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VALIDATION DOCUMENT OF MOBFLOW

MOBILE BOUNDARY FLOW MODEL WITH SEDIMENT SORTING

RELATÓRIO 305/94 - NHHF

Lisboa, October 1994

Software Validation Task Force

I&D
HIDRÁULICA

NOTA PRÉVIA

O presente relatório refere-se à validação do MOBFLOW, Versão 1.1, modelo matemático para o estudo de escoamentos variáveis em cursos de água de secção irregular, com leito fixo ou móvel.

Este trabalho foi efectuado no âmbito do Processo de Estudo LNEC 602/19/10648, incluído na investigação programada para o quatriénio 92 - 95, tendo sido financiado parcialmente pela Junta Nacional de Investigação Científica e Tecnológica (JNICT), mais precisamente por verbas dos projectos PEAMB/C/APR/7/91, PBIC/C/CEG/1354/92 e STRDA/C/AMB/14/92.

A validação do MOBFLOW integrou-se num estudo mais vasto com o objectivo de estabelecer regras para a validação de modelos computacionais. Este estudo envolveu investigadores das seguintes instituições europeias:

- **Centro de Estudios y Experimentación de Obras Publicas** (CEDEX), Espanha
- **Danish Hydraulic Institute** (DHI), Dinamarca
- **DELFT HYDRAULICS**, Holanda
- **HR Wallingford Limited** (HR Wallingford), Reino Unido
- **Laboratoire Hydraulique de France** (LHF), França
- **Laboratoire National d'Hydraulique** (LNH), França
- **Laboratório Nacional de Engenharia Civil** (LNEC), Portugal
- **Norwegian Hydrotechnical Laboratory** (NHL), Noruega
- **SOGREAH Ingénierie**, França
- **Water Resources Research Centre** (VITUKI), Hungria

e conduziu à publicação pela International Assotiation of Hydraulic Research (IAHR) das:

Guidelines for documenting the validity of computational modelling software, IAHR, June 1994

O relatório que agora se apresenta pretende ilustrar a aplicação dessas regras a um caso complexo de modelação matemática, constituindo a primeira etapa do processo de validação do MOBFLOW, ainda sem se efectuar qualquer referência à sua interface gráfica, presentemente em fase de desenvolvimento.

SUMMARY

MOBFLOW V 1.1
Validation document

This document concerns the validation of MOBFLOW, Version 1.1. Following a brief overview, all the information pertaining to the validation of the computational core of the model is summarized. This includes the assumptions and approximations that were introduced during the design and implementation of the model. It further includes claims about the applicability and/or accuracy of the model, as well as some statements about the substantiation of those claims and validation studies. The technical description and the mathematical formulation of all MOBFLOW components can be found in a separate document.

RESUMO

MOBFLOW V 1.1
Documento de Validação

Este documento refere-se à validação do MOBFLOW, Versão 1.1. Depois de uma breve apresentação, toda a informação relativa à validação do núcleo computacional do modelo é resumida. As hipóteses e aproximações consideradas durante o desenho e implementação do modelo são inventariadas e descritas. São também apresentadas as condições de aplicabilidade e/ou a precisão dos diversos procedimentos, bem como os estudos necessários para validar as afirmações efectuadas. A descrição técnica dos diversos componentes e a formulação matemática do MOBFLOW são apresentadas num outro documentado em preparação.

RESUMÉ

MOBFLOW V 1.1
Document de Validation

Ce document rapport la validation du MOBFLOW, Version 1.1. Après une breve présentation, toute la information concernant la validation du coeur computational du model est résumée. Les hypothèses et approximations considerées pendent le dessin et l'implentation du model sont énumérées et décrites. Les conditions d'applicabilité et/ou precision de touts les procédés, comme les études nécessaires pour faire la preuve correspondente sont aussi presentées. La description technique des plusieurs modules et la formulation mathématique du MOBFLOW sont presentées dans un autre document en préparation.

MOBFLOW, VERSION 1.1

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SUMMARY

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This document concerns the validation of MOBFLOW, Version 1.1. Following a brief overview, all the information pertaining to the validation of the computational core of the model is summarized. This includes the assumptions and approximations that were introduced during the design and implementation of the model. It further includes claims about the applicability and/or accuracy of the model, as well as some statements about the substantiation of those claims and validation studies. The technical description and the mathematical formulation of all MOBFLOW components can be found in a separate document.

TABLE OF CONTENTS

PREFACE

The subject of this document is the validation of a computational model. The term *computational model* refers to software whose primary function is to model a certain class of physical systems and includes pre- and post-processing components and other necessary ancillary programs. *Validation* applies primarily to the theoretical formulation and to the computational techniques that form the basis for the numerical and graphical results produced by the software. In the context of this document, validation of the model is viewed as the formulation and substantiation of explicit claims about applicability and accuracy of the computational results.

This preface explains the approach that has been adopted in organizing and presenting the information contained in this document.

Standard validation documents

This document conforms to a standard system for validation documentation, prepared and adopted by a number of hydraulic research institutes. The standard documentation system has been developed in order to address the need for useful and explicit information about the validity and accuracy of computational results. This type of information is presented in form of a validation document, which accompanies the technical reference documentation associated with a computational model.

Conform the standard, this validation document meets the following requirements:

1. This validation document has a prescribed table of contents, based on a well-defined framework that allows separate quality issues to be clearly distinguished and described.
2. This validation document contains a comprehensive list of the assumptions and approximations that were made during the design and implementation of the model.
3. This document contains claims about the performance of the model, together with statements addressing the available substantiating evidence for these claims.
4. Claims about the model included in this document are substantiable and bounded. This means that feasible experiments (physical or computational) and/or analyses can be devised which will test the claims.
5. Claims are substantiated in this validation document by means of evidence contained within the document itself, or by specific reference to accessible publications.
6. This validation document will be updated as the process of validating the model progresses.
7. Results of validation studies, included or referred to in the validation document, are reproducible. Consequently the contents of this document are up to date and consistent with the current version of the software.

Organization of this document

Chapter 1 contains a short overview of the computational model and introduces the main issues to be addressed by the validation process. The model overview includes information about the purpose of the model, about pre- and post-processing options and other main features, and version information. Validation priorities and approaches are briefly described, and a list of related documents is included.

Chapter 2 describes the validity of the computational core of the model. In this chapter, claims are made about the range of applicability of the model and about the accuracy of computational results. Each claim is followed by a brief statement about its substantiation. This statement indicates the extend to which the claim has in fact been substantiated and points to the available evidence.

Chapter 3 contains such evidence, in the form of brief descriptions of relevant validation studies. Each description includes information about the purpose and approach of the study, and a summary of main results and implications.

A glossary and a complete list of references are contained in appendices.

A word of caution

This document contains information about the quality of a complex modeling tool. Its purpose is to assist the user in assessing the reliability and accuracy of computational results, and to provide guidelines with respect to the applicability and judicious employment of this tools. This document does not, however, provide mathematical proof of the correctness of the results for a specific application. The reader is referred to the Licence Agreement for pertinent legal terms and conditions associated with the use of the software.

The contents of this document attest to the fact that computational modeling of complex physical systems requires great care and inherently involves a number of uncertain factors. In order to obtain useful and accurate results for a particular application, the use of high-quality modeling tools is necessary but not sufficient. Ultimately, the quality of the computational results that can be achieved will depend upon the adequacy of available data as well as a suitable choice of modeling parameters.

1. INTRODUCTION

1.1. Model overview

1.1.1. Purpose

MOBFLOW is a one-dimensional mathematical model aimed at studying the morphological evolution of streams with highly variable flow regime, irregular cross-section and graded bed material. This model provides the means to analyze transient and long-term river phenomena. It was designed so that it can be used either with rigid boundary channels or with mobile boundary channels. A large number of other alternative procedures were also implemented, allowing the user to select the more appropriate set of parameters to run a particular case study.

1.1.2. Pre- and post-processing and other features

The model MOBFLOW runs under MS-DOS (V 3.3 or later), preferably in a PC with more than 640 kbytes RAM and hard disk of more than 20 Mbytes. At this moment, the output is limited to ASCII data files stored in devices such as hard and floppy disks, and to the standard monitors and printers connected to the system.

The interface of the mathematical model is constituted by three different programs, namely, GEOSED, GEOTAB and GERDAT. These programs allow the user to construct data files for geometry and sediment, and an input file with general information, including boundary and initial conditions.

MOBFLOW does not require a calibration of the flow resistance coefficients, this being a unique feature of the mathematical model. However, the calibration process can also take place if the stream bed behaves like a fixed boundary, i. e., rather independent on flow properties, and/or if there is enough information about the roughness characteristics of the stream.

The model presents different methods of calculating the resistance to flow and the sediment load and of solving the system of differential equations. Some procedures were included in MOBFLOW to deal with special situations, such as those that occur in channels with transitions from lower to upper flow regimes, and vice-versa, very irregular flow regime, with irregular geometry and/or bed-rock near the bottom surface.

1.1.3. Version information

The present version of MOBFLOW (V 1.1) is an upgrade of a previous version dated from June 1992. It responds to a challenge of modeling realistic situations with the maximum complexity allowed by the limitations related to scientific knowledge and PC hardware technology. At the moment, it is still a prototype version rather than a well developed product. Nevertheless, the most recent improvements in both computational methods and fluvial hydraulics have been employed, allowing a fast convergence towards an industrial code.

1.2. Validation priorities and approaches

The following items are of great importance to the validation process:

1. The uncertainties arising from the use of sediment transport predictors and friction factor relationships are considered to be more important than the inaccuracies introduced by the numerical scheme. Thus, the uncoupled nature of the model is not a major handicap, rendering its structure flexible and easy to adapt to a particular case study (see 2.4.1, 3.1, 3.4).
2. Despite its complexity, the model must be able to represent very simple situations, namely, those with uniform flow regime and equilibrium between the sediment that is eroded and deposited over a certain period of time (see 3.3).
3. The model must represent the classical non-equilibrium situations of deposition upstream of a dam and erosion downstream of a dam, under both steady and unsteady flow conditions (see 3.1, 3.2, 3.4).
4. The highest level of complexity of the model corresponds to the representation of low flow simulations, selective transport of bed material and the formation of an armor coat. The relative importance of these effects must be quantified, at least as a first estimate, when the geometry of the channel is regular and the flow is steady and when its geometry is irregular and the flow is unsteady (see 3.5, 3.6).
5. Finally, the ability of the model to deal with complex case studies must be tested using appropriate field data in order to evaluate the prediction of discharges, water and bottom levels, and sediment characteristics (see 3.6).

The validation process can be viewed as the demonstration of the capabilities of the model to deal with different problems in the context of the items referred to above. Proof-of-concept tests, numerical tests, experimental and field studies can be designed and chosen to assess this objective.

1.3. Related documents

Some documents can give some insight upon different aspects of the model. Among the most relevant, the following were selected:

Belo, J. M. (1992) *Mathematical modeling of unsteady flow with mobile boundaries and non-uniform sediment*. MSc Thesis (in Portuguese), IST, Technical University of Lisbon, Portugal.

Belo, J. M. (1993) *A technical description of the mathematical model MOBFLOW*. Report ITH, LNEC, Laboratório Nacional de Engenharia Civil, Lisbon, Portugal (in preparation).

Correia, L. P. (1992) *Numerical modeling of unsteady channel flow over a mobile boundary*. Ph.D Thesis No. 993 presented at EPFL, Lausanne, Switzerland.

Krishnappan, B. G. (1981) *MOBED: unsteady, non uniform mobile boundary flow model. User's manual*. Hydraulics Division, National Water Research Institute, Burlington, Ontario, Canada.

Krishnappan, B. G. (1981) *MOBED user's manual. Update II*. Hydraulics Division, National Water Research Institute, Burlington, Ontario, Canada.

2. MODEL VALIDITY

2.1. Physical system

The physical system under consideration corresponds to a stream with tributaries and lateral storage basins, subject to time varying stages and/or discharges at boundaries. Both liquid and solid phases and their interactions are taken into account. Tributaries are viewed as concentrated input fluxes of water and sediment or, alternatively, as distributed input fluxes along a certain length of the main stream. The main goal of lateral storage basins is to smooth the peak of floods. In MOBFLOW, whenever the change in flow momentum could be neglected, they may also be employed to represent the effect of flood plains on the flow at the main stream.

2.2. Model functionality

2.2.1. Applications

The model is applicable to a wide range of practical problems. Both short- and long-term responses of natural and/or man-made changes imposed on a stream can be modeled. Examples of possible applications of the model MOBFLOW are summarized in the following paragraphs:

1. **Flood and sediment routing**
   This is a problem of water and sediment waves propagation. In MOBFLOW, the channel can be of regular type (rectangular or trapezoidal cross-section) or can have irregular geometry, defined by points belonging to a plane normal to the stream direction. The input boundary discharges and levels can be constant or time-dependent. Stage-discharges curves are also allowed at boundaries. The model evaluates the evolution of the geometry of the stream channel (bed level) and of the hydraulic properties of the flow (solid and liquid discharges, water levels) for the entire simulation period (see 3.4 and 3.6). Long-term evolution of a bed river can be considered as a special case of flood and sediment routing. It is usually analyzed using large mesh sizes and large time steps (see 3.3).
2. **Flood and sediment routing with sediment mixtures and bed-rock influence**
   If, in adition to the flow and sediment routing the study includes the evolution of the graded bed material, additional data is needed to run MOBFLOW. These data comprise: granulometric bed material curves at each cross section; the granulometric curve of the sediment inflow; and, the erodible depth above the bed-rock at each cross-section. In addition to geometric and flow variables, the model calculates the granulometric curves of bed and transported material, and accounts for the effects of the bed-rock in limiting erosion and changing boundary roughness (see 3.6).
3. **Deposition upstream of a dam**
   A dam at the downstream end of a river reach promotes bed aggradation due to the excess of sediment inflow above the capacity of water to transport solid material (see 3.1). MOBFLOW gives information about the time evolution of the geometry, water and sediment discharges, water and bed levels, bed material diameters, and other hydraulic and sediment characteristics along the reach. The aggradation process of sediment mixtures may easly be followed, considering the evolution of computed granulometric curves at three different strata of bed material (see 2.3.1 v- and 3.5).

