VARIANCE OF INTACT ROCK STRENGTH DETERMINED BY TRIAXIAL TESTS

*J. Muralha
National Laboratory for Civil Engineering (LNEC)
Lisbon, Portugal
*(*Corresponding author: jmuralha@.lnec.pt)*

L. Lamas
National Laboratory for Civil Engineering (LNEC)
Lisbon, Portugal

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ABSTRACT

Given the results of triaixial tests, regressions are commonly used to evaluate the parameters of failure criteria that model intact rock strength and to perform statistical inferences used to evaluate characteristic values defined in EC7. Statistical robustness of these inferences is affected if the basic hypothesis underlying regression are not met, namely homoscedasticity (constant variance of the errors). This paper presents analyses of 23 sets of triaxial tests, starting by the evaluation of the regression parameters of the Mohr-Coulomb linear criterion and of the Hoek-Brown non-linear criterion, followed by carrying several statistical tests to check the homoscedasticity null hypothesis of independence between the variance of intact rock strength and the confining stress.

KEYWORDS

Triaxial tests, rock strength, failure criterion, regression, homoscedasticity, statistical tests

INTRODUCTION

Though rock engineering design is clearly included in the scope of Eurocode 7(EN1997-1), its implementation raised a wide discussion regarding the applicability of structural safety concepts to geotechnical design of construction works in or on rock masses. The basis of the limit state design philosophy adopted in EC7 is that, for each particular design situation, all the possible limit states for a structure, or part of it, shall be considered and that it shall be demonstrated that the likelihood of any limit state being exceeded is sufficiently small. Though EC7 allows the limit states to be verified by one or a combination of methods, use of calculations is by far the most used, and it is often confused with the EC7 itself. It involves using characteristic values of actions, ground properties and geometrical data, as well as obtaining their design values by the partial factor method.

Characteristic value of a given material property is defined in Eurocode (EN 1990) as: “value of a material or product property having a prescribed probability of not being attained in a hypothetical unlimited test series. This value generally corresponds to a specified fractile of the assumed statistical distribution of the particular property of the material or product”. As regards the design value of a material or product property, according to the Eurocode it is a “value obtained by dividing the characteristic value by a partial factor”.

EC7 provides generic rules for obtaining characteristic values of geotechnical parameters, which take into consideration that geotechnical design does not deal with manufactured materials, with relatively well controlled parameter values, but with a wide diversity of natural materials regarding their origin and the conditions in which they are found in nature. EC7 defines how characteristic values of geotechnical parameters are obtained. Characteristic values “shall be selected as a cautious estimate of the value affecting the occurrence of the limit state”. This value depends on the zone of ground governing the behaviour of the geotechnical structure. Usually it is much larger than the volume affected in an in situ or laboratory test, and the characteristic value should be “a cautious estimate of the mean value” or of the range of values covering that whole zone of ground. However, if the behaviour of the geotechnical structure is governed by the lowest or highest value of the ground property, the characteristic value should be “a cautious estimate of the lowest or highest value”. If statistical methods are used, the characteristic value is “a selection of the mean value of the limited set of geotechnical parameter values, with a confidence interval of 95%”, in the first case, or “a 5% fractile” in the second case.

Also available in EC7 are recommended values of the partial factors to use for some specific ground parameters: angle of shear resistance, effective cohesion, undrained shear strength, unconfined strength and weight density. It is easy to recognise that these parameters were chosen having in mind soil properties. Moreover, statistical analyses of rock engineering properties, such as intact rock strength, joint shear strength, rock mass deformability, are not frequent. These present circumstances account for some of the justifications forwarded by those that do not think that EC7 is applicable to rock engineering design.

In order to try to bridge this gap, using the results of several sets of tests performed by LNEC, Muralha & Lamas (2014) presented in a previous paper the statistical study of the parameters of Mohr-Coulomb and Hoek-Brown strength criteria aimed at calculating the characteristic failure envelopes for intact rock. These parameters are generally evaluated using regression techniques, linear in the case of the former and non-linear in the case of the latter. Regression can only be applied under several assumptions, namely homoscedasticity that requires the variance of the maximum principle stress at failure σ_1 (dependent variable) to be constant along the whole range of confining stresses σ_3 (independent variable). In this paper, results of several sets of triaxial tests are analysed. Some of the sets are from tests performed by LNEC, whilst others were obtained from a literature survey. The purpose is to verify if the variance of the maximum principle stress remains constant over the range of applied confining stresses, and consequently ordinary statistical analyses can be used to calculate characteristic values according to EC7 definitions.

