Measurement: Sensors 18 (2021) 100296

Contents lists available at ScienceDirect

Measurement: Sensors

journal homepage: www.sciencedirect.com/journal/measurement-sensors

A numerical study of measurement uncertainties for wave gauges

ARTICLE INFO

Keywords
Wave gauges
Wave measurement
Physical modelling
Instrumental target measurement uncertainty

ABSTRACT

This study describes the development of a Monte Carlo based numerical analysis method to determine the instrumental uncertainty of dimensional measurements with wave gauges. The paper presents the adopted calculation procedure, namely, the mathematical model used in the determination of the amplitude and frequency of the wave gauge measurements. The results show a relative expanded (95%) target instrumental measurement uncertainty between 0.067 7 m and 0.068 2 m for the amplitude and between 0.592 4 Hz, and 0.592 8 Hz for the frequency.

The study carried out allows for the conclusion that the use of a Monte Carlo-based numerical approach is suitable to evaluate the measurement uncertainty of parameters fitted to wave displacement analysis.

1. Introduction

A wide range of coastal engineering studies rely heavily on physical modelling, both for numerical simulations validation and to support the design of marine structures. In this field of study wave measurements are ubiquitious and rely, for the most part, on the use of wave gauges.

Laboratory facilities for this purpose involve wave flumes or wave tanks, where water waves are mechanically generated by moving paddles to simulate specific sea wave conditions.

Physical modelling of wave-induced processes depends on the measurement of both the amplitude and frequency of the observed waves to verify that the wave characteristics are generated according to prescribed specifications.

Measurement of the reflected wave is also needed for active wave absorption on the paddle, for accurate wave generation. In fact, active wave absorption methodologies are implemented in order to cancel the reflected waves from a tested structure and to adequately reproduce the desired or prescribed sea states in a laboratory environment. For active wave absorption implementation, rigorous wave measurements are mandatory in order to adequately implement strategies of hydrodynamic feedback for wave reflection cancelation [1]. In active absorption systems typically, the hydrodynamic feedback is implemented with incident and reflected waves separation methods. However, most of the authors acknowledge accuracy problems in the separations process related to noise in the wave measurements [2–4].

Moreover, in active absorption systems, the amplitude deviations affect proportionally the magnitude of the reflected waves while frequency deviations are responsible for generating long waves in waves flumes and tanks, which is not desirable.

In response to the increasing demands of the scientific community, the laboratory facilities have become increasingly more sophisticated in the last decades, with new equipment being developed to measure water waves over time [5], new methodologies for active absorption systems [6] and new methods for wave separation [7].

In spite of the plethora of devices applied to water wave measurements, the most widely and commonly used include resistive wave gauges, also adopted in this study.

For wave measurements the usual procedure is to place one or more wave gauges at reference fixed points along the test facility (wave tank or flume). The measurement principle is based on the detection of the surface water displacement. The gauge is partially submerged in water and the measurement is proportional to the change of the water surface in time due to local surface displacement as the waves propagates.

Since the physical modelling is normally conducted at reduced scales, typically 1:20 or 1:50, the dimensional measurements of amplitude and frequency accuracy of at least one order of magnitude is mandatory.

The main objective of this study is to develop a numerical approach able to estimate the measurement uncertainty of amplitude and frequency in wave gauge measurements.

2. Approach and methodology

The amplitude and frequency of water waves in a flume are imposed by a mechanical wave generator. In this study, a regular wave with amplitude of 0,07 m and nominal frequency of 0.6 Hz was generated. The data from a wave gauge was recorded for 200 seconds with a sampling frequency of 128 Hz.

From this experimental data, the first 5 seconds (640 samples) are used in this study for amplitude and frequency estimation, considering that it provides a less complex combination of signals.