4. **Erosion downstream of a dam**
   A dam at the upstream end of a reach promotes stream bed degradation due to the imbalance between the sediment inflow and the capacity of water to transport solid material. The model gives, basically, the same information as in the previous example (see 3.2). If the granulometric curve of bed-material is enough graded, the model is expected to describe the formation of a superficial armor coat, diminishing the amount of bed erosion (see 3.5).

5. **Flow diversion**
   Physically, the impact of flow diversion in a stream is analogous to the effect of deposition upstream of a dam. However, the boundary conditions are different and the decrease of flow capacity to transport sediments is related to the reduction of the liquid discharge. The diversion outflow is simulated by a local negative concentrated discharge. No tests were performed so far to substantiate this claim.

6. **Sand and gravel mining**
   Sand and gravel are important mineral resources used as construction materials. However, their mining significantly distort the natural equilibrium of stream channels, thereby inducing stream channel changes, which may not be environmental or economically acceptable. MOBFLOW allows to assess the impact of sand and gravel mining. The volume of bed material extracted in a certain reach can be considered through a local sediment outflow. The simulation process, beginning with the conditions that prevail before the excavation, will show how the stream will adjust itself, subject to historic or hypothetical floods. The model information about the evolution of the sediment characteristics can assist the decision process and planning of sand and gravel mining. No tests were performed so far to substantiate this claim.

7. **Behavior of small streams in dry seasons**
   The knowledge of velocities and water depths in small streams during low flow seasons can be of great importance in environmental studies to characterize natural habitats. In this context, the selective transport of bed material and the formation of a superficial armor coat are relevant issues. The evaluation of weak solid transport, normally irrelevant to the temporal bed deformation, is a valuable indicator of the transport of pollutants. The model MOBFLOW gives information on all of these topics, specifically, on their physics (see 3.5, 3.6).

8. **Evolution of highly irregular and low-land streams**
   Highly irregular streams usually exhibit steep slopes, coarse bed material, and very irregular flow discharge with flash floods. At the other hand, low-land rivers have gentle slopes, fine bed material and floods with long wave lengths. In some natural rivers, both of these flow types can be found along their longitudinal profiles. MOBFLOW allows the representation of such rivers automatically, without any segmentation of the stream profile. The results are of the same type of those mentioned previously (see 3.6).

2.2.2. Processes

The MOBFLOW model was designed to represent the following processes, which can be assembled to give realistic situations:

1. Steady and unsteady flow in a stream with irregular geometry subject to varying discharges, varying stages or stage-discharge curves at the boundaries;
2. Alluvial bed roughness as a function of flow intensity;
3. Stream response to rapidly changing discharges;
4. Influence of tributaries on the flow of the main stream;
5. Influence of storage basins on the flow of the main stream;

6. Transport of sediment mixtures, that is, transport of graded bed material, implying the representation of stratified deposits and armor-coat phenomena;
7. Bed-rock influence on the limitation of erosion and on the resistance to flow;
8. Bed aggradation and deggradation under steady and unsteady flow conditions;
9. Stream bed evolution under strong non-uniform transport of sediment.

2.3. Conceptual model

2.3.1. Assumptions and approximations

This section contains a short description of the conceptual model and the presentation of assumptions and approximations made during its development. A more complete description of the conceptual model can be found in Belo (1993).

The free-surface water sediment-mixture unsteady flow is represented by two types of mathematical formulations: the conservation equations and empirical laws. The conservation equations include the St. Venant equations and a mass-balance equation. This system of three partial differential equations is closed by empirical relationships for the sediment load and for the frictional slope of the energy grade line.

The frictional slope is calculated by the generalized equation of resistance to flow presented by Krishnappan (1981), where different resistance criteria can be chosen. The criteria of Manning, Griffiths, Brownlie and Kishi-Kuroki were coded in the present version of the model, covering most of the conditions that can be found in *lower, medium and upper parts of the river profiles.*

A power-law (Chen 1973) and the formulas of Ackers-White, Brownlie, Engelund-Hansen, Karim-Kennedy, Meyer-Peter Müller or Smart-Jaeggi can be used, alternatively, to evaluate the *sediment transport capacity* (total load or bed load) at each cross-section. The suspended load is obtained by one of the following methods: Einstein-Rouse, Van Rijn (simplified), or Celik-Rodi.

The formulas of Engelund-Hansen, Karim-Kennedy and Smart-Jaeggi are used together with the Iwagaki's criterion for the beginning of sediment motion. The calculation of the suspended load is only performed if the shear velocity exceeds the critical value for the initiation of suspension (Van Rijn 1984).

The *effective sediment transport* at each cross-section can be evaluated by a loading-law, as suggested by Rahuel (1988), or, alternatively, it can be considered equal to the transport capacity calculated by the chosen sediment load formulas.

i) Conservation equations

The fundamental hypotheses made in the derivation of the equations of Saint-Venant are:

1. The flow is one-dimensional: the flow velocity is uniform over the cross-section and the water level across the section is horizontal;
2. The pressure distribution is hydrostatic: the streamline curvature and the vertical acceleration are negligible (shallow water theory);
3. The boundary friction and turbulence can be evaluated by the same resistance laws used in steady flows;

4. The average slope of the bottom of the channel is small ($S_x \leq 10$ %).

Furthermore, it is assumed that the effect of secondary currents is negligible and, therefore, sediment transport is one-dimensional. The bulk density of the sediment-water mixture is assumed to be constant along the channel, unaffected by the movement of sediment particles.

Liggett (1975) studied the stability of the mathematical description of unsteady flow and concluded that there exists a relation between the Vedernikov number (V_e) and the formation of bores and roll waves. If $V_e^2>1$, the water surface becomes unstable and the mathematical description is not valid anymore.

At the cross sections where hydraulic structures or other flow disturbances can be identified, the hypothesis of Saint-Venant equations might be not fulfilled and it is necessary to consider supplementary relations to calculate the flow variables. These relations, internal boundary conditions, are highly dependent on the problem under consideration. For the moment, the model allows the analysis of storage basins and river confluences.

ii) Resistance to flow

The equation for the generalized slope of the energy grade line is based upon the hypothesis that the solid boundary roughness can be represented by the sum of a roughness due to grain with a roughness due to bed-forms, expressed by the sum of the corresponding shear stresses. The domains of validity for the resistance criteria available in MOBFLOW are presented in Table 1.

Table 1 – Validity of the friction factor relations in MOBFLOW

Formula	Flow type	Range for formula
Brownlie	Lower regime: $F_r < 1$ Upper regime: $F_r \geq 1$	$0.088 < d_{50}$ (mm) < 2
Kishi-Kuroki	Dune I Dune II Transition I Flat bed Anti-dunes	$0.375 < d_{50}$ (mm) < 3.6
Griffiths	Gravel bed: $F_r < 1$	$1 < d_{50}$ (mm) < 152
Manning	Rigid bed	—

These criteria where chosen among the best selected by different authors (Brownlie 1981, Oliveira and Cardoso 1990).

The criteria of Brownlie and Kishi-Kuroki are appropriate for sand and fine gravel bed rivers. The criterion of Griffiths is suited for gravel bed rivers with sediment transport and the criteria of Manning applies to flows with fixed boundaries. This last criterion can be used considering that the resistance coefficient is: *a)* constant; *b)* variable from section to section, in order to reproduce the effects of stream geometry on flow; and *c)* variable with flow discharge in a tabular form.

iii) Sediment transport

Studies have been carried out to establish the validity and reliability of several solid discharge formulas. For lowland rivers the works of White et. al. (1973) and Brownlie (1981) can be mentioned. For mountain rivers, with steep slopes and wide particle size ranges, an analysis of the most significant formulae were made by Bathurst et al. (1987), and Gomez and Church (1989).

These studies led to conclude that there is no solid transport formula valid for all classes of natural conditions. Moreover, all the formulas are semi-empirical and non-linear, and most of their coefficients were determined from laboratory experiments with steady uniform flow.

The domain of validity of the sediment load relationships implemented within MOBFLOW is shown in Table 2.

The formulas in Table 2 cover most of the conditions that can be found in the lower, medium and upper parts of river profiles, allowing the user to choose an appropriate formula for a particular set of flow conditions.

Nevertheless, all of them were obtained assuming flow equilibrium, that is, steady flow (usually, uniform) in liquid phase and sediment balance between erosion and deposition over a relative long period of time. In order to use these formulas, the granulometric distribution of bed material is represented by a single size diameter.

Table 2 – Validity of sediment-transport formulae available in MOBFLOW

Formula	**Range of validity**			
	Slope (%)		**Diameter (mm)**	
	Min.	**Max.**	**Min.**	**Max.**
Power law	–	–	–	–
Ackers-White$^{(*)}$	–	–	0.040	4.00
Brownlie	0.003	3.7	0.012	2.80
Engelund-Hansen	0.43	8.2	0.190	0.93
Karim-Kennedy	0.15	24.3$^{(**)}$	0.177	28.65
Meyer-Peter Müller	0.25	2.0	0.400	29.00
Smart-Jaeggi	0.40	20.0$^{(**)}$	0.400	29.00
Einstein-Rouse$^{(*)}$	–	–	–	–
van Rijn (simplified)$^{(*)}$	–	–	0.200	2.00
Celik-Rodi	–	–	0.005	0.60

$^{(*)}$ Slope unspecified but formula appropriate for lowland rivers ($S_x < 0.1‰$)

$^{(**)}$ In MOBFLOW, slope is limited to 10 % (see 2.3.1)

iv) Influence of the bed rock

An important aspect, which limits the amount of sediment available for transportation, is the existence of a rigid bed layer (bed-rock) close to the sandy bottom. This layer can be exposed to flow by the erosion of the upper layers. Numerically, when this situation occurs, the bed lowering is stopped and the roughness characteristics of solid boundaries are changed to be represented by a

Strickler's type coefficient, K_r, specified by the user at the beginning of the simulation. Under these circumstances, the Manning formula is employed to perform the resistance to flow calculations.

v) Non-uniformity of sediment and grain-sorting

The present model accounts for the effects of wide grain size ranges by means of a sorting procedure. However, only a single grain-size is used to evaluate the sediment transport capacity and the initiation of motion. It is assumed that all phenomena concerning sorting and armoring can be described by monitoring a bed layer, including an active layer, a buffer layer and a substratum. The active layer consists of a *transport layer*, where sediment moves as bedload, and of a *mixture layer*, to allow vertical sediment equilibrium with substratum. The *buffer layer* is only considered either during deposition processes or when erosion is preceded by deposition. An armored layer is considered to occur as a consequence of changes in the mixture layer composition, being *an asymptotic case of a degrading river bed subject to selective sediment transport.*

2.3.2. Claims and substantiations

i) Conservation equations

The one dimensional form of the full Saint-Venant equations are appropriate for the simulation of wave propagation in: *a)* gradually-varied unsteady flows; and *b)* shallow water situations. Both cases can be analyzed by considering an initial situation and its temporal modification due to changes in boundary conditions.

Santos (1989), and Abreu and Santos (1989) compared different approximations of the equations for the wave movement in shallow water and concluded that the Saint-Venant equations are only valid if the effects of wave dispersion are of minor importance, when compared with the non-linear effects. However, all the approximations studied, including the linear non-dispersive (kinematic model), gave good results until a certain instant of time in the simulation occurs (Abreu and Santos 1989). In the case of dynamic models, this instant corresponds to the one at which the wave becomes steeper and tends to break. When the vertical accelerations are of great importance, an alternative approach is proposed by Santos (1989), based on Green and Naghdi (1976) equations.

As the model MOBFLOW is designed to simulate the propagation of floods in natural channels, where wave characteristic lengths are much greater than amplitudes (shallow water theory) and frictional resistance is of great importance in peak flow attenuation (non-linear effects), the Saint-Venant equations seem to be a good approximation to the physics of the phenomena. The model allows the user to choose between the complete dynamic model or the kinematic approach, based on simplifications introduced in the equation for the flow momentum. The equations for the continuity of the flow and solid phase are generally valid in all the approximations made to the description of wave movements.

ii) Resistance to flow

MOBFLOW assumes that resistance in unsteady flows can be properly described by the relationships suitable for steady flows. Since, in response to flow disturbances, the flow depth, the sediment rate and the bed-form geometry do not adjust immediately to new stream conditions, the mentioned assumption could be questionable. Nevertheless, *a)* the information about temporal lags is very scarce and based on empiricism, preventing the implementation of a general approach to deal with resistance in unsteady flows; and, *b)* the effect of flow unsteadiness seems to be irrelevant when compared to the uncertainties of the friction coefficients in real situations.

Among the models that solve the complete unsteady flow equations, MOBFLOW is one of the few that accounts for the changing alluvial bed-roughness as a function of the flow intensity (see also 2.4.1 i-). The energy-loss term in the momentum equation is thus correctly evaluated (Correia 1992). Friction formulas for rigid and mobile beds may be used, as well as empirical laws adjusted to field data (Belo 1993).

iii) Sediment transport

Two major sources contributing to the non-uniformity of the sediment transport may be considered in the study of the propagation of sediment waves: *a)* unsteadiness due to the direct effect of unsteady flow (studied by Suszka and Graf 1987, Tsujimoto 1988); and *b)* non-equilibrium due to time and spatial lags in sediment transport response (studied by Daubert and Lebreton 1967, Philips and Sutherland 1989).

