Statistical analysis of mechanical properties of prestressing strands

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ABSTRACT: A statistical analysis of the mechanical properties of prestressing strands used over the last decade in Portugal is presented. Data was collected from a number of tensile tests carried out in *Laboratório Nacional de Engenharia Civil* (LNEC) during the process of certification of those products. Tests cover a period from 2001 to 2009 and involve manufacturers from several countries. This study examines the variability of the most important mechanical properties, compares the results obtained with the recommendations of the *Probabilistic Model Code*, as well as the Eurocodes, and proposes probabilistic models for those properties.

1 INTRODUCTION

This study deals with the statistical analysis of three groups of samples of strands with nominal diameters of 13.0, 15.2 and 15.7 mm, which correspond to the nominal cross sectional areas of 100, 140 and 150 mm^2, respectively. All strands have nominal tensile strength of 1860 MPa and are all composed by 7 wires. These have been the most commonly used in Portugal.

The samples refer to tensile tests performed between 2001 and 2009 in *Laboratório Nacional de Engenharia Civil* (LNEC), Portugal. During this period, over 500 tensile tests were carried out for the 3 families of strands mentioned above. However, several of these tests refer to strands produced from the same heat. As it is well known, the variability within a single heat is lower than the variability between different heats. Thus, for the purpose of statistical analysis, only one test from each heat was chosen (at random), which reduced the sample to 131 tests.

For each of the 3 families of strands, the following mechanical properties were studied: tensile strength (or maximum stress), 0.1% proof stress, total elongation at maximum force and modulus of elasticity. However, it was found that the difference in the mean of those properties between families was of the same order of magnitude of its standard deviations, which allowed us to consider the 3 families as belonging to the same population. The 3 families were thus merged into a single population.

The strands tested came from manufacturers of different countries, including Portugal, Spain, Thailand and Italy. However, as it will be seen, the variability of the studied mechanical properties is relatively small, not justifying a separated analysis by manufacturer.

Figure 1 shows a stress-strain diagram for a typical prestressing strand. As usual, the 0.1% proof stress will be denoted by $f_{p0.1}$, the tensile strength by f_p, and the corresponding elongation (total elongation at maximum force) by ε_u. The modulus of elasticity will be denoted by E_p. As shown in Figure 1, prestressing strands do not exhibit a distinct yield point, which is typical of high strength steels, presenting, however, a slight inflection corresponding to the hardening of the steel, before reaching the maximum stress.

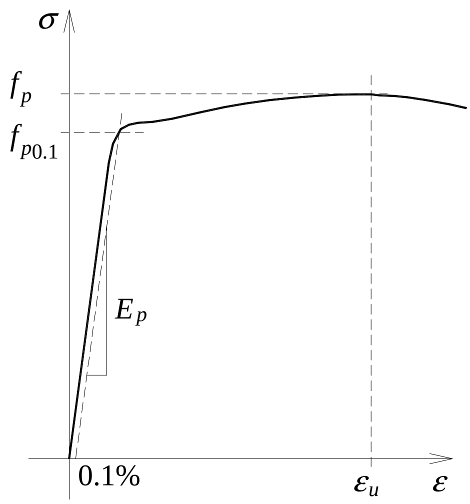


Figure 1. Typical stress-strain diagram for a prestressing strand.

As said above, the studied strands are all of the Y1860 grade, which has been clearly the most commonly used in Portugal. The value 1860 (which designates the grade) is termed *nominal tensile strength*, expressed in MPa (prEN 10138-1, 2009) and can be interpreted as the quantile 0.05 of the probability distribution of f_p, known generally by characteristic value and denoted usually by f_{pk}.

The purpose of this study is to analyze the variability of the most important mechanical properties of the prestressing strands and compare it with the corresponding recommendations of the *Probabilistic Model Code* (JCSS, 2001). The following section briefly examines those recommendations.

