 1.8 kN, which corresponds to an expanded relative measurement uncertainty between 1.5 % and 1.2 %. In the studied rotation frequency interval, a geometrical shape evolution of the output probability density function was noticed (see figures 6 and 7), from a Gaussian geometrical shape (near 5 Hz) to a trapezoidal geometrical shape (for the remaining rotation frequencies, up to 15 Hz).

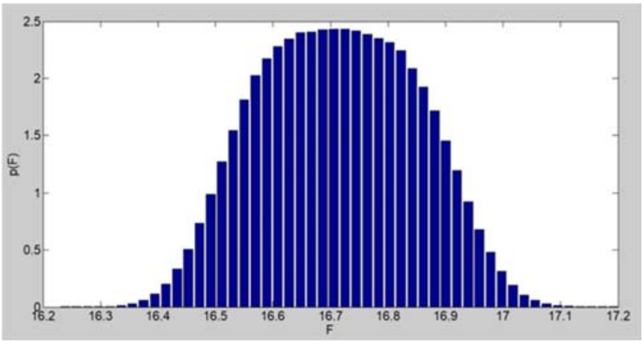


Figure 6. Output probability density function of the force – case B – rotation frequency 5 Hz.

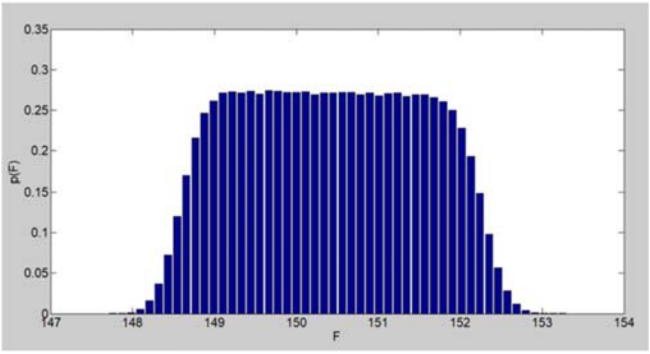


Figure 7. Output probability density function of the force – case B – rotation frequency 15 Hz.

In case B, the sensitivity analysis revealed the rod diameter measurement uncertainty as the main contribution for the excitation force dispersion of values, varying approximately between 75 % at a rotation frequency of 5 Hz and 95 % at a rotation frequency of 15 Hz, as shown in figure 8.

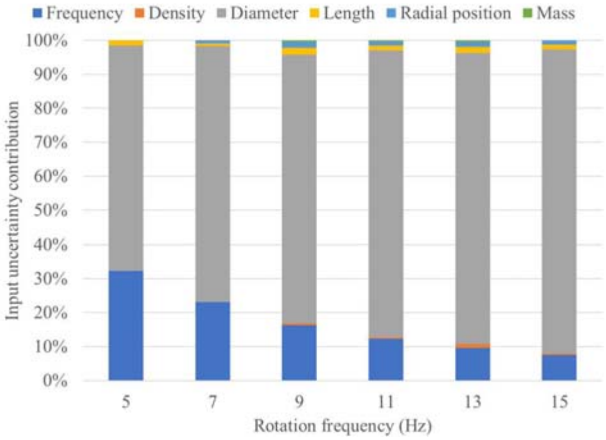


Figure 8. Measurement uncertainty contributions for case B.

Except for the rotation frequency, the remaining input uncertainty contributions are considered negligible. This result justifies the geometrical shape evolution of the output probability density function previously mentioned, since the contribution of the rotation frequency (characterized by a Gaussian distribution) reduces with the increasing rotation frequency, while the contribution of the rod diameter (represented by a uniform distribution) increases.

5. Conclusions

This study allowed evaluating the metrological quality of the excitation force of an eccentric masses vibration generator used by LNEC in the experimental context of concrete dams force vibration. The obtained knowledge is essential to assure confidence and rigorous knowledge about the applied excitation force, namely, in extreme situations near dynamical structural safety limits of the observed concrete dam and of the used testing equipment.

In addition to the establishment of SI traceability of relevant input quantities, such as the mass of the applied weights and the rotation frequency, the increase of confidence in the excitation force applied in the observed concrete dam was also obtained from the measurement uncertainty determination in two configurations (case A and B) of the vibration generator.

In case A (set of five weights), the excitation force estimates and expanded measurement uncertainties (considering a 95 % confidence interval) varied between 3.55 kN ± 0.14 kN and 127.68 kN ± 0.91 kN, being rotation frequency the major contribution for the obtained dispersion of force values. In case B (single weight), the excitation force estimates and expanded measurement uncertainties varied between 16.71 kN ± 0.25 kN and 150.4 kN ± 1.8 kN, being the generator's rod diameter the main contribution for the output measurement uncertainty.

Based on the results obtained from the sensitivity analysis, the reduction (if required) of the excitation force measurement uncertainty should be focused on the measurement uncertainty related to the input quantities rotation frequency (especially for configurations characterized by a high number of weights) and rod diameter (for a reduced number of weights applied in the vibration generator).

Although the studied configurations (A and B) correspond to the two limits of application of the vibration generator, intermediate configurations (different number and radial positions of the weights in the vibration generator) can be additionally studied, based on the described measurement uncertainty evaluation functional approach by a Monte Carlo method.

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