

# COMPARISON BETWEEN TIME, SPECTRAL AND WAVELET ANALYSIS ON WAVE BREAKING AND PROPAGATION

J.M.P. CONDE<sup>(1)</sup>, R. LEMOS<sup>(2)</sup> & C.J.E.M. FORTES<sup>(2)</sup>

<sup>(1)</sup> UNIDEMI, FCT, Universidade Nova de Lisboa (UNL), Monte de Caparica, Portugal  
jpc@fct.unl.pt

<sup>(2)</sup> National Laboratory for Civil Engineering (LNEC), Lisbon, Portugal  
rlemos@lnec.pt, jfortes@lnec.pt

## Abstract

The present work focuses on the comparison between three types of data analysis (time analysis; Fourier transform spectral analysis; and wavelet transform spectral analysis) applied to different types of waves (monochromatic, bichromatic and irregular). The main objective is to study experimentally wave shoaling and breaking over a set of different gentle slopes for several incident waves.

The data and results from time, Fourier and wavelet analysis presented include: free surface elevation along the flume and the corresponding amplitude spectra; significant wave height,  $H_s$ , and significant wave period,  $T_s$ , along the flume.

Keywords: Time analysis; Fourier analysis; Wavelet analysis; Wave shoaling; Wave breaking.

## 1. Introduction

The knowledge of wave transformation and wave breaking characteristics near the coastline is essential for nearshore hydrodynamics studies (e.g., coastal sediment dynamics) and for the design of coastal and port structures. Physical models and laboratory experiments are an important part of the research methodology for acquiring a better knowledge and characterization of these phenomena. Also, the validation of the numerical models depends greatly on accurate and reliable experimental data.

Following this reasoning, a wide range of wave flume tests was performed at the National Laboratory for Civil Engineering (LNEC) to study wave transformation and wave breaking considering different incident conditions.

Okamoto *et al.*, 2010, Endres *et al.*, 2011, Neves *et al.*, 2011, 2012, and Conde *et al.*, 2012, performed a set of experimental tests for incident regular wave conditions with and without wave breaking, considering different bottom slopes. Conde *et al.*, 2013a, 2013b, 2013c, followed the methodology of these previous works, considering incident bichromatic and irregular waves.

The present work follows the previous works but focuses on the comparison between three types of data analysis (time analysis; Fourier transform spectral analysis; and wavelet transform spectral analysis) applied to the different types of waves (monochromatic, bichromatic and irregular) tested experimentally.

This paper starts with a short description of the experimental layout, the incident wave conditions and the experimental procedures. Then, results from time and spectral (Fourier and wavelet) analysis of the free surface elevation along the flume, obtained for three different types of waves (regular, bichromatic and irregular), are presented and discussed. For each wave, time analysis results consist on the determination of the significant wave height,  $H_s$ , and significant wave period,  $T_s$ , along the flume, while spectra analysis consist on the Fourier and wavelet energy spectra.

## 2. Experiments

The experimental tests were performed in the wave flume presented in Figures 1 and 2. This wave flume, build in the fifties, was designed with a reduction of width to improve its hydraulic behavior, by preventing unwanted transversal waves, and, at the same time, to enable an increase of the regular wave heights (due to shoaling at the 1/11 bottom slope) produced by the limited capabilities of the original wave paddle. Nowadays, the flume is equipped with a piston-type irregular wave-maker system controlled by an A/D converter and a personal computer. This wave-maker can generate regular or irregular waves. A 10 m long 1/22 slope beach profile, followed by a 10 m horizontal zone were constructed. This bottom was made out of concrete so there is no permeability. At the end of the flume there is a 1/20 slope concrete bottom followed by a 1/2 slope gravel beach. Porous blankets (horsehair sheets) were installed over the 1/20 slope to reduce the reflected wave energy.



Figure 1. Wave Flume: side view (left), wave maker (middle), view from above (right).

The experiments considered for the present paper were:

1. Regular waves with a 1.5 s wave period ( $T$ ) and a 0.14 m wave height ( $H$ );
2. Bichromatic waves considering a combination of two wave periods, 1.1 s and 1.5 s, and a 0.08 m wave height;
3. Irregular waves (JONSWAP spectrum) with  $T_p=1.5$  s and  $H_s= 0.14$  m.