2 RECOMMENDATIONS OF THE *PROBABILISTIC MODEL CODE*

Table 1 shows the recommendations of the *Probabilistic Model Code* (PMC) (JCSS, 2001) concerning the tensile strength f_p, modulus of elasticity E_p and the total elongation at maximum force ε_u. As observed, PMC presents two expressions for the mean of f_p, one of which assumes constant coefficient of variation and the other constant standard deviation. This document gives no indication about which expression one should use, being implicit that it should be used the one whichever is in the safety side.

With regard to the 0.1% proof stress, PMC recommends the model: $f_{p0.1} = 0.85 f_p$, which assumes a perfect correlation between f_p and $f_{p0.1}$. As it will be seen, this model deserves some reservations.

Table 1. Recommendations of the *Probabilistic Model Code* (JCSS, 2001).

Variable		Mean	Std. dev.	V*	Distribution
f_p		$1.04 f_{pk}$ or f_{pk} + 66 MPa	– 40 MPa	0.025 –	Normal
E_p	Wires	200 GPa	–	0.02	Normal
	Strands	195 GPa	–		
	Bars	200 GPa	–		
ε_u		0.05	0.0035	–	Normal

* Coefficient of variation

3 STATISTICAL ANALYSIS

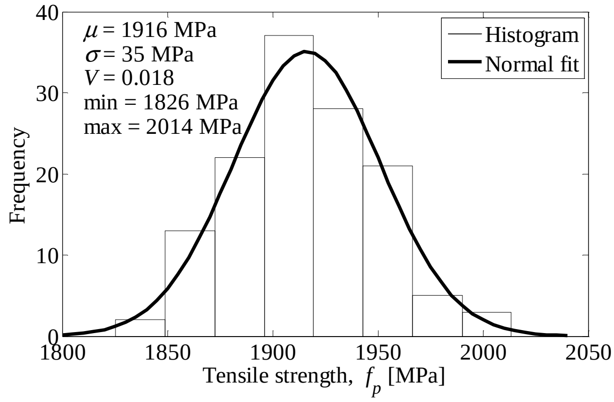
This section presents the statistical analysis of experimental values of tensile tests and produces some considerations about its relevance for the structural safety. It must be said that the stresses were computed for all cases dividing the forces obtained from tensile tests by the actual cross sectional areas of the strands and not by the nominal ones. In this way, the uncertainty in stresses and in cross sectional areas could be separated.

3.1 *Tensile strength*

Figure 2 shows the histogram of the tensile strength f_p (with 8 bins) of the 131 tests mentioned previously, as well as the values of f_p by year. As it can be seen, the normal model fits well the histogram, which agrees with the PMC recommendations and the prEN 10138-1 (2009). The coefficient of variation obtained is very low, $V = 0.018$, and lower than the one proposed by PMC (JCSS, 2001). PMC proposes σ = 40 MPa, which is 14% higher than the value obtained in this study.

According to the parameters obtained (μ = 1916 MPa; σ = 35 MPa), the characteristic value of f_p can be estimated by $f_{pk} = 1916 - 1.645 \times 35 = 1858 \approx 1860$ MPa, which satisfies the specified value for the Y1860 grade. The estimate of f_{pk} using directly the empirical distribution is 1861 MPa.

Regarding the graphic (b) in Figure 2, it can be concluded that there is no trend on the tensile strength f_p during the period observed (2001 to 2009). This figure also suggests that the sample is free of outliers.



(a)

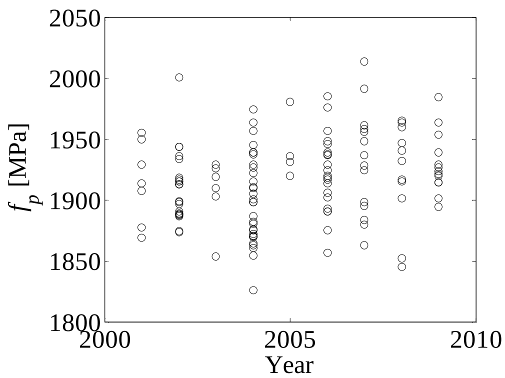


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Figure 2. Tensile strength f_p. (a) Histogram. (b) Values of f_p by year. Each dot corresponds to a tensile test.