The major challenge in the estimations of the measurements lies in the frequency estimation. Although the gauge measurements are over sampled, using a 128 Hz sampling frequency, traditional spectrum estimators rely on discrete frequency analysis. However, by fitting a mathematical model to the measured data it is theoretically possible to estimate a continuous frequency in the vicinity of the nominal frequency.

https://doi.org/10.1016/j.measen.2021.100296

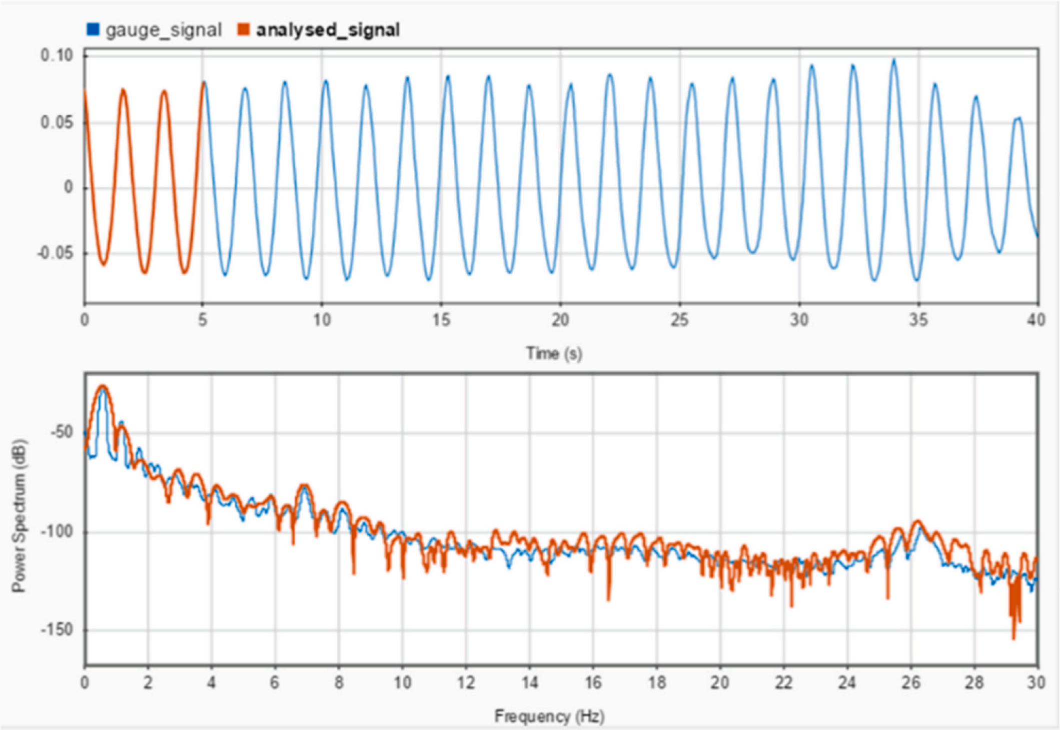

Fig. 1. Measured signal time series and spectrum: wave gauge measurements signal(blue) and analysed part of the signal (red). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