The first effect is assumed to be well described by the structure of the mathematical model, that is, errors in results are, at most, of the same magnitude of the errors in input data. This is expected, since the model uses the full Saint-Venant equations to obtain discharges and flow depths along the modelled reach of a stream (see 3.4).

The non-equilibrium of sediment can be considered by the use of a loading-law. Nevertheless, Correia (1992) argues that: *a)* the practical use of a loading law is limited to cases where the bottom and cross-sections are known with a good precision; *b)* this feature is used by modelers to dissipate numerical oscillations; *c)* the loading law has a degenerated form, modifying the nature of the system to be solved; and *d)* the empiricism of the approach reduces its applicability to a general case where no calibration values are available.

In MOBFLOW, the user can freely choose between using or not a loading-law, with constant parameters or parameters obtained automatically for bed-load and suspended-load (see 3.4).

iv) Influence of the bed rock

The influence of the bed-rock on flow properties is not well understood. In MOBFLOW it is viewed as a change in boundary roughness and sediment transport. When the bed-rock is reached during the erosional process: *a)* the flow resistance calculations change to use a formula suitable to model macro-roughnesses, instead of mobile bed situations; and *b)* the detachment of solid particles from the bed is stopped. The sediment transport is computed taking into account the solid material entering the reach from upstream and the local transport capacity. The characteristics of the material remaining on the bed in a pure erosional process are considered to be those at the beginning of the simulation, based on the assumption that this material is protected against selective transport by geometric constraints. If deposition follows erosion, it is assumed that the sediment inflow mixes with the bed material. These assumptions will hardly be proved due to the lack of field data.

v) Non-uniformity of sediment and grain-sorting

Steep rivers have usually graded bed material. The granulometric curve of solid material at each cross-section changes with time, depending on flow and sediment properties, which can be represented by the relation between flow transport capacity and sediment availability. The sediment load formulas only give good results if the available sediment is below the flow capacity to transport them (which occurs during erosion of the stream bed).

Thus, a procedure must be implemented to correct sediment transport capacity and to model selective transport and armor-coat formation. The sediment transport of different size classes, though appearing to be physically sound, was not included in the MOBFLOW model due to the following reasons (Correia 1992):

1. There is no experimental or theoretical research that may be used as a reliable framework to evaluate the accuracy of predictions based on the separate transport of different grain-sizes;
2. The understanding of the physical processes involved is very limited, especially in the prediction of bed-load discharge at weak transport rates and under conditions of limited sediment availability, where external supply and sediment size-distribution effects are important;
3. If the transport in different grain-sizes is to be introduced, the complexity of the model and, consequently, the computational time is highly increased in order to gain an extra accuracy that is impossible to measure;
4. Input data requirements also increase greatly.

The procedure implemented in MOBFLOW utilizes only one single grain-size taken as representative to evaluate sediment load. The non-uniformity of sediment and grain sorting are considered by the deformation of granulometric curves. This procedure is an improved version of the ones presented previously by Krishnappan (1985) and Correia (1992).

2.4. Algorithmic implementation

2.4.1. Assumptions and approximations

This section contains a description of the algorithmic implementation of the conceptual model. The discretization procedure of the equations and the solution methods are mentioned, and the procedures for highly variable and low flows are presented and discussed. A more complete description of the algorithmic implementation can be found in Belo (1993).

i) Discretization of equations and solution procedure

The mathematical model solves the system of the conservation equations of flow continuity and momentum assuming that:

1. the local variation of the bottom level is less important than the temporal variation of the water depth; and
2. the mean flow velocity is approximately equal to the component of the lateral inflow velocity in the longitudinal direction (concentrated and distributed lateral input discharges).

The solution of the Saint-Venant equations may then be decoupled from the solution of sediment continuity equation; at each time step, continuity and momentum equations for the water-sediment mixture are first solved implicitly and then, using the just obtained hydraulic results, the sediment continuity is solved and the stream geometry is updated.

The generalized Preissmann scheme (Lyn and Goodwin 1987, Correia 1992) is used to discretize the equations of Saint-Venant which are solved, after linearization, by the double-sweep technique (DSP) or by an alternative sweep technique (ASP). The backward Euler scheme is used to discretize the Exner's equation in a way that led to obtain all the changes in the bottom levels, once the flow conditions at two consecutive time steps are known (Belo 1992, 1993).

In the hyperbolic system of fundamental equations it is necessary to consider three boundary conditions and proper initial conditions. The two boundary conditions for the water-sediment mixture are chosen among the following: discharge hydrographs, stage hydrographs and stage-discharge curves. The third boundary condition refers to the sediment input hydrograph (imposed upstream in subcritical flows). The initial conditions correspond to the discharge, the water depth and the bottom level; they can be obtained using a facility offered by the model MOBFLOW, which performs the *stabilization* of the solution before starting the simulation process.

ii) Highly variable flow simulation

When the flow is highly variable (in time) it is usually difficult to reproduce the river behavior due to the linearization procedures and other simplifications introduced in the numerical model. Usually, an adequate solution of the Saint-Venant equations is obtained by reducing the time step for *all* the simulation period. An algorithm was developed which allows to detect extreme variations in hydrographs and split the original time step only when these variations occur. The difference between discharges in consecutive times is known from discharge hydrographs specified at the boundaries. If, *a)* the absolute value of this difference is greater than a maximum allowed value specified initially by the user or, *b)* if the absolute value of the difference is greater than a specified percentage of the inflow discharge, the original time step will be divided in minor intervals. The number of divisions depend on the criteria adopted (*a* or *b*), and on the amplitude of the discharge variation. The output results are only given for the original time intervals.

iii) Low flow simulation

When flow discharges are low, the role of boundary roughness associated with irregular geometry gains an extra importance. The problem becomes very complex when flow changes from subcritical to supercritical, and vice-versa.

If the flow regime remains subcritical, as suggested by Cunge et al. (1988), the procedure implemented in MOBFLOW leads to the calculus of the space weighting factor of Preissman scheme as function of the local Froude number. The independent coefficient of the discretized form of the momentum equation is then bounded in order to inhibit parasitic oscillations. The limits for the variation of this coefficient depend on the geometry of the cross-sections. After a sensitive analysis concerning the prediction of water depths, a relationship for the reduction coefficient was found to give good results (Belo 1993).

2.4.2. Claims and substantiations

i) Discretization of equations and solution procedure

The Preissmann scheme is *a)* compact and, *b)* implicit. Once it is compact, it adjusts easily to complex limiting conditions, allowing the use of nonuniform spaced grids. Once it is implicit, it has an wide region of stability, allowing to reduce the computer time required for the calculations by a convenient choice of the space and time increments. Thus, the time interval is not limited by the Courant-Friedrichs-Lewy condition (Cunge et al. 1988), as in explicit numerical schemes, depending on the nature of the problem to be solved. However, since MOBFLOW is an uncoupled model, its solution procedure is not implicit for the full system of governing equations, but only for the solution of the Saint-Venant equations.

The generalized Preissmann scheme was studied by Lyn and Goodwin (1987), and by Correia (1992). These authors present its characteristics of consistency, stability and convergence based in techniques that require the simplification of the system of conservation equations. In MOBFLOW, the full Saint-Venant equations are used. Thus, the analyses of Lyn and Goodwin (1987) and Correia (1992) are merely indicative about the behavior of the Preissmann scheme.

The backward Euler scheme, presented by Krishnappan (1981), is used to discretize the sediment continuity equation. The scheme is dissipative, inhibiting the growth of parasitic oscillations in bed waves originated by the disparate scales of both bed and flow phenomena.

The discretization of the term responsible for the energy-loss in the momentum equation is usually one of the major difficulties encountered by modelers. It is not just a matter of introducing different semi-empirical friction formulas into the model, but to ensure that friction is consistently expressed in the discretized momentum equation. In MOBFLOW, the energy-loss term was treated implicitly through the generalized equation for the energy grade line (Belo 1993). Thus, its discretization follows the criteria adopted for the other terms of the momentum equation, diminishing the potential sources of numerical instabilities.

After discretization, the system of conservation equations is linearized and solved by the DSP or by the ASP procedures, as mentioned in 2.4.1 i-. The linearization affects the variable increments of order superior to two, which are considered to be not important to represent the phenomena, the errors being, at most, of the same magnitude of those associated with the empirical formulas.

Problems affecting the efficiency of the double-sweep procedure (DSP) have been reported in literature: *a)* Henriques (1986) found that the mass balance is not adequately verified; and *b)* when the step-by-step recurrence process of the DSP is analyzed, it is found that there is a fast growth of some coefficients with increasing section number, from upstream to downstream, while other coefficients practically remain constant. A solution for this second problem was devised by Holly and Rahuel (1990). However, two another important problems have to be solved when using the DSP: *a)* to initiate the procedure it is necessary that at least one of the boundary values is not zero; and *b)* it is not possible to use $\phi = 1$ in Preissmann scheme, once the determinant of the matrix to be inverted will turn equal to zero.

The alternative sweep procedure (ASP) does not suffer from these problems, as shown by Correia (1992), being easily applicable in real situations. When using this procedure, upstream and downstream boundaries are linked and the solution of the system can be found without iterations. However, once DSP remains as a standard for the solution of Saint-Venant equations, both methods, ASP and DSP, were coded in MOBFLOW.

ii) Highly variable flow simulation

With the procedure implemented in MOBFLOW for highly variable flow, the user is requested to specify (initially) the parameters that regulate the partition of time interval. Then, the calculus can proceed without direct interferences. The time step splitting occurs occasionally, following the behavior of the inflow hydrograph, and, by consequence, all computation time is optimized.

This procedure must only be used after, unsuccessfully, trying different combinations of flow resistance and sediment transport formulas because the accuracy of the simulation becomes difficult to control. However, the splitting of the time increment generally conducts to a better agreement with the basic hypothesis of the conservation equations (namely, with the hypothesis of shallow water theory), diminishing numerical oscillations.

iii) Low flow simulation

The procedure implemented in MOBFLOW assumes that the flow remains subcritical during the low flow regime. Thus, some precautions have to be taken into account to assure that the modification made in the original system does not lead to large differences between the obtained solution and that of the original system. If the flow regime changes from subcritical to supercritical, the time step should be reduced and the model rerun. If the flow regime still changes, the results with Froude numbers great than unity should be carefully considered.

2.5. Software implementation

2.5.1. Implementation techniques

The modules of the model MOBFLOW are written in standard FORTRAN 77. The numeric variables are represented in single precision IEEE (4 bytes). The model runs virtually in all IBM PC compatible machines, having a minimum of 640 kbytes of RAM. When the problem at hand is very time consuming, it is desirable to use a machine with, at least, a 80386 architecture. In these situations, a co-processor is also recommended. The model was designed to run in a MS-DOS environment. However, it is also possible to run MOBFLOW without any modifications under MS-WINDOWS and other operating systems that allow DOS compatibility.

The concepts of local and global variables were introduced to distinguish between the variables used only by one subroutine from those used by two or more subroutines. The names of local variables differ from those of global type, and these last ones are shared among subroutines through the use of COMMON blocks, thus assuring that they are not corrupted.

The input files of the model are ordinary ASCII data files. They can be written using a DOS or Windows based word-processor or may be built by the programs that constitute the interface, namely, GEOSED, GEOTAB and GERDAT. A complete description of the structure and contents of data files can be found in Belo (1993).

After reading the input files, the mathematical model establishes the framework for all the operation and enters into a main loop (see Fig. 2.1). This loop is executed for each time interval of the simulation period, beginning with the calculus of the boundary conditions and ending with the output of the numerical results. The nested loop is related to the division of time step to allow the computation of highly variable flows (see 2.4.1).

In Fig. 2.1, the names or sentences in rectangles correspond to subroutines. OUTPUT is also a subroutine, while the remaining symbols correspond to actions and decisions implemented in the main program. The names of the routines are quite self-explanatory; for instance, NEWDELTAT is the routine where the new time step is computed.

The standard output of MOBFLOW consists of flow parameters (namely, flow rate, flow depth, sediment rate, Froude number, bottom elevation, surface width, flow area, friction factor and hydraulic radius) and percentages of sediment by weight in different size classes, the d_{35}, d_{50}, d_{65} values, the total bed depth, the mixture layer thickness, the type of bed forms present at each node, the friction parameters at each node and the ratio between the suspended sediment load and the total load at each grid node. The average energy slope of the reach, the average bed slope and the average sediment transport rate are also printed out.

The results of the mathematical model are written in tables, for each time step. The user can modify the standard output by choosing their features. This way, the model can print out only the hydraulic and geometrical variables, only the sediment characteristics at each cross-section, only the resistance to flow variables, or combinations of these types of output. See Belo (1993) for a complete description of output options and file contents.