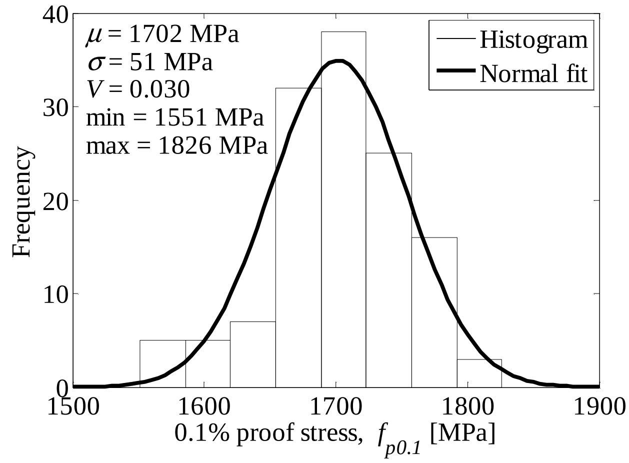
3.2 *The 0.1% proof stress*

The 0.1% proof stress $f_{p0.1}$ is a parameter of great importance for the structural safety and, to some extent, more decisive than the tensile strength itself. In fact, the tensile strength is only achieved for large strains, hardly reached even for ultimate limit states.

Figure 3 shows the histogram of the 0.1% proof stress and its temporal variation (131 tests).

As it can be seen, the 0.1% proof stress has greater variability ($\sigma_{fp0.1}$ = 51 MPa) than the tensile strength (σ_{fp} = 35 MPa). In fact, the 0.1% proof stress is more sensitive than tensile strength, because it depends on the measured modulus of elasticity and even the curvature of the stress-strain diagram where yielding starts. This finding raises a comment on the model $f_{p0.1} = 0.85 f_p$ proposed by PMC. In fact, according to this model, the standard deviation of the 0.1% proof stress is smaller than that of the tensile strength, contrarily to the results obtained. Later in this paper, a model for obtaining $f_{p0.1}$ from f_p based on regression analysis will be proposed, which allows overcoming this issue.

According to the above results, the characteristic value of $f_{p0.1}$ can be estimated by $f_{p0.1k} = 1702 - 1.645 \times 51 = 1618$ MPa. The ratio between the mean value of $f_{p0.1}$ and the mean value of f_p is 1702/1916 = 0.89 and the ratio between their characteristic values is 1618/1860 = 0.87. This ratio is important in characterizing the steels ductility and is in accordance with the standard prEN 10138-3 (2009).



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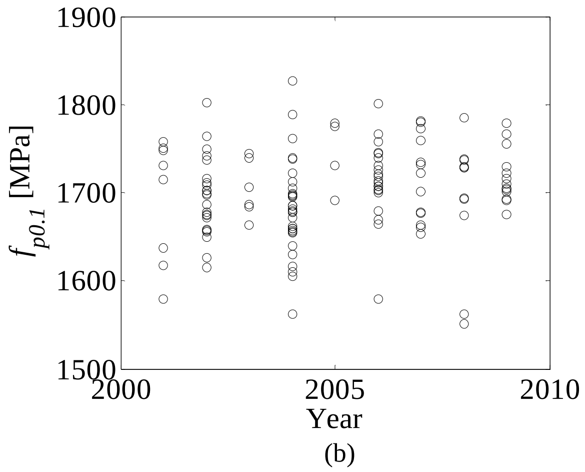

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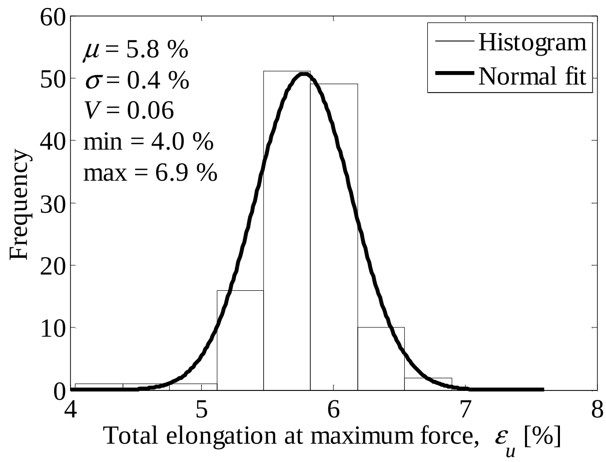
Figure 3. The 0.1% proof stress $f_{p0.1}$. (a) Histogram. (b) Values of $f_{p0.1}$ by year. Each dot corresponds to a tensile test.