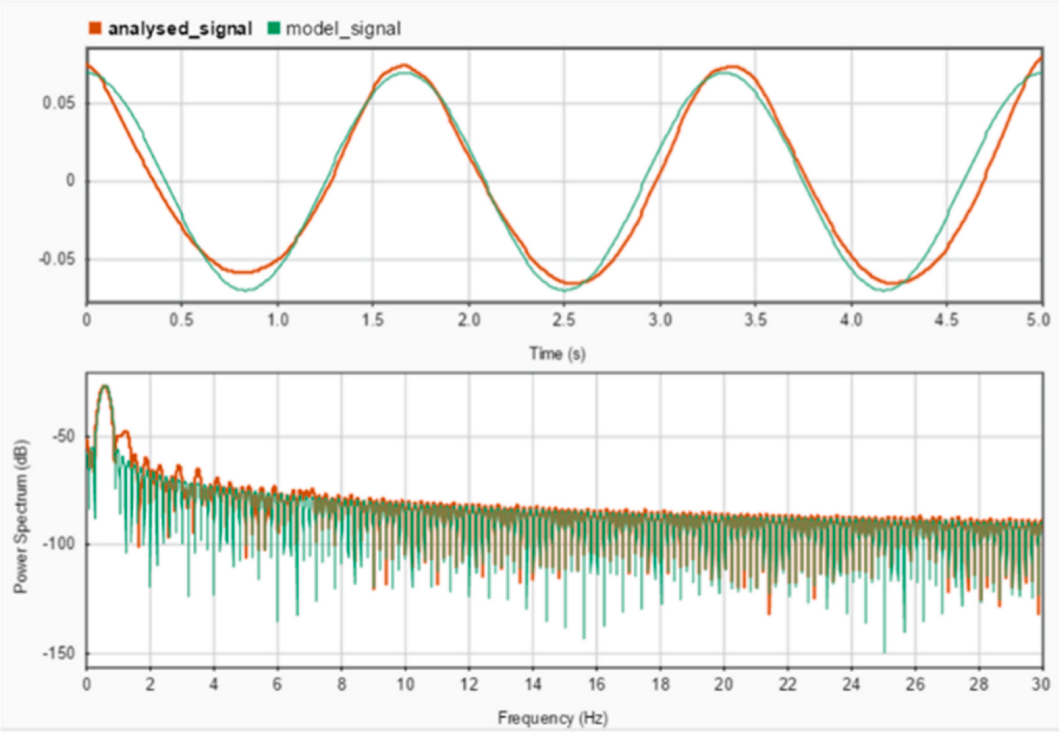

Fig. 2. Measured (red) and model (green) signal time series and spectrum. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

The amplitude and frequency estimations are calculated by a fitting procedure using the least squares regression analysis with a nonlinear model. The applied model corresponds to a sine wave with arbitrary amplitude A and frequency F, described by

$$g(t) = A\ sin(2\pi Ft + \varphi) + B \tag{1}$$

In this study the model in (1) is used to estimate only the parameters amplitude A and frequency F, with a fitting procedure. The parameters φ

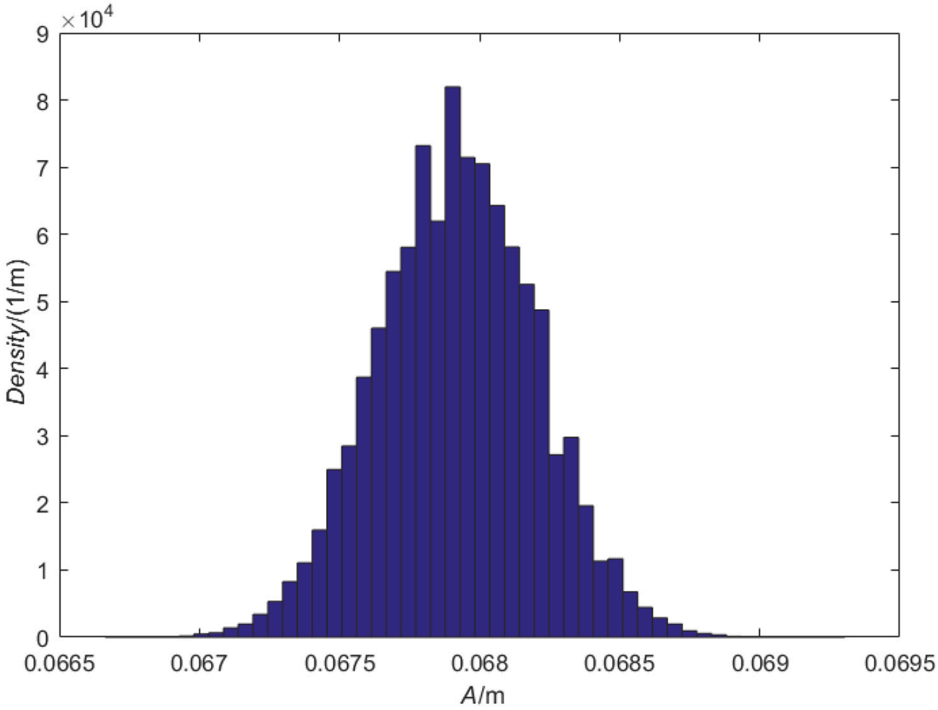

Fig. 3. PDF for parameter A with input displacement standard uncertainty of 0.005 m.

and B are considered constant in this study.

