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CALCULATION OF THE ACCELERATED BREAKING-ROLLER PROPAGATION SPEED AND WAVE ENERGY TRANSFER TO THE ROLLER

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ABSTRACT

Wave breaking has different physics from the potential flow wave motion. The roller model introduced by Svendsen [1] illustrates the separation of the wave motion and the roller. The roller propagation speed, therefore, is a very important factor for the energy calculation of the bore.

The wave celerity data collected at the wave tank displays that the maximum roller propagation speed occurs when the wave has already decayed due to the breaking. This fact clearly displays that the bore energy cannot be calculated only from the wave height as it is done for non-breaking waves.

It is certain that most the energy is dissipated through the roller formation in the outer surfzone, but a certain amount of energy is transferred to the roller at the same time and it accelerates the bore speed. Slow decay of the roller propagation speed indicates that the excess energy left in the roller dissipates in the inner surfzone at much slower rate than in the outer surfzone. Therefore, these two zones have to be clearly separated, but the amount of energy transferred into the roller is unknown.

In this paper, we focus on the examination of the peak roller propagation speed that appears at the border of the outer and the inner surfzone by using the experimental data collected at the wave tank. In that way, the initial condition of roller propagation speed can be determined for the inner surfzone. The energy conservation between the wave motion and the roller kinetic energy derives an equation to calculate the roller propagation speed. The energy transfer rate is estimated by adjusting the value given by the full energy conversion with the observed roller propagation speed. It is found that about half of the energy is transferred into the roller. The model successfully illustrates the peak bore propagation speed which existing formulae cannot explain.

INTRODUCTION

Since wave breaking has different physics from the potential flow wave motion, it is very difficult to formulate the energy dissipation through the wave breaking. The roller model introduced by Svendsen [1] is one of the well-accepted concepts to illustrate the structure of breaking waves. It separates the roller from the wave motion by treating it as a water mass propagating with the wave celerity. The particle velocity in the roller is considered to be equal to the roller propagation speed.

The roller propagation speed, therefore, is a very important factor for the energy calculation of the bore. It is well known that the bore propagation speed becomes much faster than the wave celerity given by the wave theory. Therefore, for example, Schäffer et al. [2] made the bore propagation speed 30% faster than the linear shallow water wave celerity for their energy dissipation calculation. However, existing formulae are only based on the observation of the wave breaking at the shoreline where the water depth becomes zero at the end. Consequently, these formulae are not suitable when wave breaking terminates at offshore locations with finite water depth. Wave breaking on offshore bar is the one of those examples.

Okamoto et al. [3] revealed that the roller propagation speed always decreases after it is accelerated by the wave breaking regardless of the local water depth change. And, it decreases towards the value given by the mixed state of harmonics. This result illustrates two problems of existing bore propagation speed theories: 1) Existing bore propagation theories cannot predict the bore propagation speed correctly when the water depth increases as the wave propagates. Since those are function of local water depth, the roller propagation speed also increases under those conditions; 2) Existing bore propagation speed theories always provide higher values than the one suggested by linear wave theory. However, it becomes slower than the linear wave celerity due to the influence of higher harmonics propagating with slower speed than the main frequency component. This effect can be ignored only when the water depth becomes very shallow so that the shallow water approximation can be applied to all the notable higher harmonics.

It can be concluded that existing formulae are only applicable for wave breakings at the shoreline. However, since wave breaking occurs not only at the shoreline but also in the offshore where it has considerable local water depth, the establishment of a new model to calculate the roller propagation speed is needed for those cases.

Wave energy calculation based on the potential flow theory also has a problem. It is basically a function of wave height. So, if the wave height is higher, the wave energy is greater. However, the wave celerity data collected at the wave tank shows that the maximum roller propagation speed occurs when the wave has already decayed due to breaking. This fact clearly states that the energy of the bore cannot be calculated only from the wave height as we do for non-breaking waves. Therefore, different formula to calculate the bore propagation speed and the roller energy is needed.