3.3 *Total elongation at maximum force*

Total elongation at maximum force, undoubtedly an important parameter for the safety of structures, does not generally raise concerns since typical values of this parameter (mean value above 5%, as shown in Figure 4) provide a rotation capacity of concrete sections in plastic domain higher than what is normally required in plastic analysis. Indeed, even for strains relatively high during tensioning (for example strains of about 0.7%), the increase in strain necessary to bring the steel to rupture would be 5% – 0.7% = 4.3%, which would correspond to very high levels of cracking and deformation in concrete.

Figure 4 shows the histogram of the ε_u for the sample of 131 tests. Comparing the values obtained (mean and standard deviation) with the recommendations of the PMC, it can be concluded that these recommendations seem reasonable. The histogram, which appears relatively symmetrical, supports the recommendation of PMC that suggests a normal distribution. The graphic (b) shows no temporal trend. On the other hand, the minimum and maximum values observed did not seem to be outliers. It is noted that the available sample (131 tests) satisfies the requirement $\varepsilon_u \geq 3.5\%$, adopted in Portugal (E453, 2002), which is also specified in prEN 10138-1 (2009).



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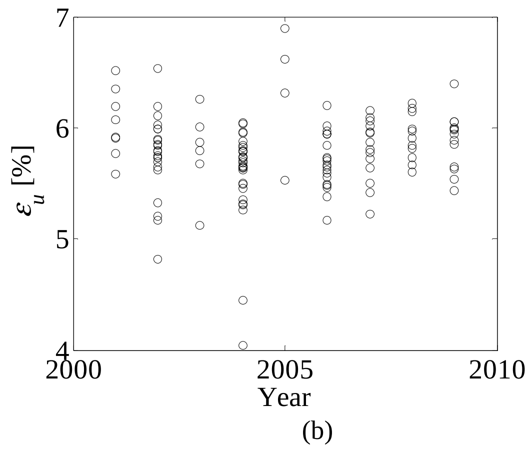


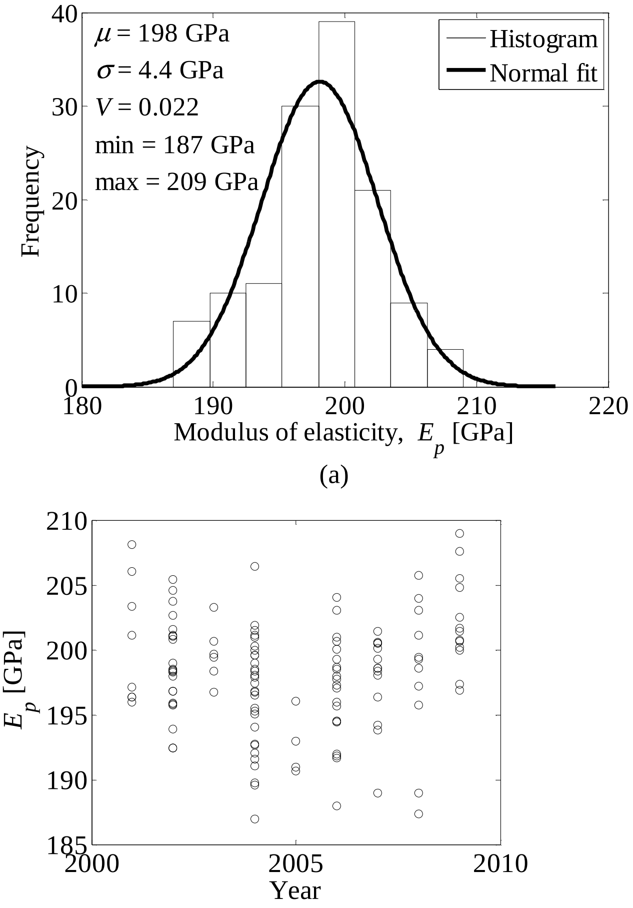
(b)

Figure 4. Total elongation at maximum force ε_u. (a) Histogram. (b) Values of ε_u by year. Each dot corresponds to a tensile test.