STATISTICAL ANALYSIS OF TRIAXIAL TESTS RESULTS

The prevalent strength criteria used to describe the strength of intact rock under triaxial conditions are the well-known Mohr-Coulomb and Hoek-Brown criteria. The Mohr-Coulomb criterion can appropriately model the relation between the principal stresses at failure using a linear relation, with the parameters c and ϕ (cohesion and internal friction angle), as long as relatively small ranges of the confining stresses are involved. Hoek & Brown (1980) developed a nonlinear relationship between the principal stresses at failure characterized by the parameters m_i and σ_{ci} (uniaxial compressive strength), where the index i stands for intact rock. Though strength parameters estimates are available in the literature for a variety of rock types, important projects require specific triaxial tests to be performed to evaluate the actual values. For this purpose, a statistically significant set of triaxial tests should be performed, under confining stresses that cover the expected range of confining stresses. In order to assess parameter variability, the rock specimens to be tested under all the confining stresses should be prepared from a homogeneous sample of rock cores. Results presented in this paper come from tests performed according to ASTM D7012-07 (ASTM, 2007) or to the standard "type I" test of the ISRM Suggested Method (ISRM, 2007).

In this section, an example of the statistical analysis of the results of a chosen set of triaxial tests will be presented. This particular set comprises 21 results of medium-grained granite that were tested under the following confining stresses σ_3: 0, 2, 5, 10 and 15 MPa. To estimate the parameters of the Mohr-Coulomb criterion (c and ϕ), it is firstly necessary to perform a linear regression of the maximum principal stress σ_1 versus the confining stress σ_3 (and also minimum principal stress). Figure 1 displays the results of the triaxial tests (dots) and their best fit straight line in the $\sigma_2 = \sigma_3$ plane of the principal stress space. Then, from the slope of the straight line (tan β) and the y-axis intercept, that in this case is the uniaxial compressive strength of the intact rock σ_{ci}, it is possible to calculate the internal friction angle and the cohesion for the mean Mohr-Coulomb failure envelope. Any kind of regression in the Mohr diagram is very difficult since it means evaluating the "best" tangent straight line to a set of Mohr circles each representing a triaxial test result. As a consequence, all variability analyses have to be performed in terms of σ_1 versus σ_3, which are the direct results of the triaxial tests.

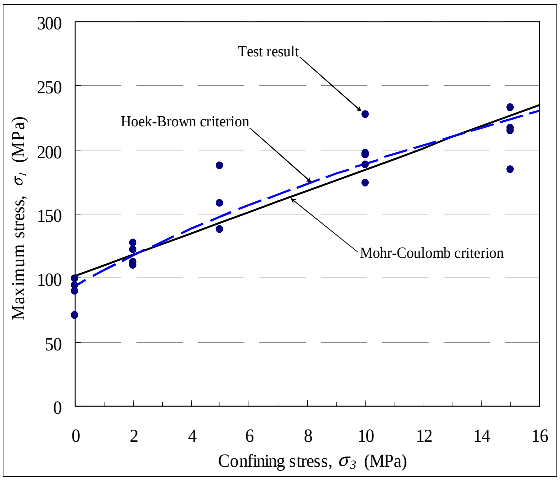

Figure 1. Mohr-Coulomb and Hoek-Brown failure criteria in the $\sigma_2 = \sigma_3$ plane of principal stress space.

Test results displayed in Figure 1 show that a curved function may yield a better relation between σ_1 and σ_3. To consider this negative curvature several types of rocks often display, Hoek-Brown's (1980) non-linear criterion is also represented in Figure 1 (dashed line). The criterion parameters σ_{ci} and m_i have to be determined from triaxial tests results by non-linear regression.