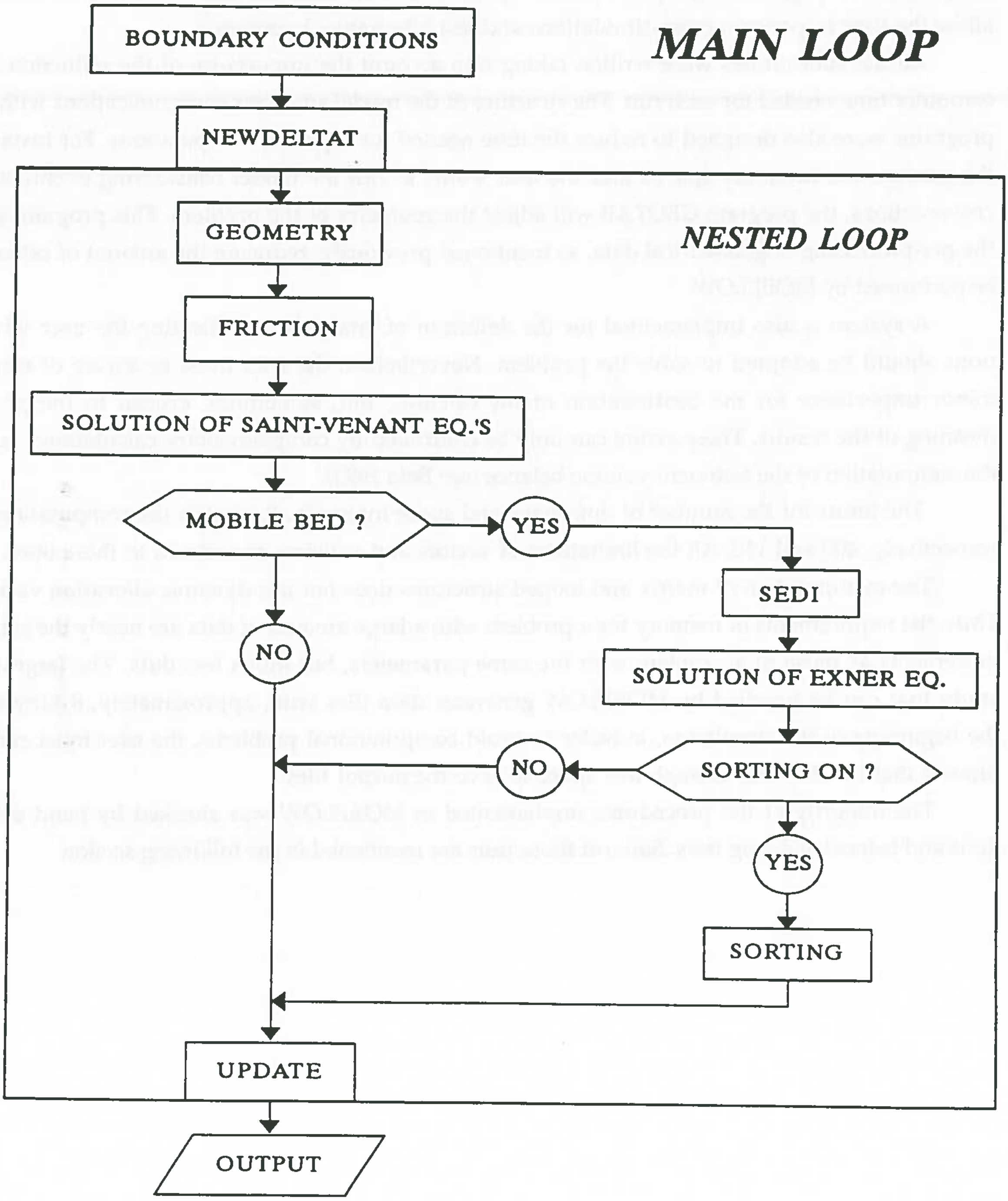

Fig. 2.1 – Flow chart for the MAIN LOOP of the mathematical model

2.5.2. Claims and substantiations

MOBFLOW is programmed in a structured and modular way. Therefore, any new physical relationships can be added to the program through the subroutines without changing the existing structure. All the processes, such as morphological (bed) evolution and grain sorting, which are not coupled with the governing equations, are optional. The modularity and versatility of the model allow the user to perform cross-simulations and test alternative hypothesis.

All the subroutines were written taking into account the importance of the reduction in the computer time needed for each run. The structure of the model and their communications with other programs were also designed to reduce the time needed for repeated computations. For instance, if the sections are unevenly spaced and the user wants to run the model considering evenly spaced cross-sections, the program GEOTAB will adjust the geometry of the problem. This program allows the pre-processing of geometrical data, as mentioned previously, reducing the amount of calculus to be performed by MOBFLOW.

A system is also implemented for the detection of fatal errors, indicating the user what actions should be adopted to solve the problem. Nevertheless, the user must be aware of errors of minor importance for the continuation of the calculus, but, sometimes, crucial to the physical meaning of the results. These errors can only be controlled by complementary calculations, such as the computation of the sediment volume balance (see Belo 1993).

The limits for the number of time steps and space intervals allowed in the computations are, respectively, 400 and 110. All the limitations of vectors and matrices are related to these ones.

The manipulation of matrix and looped structures does not use dynamic allocation variables. Thus, the requirements of memory for a problem with a large amount of data are nearly the same requirements as those of a problem with the same parameters, but much less data. The largest case study that can be handled by MOBFLOW generates data files with, approximately, 8 Mbytes. At the beginning of the simulation, in order to avoid computational problems, the user must certified himself that the disk has enough free space to save the output files.

The integrity of the procedures implemented in MOBFLOW was checked by hand calculations and extensive debug tests. Some of those tests are mentioned in the following section.

3. VALIDATION STUDIES

The studies presented in this section are aimed at substantiating some of the claims relative to the conceptual model and its algorithmic implementation. The following table summarizes some features of these examples.

TEST	Souce of data	Type of flow	Sediment transport	Friction factor	Date of study	Date of summary	Version of model	Remarks
3.1. Deposition upstream of a dam	theor	steady	Brownlie Power law	Brownlie	Jan.93	Feb.93	V 1.1	Comparison with published results (Lyn, Correia)
3.2. Erosion downstream of a dam	theor	steady	Ackers and White	Brownlie	Jan.93	Feb.93	V 1.1	Efficiency of the ASP
3.3. Long term evolution of bed river	theor	uniform	Ackers and White	Uniform	Jan.93	Feb.93	V 1.1	$Q_s(1)$ = sediment transport capacity
3.4. Erosion and deposition in unsteady flow	theor	unsteady	Ackers and White Power law	Kishi and Kuroki Manning	Feb.93	Feb.93	V 1.1	Effect of: Variable roughness Loading law Power law Restricted erosion Change in sediment supply
3.5. Evolution of a bed river with transport of sediment mixtures	theor	steady	Ackers and White	Griffiths	Feb.93	Feb.93	V 1.1	Effect of sorting
3.6. Evolution of river Mondego	field	unsteady	Ackers and White	Griffiths	Jun.92	Nov.93	V 1.0	Comparison with field data

theor – theoretical data source
field – field data source

3.1. Deposition upstream of a dam

Purpose

The purposes of this example are: *a)* to test MOBFLOW for reaches subject to strong deposition, what happens whenever the solid discharge inflow is far greater than the capacity of flow to transport sediments; and *b)* to compare the results with published cases, accounting for the differences in the formulations of resistance to flow and sediment transport.

Approach

This case study was first studied by Lyn (1987). The sediment and the geometric characteristics of the reach used in the computations were:

- flow rate per unit channel width, $q = 5\ m^3/s/m$;
- length of the reach, $L = 13.6$ km;
- initial bed slope, $S_x = 0.0005$;
- median grain size of the sediment, $d_{50} = 0.4$ mm;
- geometric standard deviation of grain size distribution, $\sigma_g = 1.0$;
- volume of sediment per unit volume of bed-layer, $\vartheta = 1.0$;
- initial upstream flow depth, $h(1)_{initial} = 3.0$ m;
- constant downstream flow depth, $h(n) = 5.0$ m; and
- constant sediment input rate per unit channel width, $q_s(1) = 5.0$ kg/s/m.

Lyn solved this problem using the equations of Brownlie for the friction factor and for the sediment load calculations; a uniform spatial grid with $\Delta x = 340$ m; a time step of $\Delta t = 46260$ s (12.85 hours); and the standard Preissmann scheme ($\phi = 0.5$) with a time weighting factor of $\theta = 0.75$.

The prediction using the model MOBFLOW was carried out using the same values of the mentioned parameters. The sorting procedure was not used since the sediment is uniform ($\sigma_g = 1.0$). Correia (1992), in testing FCM (Fully Coupled mobile boundary flow Model), considered the same problem of deposition upstream of a dam.

Results and discussion

Fig. 3.1 shows the bed-growth profile calculated by MOBFLOW after an elapsed time of 514 hours. The ASP (alternative sweep procedure) was used to performed the computations. The abscissa was made dimensionless using the length of the reservoir and the ordinate was normalized using the critical depth corresponding to a unit discharge of $5\ m^3/s/m$, that is $h_c = 1.35$ m.

The results of Lyn (1987), pp. 10, and the results of Correia (1992), pp. 87, are also presented in Fig. 3.1. The dimensionless change in bottom level calculated by the model MOBLFOW follow fairly those presented by Lyn and by Correia, except when using the Brownlie's formula (dashed curve). This discrepancy may be related to the use of a sediment transport predictor not suitable for deposition, once the amount of deposited sediment depends mainly on upstream conditions, instead of local conditions (Belo 1993).

When studying problems similar to this one, Chang and Richards (1971), and Chen (1973) considered that the sediment transport under deposition can be described by the power-law:

$$C_s = KU^m h^n \tag{3.1}$$

where $C_s = Q_s/Q$ is the cross-averaged sediment concentration; $K = k/(gw)$ is a constant; g is the gravitational acceleration; w is the mean fall velocity of grains; U is the mean flow velocity; h is the depth of flow; and m and n are constants. In their examples, they used k = 7.55E–5, m = 3, n = –1, and w = 0.002 m/s. Fig. 3.1 shows the prediction of MOBFLOW considering a power-law formula for the sediment load identical to Eq. (3.1), with k = 9E–5, m = 3, n = –1, and w = 0.01276 m/s. This bed-growth profile follows closely the results of Correia (1992).

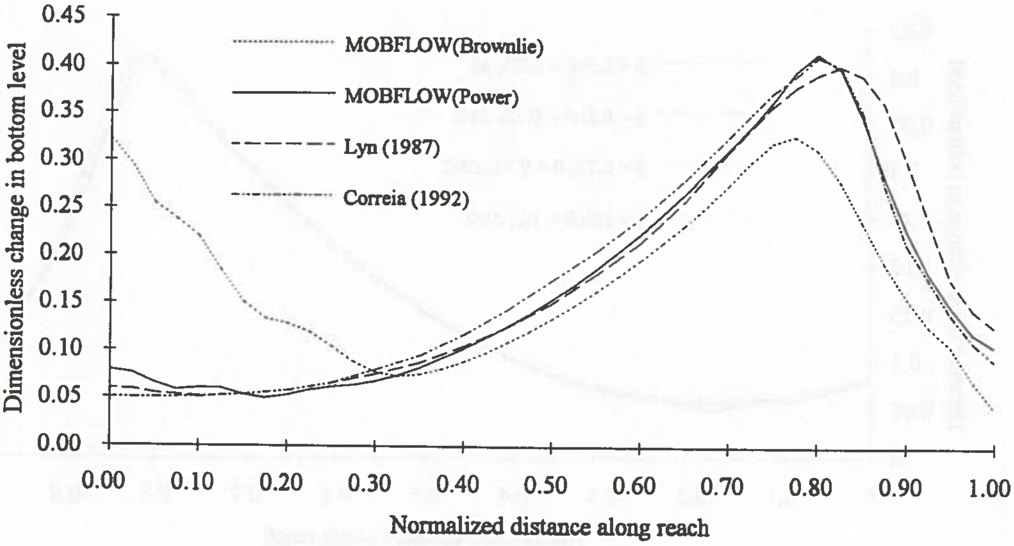

Fig. 3.1 – Bed-growth profiles for the deposition upstream of a dam

The good results obtained by Lyn and Correia, shown in Fig. 3.1, can be attributed to a supplementary imposition on the bottom level at the upstream section, which is propagated downstream due the coupled nature of the models. In fact, both these authors specify a boundary condition for the upstream bottom level, through a relation involving, among other variables, the sediment input to the reach and a calibration parameter. That is:

$$\Delta y_{bu} = \frac{\left(Q_{su} - Q_{se}\right) \Delta t}{c B_u \vartheta \Delta x} \tag{3.2}$$

where Q_{su} is the upstream sediment input; Q_{se} is the equilibrium sediment transport rate obtained from a sediment transport equation; B_u is the upstream width; and c is a constant that takes into account the distance along which the exchanges between the flow and the bed occur (Correia 1992). In practice, the value of c must be set before running the model, *constituting a calibration parameter*.

In Fig. 3.1, the differences between the net input of sediment into the modeled reach and the accumulated sediment within the reach are equal to 0.12 %, when the equation of Brownlie for the sediment load is used, and equal to –1.26 %, when the power-law is used. A better result for the sediment-mass balance could be obtained by modifying slightly the value of k in the power-law formula.

The sensitivity of the solution obtained by MOBFLOW to modifications in the parameters θ and ϕ of the Preissmann scheme, and to the time step is shown in Fig. 3.2.

The upwinding of the numerical scheme, that is, the increase of the value of the space-weighting factor ϕ introduces numerical dissipation into the solution. The results of MOBFLOW remain almost unaltered, while the results of FCM differ from those obtained without upwinding the scheme (Correia 1992, pp. 87).