The water depths in the lee part of the bar (Figure 2) was  $d=0.1$  m, in order to have wave breaking conditions.

Free surface elevations were measured using an eight gauge mobile structure (Figure 3 - left). This structure was placed at different locations along the flume, and measurements were taken from  $x=-10$  m up to  $x=10$  m. A reference wave gauge was installed near the wave-maker at  $x=-10.8$  m, in order to verify the input wave height (Figure 3 - right). The gauges in the mobile structure were placed 0.2 m apart and measurements were taken along the covered area with a 0.1 m or 0.2 m interval.

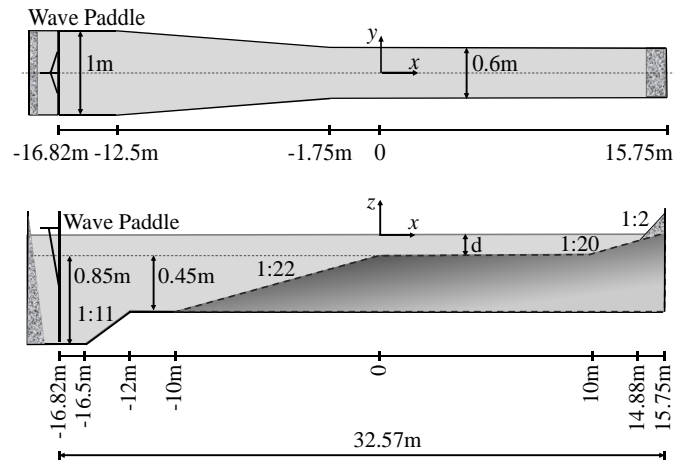


Figure 2. Wave flume's plan and longitudinal-section views.



Figure 3. Eight wave gauge mobile structure (left) and wave gauge near the wave maker (right).

### 3. Results and discussion

For each incident wave condition (regular, bichromatic and irregular), three free surface elevations data analysis along the flume were performed: Time analysis, Fourier transform spectral analysis, and wavelet transform spectral analysis.

#### 3.1 Time analysis

Free surface elevation time analysis along the flume allowed us to obtain the significant wave height,  $H_s$ , and significant wave period,  $T_s$ , values based on the down-crossing method, for each incident wave conditions. Figures 4 and 5 present these values along the flume.

Figure 4 shows that there is a similar behaviour of  $H_s$  for the different types of incident waves. In fact, there is an increase of the significant wave height as the waves propagate along the flume due to wave shoaling, until a certain point (regular wave:  $x=-3.3$  m, bichromatic wave,  $x=-4.1$  m and irregular wave,  $x=-2.6$  m). Then, the wave breaks and there is a significant decrease of  $H_s$  until  $x=0.5$  m, approximately; while after,  $H_s$  values are almost constant and quite similar whatever the incident wave considered. Notice that regular wave  $H_s$  results show higher oscillations than the bichromatic one and especially the irregular one.

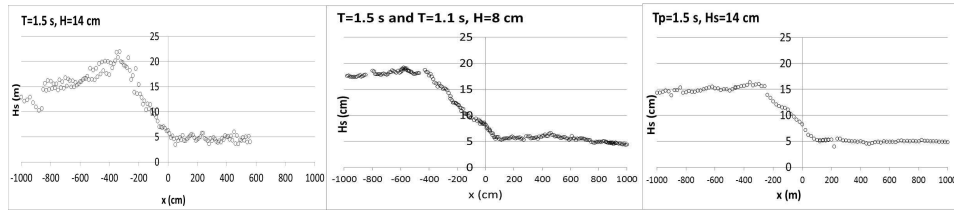


Figure 4. Significant wave height,  $H_s$ . From left to right: regular, bichromatic and irregular waves.