For both failure criteria, regression was used to calculate the criterion parameters. It is a common statistical procedure for fitting data to any selected equation by minimizing the residual sum of squares *RSS*. In addition, several statistical methods can be used to quantify goodness of fit. Generally, all take into account s^2, an unbiased estimator of the variance of the residuals, also known as the residual mean square, given by:

$$s^2 = \frac{\sum_i r_i^2}{n-p} = \frac{\sum_i (\sigma_{i1} - \hat{\sigma}_{i1})^2}{n-p} \tag{1}$$

where r_i are the residuals, σ_{1i} are the test results and $\hat{\sigma}_{i1}$ are the corresponding model predicted values, n is the number of experimental values, and p is the number of parameters in the model (two in both cases). In sequence, standard errors of the regressions s can be easily calculated. In linear regressions (Mohr-Coulomb criterion), this estimator and appropriate values of the Sudent's t distribution, allow predicting, for a given confining stress σ_3, 95% confidence limits for the true mean value of the maximum principal stress $\hat{\sigma}_1$, and 5% fractiles:

$$\sigma_1^{95\%} = \hat{\sigma}_1 - s\left(\frac{1}{n} + \frac{(\sigma_3 - \bar{\sigma}_3)^2}{\sum_i (\sigma_{3i} - \bar{\sigma}_3)^2} \right) t_{(n-p,0.95)} \tag{2}$$

$$\sigma_1^{>5\%} = \hat{\sigma}_1 - s\left(1 + \frac{1}{n} + \frac{(\sigma_3 - \bar{\sigma}_3)^2}{\sum_i (\sigma_{3i} - \bar{\sigma}_3)^2} \right) t_{(n-p,0.90)} \tag{3}$$

In non-linear regression, exact definition of confidence intervals is seldom possible. So, to determine the 95% confidence limits and the 5% fractiles for the Hoek-Brown criterion the bootstrap method (Efron 1979) was used. This procedure considers the sample as the population, and performs draws with replacement samples with the same size n. A sufficiently large number of draws will allow the bootstrap estimates to asymptotically tend to the correct values, and almost all statistical inference calculations can be carried out.

Evaluating equations (2) and (3) and using the bootstrap method for any required σ_3 value within the range of applied confining stresses, 95% confidence intervals for the fitted values and 5% failure envelopes were determined. They are displayed in Figure 2 for the Mohr-Coulomb and Hoek-Brown criteria, respectively.

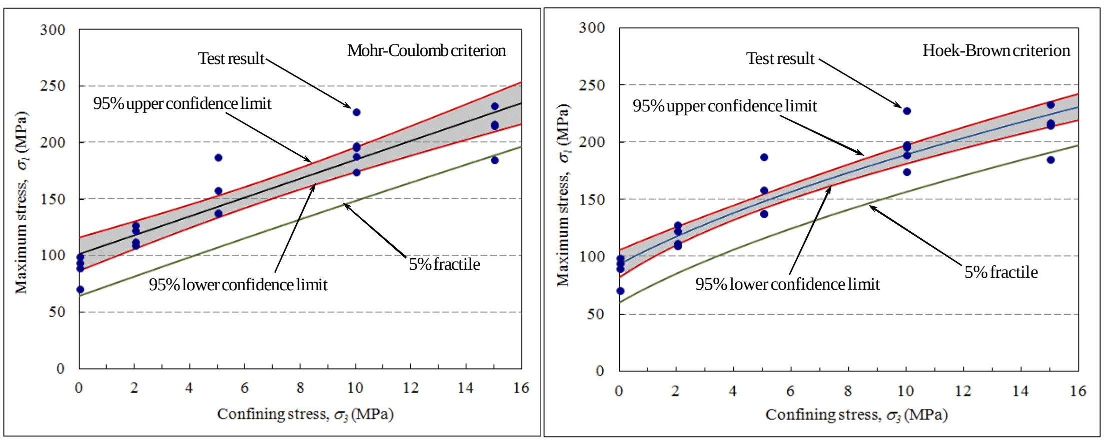

Figure 2. 95% confidence intervals and 5% failure envelopes for both failure criteria.

These plots show that, in this particular case, the Hoek-Brown criterion provides a better fit to the test results, since its 95% confidence interval is clearly narrower. This conclusion can also be recognized by the regression standard error values *s*: 20.4 MPa and 18.2 MPa, for the Mohr-Coulomb and Hoek-Brown models, respectively. The plots also reveal that the 95% confidence limits and the 5% envelope are neither straight lines in the case of the Mohr-Coulomb criterion, nor parabolas in the case of the Hoek-Brown criterion. However, both criteria could still provide good approximations to these curves. As could be expected from the analysis of equations (2) and (3), the curves are closer to the models when the confining stress is equal to its average value $\bar{\sigma}_3$ (6.57 MPa in this example).