3.4 *Modulus of elasticity*

Accurate knowledge on the actual value of the modulus of elasticity is important especially during tensioning, since one of the criteria for controlling the actual prestressing force applied is based on the comparison between measured and calculated elongations, which, of course, depends on the modulus of elasticity. Regarding safety checking, this is a parameter of some importance only with respect to serviceability limit states, namely decompression limit state and crack widths, having little effect on ultimate limit states, because when these are reached the steel are in general in plastic domain.

Figure 5 shows the histogram of the modulus of elasticity, as well as the temporal variation in the observed period (2001-2009). The histogram suggests that the normal model, as recommended by the PMC, is adequate to describe E_p. For strands both the PMC and EN 1992 (2004) recommend an average value of 195 GPa. The mean of the 131 tests is higher than this value, although the difference is small. For the coefficient of variation, the PMC recommends 0.02, which corresponds to a standard deviation 11% lower than the value obtained in this study (4.4 GPa). Thus, maintaining the usual recommendation for the mean value equal to 195 GPa, the results show that higher standard deviation should be adopted, closer to 5 GPa.



(a)

(b)

Figure 5. Modulus of elasticity E_p (a) Histogram. (b) Values of E_p by year. Each dot corresponds to a tensile test.

3.5 *Cross sectional area*

According to prEN 10138-1 (2009), the accepted tolerance for the cross sectional area of strands is ±2% of the nominal value. The histogram presented in Figure 6 refers to a sample of 257 strands of 15.2 mm, which meets this requirement. Note that in order to study the variability of the cross sectional area there is no inconvenient in merging strands from the same heat, because the diameter depends mainly on the work performed by drawing plants. Although the histogram refers to strands of 15.2 mm, the coefficient of variation obtained for other strands, namely strands with diameters of 13.0 and 15.7 mm, was very similar.

In face of these results and the quality control normally carried out by manufacturers and costumers, it seems reasonable to adopt a normal model with mean equal to the nominal area and coefficient of variation of 0.01.

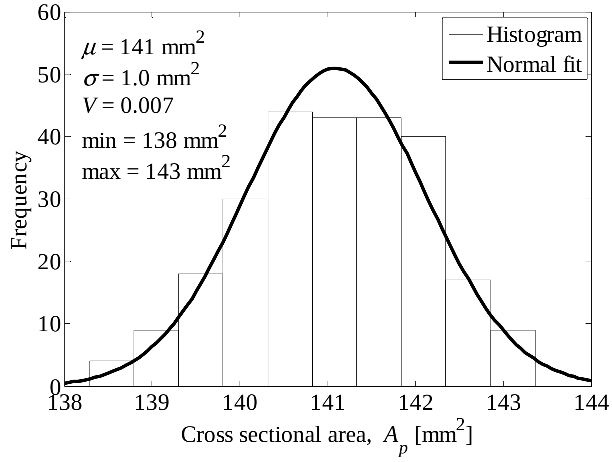


Figure 6. Cross sectional area histogram for strands with nominal diameter of 15.2 mm.

3.6 *Correlation analysis*

3.6.1 *Correlation between 0.1% proof stress and tensile strength*

Figure 7 shows the scatter diagram of points (f_p, $f_{p0.1}$) regarding the sample of 131 tensile tests. A linear regression analysis was performed and the following regression parameters were obtained (Ang & Tang, 2007):

$$\hat{\beta}_0 = -543 \text{ MPa};\ \hat{\beta}_1 = 1.17;\ \hat{\sigma} = 32.1 \text{ MPa}.$$

The coefficient of determination is $R^2 = 0.635$, which corresponds to the coefficient of correlation $\rho = 0.80$ and indicates very good correlation between those two parameters.