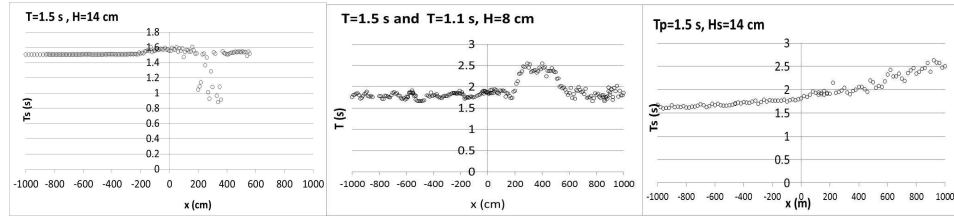


Figure 5. Significant wave period,  $T_s$ . From left to right: regular, bichromatic and irregular waves.

In what concerns to  $T_s$  values, Figure 5, for each incident wave type, the behaviour varies, as the waves propagate along the flume. For the regular waves case,  $T_s$  values are practically constant, as expected, up to  $x=-2$  m and slightly increases up to  $x=2$  m, behaving randomly between  $x=2$  m and  $x=4$  m. After  $x=5$  m it returns to the previous values of approximately 1.5 s.

Regarding bichromatic waves, the significant wave period exhibit almost constant values up to  $x=2$  m and increasing values from  $x=2$  m up to  $x=6$  m, where  $T_s$  values stabilized, returning to the previous values. In real non-linear bichromatic wave, some waves don't cross the mean water level and, for that reason, are not identified as individual waves by the wave identification methods, e.g., zero up-crossing or down-crossing methods. As a consequence of this, higher period waves are identified and a higher significant period is obtained (Conde *et al.*, 2013b).

Irregular waves present almost constant  $T_s$  values, up to  $x=2$  m. After this position  $T_s$  values increase and present an oscillating behaviour. This can be due to the same reason as in bichromatic waves.

### 3.2 Fourier spectral analysis

Figures 6 to 8 present the energy spectra at  $x=-10$ ,  $-5$ ,  $0$  and  $2$  m, for the different types of waves (regular, bichromatic and irregular). These energy spectra were obtained by a Fast Fourier Transform (FFT) using the software SAM (Capitão, 2002). This analysis allowed the characterization of the nonlinear phenomena and the harmonic generation due to shoaling.

In general, for all the incident wave types tested, the behaviour is similar, e.g., there is the generation of harmonics as the waves propagates along the flume. There is a clear transference of the main frequency at  $x=-10$  m for other higher and lower frequencies, as the wave propagates. After the wave breaks there is a clear decrease of the wave energy and the generation of more harmonics.

For monochromatic wave, Figure 6, it is clearly visible the increase of the wave energy as the wave propagates from  $x=-10$  m until  $x=-5$  m as well as the generation of harmonics. After that, at  $x=0$  and 2 m, the wave has already broken and there is a significant energy decrease.

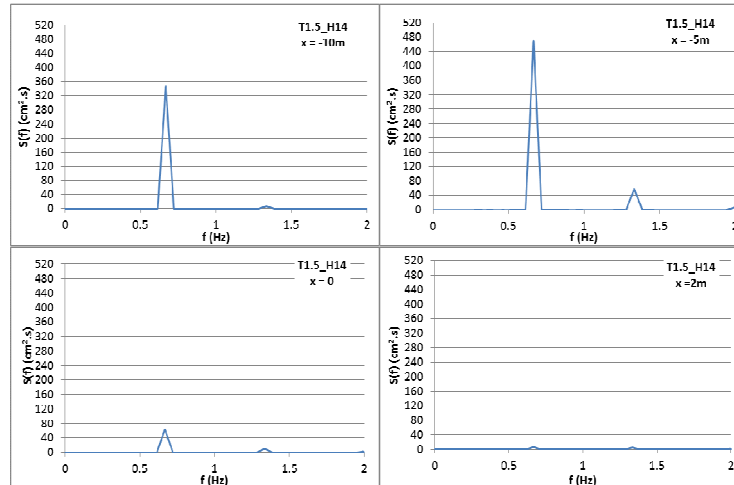


Figure 6. Regular waves energy spectra at different sections along the flume.