For the Mohr-Coulomb (linear) criterion, results displayed in Figure 2 can be easily transferred to the space of the estimated parameters (tan β–σ_{ci}) by evaluating the tangents to each curve for any given σ_3 value (Muralha & Lamas, 2014). In Figure 3, the red ellipse corresponds to all tangents to the 95% confidence limits. The black dot represents the mean Mohr-Coulomb envelope determined by the linear regression (tan β = 8.34 and σ_{ci} = 101.5 MPa), and therefore defines the centre of the ellipse. The green ellipse segment represents the 5% fractiles for positive σ_3 values. The remaining part of this ellipse corresponds to the 95% fractile, and therefore it was not plotted.

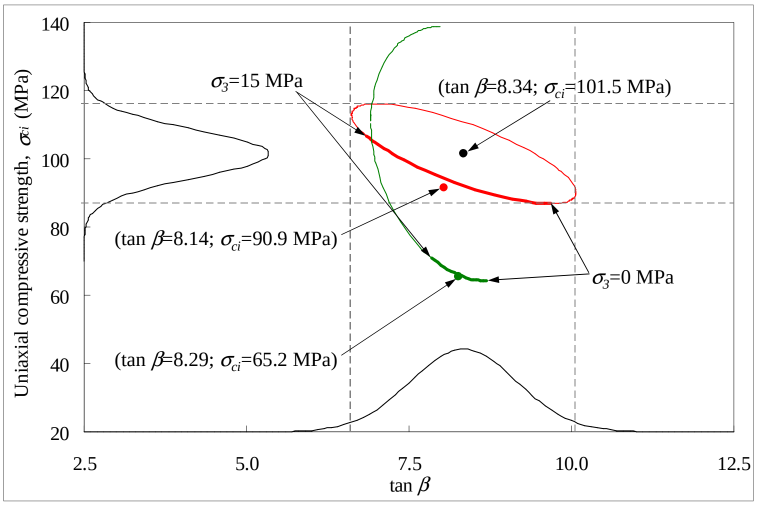

Figure 3. Confidence limits and 5% fractile for tan β and σ_{ci}.

Though the failure criterion is linear, the 95% confidence limits and the 5% fractiles are not (Figure 2). So, in order to assess the reduction from the mean value they lead to, linear approximations were calculated within the range of the triaxial tests (0-15 MPa, thicker parts of the ellipses). These averaged values are (tan β=8.14; σ_{ci} = 90.9 MPa) and (tan β = 8.29; σ_{ci} = 65.2 MPa), respectively, and are also represented in Figure 3. Student's *t* probability distributions of the mean values are also plotted for both parameters showing, that the horizontal and vertical tangents to the 95% confidence limits ellipse define 2.5% tail areas of the independent probability distributions of each parameter.

Though tan β and σ_{ci} are the intrinsic regression parameters, they are not commonly used. So, it is essential to define the same results in terms of internal friction angle (tan β) and cohesion *c*. It should be noted that 95% confidence limits no longer yield an ellipse and the mean values are not in the centre, meaning that the joint distribution of tan β and *c* is skewed.

For the Hoek-Brown criterion, a similar approach can be followed. In this case, since it is a non-linear criterion, the 95% confidence limits for the mean values of the parameters, σ_{ci} and m_i, do not produce an ellipse but an elongated closed curve as shown by Muralha & Lamas (2014).

VARIANCE ANALYSIS OF TRIAXIAL TESTS RESULTS

As already mentioned, all these statistical analyses, and particularly inferences, are strongly influenced by the assumptions underlying regressions. So, it seems appropriate to discuss the applicability of these assumptions to the case of the analysis of triaxial tests results. Classical regression assumes the following hypotheses are fulfilled: model adequacy, no linear dependence, strict exogeneity, homoscedasticity and uncorrelated errors (Draper & Smith, 1998; Weisberg, 2005).