Based on the above regression model, in case it is necessary to model simultaneously $f_{p0.1}$ and f_p, the following probabilistic model is proposed:

$$f_{p0.1} = -500 + 1.15 f_p + 30Z \qquad (1)$$

where $Z \sim N(0, 1)$.

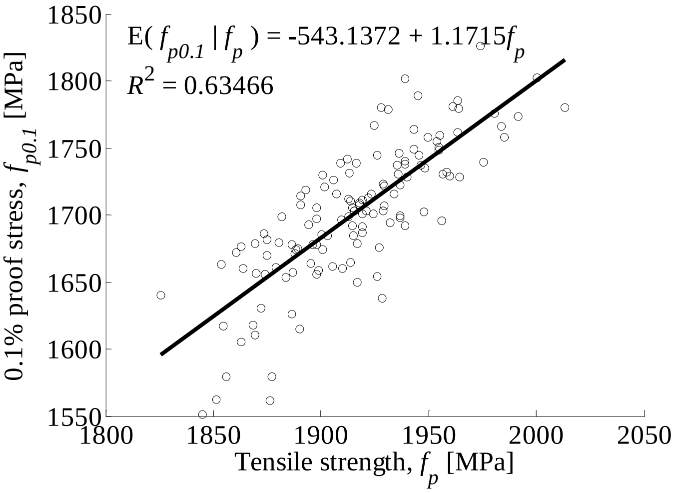


Figure 7. Scatter diagram of points (f_p, $f_{p0.1}$).

3.6.2 *Correlation between total elongation at maximum force and tensile strength*

Figure 8 shows the scatter diagram of points (f_p, ε_u). As indicated, the coefficient of determination is $R^2 = 0.024$, which corresponds to the coefficient of correlation $\rho = 0.15$. From a practical point of view, these results show that ε_u and f_p can be considered independent.

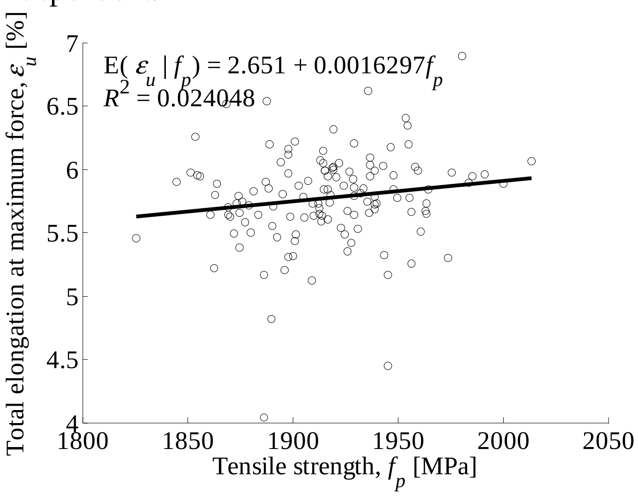


Figure 8. Scatter diagram of points (f_p, ε_u).

4 BAYESIAN ANALYSIS OF THE TENSILE STRENGTH

Tensile strength f_p and its characteristic value f_{pk} were analyzed in sub-section 3.1. Since it was assumed that f_p follows a normal distribution, i.e. $f_p \sim N(\mu, \sigma)$, an estimate of f_{pk} was computed using the following expression:

$$f_{pk} = \mu - 1.645\sigma \quad (2)$$

Remember that considering point estimates for μ and σ, the estimate f_{pk} = 1858 MPa was obtained. According to the Bayesian paradigm the parameters μ and σ are modelled as random variables (Bernardo & Smith, 2000). Since f_{pk} is a function of μ and σ, it follows that f_{pk} is also a random variable. The standard deviation of f_{pk} can be seen as a measure of the error in the estimate f_{pk} = 1858 MPa. Let us compute this error.