For bichromatic wave, Figure 7 shows the wave transformation, which becomes more visible after wave breaking, at approximately  $x=-5$  m. At  $x=-10$  m, the fundamental frequencies ( $f_1$  and  $f_2$ ) amplitudes are slightly different and inferior to the imposed value of 0.04 m. This is due to the interchange of energy between these frequencies and its harmonics and harmonic combinations that occurs from the wave maker up to this section.

For irregular wave, from  $x=-10$  m up to  $-5$  m, Figure 8 shows the crescent increase of harmonics as the waves propagates along the flume, due to the increase of nonlinear wave characteristics. There is a clear transference of energy from the main frequency at  $x= -10$  m for other higher and lower frequencies, as the wave propagates. At  $x=0$ , there is a clear decrease of the wave energy because the wave had already broken. This decrease is more clear at  $x=2$  m.

### 3.3 Wavelet spectral analysis

Wavelet analysis has become a common tool for the analysis of localized variations of power within a time series. By decomposing a time series into time–frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time.

Figures 9 to 11 presents the Morlet wavelet power spectrum using regular, bichromatic and irregular waves data acquisition along the flume, at sections  $x=-10$ ,  $-5$ , 0 and 2 m.

With the wavelet technique, the conclusions made with the Fourier analysis are the same. It is clear that there is an increase of wave energy as the wave propagates along the flume and there is a clear transference of the wave energy from the main frequency to the others harmonics.

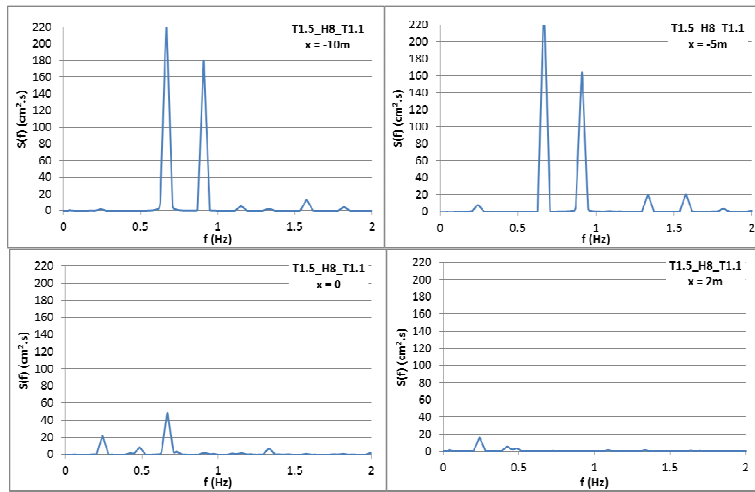


Figure 7. Bichromatic waves energy spectra at different sections along the flume.

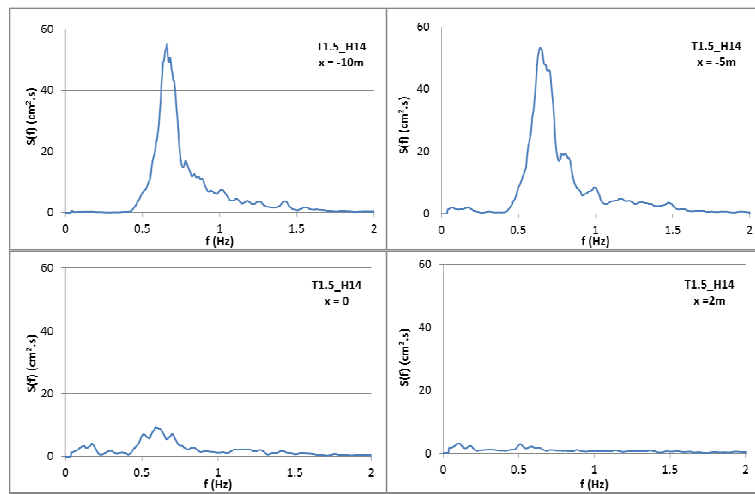


Figure 8. Irregular waves energy spectra at different sections along the flume.

With the wavelet technique is also possible to see the energy distribution associated with each frequency over the course of the experiment. It's clearly visible that the energy associated to each frequency is not constant in time. Apart from the monochromatic wave, the remaining waves present variations even for the  $x=-10$  m position.