Regarding homoscedasticity, which means that the error term has the same variance in each observation, or that variability of the results is uncorrelated with the independent variables, it plays a key role in ordinary least squares regressions. This assumption implies that every observation of the dependent variable contains the same amount of information and, consequently, the same weight. Heterogeneous variance is commonly associated with an increase or decrease of the variability with an increase of a given independent variable. In the case of triaxial tests, it could be foreseen that weaker samples would displays smaller values of both uniaxial compression strength and internal friction (or m_i) than stronger ones. This would lead to an increase in maximum stress variance with increasing confining stress, implying that the homoscedasticity hypothesis would not hold and that regressions would not provide consistent variance estimates, thus disallowing usual inference procedures with t and F tests and statistics.

Graphical analyses of the residuals are usually quite easy to do, especially in regressions with a single independent variable as in this case, and are very revealing when assumptions are infringed, because residuals can be looked as model error estimates. Plots of the residuals r_i versus the fitted values of the dependent variable $\hat{\sigma}_1$ are particularly useful (Figure 4, left). If assumptions are satisfied, a random scattering of the points above and below the line $r_i = 0$ with nearly all the data points being within the band defined by $r_i = \pm 2s$ is expected. Any pattern in the magnitude of the dispersion about zero associated with changing $\hat{\sigma}_1$ suggests heterogeneous variances of r_i. Particularly, fan-shaped patterns are the typical pattern when variance increases (or decreases) with the mean of the dependent variable. Any asymmetry of the distribution of the residuals about zero suggests a problem with the model or the basic assumptions. A majority of relatively small negative residuals and fewer but larger positive residuals would suggest a positively skewed distribution of residuals instead of the assumed symmetric normal distribution. A preponderance of negative residuals for some regions of $\hat{\sigma}_1$ and positive residuals in other regions suggests a systematic error in the data or model inadequacy with an important variable missing from the model. An outlier residual would appear in any of the plots of r_i as a point well outside the band containing most of the residuals. However, an outlier in $\hat{\sigma}_1$ will not necessarily have an outlier residual. Plots of the residuals against the independent variable σ_3 (Figure 4, right) have interpretations similar to plots against $\hat{\sigma}_1$. Differences in magnitude of dispersion about zero also suggest heterogeneous variances. In this kind of plots, missing higher-degree polynomial terms for the independent variable and outlier should residuals will be evident. Moreover, they enable to detect potentially influential observations, that appear as isolated points at the extremes of the σ_3 scale, though they will tend to have small residuals due to their high leverage.

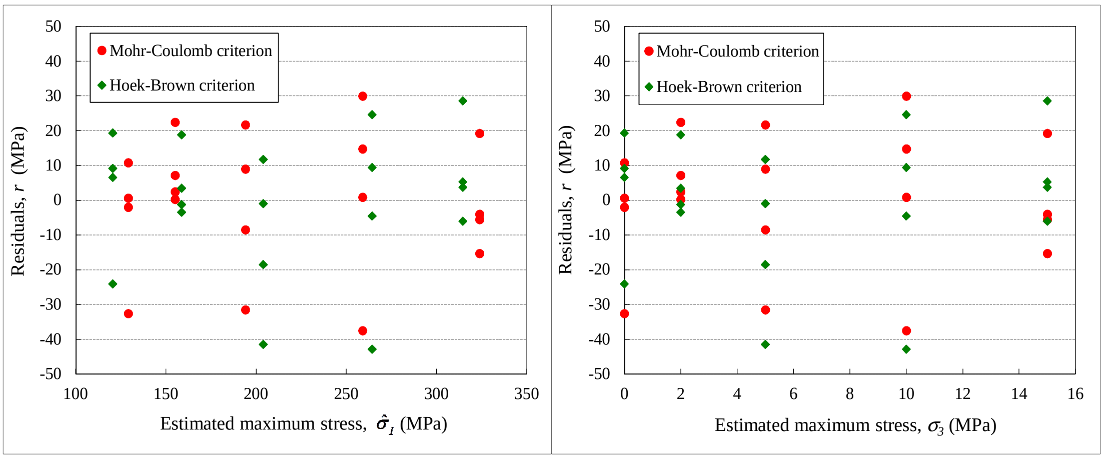

Figure 4. Example of relations between residuals and the predicted variable ($\hat{\sigma}_1$) and the independent variable (σ_3), for a set of triaxial test on granitic rock samples.