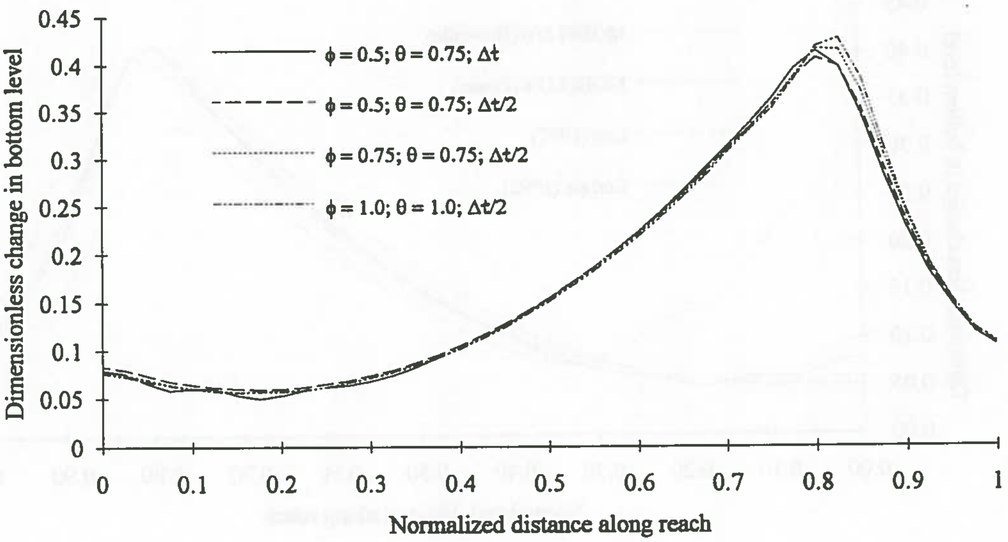

Fig. 3.2 – Sensitivity analyses of bed-growth profiles

Conclusions

After changing the sediment transport formula, the results of MOBFLOW follow closely the results of FCM (Correia 1992). Both models are adequate to simulate the deposition upstream of a dam. This test serves to demonstrate the importance of the choice of the sediment transport formulation. The robustness of MOBFLOW remains practically unaffected by changes in the parameters of the generalized Preissmann scheme and in the computational time step (see Fig. 3.2).

3.2. Erosion downstream of a dam

Purpose

Suppose a dam trapping all the incoming sediment. The water releases, free of sediment, will promote bed degradation downstream of the dam. To avoid erosion, the first downstream section is protected by a concrete layer (see also 3.4). The model MOBFLOW is used to calculate the bed evolution of the reach, assuming steady flow and a numerical grid of 101 points (cross-sections). This way, the ability of the model to reproduce localized erosion and to handle large spatial grids are tested.

Approach

Assume that:

- the flow rate per unit channel width released by the dam is constant, $q = 5\ m^3/s/m$;
- the channel has a length, $L = 20.0$ km; and initial slope, $S_x = 0.00025$;
- the sediment has $d_{50} = 0.4$ mm, $\sigma_g = 1.0$ and $\vartheta = 1.0$;
- $h(1)_{initial} = 3.6$ m; and
- $h(n) = 4$ m, for all the simulation period.

The meaning of the symbols is identical to those of 3.1. By hypothesis, $Q_s(1) = 0.0$ kg/s. The other parameters are: $\Delta x = 200.0$ m; $\Delta t = 10800$ s (3 hours); and $\theta = \phi = 0.75$. The friction factor and the sediment transport rate computations are made using the formulas of Brownlie, and Ackers-White, respectively. The ASP is used to solve the linearized system of Saint-Venant equations. The DSP cannot be used, once the increments of the dependent variables are nil ($\Delta Q = 0$; $\Delta h = 0$) (see 2.4.2 i-).

Results and discussion

The results obtained by MOBFLOW are shown in Fig. 3.3. The bottom profiles for elapsed times of 10 days and of 20 days and the bottom level at the upstream boundary remain stable. The erosion takes place just downstream of the first section of the modeled reach, as expected. The water surface remains almost unaltered during the simulation period. These results agree with the results obtained by Correia (1992), pp. 91 (not shown here because the results of both MOBFLOW and FCM are almost superimposed).

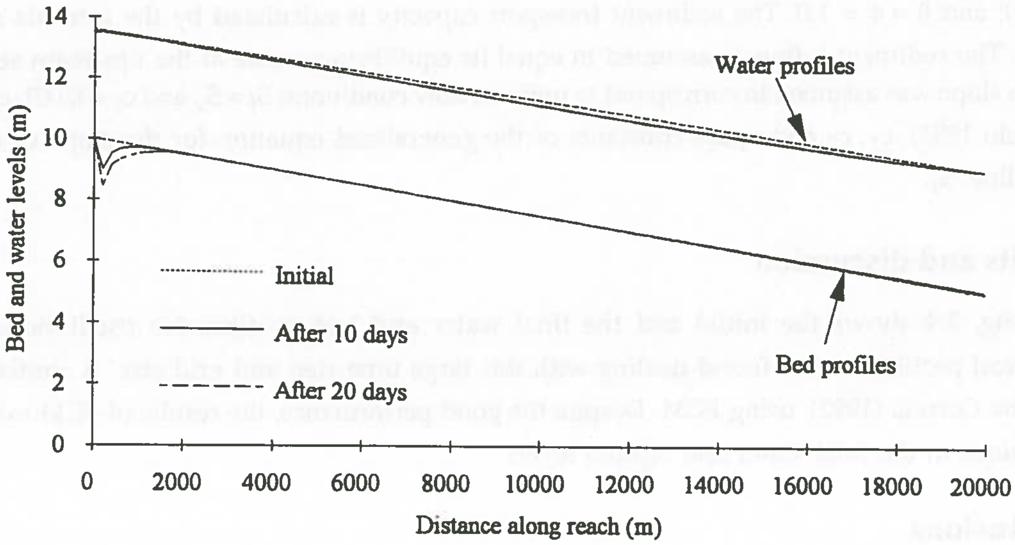

Fig. 3.3 – Erosion downstream of a dam using the alternative sweep procedure

Conclusions

The ASP is adequate to solve the system of Sain-Venant equations: a) the solution remains stable even when large spatial grids are used; and b) the calculus does not depend on the existence of non-nil increments of the dependent variables at boundaries. MOBFLOW is appropriate for the simulation of localized erosion downstream of a dam, under steady flow.

3.3. Long-term evolution of bed river

Purpose

To simulate the long term evolution of bed river is generally convenient to use large mesh sizes and large time steps. This case study is intended to give some insight into the ability of MOBFLOW to use large space and time steps, and to simulate uniform flow regimes with equilibrium sediment transport.

Approach

Assume that:

- the flow is constant over all the simulation period (5 years), and $Q = 1000\ m^3/s$;
- the channel has a bottom width, $B = 100$ m; a length, $L = 50.0$ km; and slope, $S_x = 0.001$;
- the sediment has $d_{50} = 0.4$ mm, $\sigma_g = 1.0$ and $\vartheta = 1.0$;
- $h(1)_{initial} = 5.0$ m; and
- $h(n) = 5.0$ m.

To perform the computations, the following parameters were chosen: $\Delta x = 2500$ m; $\Delta t = 86400$ s (1 day); and $\theta = \phi = 1.0$. The sediment transport capacity is calculated by the formula of Ackers-White. The sediment inflow is assumed to equal its equilibrium value at the upstream section. The friction slope was assumed to correspond to uniform flow conditions: $S_f = S_x$ and $c_1 = 0.001$, $c_2 = c_3 = 0.0$ (see Belo 1993). c_1, c_2 and c_3 are constants of the generalized equation for the slope of the energy grade line, S_f.

Results and discussion

Fig. 3.4 shows the initial and the final water and bed profiles. No oscillations or other numerical problems were found dealing with this large time step and grid size. A similar test was made by Correia (1992), using FCM. Despite the good performance, the results of FCM exhibit some oscillations in the final water and bottom levels.

Conclusions

The model MOFLOW proved to be sufficiently robust when using large mesh sizes and large time steps. However, it must be kept in mind, that the space and time steps should be chosen according to the physical scale of the phenomenon to be represented.

MOBFLOW also proved to correctly simulate schematic situations, like uniform flow regime in a rectangular channel in equilibrium, that is, a channel where deposition balances erosion, over a certain period of time.

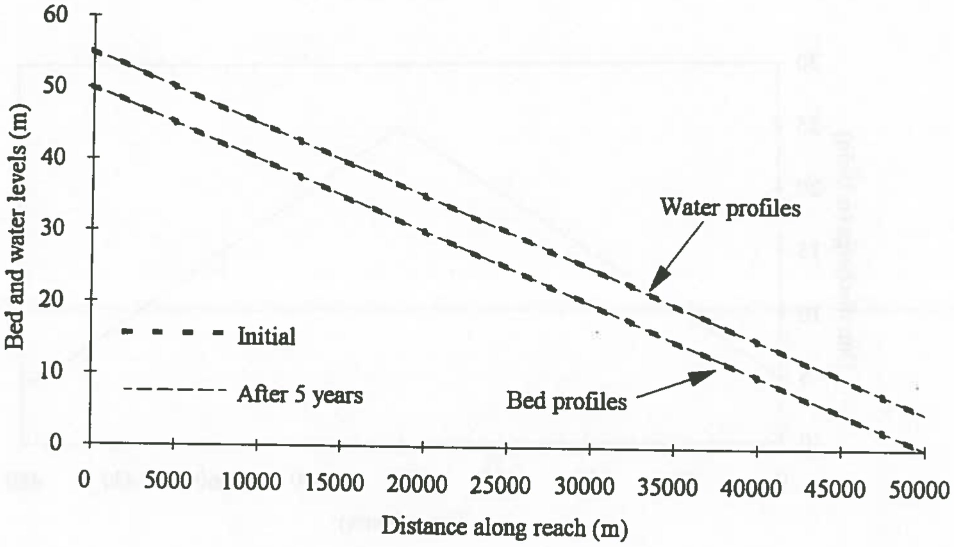

Fig. 3.4 – Long term simulation under uniform flow and sediment equilibrium conditions

3.4. Erosion and deposition under unsteady flow

Purpose

The inflow discharge of the reach characterized in 3.1 is no more constant. The inflow hydrograh contributes to change the pattern of the stream evolution: at the entrance, it is expected that erosion will take place, while, near the downstream end of the reach, some slightly deposition is expected. The purpose of this test is to evaluate and compare the water and bed profiles at the end of the simulation period when: *a)* the roughness is either variable or constant; *b)* a loading-law is used; *c)* a solid transport formula suitable for deposition is employed; *d)* the erosion at the upstream section is artificially restricted; and, *e)* when the sediment supply is either constant and greater than zero, or nil, or equal to the transport capacity.

Approach

Suppose that the hydrograph shown in Fig. 3.5 corresponds to the inflow discharge for the simulation period (20 days). The downstream water depth is assumed to equal 5.0 m when $q = 5$ $m^3/s/m$, and to increase (decrease) linearly up to (from) 8.0 m, which corresponds to the peak of the flow hydrograh ($q = 25\ m^3/s/m$). The sediment inflow is constant, and $q_s = 5.0$ kg/s/m. The other parameters are: $L = 13600$ m; $\Delta x = 340$ m; $\Delta t = 5400$ s (1.5 day); and $\phi = \theta = 0.75$.

The sediment transport capacity is obtained by the formula of Ackers-White, the effective transport is obtained by a loading-law with parameters calculated automatically by MOBFLOW (Chang 1988, pp. 176, Belo 1993), and the resistance to flow is calculated using the relationships of Kishi-Kuroki. The ASP is used to solve the system of Saint-Venant equations.

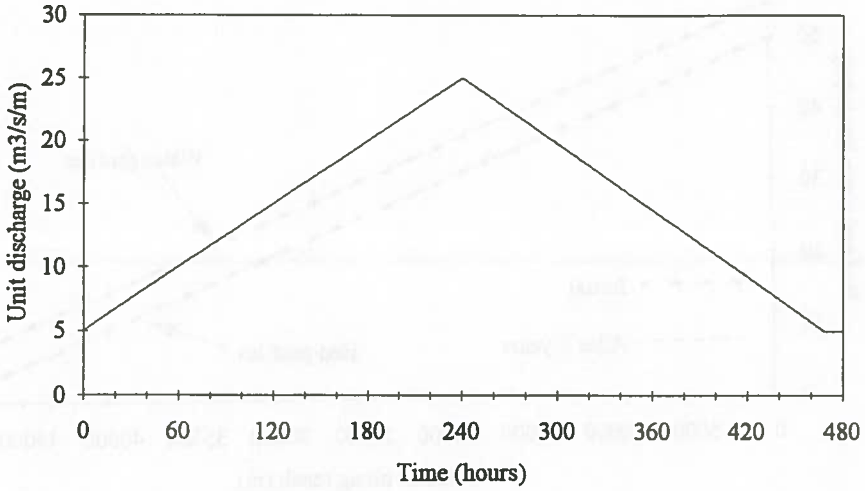

Fig. 3.5 – Flow hydrograph

Results and discussion

a) Variable and constant roughness

In the method of Kishi-Kuroki, resistance to flow is considered to change with the flow properties, namely, with $Z = R/d_{65}$ and $Y = \gamma R S_f / \left[(\gamma_s - \gamma) d_{50} \right]$. Z and Y are dimensionless parameters; R is the hydraulic radius of the cross-section; and, γ and γ_s are the specific weight of water and sediment, respectively. The formula of Manning considers a constant roughness. MOBFLOW was run considering both the formulations of Kishi-Kuroki and Manning for the calculation of the friction slope. The Manning formula was used with a constant coefficient of $K_r = 40\,m^{1/3} s^{-1}$. The obtained results are presented in Fig. 3.6.

When the method of Kishi-Kuroki is employed, MOBFLOW predicts more erosion (upstream) and more deposition (downstream). The water profile calculated with the Manning formula remains bellow the profile obtained with the method of Kishi-Kuroki. Thus, the calculations made using the Manning formula underpredict the bed movements and the water elevations.