The monochromatic and bichromatic waves, due to its regular nature, present the transfer of energy between frequencies with a periodicity even after wave break, Figures 9-10. On the other hand the irregular wave, Figure 11, doesn't present a clear pattern of repeatability over the course of the experiment.

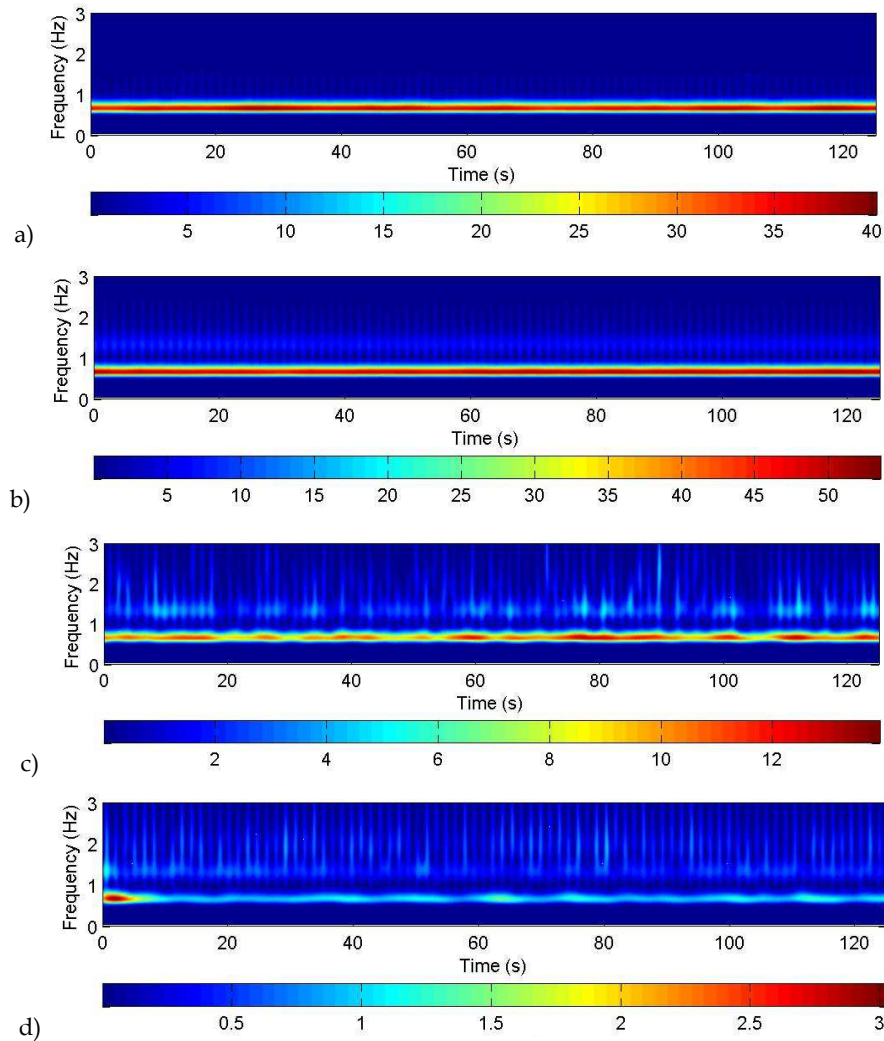


Figure 9. Wavelet analysis for regular waves at different sections along the flume:  $x=-10$  m (a),  $x=-5$  m (b),  $x=0$  (c) and  $x=2$  m (d).

#### 4. Conclusions

In this paper, recent physical modelling tests on a wave flume at the National Laboratory for Civil Engineering (LNEC), Lisbon, Portugal, were presented. This is contribution for the study of the wave propagation hydrodynamics in varying sloping beaches. The main objective was to study in detail all the process related with the wave propagation and breaking in complex bathymetries. The present work focuses on the comparison between three types of data analysis (time analysis; Fourier transform spectral analysis; and wavelet transform spectral analysis) applied to different types of waves (monochromatic, bichromatic and irregular). The main objective is to study experimentally wave shoaling and breaking over a set of different gentle slopes for several incident waves.