These simple plots allow a qualitative appraisal of the variability of the regression residuals. However, they do not enable any robust decision on whether the variance remains constant and enables predictions with the model results. For this purpose, several statistical tests for the presence of heteroscedasticity are available. Some heteroscedasticity tests address the null hypothesis that the error variances are all equal versus the alternative that the error variances are a function of one or more variables. Since error variances are unknown, the squared residuals are used as estimates. Breusch-Pagan (1979) test will detect linear forms of heteroscedasticity, White (1980) test allows for nonlinearities by using squares and cross-products of the variables, and the simplest type of the Park (1966) test considers a logarithmic relation. For grouped data, similar to the triaxial tests results displayed in the previous section, regular homoscedasticity tests such as Bartlett's test and Levene's test check data sets for homogeneity of variances (NIST, 2012). The Levene test is an alternative to the Bartlett test, as it is less sensitive to departures from normality. Levene's original test is aconventional one-way analysis of variance of a transformed response variable defined as absolute the deviations from the mean of each data group. Brown and Forsythe (1974) extended Levene's test to use either the median or the trimmed mean in addition to the mean. Although primarily designed as an outlier test for variances, the Cochran's C and G tests can also be used as a simple alternative (Lam, 2013). The Cochran C test is a one-sided outlier test that will identify deviant standard deviations. It only applies to data sets of equal size. Lam (2013) transformed the C test into a more general G test, deriving expressions to calculate upper limit as well as lower limit critical values for data sets of equal and unequal size at any significance level.

To establish whether the variance of triaxial tests results remains constant or not as the confining stress increases, statistical tests were applied to some sets of results. These sets of triaxial tests results can be divided in two main groups: tests performed at LNEC, where tests were done at given confining stresses, and tests obtained from literature, where the confining stresses take up different values along a given range. The former are considered tests with grouped data, while the latter are not, and so some of the above described statistical tests cannot be applied to them.

For all 12 sets of LNEC tests, Table 1 includes the set reference, the rock type, the confining stresses used for the tests, the number of tests, the mean parameters calculated for the Mohr-Coulomb and the Hoek-Brown criteria, and the standard deviations of both regressions.

Table 1. Mean regression parameters of LNEC tests

Set	Rock type	Confining stresses (MPa)	n	Mohr-Coulomb					Hoek-Brown		
				tan β	σ_{ci} (MPa)	s (MPa)	ϕ (°)	c (MPa)	m_i	σ_{ci} (MPa)	s (MPa)
G_{1pic}	Granite	0.5; 1; 2; 5	21	10.80	63.8	15.3	56.2	9.7	29.5	60.6	15.2
G_{2s}	Granite	0; 1; 2; 5; 10	25	12.30	110.4	22.1	58.2	15.7	34.2	106.7	22.7
G_{3w1}	Granite	0; 1; 2; 5; 10	24	9.44	119.0	16.0	53.9	19.4	23.7	115.6	15.4
$G_{4w2/3}$	Granite	0; 1; 2; 5; 10	22	4.78	81.1	12.4	40.8	18.5	9.4	80.2	12.4
G_{5be7}	Granite	0; 2; 5; 10; 15	22	6.69	139.9	27.9	47.7	27.0	15.1	137.3	27.8
G_{6be8}	Granite	0; 2; 5; 10; 16	21	8.34	101.5	20.4	51.8	17.6	24.9	93.6	18.2
G_{7par}	Granite	0; 2; 5; 10; 17	20	12.98	129.2	19.1	59.0	17.9	41.6	120.6	19.9
P_{1par}	Pegmatite	0; 2; 5; 10; 18	20	7.80	88.4	39.3	50.6	15.8	24.3	80.1	38.1
G_{8bem}	Granite	0; 0.5; 1; 2; 5	30	8.59	52.6	17.6	52.3	9.0	22.6	50.2	17.4
M_{1bem}	Migmatite	0; 0.5; 1; 2; 5	34	4.53	38.4	14.8	39.7	9.0	12.6	34.3	15.0
S_{1alv}	Schist	0; 1; 2; 5; 10	23	4.10	20.2	19.5	37.4	5.0	16.1	15.4	18.8
Gr_{1alv}	Greywacke	0; 1; 2; 5; 10	28	3.33	59.6	17.3	32.6	16.3	5.8	59.0	17.2