The gauge signal time series and power spectrum are shown in Fig. 1 in blue. The analysed 640 samples of the corresponding signals are shown in red. Even though the signals correspond to regular water wave generation, it is noticeable the amplitude variation along time and the spectral content besides the expected fundamental frequency of 0.6Hz.

The first procedure is to compare the analysed signal with the theoretical model using the nominal parameters ($A = 0.07$ m and $F = 0.6$ Hz) (in green, Fig. 2). This procedure can confirm that the established model explains the fundamental frequency spectral content of the measured signal, as show in Fig. 2.

3. Evaluation of measurement uncertainty of fitted sine waves parameters using a Monte Carlo method

The fitting of sine waves to the experimental data using any classical method yields estimates of two variables (wave amplitude and frequency). Because of the uncertainty inherent to gauge measurements, the estimated values of these variables are plagued with uncertainty. In this context, the evaluation of the uncertainty of these parameters (including the knowledge about the probability distribution functions – PDF – shape and statistical parameters) becomes of interest as added information to be applied in the processes of monitoring and modelling used in hydraulics studies.

As a first step in this approach, it is required to have information regarding the traceability of the measuring instruments. This information is usually obtained from calibration certificates. For this purpose, measurement standard uncertainties of 2 mm (0.002 m) and 5 mm (0.005 m) are used in the present work, allowing to assess the impact of these measurement uncertainties in the fitted model parameters.

A common method used for the evaluation of measurement uncertainty, described in Ref. [8], is known as GUM (Guide to the Expression of Uncertainty in Measurement), first published by ISO, IEC and other organizations in 1993. The method assumes that given a functional relation f described by

$$y = f(x_1, \cdots, x_n) \tag{2}$$

where y is the output quantity obtained from n input quantities, x_i, using the development of the function f as a 1st order Taylor series, a formulation for the measurement standard uncertainty of the output quantity, $u(y)$, is given in terms of the Law of Propagation of Uncertainties

$$u^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + \sum_{i=1}^{n-1} \sum_{j=2}^{n} \left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right) u(x_i) u(x_j) \tag{3}$$

where $u^2(y)$ is the output standard uncertainty squared, having the dimension of a variance.

In this equation, the first part of the second term is related to the variance of each input quantity, while the second part of the second term is related to the contributions due to the correlation between input quantities.

It should be reminded that this general method gives an exact solution for linear functions but is only an approximate solution for the nonlinear case and more complex functions, as is the case of the fitting process used for this study. In this case, Supplement 1 of GUM [9] was published recently introducing the description of the use of Monte Carlo method to more complex functional relation. This was the method adopted in this study to evaluate the measurement uncertainty of the PDF's parameters.

The Monte Carlo method was implemented using MatLab © tools, by performing numerical simulation for 10^5 and 10^6 runs, using the software intrinsic function to generate series of Gaussian pseudo-random numbers. Fit functions based on the MatLab © routine *fminsearch*, as well as the statistical tools to obtain the percentiles needed to achieve the limits of centered expanded measurement uncertainty intervals are also applied.

The steps for the implementation of the numerical simulation using a Monte Carlo method, were as follows:

a. for each PDF considered in the study, a matrix nData($\boldsymbol{p}$, $\boldsymbol{p}_{\mathbf{num}}$) was created with k (640) rows for each wave gauge measurements values, p_k, and the 10^6 columns, i, of Gaussian pseudo-random number generated series, $p_{\mathrm{num}}(k,i)$, obtained for each experimental value k;