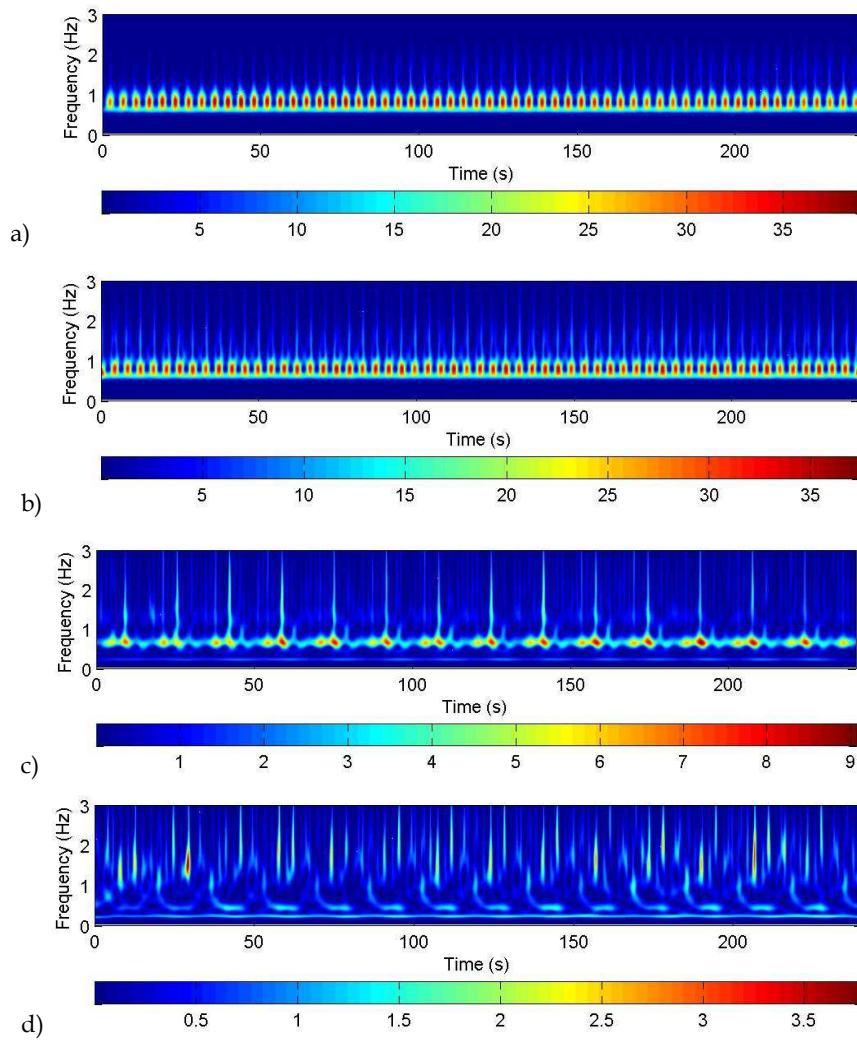


Figure 10. Wavelet analysis for bichromatic waves at different sections along the flume:  $x=-10$  m (a),  $x=-5$  m (b),  $x=0$  (c) and  $x=2$  m (d).

The data and results from time, Fourier and wavelet analysis presented include: free surface elevation along the flume and the corresponding amplitude spectra; significant wave height,  $H_s$ , and significant wave period,  $T_s$ , along the flume.

The time analysis allowed us to conclude that there is similar behaviour of  $H_s$  for the different types of incident waves. In fact, there is an increase of the significant wave height as the waves propagate along the flume due to wave shoaling, until the beginning of the wave break. Then, the wave breaks and there is a significant decrease of  $H_s$ , after the end of the wave break  $H_s$  values are almost constant and quite similar whatever the incident wave considered. In what concerns to  $T_s$  values, for each incident wave type, the behaviour is distinct as the waves propagate along the flume.