This example shows the importance of varying roughness with the flow properties, which is the case when using the relationships of Kishi-Kuroki.

b) Loading-law

The use of a loading-law (see 2.3.2 iii-) allows a better distribution of the eroded/deposited sediment along the stream bed profile. As it can be seen from Fig. 3.7, the loading-law implemented in MOBFLOW, with parameters calculated automatically by the model, works like a filter to parasitic oscillations, allowing to use greater time increments in the simulation process. This way, it is possible to diminish the overall computer time needed for the simulation, without having stability problems. In the legend of Fig. 3.7, *No lag* corresponds to the predictions of MOBFLOW without using a loading-law, that is, without considering any spatial-lag in sediment transport.

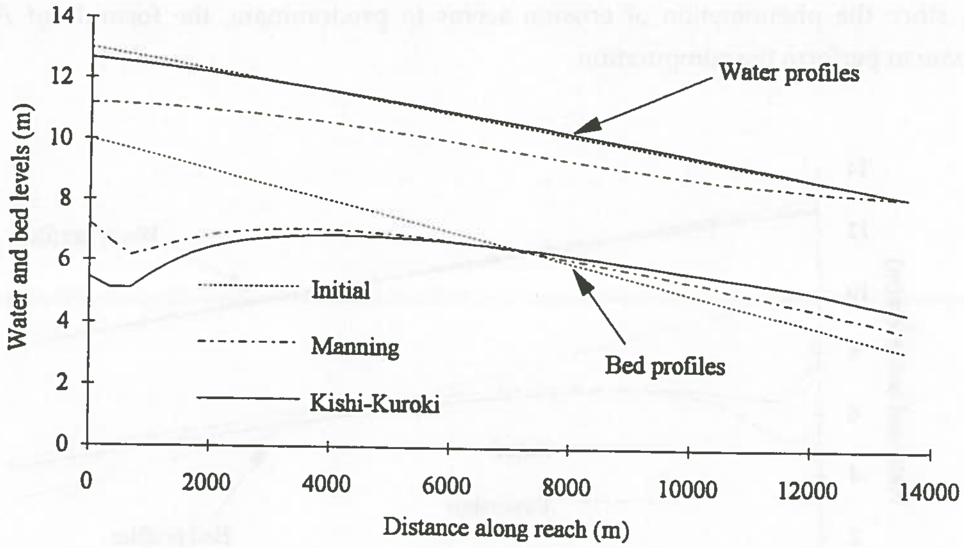

Fig. 3.6 – Bed and water profiles with constant and changing alluvial roughness

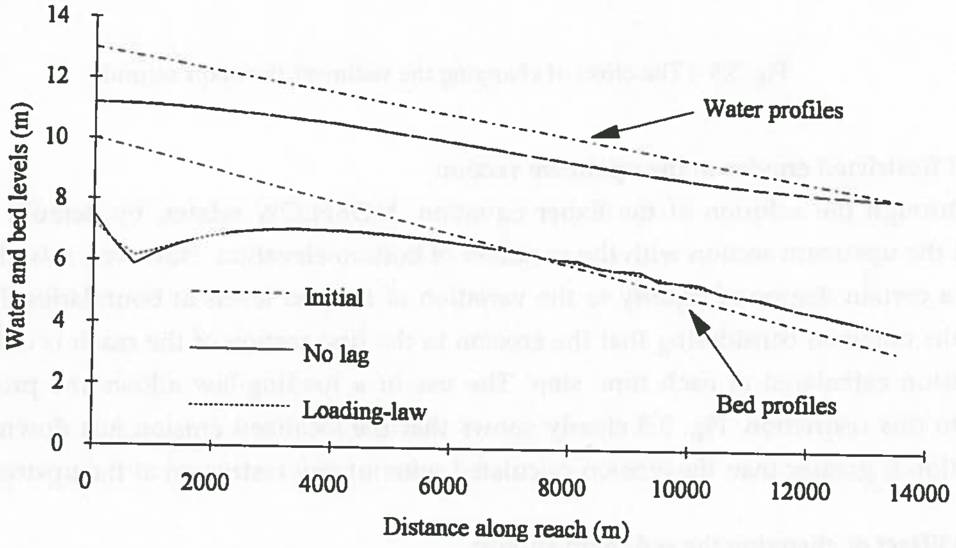

Fig. 3.7 – The effect of using a loading-law

c) Sediment transport formula suitable for deposition

In the example 3.1, the power-law equation for the calculus of the solid discharge proved to give better results than the Brownlie formula. Thus, assuming the nature of the problem identical to that of the problem 3.1 (that is, deposition), the power-law equation is used to perform the computation of sediment transport capacity. The results, shown in Fig. 3.8, indicate that the water profile remains practically unaltered, while the bed profile underpredicts sand movements. If deposition is the prevailing phenomenon, it seems, in spite of the absence of experimental evidence, that the bed profile at his downstream part should exhibit bigger elevations than those calculated by the formula of Ackers-White (see comments in 3.1 relative to the usage of Brownlie's formula). Thus, since the phenomenon of erosion seems to predominate, the formula of Ackers-White is adequate to perform the computation.

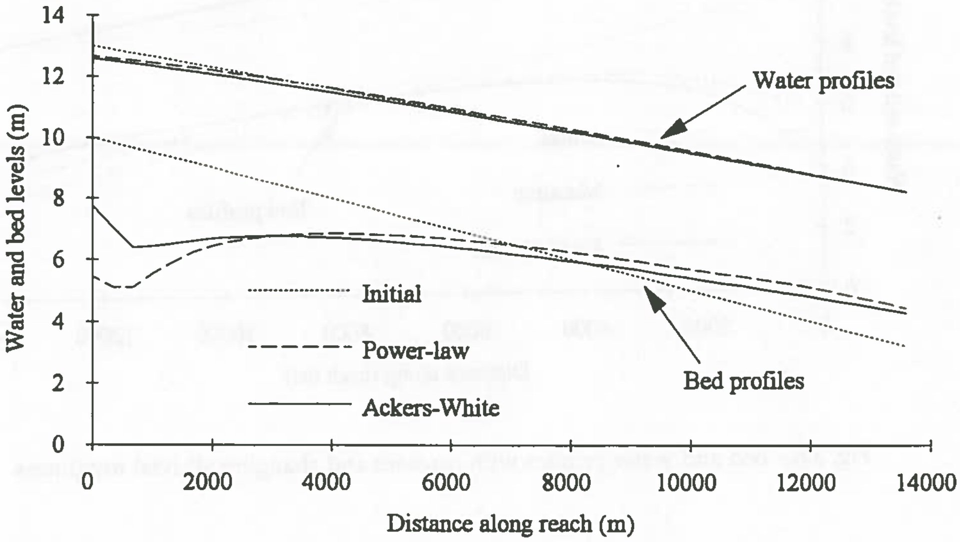

Fig. 3.8 – The effect of changing the sediment transport formula

d) Restricted erosion at the upstream section

Through the solution of the Exner equation, MOBFLOW relates, by default, the sediment input at the upstream section with the variation of bottom elevation. However, it is also possible to impose a certain degree of rigidity to the variation of the bed levels at boundaries. Fig. 3.9 shows the results obtained considering that the erosion in the first section of the reach is only 10 % of the free erosion calculated at each time step. The use of a loading-law allows the profile to adjust locally to this restriction. Fig. 3.9 clearly shows that the localized erosion just downstream of the first section is greater than the erosion calculated without any restriction at the upstream end.

e) Effect of changing the sediment supply

Fig. 3.10 and Fig. 3.11 show how the predicted bed profile varies, when the sediment supply changes from constant and equal to 5 kg/s/m to nil (Fig. 3.10), and to its equilibrium value calculated by the formula of Ackers-White (Fig. 3.11).

As expected, the erosion at the upstream end of the reach grows with nil sediment supply. The equilibrium sediment supply promotes bed aggradation, since the downstream boundary condition (that is, water depths ranging from 5.0 m to 8.0 m) imposes a decrease in the flow velocity, leading to deposition of the transported sediment.

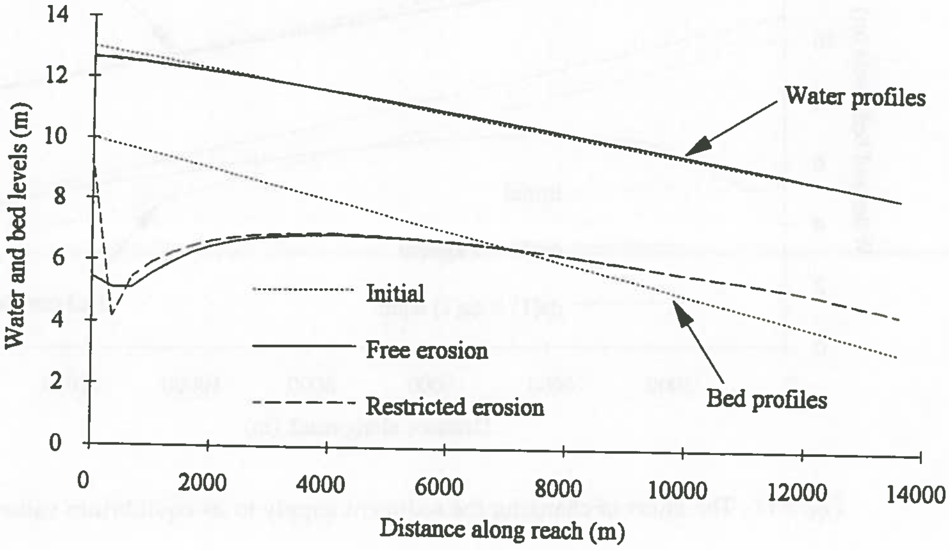

Fig. 3.9 – The effect of limiting the mobility of stream bed at the upstream boundary

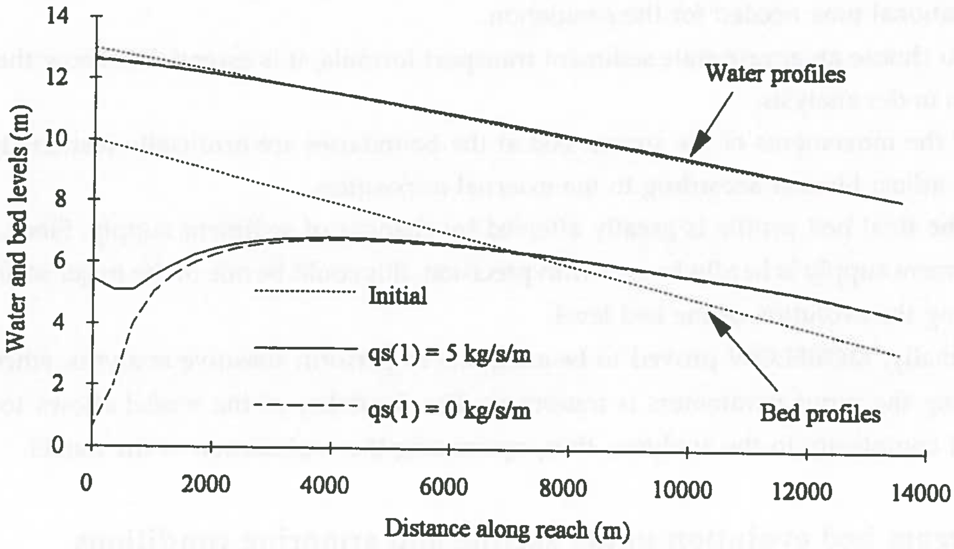

Fig. 3.10 – The effect of changing the sediment supply to nil

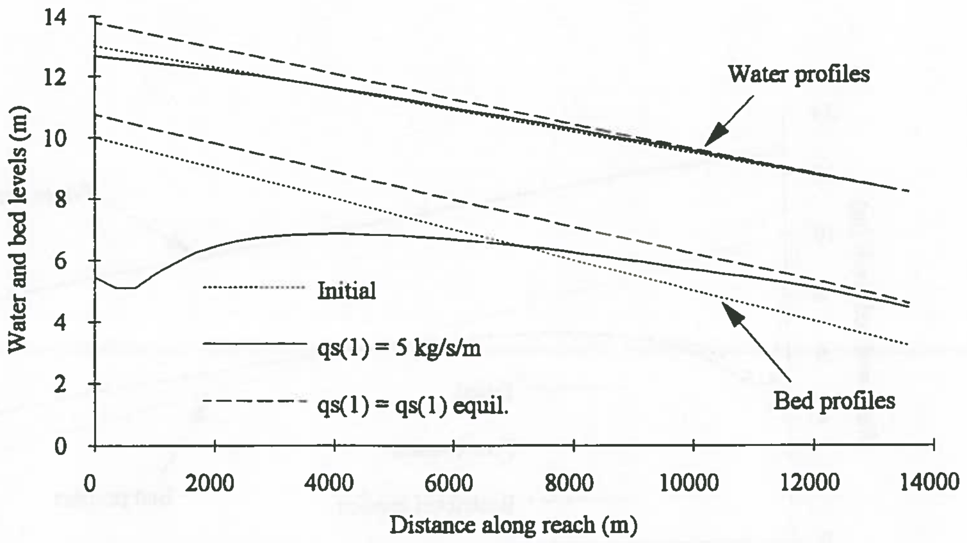

Fig. 3.11– The effect of changing the sediment supply to its equilibrium value

Conclusions

When studying problems related to the evolution of streams with mobile boundaries, roughness must be considered to depend on flow and sediment properties, since the prediction using constant roughness coefficients could result in lower water depths and underpredict erosion and deposition.

A loading-law is simply a filter, allowing to use greater time steps, thus diminishing the computational time needed for the simulation.

To choose an appropriate sediment transport formula, it is essential to know the nature of the problem under analysis.

If the movements of the stream bed at the boundaries are artificially restricted, the bed profile will adjust himself according to the external imposition.

The final bed profile is greatly affected by changes of sediment supply. Since, in real cases, the sediment supply is hardly known with precision, this could be one of the major sources of error in predicting the evolution of the bed level.