Posterior probability distributions for μ and σ can be found in texts such as Bernardo & Smith (2000) and Paulino *et al*. (2003). According to those references, the parameter μ is *t*-distributed and σ^2 follows a inverted gamma distribution. Using those distributions and assuming independence between μ σ^2, a sample of f_{pk} was generated (Monte Carlo Method) from which the mean and the standard deviation was computed. The mean of f_{pk} is 1858 MPa and the standard deviation is 4.8 MPa, which yields a relative error of 4.8/1858 = 0.3%. Since it is a very small error, this means that the value f_{pk} = 1858 MPa can be considered very close to the true value.

The quantile 0.05 of f_{pk} was also computed and the value 1850 MPa was obtained. So, the probability that the true value of f_{pk} is above 1850 MPa is 0.95. Since 1850 is close to 1858, this shows that the distribution of f_{pk} is quite narrow or that the uncertainty in f_{pk} is small. This can be appreciated in Figure 9, which plots the posterior distribution of the characteristic tensile strength f_{pk} together with the predictive distribution of the tensile strength f_p.

It is interesting to note that the Bayesian 0.05-quantile of f_{pk} (1850 MPa) coincides with the corresponding classical lower limit of the one-sided tolerance interval — confidence level of 0.95 and probability coverage of 0.95 (Montgomery & Runger, 2007; ISO 12491, 1997).

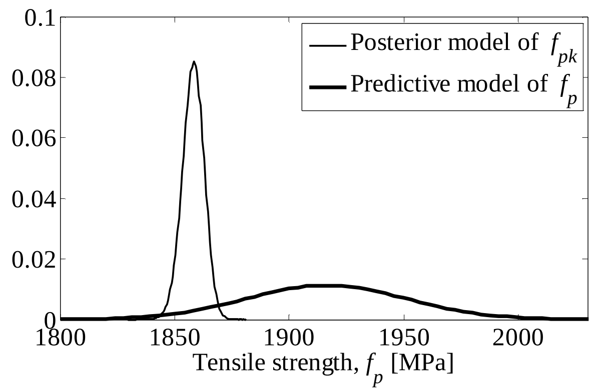


Figure 9. Bayesian probabilistic models for f_{pk} and f_p.

5 CONCLUSIONS

The present study made it possible to appreciate the low variability of the mechanical properties of prestressing strands, which, of course, benefits the safety of structures. The highest variability was obtained for the elongation at the maximum force, which revealed a coefficient of variation of about 0.06. For the remaining properties, the coefficient of variation obtained was lower than 0.03.

The Bayesian analysis showed that the estimate of the characteristic tensile strength can be considered accurate. Since the standard deviations for other mechanical properties are also small, this conclusion can be also applied to those properties. In addition, it is believed that the sample at hand has a reasonable representativeness, so that it can be proposed probabilistic models for the main mechanical properties of prestressing strands. Table 2 summarizes the models proposed.

Table 2. Proposed probabilistic models for prestressing strands of the Yf_{pk} grade (f_{pk} = characteristic tensile strength).

Variable	Unit	Mean	Std. dev.	V	Distrib.
f_p	MPa	f_{pk} + 1.645×35	35	–	Normal
$f_{p0.1}$	MPa	0.89μ_{fp} *	50	–	Normal
ε_u	–	5%	0.4%	–	Normal
E_p	GPa	195	5	–	Normal
A_p	any	Nominal value	–	0.01	Normal

* When it is necessary to model simultaneously $f_{p0.1}$ and f_p, Eq. (1) can be used.

The proposed models were based on the results obtained for strands of the Y1860 grade. However, taking into account the quality control typical for this kind of product, we believe the same model can be applied to strands of other grades.

It was demonstrated that the correlation between 0.1% proof stress and tensile strength is strong. In fact, these parameters cannot be considered independent from each other. On the other hand, the correlation between tensile strength and total elongation at maximum force can be neglected.

Finally, as a last comment, it should be emphasized that the proposed models were the result of tests performed between 2001 and 2009. During this period the mechanical properties studied did not show any trend. However, for purposes of assessment of existing structures, the models should be verified, especially if the steel have been produced in a period outside the period analyzed.

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