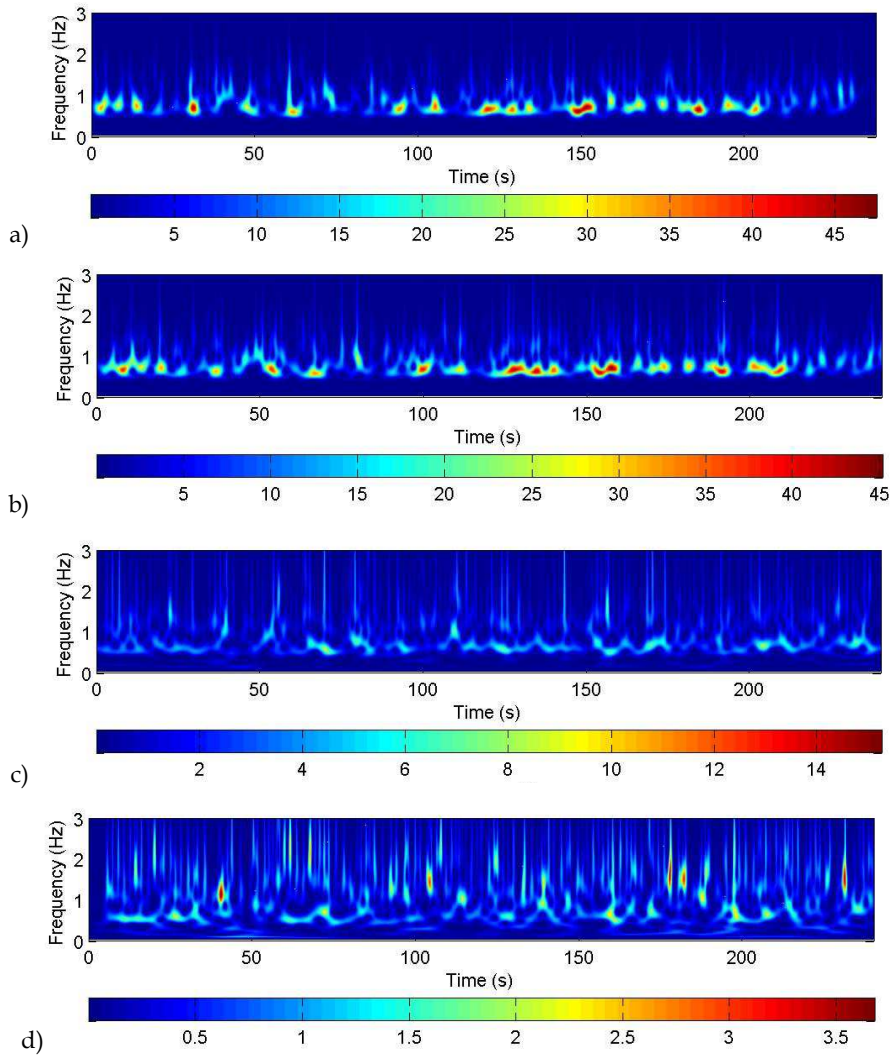


Figure 11. Wavelet analysis for irregular waves at different sections along the flume:  $x=-10$  m (a),  $x=-5$  m (b),  $x=0$  (c) and  $x=2$  m (d).

The Fourier analysis allowed the characterization of the nonlinear phenomena and the harmonic generation due to shoaling and wave breaking. In general, for all the incident wave types tested, the behaviour is similar, e.g., there is the generation of harmonics and a transfer of energy from the main frequency to the harmonics and among them as the waves propagate along the flume. Post wave breaking there is a clear decrease of the wave energy and the generation of more harmonics. Due to its characteristics, each incident wave presents a different behaviour.

With the wavelet technique, the conclusions made with the Fourier analysis are the same. But with the wavelet technique important information related with the energy distribution associated with each frequency over the course of the experiment, are obtained.

The monochromatic and bichromatic waves, due to its regular nature, present the transfer of energy between frequencies with a periodicity even after wave break. On the other hand the irregular wave doesn't present a clear pattern of repeatability over the course of the experiment.

As a final conclusion one may state that each of these techniques has its own advantages and drawbacks. None of them may be considered as a substitute of the other, instead each should be used as a complement of the others.

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