Table 2 comprises a similar description of the 11 sets of triaxial tests results gathered from various references found in the literature. Numerous tests come from the fundamental paper from Franklin and Hoek (1970). The limestone I tests attributed to Schwartz (1964) were reported by Singh, Raj & Singh (2011), and the granite LdB tests were presented by Suorineni, Chinnasane & Kaiser (2009) but refer to Martin's (1993) tests. Some adjustments to the results were introduced prior to the evaluation of the regression results presented in Table 2. Some outliers were removed. Tests were performed along a wide range of confining stresses, but some sets contained various tests under null confining stress (uniaxial tests). In these cases, in order not to oversample that specific value (σ_3=0), just a single average value was considered. On the other hand, the andesite tests, which could also be considered as grouped data, included a large quantity of results from uniaxial tests. In this particular case, nine results (mean value of tests of each data group) were randomly picked and included in the regressions.

Residuals of the Mohr-Coulomb and Hoek-Brown criteria from the triaxial tests results with grouped data (LNEC) were checked for heteroskedasticity using the following statistical tests: Levene (using the mean), Brown-Forsythe (using the median), Bartlett, White (both the F and LM statistics), Breusch-Pagan (both the F and LM statistics), and Cochran. Mostly, the statistical tests accepted the null hypothesis that the variances are equal, except for the pegmatite set P_{1par} and the schist set S_{1alv}, as displayed in Tables 3 and 4 (✓meaning the null hypothesis is accepted, and ✕ that it is rejected). It is interesting to note that the pegmatite set exhibits regression standard deviations s much larger than all the rest.

Table 2. Mean regression parameters of literature tests

Reference	Rock type	σ_3 range (MPa)	n	Mohr-Coulomb					Hoek-Brown		
				tan β	σ_{ci} (MPa)	s (MPa)	ϕ (°)	c (MPa)	m_i	σ_{ci} (MPa)	s (MPa)
Franklin & Hoek (1970)	Dolerite	0 – 44.1	24	5.97	298.8	22.7	45.5	61.2	288.9	13.8	21.7
Franklin & Hoek (1970)	Granite	0 – 38.1	32	7.96	225.3	25.7	51.0	39.9	216.6	19.9	26.4
Franklin & Hoek (1970)	Limestone 1	0 – 46.6	29	3.24	100.9	18.9	31.9	28.0	92.5	7.4	18.2
Franklin & Hoek (1970)	Limestone 2	2.9 – 36.6	33	2.86	60.2	16.0	28.8	17.8	51.4	7.0	15.6
Franklin & Hoek (1970)	Marble	0 – 51.7	12	3.54	96.3	9.2	34.0	25.6	90.6	8.4	11.7
Franklin & Hoek (1970)	Sandstone 1	0 – 51.7	21	4.06	84.0	19.3	37.2	20.8	61.8	15.9	14.8
Franklin & Hoek (1970)	Sandstone 2	0 – 52.8	22	4.45	102.0	12.6	39.3	24.2	78.1	16.6	8.4
Franklin & Hoek (1970)	Sandstone 3	0 – 50.1	27	5.00	219.4	15.1	41.8	49.1	205.8	12.4	11.3
Sari (2012)	Andesite	0 – 25	128	4.04	77.9	13.8	37.1	19.4	77.0	8.9	14.2
Schwartz (1964)	Limestone I	0 – 68.4	11	1.52	59.7	7.9	12.0	24.2	56.5	1.5	7.1
Suorineni et al (2009)	Granite LdB	0.5 – 29.4	28	11.35	215.9	32.7	56.9	32.0	200.7	33.7	28.5

Table 3. Statistical tests for the Mohr-Coulomb criterion of LNEC results

Set	Rock type	Levene (mean)	Brown-Forsythe (median)	Bartlett	White *F*	White *LM*	Breusch-Pagan *F*	Breusch-Pagan *LM*	Cochran
G_{1pic}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{2s}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{3w1}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
$G_{4w2/3}$	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{5be7}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{6be8}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{7par}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
P_{1par}	Pegmatite	✗	✗	✗	✓	✓	✓	✗	✗
G_{8bem}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
M_{1bem}	Migmatite	✓	✓	✓	✓	✓	✓	✓	✓
S_{1alv}	Schist	✓	✓	✗	✓	✓	✗	✗	✗
Gr_{1alv}	Greywacke	✓	✓	✓	✓	✓	✓	✓	✓