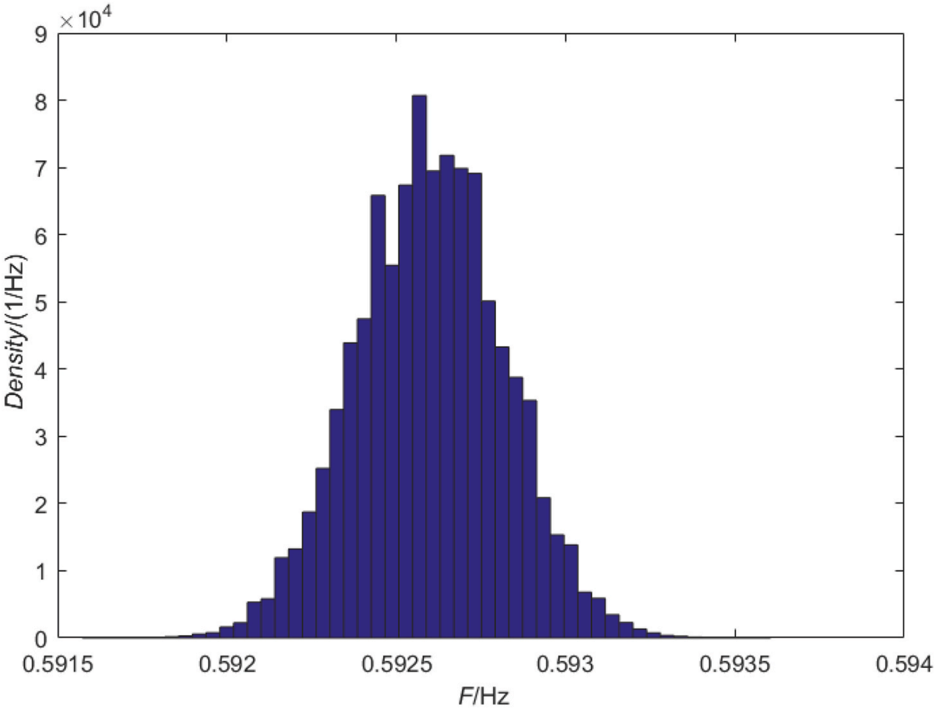

Fig. 4. PDF for parameter F with input displacement standard uncertainties of 0.005 m.

b. for each vector column of the matrix of the type [(p_1, $p_{\mathrm{num}}(1,1)$), …., (p_k, $p_{\mathrm{num}}(k,1)$)] the PDF was fitted and the parameters obtained, amplitude $A(i)$ and frequency $F(i)$. Having completed this step, the 10^6 generated numerical series of the parameters should be available (in this study, the parameters were obtained using the moments method);
c. the parameters vector series is ordered using the function *sort*; and
d. 0.025 and 0.975 percentiles of the ordered series are evaluated, allowing to obtain the (centered) limits of the expanded measurement uncertainty.

Table 1
Expanded measurement uncertainty related with the output quantities (amplitude and frequency).

Displacement standard uncertainty/m	$U_{95}(A)$/m	U_{95} (F)/Hz
0.002	[0.067 7, 0.068 2]	[0.592 4, 0.592 8]
0.005	[0.067 4, 0.068 5]	[0.592 2, 0.593 0]