Most of the energy is dissipated through the formation of the roller in the outer surfzone for sure, but a certain amount of energy is transferred to the roller at the same time. This energy accelerates the bore speed. However, the amount of energy transferred into the roller is unknown. In the inner surfzone, the roller propagation speed decreases from the accelerated speed at the beginning with much slower rate than in the outer surfzone. Slow decay of the roller propagation speed in the inner surfzone indicates that the excess energy left in the roller dissipates in the inner surfzone with much slower rate than the outer surfzone. So, the energy dissipation mechanism should also be divided between the outer and inner surfzone.

In this paper, our focus goes on the inner surfzone. More specifically, we focus on the examination of the peak roller propagation speed appearing at the border of the outer and the inner surfzone by using the experimental data collected at the wave tank. The objective is to determine the initial condition of roller propagation speed in the inner surfzone. The equation is derived from the energy conservation between the wave motion and the roller kinetic energy under the assumption of the full energy transfer between two bodies. The energy transfer rate is estimated by adjusting the value given by the full energy conversion to the observed roller propagation speed.

BORE PROPAGATION SPEED

At the wave breaking, it is known that the wave celerity becomes much faster than the one derived from the wave theory based on the potential flow, normally given as $\sqrt{gd}$ because wave breaking occurs mostly in shallow water zone. To explain this, several models have been introduced.

One of those models is called "Bore model". The construction of bore models is based on the consideration of similarity between the breaking wave and the moving hydraulic jump. Therefore, it is more related to the open channel hydraulics than the wave theory. The bore propagation speeds given by bore models are described by the combination of local water depth at the crest and the trough of the wave together with the mean water level. Svendsen et al. [4] derived the bore propagation speed formula for periodic bore from the momentum balance equation as follows:

$$C = \sqrt{\frac{1}{2}\frac{gh_1h_2}{d^2}\left(h_1 + h_2\right)} \qquad (1)$$

where g is the acceleration of gravity, d is the mean water level, h is the total water depth from the free surface to the bottom, and subscript 1 and 2 denote the location of wave trough and crest, respectively. This is basically the same as the classical bore propagation speed on no current field.

Bonneton [5] derived another bore propagation speed formula from Saint Venant shock-wave model. Several assumptions required for the classical bore propagation formula, such as locally horizontal bottom, quasi-constant wave shape, and so on, were removed by using Saint Venant shock-wave model. As a result, it can be applied to more realistic situations. The bore propagation speed is given as;

$$C = -2\sqrt{gd} + 2\sqrt{gh_1} + \sqrt{\frac{1}{2}\frac{gh_2}{h_1}\left(h_1 + h_2\right)} \qquad (2)$$

Bonneton [5] compared it with the classical formula, Eq. 1, by using the experimental data on a plane slope beach obtained by Stive [6] and concluded that Eq. 2 provides a better agreement with data than Eq. 1.

Depth inversion technique has been developed in recent years to estimate the local water depth from remote-sensing data. This technique relies on the assumption of the wave celerity being a function of the water depth. Thus, the wave celerity estimated from remote-sensing data can be converted to the local water depth. The accuracy of the estimated water depth, therefore, depends very much on the accuracy of the wave celerity equation to be used.

Catalan and Haller [7], therefore, made a comparison of wide range of wave celerity formula, from linear wave equation to the complicated, non-linear composite formula. The bore propagation formula given by [5] was also in their list. They tested those equations with data obtained on a bar-trough shaped beach in a large scale wave tank. Although the wave celerity calculated by those equations were not completely agreed with the observed data obtained by using remote sensing data, especially at the outer surfzone and the following area, they concluded that a composite model given by Kirby and

Dalrymple [8], hereafter KD86, gives the best agreement to the data among the equations they tested. The equation given by KD86 is shown as follows:

$$C^2 = g/k(1+f_1\varepsilon^2 D)\tanh(kd+f_2\varepsilon) \quad (3)$$

where k is the wave number, H is the wave height, $\varepsilon=kH/2$ and

$$D = \frac{8+\cosh(4kd)-2\tanh^2(kd)}{8\sinh^4(kd)} \quad (4)$$

$$f_1(kd) = \tanh^5(kd),\ f_2(kd) = \left(\frac{kd}{\sinh(kd)}\right)^4 \quad (5)$$

This equation has a very complicated shape but it becomes the linear wave model when the wave height gets infinitesimally small. For finite amplitude waves, it asymptotically approaches to the Stokes third order theory and Solitary wave theory for deep and shallow water waves, respectively.