Finally, MOBFLOW proved to be adequate to perform sensitive analyses, since the effort of modifying the input parameters is minimum. The flexibility of the model allows to use different levels of complexity in the analyses, thus, optimizing the exploitation of the model.

3.5. Stream bed evolution under sorting and armoring conditions

Purpose

During the low flow seasons, streams with graded-bed materials can exhibit a tendency to form coarser surface layers at the bed, which protect the underlaying finer sediments from erosion. The model MOBFLOW is used to simulate the development of this coarse surface layer. This case study is as an indicator to practical applications of the model in environmental problems.

Approach

The channel of the example 3.1 is considered. The inflow discharge is constant, $q = 2\ m^3/s/m$, and the sediment supply is nil. The initial water depth at the upstream section is $h(1) = 1.0$ m, and the downstream water depth is $h(n) = 4.0$ m, during all the simulation. The initial sediment is log-normal distributed with parameters: $d_{50} = 1.0$ mm, and $\sigma_g = 3.0$. All the bed-layers (see Belo 1993) are assumed to have sediments with identical distributions. The total load capacity is calculated by the formula of Ackers-White, the effective load is obtained by a loading-law with parameters calculated automatically by the model, and the resistance to flow calculation is performed using the method of Griffiths, once $d_{50} \geq 1.0$ mm. The other parameters are: $L = 13600$ m; $\Delta x = 340$ m; $\Delta t = 14400$ s (4 hours); and $\phi = \theta = 0.75$. The ASP is used to solve the system of Saint-Venant equations, and the sorting option is turned to on.

Results and discussion

Fig. 3.12 shows the results obtained by MOBFLOW after an elapsed time of 10 days. The results under the absence of sorting are also shown in this figure. The water profile obtained considering the selective transport of material is comparatively higher than the profile calculated assuming equal mobility of sediment, but the bottom profiles are superimposed.

Fig. 3.13 shows the longitudinal variation of the sediment diameters of the mixed layer, namely, d_{35}, d_{50} and d_{65}, after an elapsed time of 10 days. The coarsening of this layer is notorious and, since it protects the underlaying bed material of buffer layer and substratum from the direct action of flow, it is expected the bed degradation to slow progressively. The deformation of the granulometric curves may also be inferred from Fig. 3.13, since the relative differences between d_{35}, d_{50} and d_{65} are not identical along the profilew of the reach after 10 days of simulation.

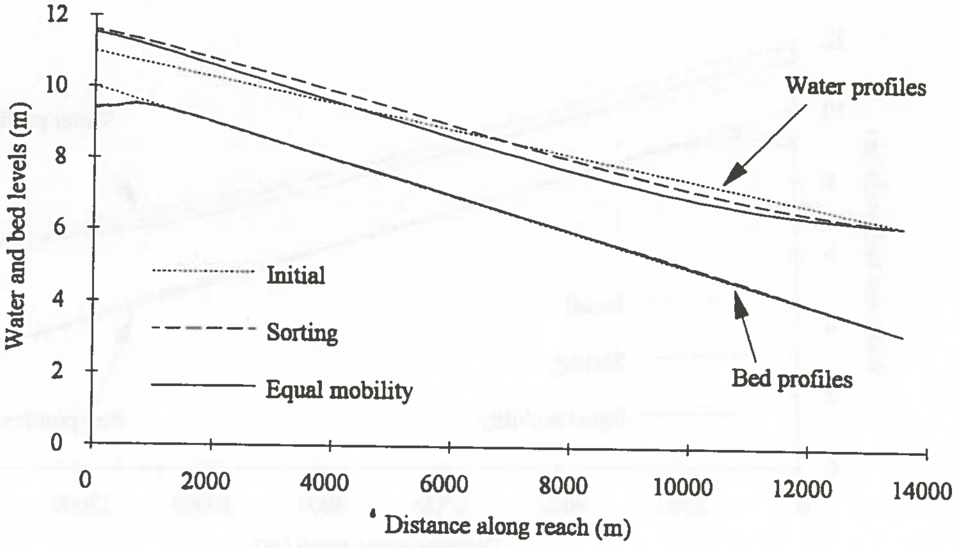

Fig. 3.12 – Water and bed profiles under sorting and non-sorting (equal mobility) conditions after 10 days

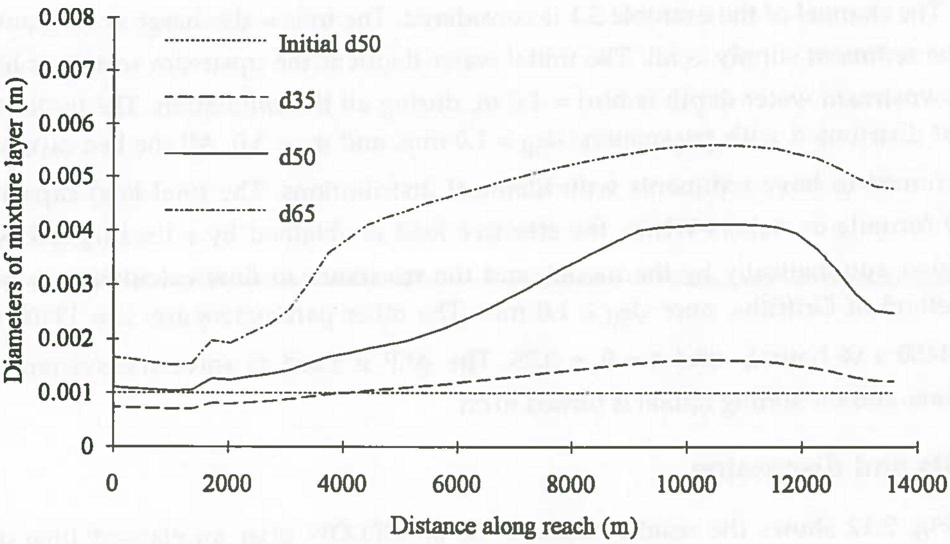

Fig. 3.13 – Longitudinal distribution of bed diameters of the mixed layer after 10 days of simulation

After an elapsed time of 25 days, a difference in bed profiles can already be observed (Fig. 3.14). The bed profile calculated under sorting conditions exhibits less erosion at the upstream sections than the profile with sediment of equal mobility. The second profile also shows some deposition at the downstream part of the reach. This tendency to deposition is also present in the first profile, though incipient. In fact, as can be seen in Fig. 3.13, the coarser sizes of sediment eroded upstream tend to deposit at the downstream part of the reach, increasing bed diameters, while the finer sizes leaves the modelled reach through his downstream boundary.

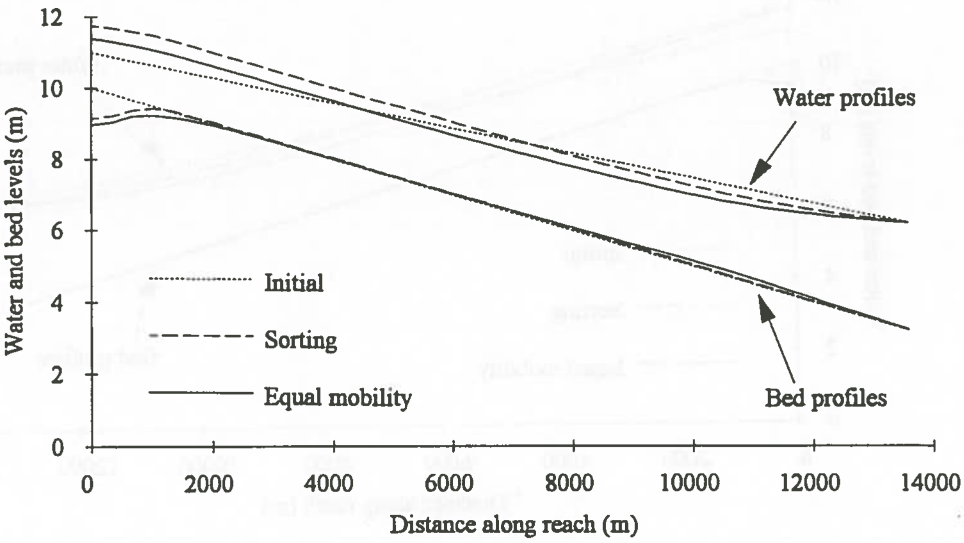

Fig. 3.14 – Water and bed profiles under sorting and non-sorting (equal mobility) conditions after 25 days

Conclusions

The model MOBFLOW predicts the evolution of the granulometric curves of the bed material in the presence of selective sediment transport. The sorting option should be used whenever the natural conditions of the stream suggest the possibility of sorting and armoring, since the water levels tend to be higher than those of streams where sediment moves with equal mobility. MOBFLOW predicts deposition of coarser materials and erosion/transport of finer materials at the same reach, which is the main problem that must be solved when dealing with the transport of sediment mixtures.

3.6. Bed evolution of the Mondego river

Purpose

The main purpose of the study was to analyze and describe the evolution of bed level and river regime changes due to dam construction and operation. It served also to test the ability of the mathematical MOBFLOW to simulate real cases of streams having non-uniform transport of sediment and highly irregular flow regime.

Approach

The model MOBFLOW was applied to a 33.2 km-long reach of river Mondego, between the Raiva's dam and Santa Clara bridge near Coimbra (see scheme in Fig. 3.15). The geometry of the cross-sections and the longitudinal profile being known at two different times, eleven years apart (namely, 1974 and 1985), allow the comparison between numerical results and field observations along the studied reach, since the hydrological data is also available for the entire simulation period (Belo 1992).

The granulometric curves, available for some cross-sections surveyed in 1985, were averaged at each cross-section. A log-law of the Sternberg type was fitted by least squares to the averaged granulometric curves to obtain the longitudinal variation of bed diameters, namely d_{35} and d_{50}.

Some preliminary runs of the mathematical model were carried out. It was verified that, under low flow conditions ($Q \leq 35\ m^3/s$), sediment transport was very weak and bed level changes were negligible. So, only discharges greater than $35\ m^3/s$ were taken for simulation purposes.

The model was run with 1-day time-step, which was divided in smaller time-steps if the difference in sequent discharges at one of the boundaries exceeded $5\ m^3/s$ (see 2.4.1 ii-), uniform spacing of sections (≈ 1 km), Griffiths flow resistance formula, Ackers-White total load formula, and Einstein-Rouse procedure for suspended-sediment load. The procedure for simulating the effects of the bed-rock was considered, assuming the Strickler coefficient of the exposed rock, K_r, equals to 25 $m^{1/3}s^{-1}$. The initial values of the erodible bed layer thickness were established on the basis of local observations.

The granulometric curves at the beginning of the simulation period were not known. Thus, assuming a log-normal distribution of bed material, three different scenarios at the beginning of the simulation period (1974) were defined: scenario a) a unique granulometric curve for the entire reach with $d_{50} = 6$ mm, and $\sigma_g = 2.5$; scenario b) different granulometric curves at each cross-section with

d_{50} given by the Sternberg's law for d_{50} in 1985, and $\sigma_g = 2.5$; and scenario c) granulometric curves with d_{50} given by Sternberg's law for d_{35} in 1985, and $\sigma_g = 3$.

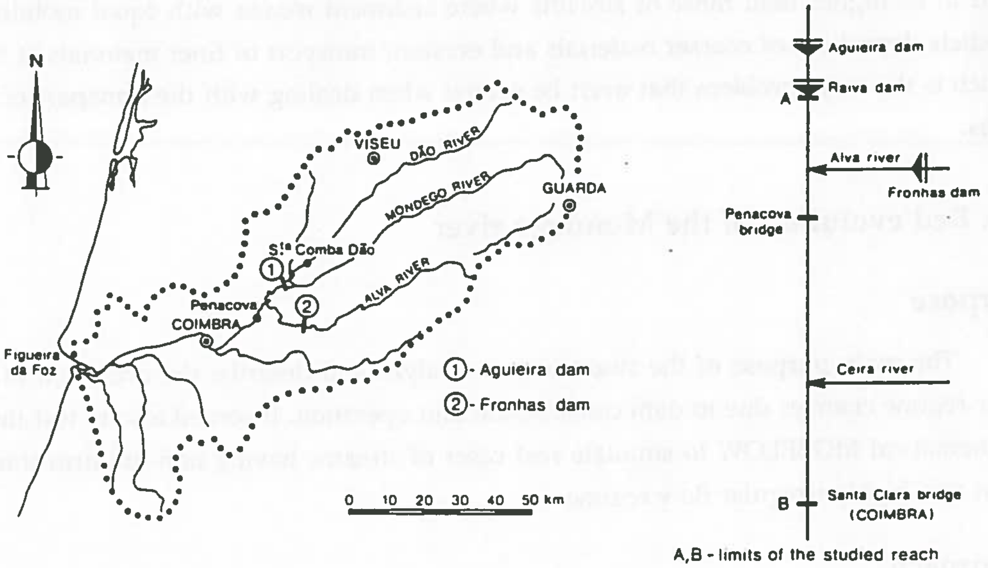

Fig. 3.15 – Mondego river basin and schematic layout of study reach

Results and discussion

Unacceptable results were obtained for scenario a). So, only the results of scenarios b) and c) are shown here, respectively, when *bed material sorting is neglected* (scenario b), and when *the influence of selective transport and armor formation are considered* (scenario c).

The final bed profiles are presented in Fig. 3.16. For *scenario b)* the profile can be split up into two distinct parts. In the first one, between 5000 and 12000 m, there is poor agreement between numerical computations and observed values, due to secondary currents and other flow and sediment features not accounted for by MOBFLOW. In the second part, x > 12000 m, there is a fairly good agreement between numerical and observed values. Despite of this agreement, it is important to notice that, between 1978 and 1984, important sand and gravel mining took place in the sub-reach from 22500 m downstream.