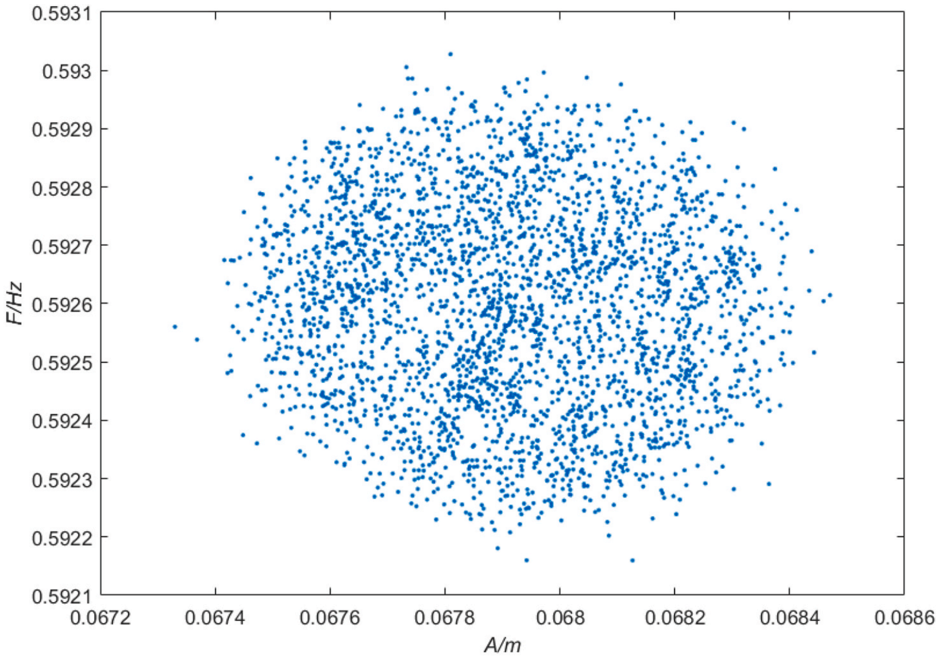

Fig. 5. Representation of the combined pairs numerically generated of A and F parameters for correlation analysis.

By applying the above procedure, we obtained the results presented and discussed in the following section.

4. Results and conclusions

The results obtained using a Monte Carlo-based approach led to the probability density functions for the parameters A and F of the PDF's studied (Figs. 3 and 4, respectively), from which statistical information was obtained. The mean values and the expanded measurement uncertainties (MU) with 95% of confidence intervals are given in Table 1 for two different input standard uncertainties.

Considering the visual shape of the PDF, it was deemed necessary to do an analysis of the series regarding symmetry and flatness using the statistical parameters of skewness and kurtosis. In both cases, the results obtained comply with the expected values for Gaussian PDF.

The estimates of the average values obtained from the output pseudo-random series generated for the quantities A and F, were also calculated, having the value of 0.067 9 m for the amplitude, A, and 0.592 6 Hz for the frequency, F.

The standard uncertainties related with these estimates, obtained for the two input displacement uncertainties, are given in Table 1.

For the two values of gauge standard uncertainty, the comparison between the uncertainty intervals obtained for A and F shows that the increase of input uncertainties has low impact on the uncertainty of the output quantities.

The statistical analysis of the parameters was completed with the study of the correlation in the cases that parameters were simultaneously obtained (PDF A and F). In this case, the variance-covariance matrices, $\mathrm{Var}(A,\ F)$, were calculated for the two input standard uncertainties of 0.002 m and 0.005 m, presented below, with the graphical representation in Fig. 5, for the latter case. The results show a low correlation of A and F parameters, indicating that model used in (1) and the fitting procedure are independent.

For displacement standard uncertainties of 0.002 m:

$$\mathrm{Var}(A,\ F) = 10^{-07}\begin{bmatrix} 0.135\,8 & -0.000\,4 \\ -0.000\,4 & 0.081\,9 \end{bmatrix}$$

For displacement standard uncertainties of 0.005 m:

$$\mathrm{Var}(A,\ F) = 10^{-07}\begin{bmatrix} 0.798\,3 & -0.021\,2 \\ -0.021\,2 & 0.474\,8 \end{bmatrix}$$

The study performed allows for the conclusion that the use of the Monte Carlo-based numerical approach is suitable to evaluate the measurement uncertainty of parameters fitted to free surface displacement analysis, although some developments and further analysis should be made regarding the output PDF's quality.

Future work should address the analysis of more complex signals, *e.g.*, irregular waves measurements, and their deconvolution, enabling the use of this approach in multivariate conditions. Also, because the dynamic behavior of the phenomena under observation requires the use of measuring instruments at different locations, *e.g.*, multi gauge measurements, the complexity due to the synchronization of different time series is also a challenge for the evaluation of uncertainty. Finally, concerning the studies under development intended to use these measurements to implement the hydrodynamic feedback for active wave absorption systems. Both amplitude and frequency of the observed waves are determinant for quality of the active absorption system implementation. Also, in the development of new methodologies for incident and reflected waves separation based on gauge arrays. The knowledge of amplitude and frequency uncertainty enables further optimizations of the separation method, where expected noise robustness of these methods is related the number of gauges and the measurements uncertainty information.

Acknowledgments

The first author acknowledges the support of Fundação para a Ciência e Tecnologia (FCT) through the Grant No. SRFH/BD/136899/2018.

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