Table 4. Statistical tests for the Hoek-Brown criterion of LNEC results

Set	Rock type	Levene (mean)	Brown-Forsythe (median)	Bartlett	White *F*	White *LM*	Breusch-Pagan *F*	Breusch-Pagan *LM*	Cochran
G_{1pic}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{2s}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{3w1}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
$G_{4w2/3}$	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{5be7}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{6be8}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
G_{7par}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
P_{1par}	Pegmatite	✗	✗	✗	✓	✓	✓	✗	✗
G_{8bem}	Granite	✓	✓	✓	✓	✓	✓	✓	✓
M_{1bem}	Migmatite	✓	✓	✓	✓	✓	✓	✓	✓
S_{1alv}	Schist	✓	✓	✗	✗	✓	✗	✗	✗
Gr_{1alv}	Greywacke	✓	✓	✓	✓	✓	✓	✓	✓

For the triaxial tests results taken from literature the statistical tests used were: White (both the *F* and *LM* statistics), Breusch-Pagan (both the *F* and *LM* statistics), and Park (*t* statistic). For the latter, uniaxial tests results were not considered. All but one statistical test accepted the null hypothesis, as displayed in Tables 5 and 6.

DISCUSSION AND CONCLUSIONS

The analysis of the statistical tests shows that the intact rock strength derived from results of triaxial tests does not display any evidence of heteroskedasticity. Subsequently, the ordinary regressions (linear and non-linear) performed to determine the mean parameters of the Mohr-Coulomb and Hoek-Brown criteria can be used; moreover, the variance statistics that they provide are statistically robust and allow to infer 95% confidence intervals and 5% fractiles, as defined in EC7. If this conclusion was precisely the opposite, the mean parameters of the Mohr-Coulomb and Hoek-Brown criteria could still be evaluated using common regressions, but standard deviations would require different statistical approaches. Beforehand, it could be anticipated that the variance of the results would increase with increasing confining stresses: a harder specimen of rock would display both higher internal cohesion and friction angle than a weaker rock, thus increasing dispersion. However, possibly due to the relatively small range of applied confining stresses, this assumption was not confirmed.

Table 5. Statistical tests for the Mohr-Coulomb criterion of literature results

Reference	Rock type	White *F*	White *LM*	Breusch-Pagan *F*	Breusch-Pagan *LM*	Park *t*
Franklin & Hoek (1970)	Dolerite	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Granite	✓	✓	✓	✓	✗
Franklin & Hoek (1970)	Limestone 1	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Limestone 2	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Marble	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 1	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 2	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 3	✓	✓	✓	✓	✓
Schwartz (1964)	Andesite	✓	✓	✓	✓	✓
Sari (2012)	Limestone I	✓	✓	✓	✓	✓
Suorineni et al (2009)	Granite LdB	✓	✓	✓	✓	✓

Table 6. Statistical tests for the Hoek-Brown criterion of literature results

Reference	Rock type	White *F*	White *LM*	Breusch-Pagan *F*	Breusch-Pagan *LM*	Park *t*
Franklin & Hoek (1970)	Dolerite	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Granite	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Limestone 1	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Limestone 2	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Marble	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 1	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 2	✓	✓	✓	✓	✓
Franklin & Hoek (1970)	Sandstone 3	✓	✓	✓	✓	✓
Schwartz (1964)	Andesite	✓	✓	✓	✓	✓
Sari (2012)	Limestone I	✓	✓	✓	✓	✓
Suorineni et al (2009)	Granite LdB	✓	✓	✓	✓	✓

Considering the standard deviations of the regressions, there is no clear difference in the fits from both strength criteria to the tests results. However, in some cases, the Hoek-Brown criterion shows better estimates.

These conclusions were derived considering both grouped and scattered data. To date, just these 23 sets of results were analysed, and it is relevant to remind that the number of tests of some sets (around 20) can be judged small in the statistical context. So, this type of analysis should continue in order to confirm the main conclusion about homoskedasticity, and, for now, it seems more appropriate to perform triaxial tests under 4 or 5 confining stresses that render grouped data, that allows easier ways to test the hypothesis that variance does not change with σ_3.

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