The final bed profile for *scenario c)* exhibits minor erosion than the profile of scenario b) in the zone where sand mining took place. The difference probably balances sand mining, but may also results from the variation of the resistance coefficients related to the evolution of bed material distribution. In the first part of the profile, the results seems to be slightly better than those of scenario b).

Fig. 3.17 shows the comparison between calculated and observed water levels at Penacova bridge (x ≈ 9600 m) for the scenarios b) and c). The mean relative errors in water level calculations are of about 0.3 % (0.012 m) and – 0.4 % (– 0.017 m), respectively. The mean relative absolute errors are 5.1 % (0.212 m) and 5.0 % (0.219 m). The values in parenthesis correspond to the differences between observed and computed in water levels. These indicators were obtained only for the period from 19th December 1978 to 5th May 1979, but were seen to be of the same order of magnitude for other periods.

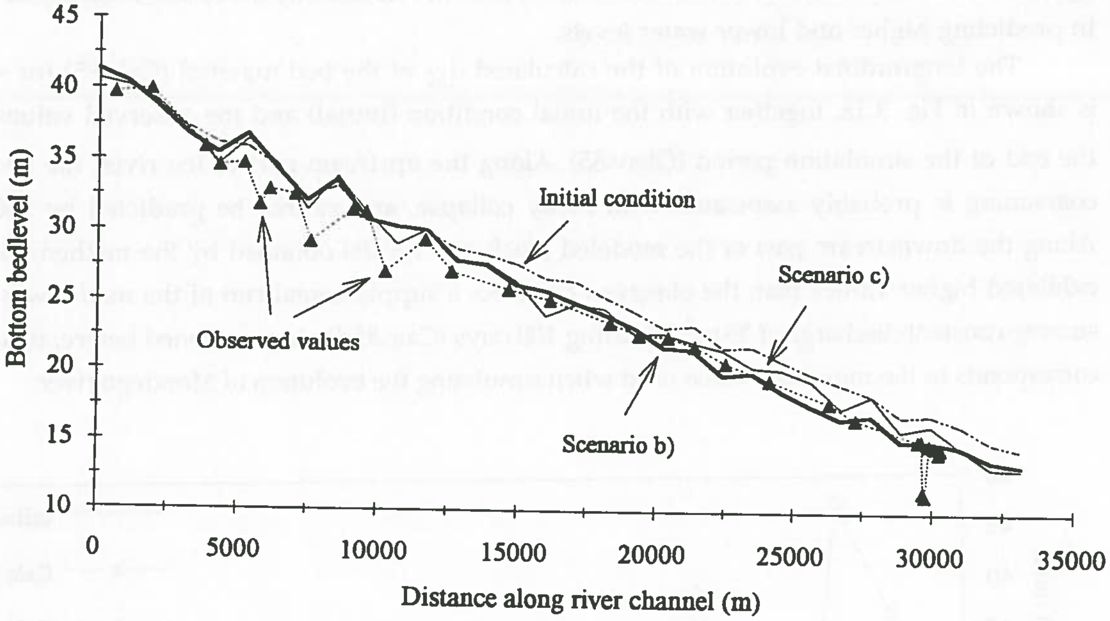

Fig. 3.16 – Bed level changes observed and calculated in 1974 and 1985

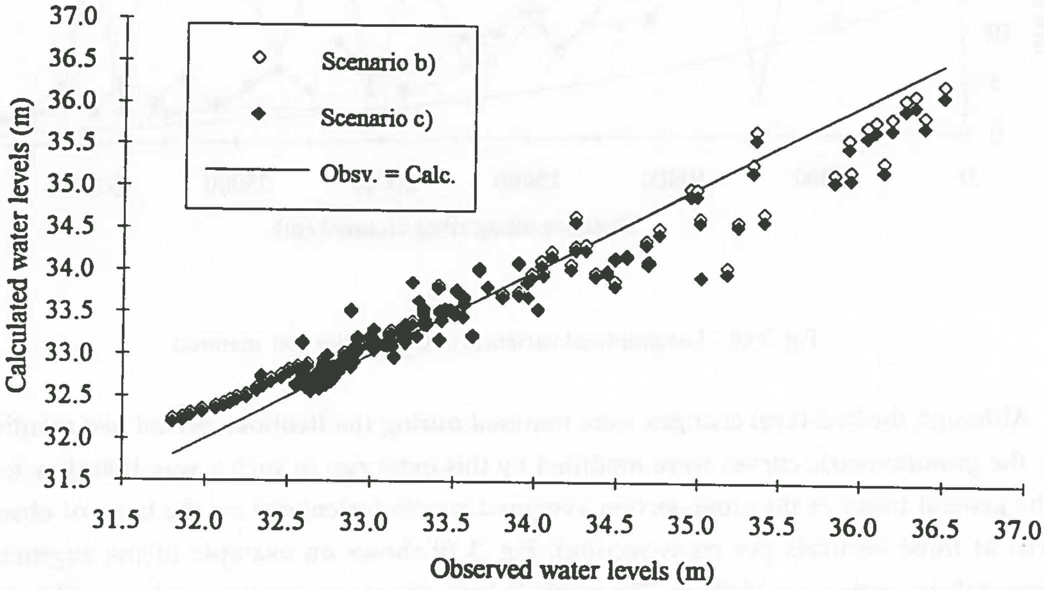

Fig. 3.17 – Observed and calculated water levels at Penacova bridge (from 1978/12/19 to 1979/5/5)

In the period considered, the calculated discharges at Penacova ranged from 49.8 m^3/s to 1068.5 m^3/s, and the averages of computed water depths were of 4.185 m and 4.415 m, respectively for scenarios b) and c). Despite of the good results, the model seems to underpredict the highest water levels and to overpredict the lower ones (see Fig. 3.17). For these extreme levels, due to the lack of field data, the rating curves at the boundaries were extrapolated and truncated. So, the less agreement between results and observed values does not necessarily mean less accuracy of the model in predicting higher and lower water levels.

The longitudinal evolution of the calculated d_{50} of the bed material (Calc.85) for scenario c) is shown in Fig. 3.18, together with the initial condition (Initial) and the observed values of d_{50} at the end of the simulation period (Obsv.85). Along the upstream part of the river, the bed material coarsening is probably associated with rocky collapse, and cannot be predicted by MOBFLOW. Along the downstream part of the modeled reach, the results obtained by the mathematical model exhibited higher values than the observed ones. So, a supplemental run of the model was done, assuming constant discharge of 35 m^3/s during 100 days (Calc.85d). As mentioned before, this discharge corresponds to the minimum value used when simulating the evolution of Mondego river.

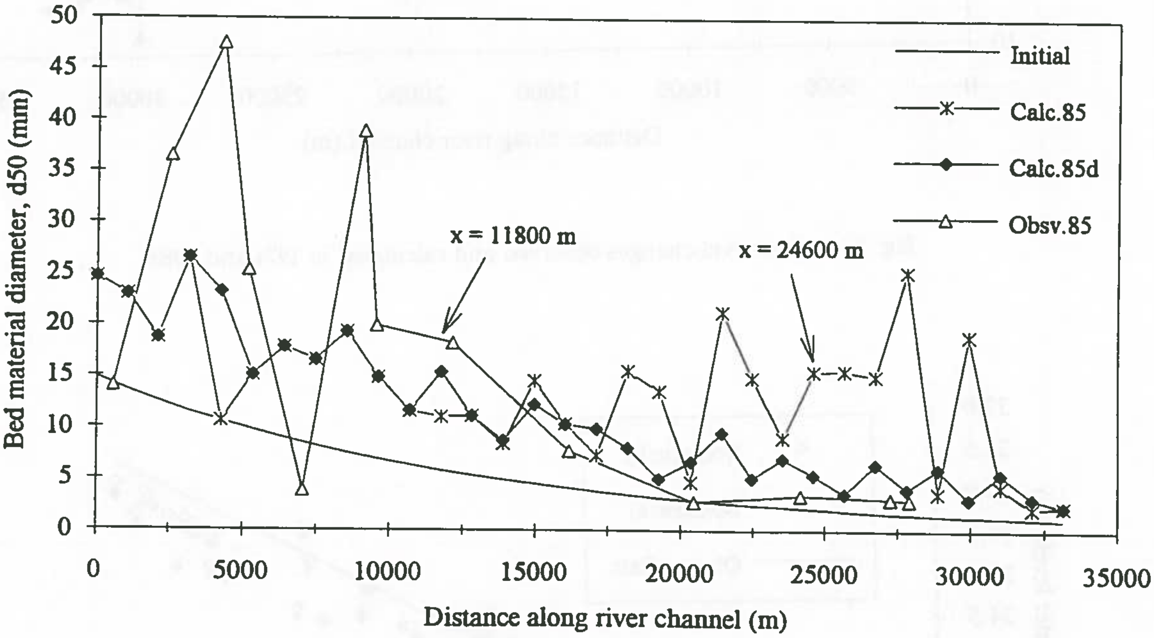

Fig. 3.18 – Longitudinal variation of d_{50} of the bed material

Although the bed level changes were minimal during the fictitious period just mentioned (100 days), the granulometric curves were modified by this extra run in such a way that they tend to follow the general trend of the cross-section averaged curves (calculated on the basis of observed bed material at three verticals per cross-section). Fig. 3.19 shows an example of the augmentation of fine materials in section x ≈ 24600 m. The symbols have the same meaning as those of Fig. 3.18.

Along the upper part of the modeled reach, the bed material distribution remains almost unaltered, while changing along its downstream part. This can be explained by the deposition of fine materials in the lower reach as they are transported at small discharges (Q < 35 m^3/s). This phenomenon seems to be well reproduced by the supplementary run for Q = 35 m^3/s.

An important percentage of the fine material referred to above is entrained by the flow at section x ≈ 11800 m. The sediment discharge at this section is comparatively higher than at neighbor sections, though it is insufficient to promote an important change on the bed level. According to the previous observations, the value of d_{50} grows at section x ≈ 11800 m, as can be seen in Fig. 3.18 and in Fig. 3.19. In other sections the results also tend to follow the observations after the considered fictitious run of 100 days with Q = 35 m^3/s.

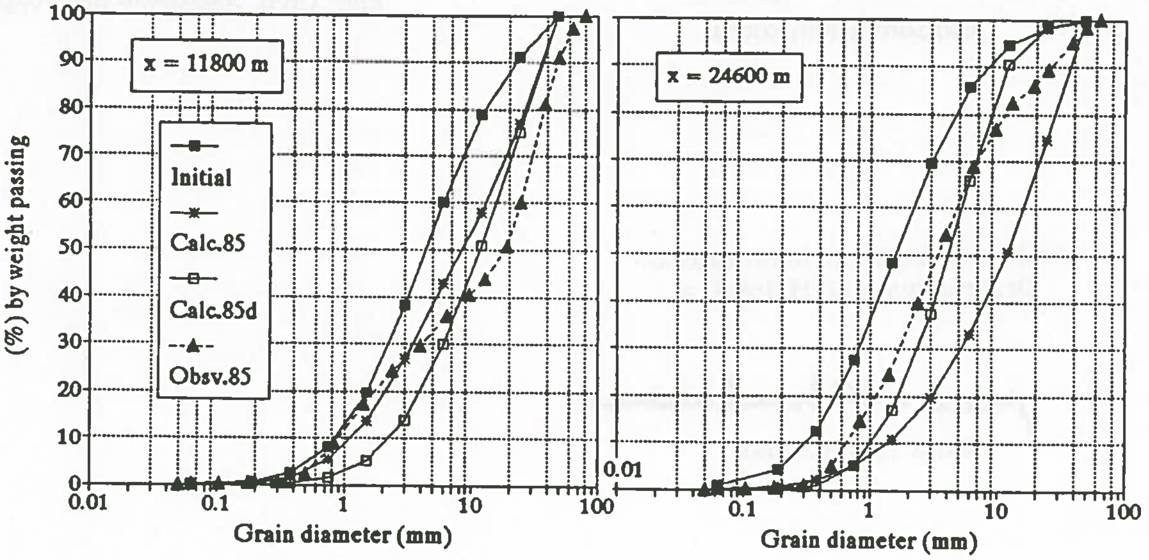

Fig. 3.19 – Granulometric curves at x ≈ 24600 m and x ≈ 11800 m

Conclusions

In the framework defined by the scenarios referred to above, the numerical results follow the water and bed level observations. The version 1.0 of MOBFLOW model also predicts reasonably well the general behavior of bed material characteristics.

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Lisboa, Laboratório Nacional de Engenharia Civil, Outubro de 1994

VISTOS

O Chefe do Núcleo de Hidrologia e
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APPENDICES

Appendix A

GLOSSARY

computational model: software whose primary function is to model a certain class of physical systems. The computational model may include pre- and post-processing features, a user interface, and other ancillary programs necessary in order to use the model in applications. However, this validation document primarily concerns the core of the computational model, consisting of the underlying conceptual model, its algorithmic implementation and software implementation.

conceptual model: a mathematical/logical/verbal representation of a physical system. This representation may involve differential equations, discrete algebraic equations, decision graphs, or other types of conceptual descriptions.

algorithmic implementation: the conversion of the conceptual model into a finite set of rules suitable for computation. This may involve spatial discretization schemes, time integration methods, solution procedures for algebraic equations, decision algorithms, etc..

software implementation: the conversion of the algorithmic implementation into computer code. This includes coding of algorithms, use of standard mathematical software, design and implementation of data structures, etc.. The term software implementation, for the purposes of this document, is limited to the computational core of the model. It does not include pre- and post-processing software, user interfaces, or other ancillary programs associated with the computational model.

Appendix B

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