All existing formulae discussed above are the dependant on the local water depth. Especially near the breaking point, which is classified as the shallow water region in most of the cases, these equations can be related to the linear shallow water wave celerity, $\sqrt{gd}$. These equations include nonlinear effects, so they predict wave celerity more accurately than the linear wave theory. However, the basic trend does not differ so much from the linear wave celerity. In other words, these formulae state that the bore propagation speed should increase when the local water depth increases.

Another thing to be discussed on the bore propagation speed is the acceleration of the wave celerity due to the wave breaking. Existing formulae discussed above include nonlinear effect of the wave, thus values given by these formulae are always faster than the linear wave celerity. Schäffer et al. [2] developed a wave breaking model for phase-resolving type, the Boussinesq equation models. For the wave celerity calculation, they made the wave celerity 30% faster than the linear shallow water wave celerity to get a better agreement with the experimental data provided by Stive [9].

However, it does not seem to be true that the bore propagation speed keeps the accelerated speed all the way to the end. Svendsen et al. [10] collected data obtained on plane slope beaches from several different papers and analyzed several different wave properties in the surfzone. In case of wave celerity, data from two papers were presented. They analyzed the wave celerity of six different cases from those two papers, and compared them with the cnoidal-bore model. The result displayed that the wave celerity becomes faster than the linear wave model as the wave becomes closer to the breaking point and it becomes about twice or more at the maximum point although the data contains large scatter. Also the peak values appeared after the location where the cnoidal-bore model gives the maximum celerity. After passing certain point, it decreases towards the linear wave model. They also suggested that the wave celerity becomes slower than the linear wave celerity in some cases.

Similar result was observed by Okamoto et al. [11] in the wave tank experiments with horizontal bottom at the breaker zone. The wave celerity becomes much faster than the linear wave celerity in the outer surfzone and it gradually decreases towards the linear wave celerity as the wave propagates and approaches to the termination of wave breaking. Furthermore, Okamoto et al. [12] conducted the wave tank experiments with simplified bar-trough profiled beaches and found that the bore propagation speed becomes much slower than the linear wave celerity. It becomes about 70% of linear wave celerity in some cases. Fig. 1 displays that the wave celerity evolution over the simplified bar-trough profiled beach. It clearly displays the acceleration of wave celerity due to the wave breaking and the deceleration in the inner surfzone.

Okamoto et al. [3] re-analyzed the experimental data and concluded that the bore propagation speed always decreases after it is accelerated by the wave breaking, even when the bottom slope has negative slope angle, i.e. the water depth increases as the wave propagates. And the wave celerity at the location of wave breaking termination can be explained by the mixture of higher harmonics generated by the wave breaking. Higher harmonics have shorter wave length. Therefore, they propagate slower than the main frequency wave unless the local water depth is very shallow. These slowly moving harmonics affect the in-situ wave celerity after the termination of wave breaking and the bore propagation speed decreases towards this in-situ wave celerity affected by the higher harmonics.

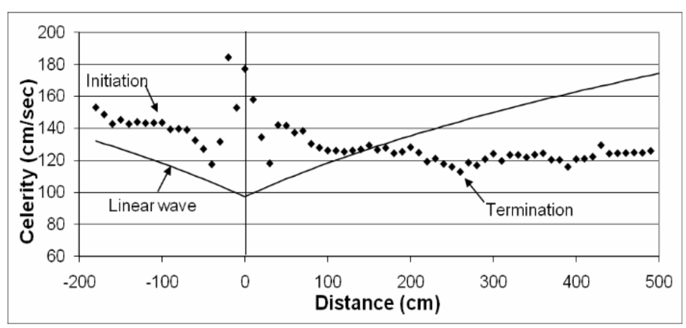


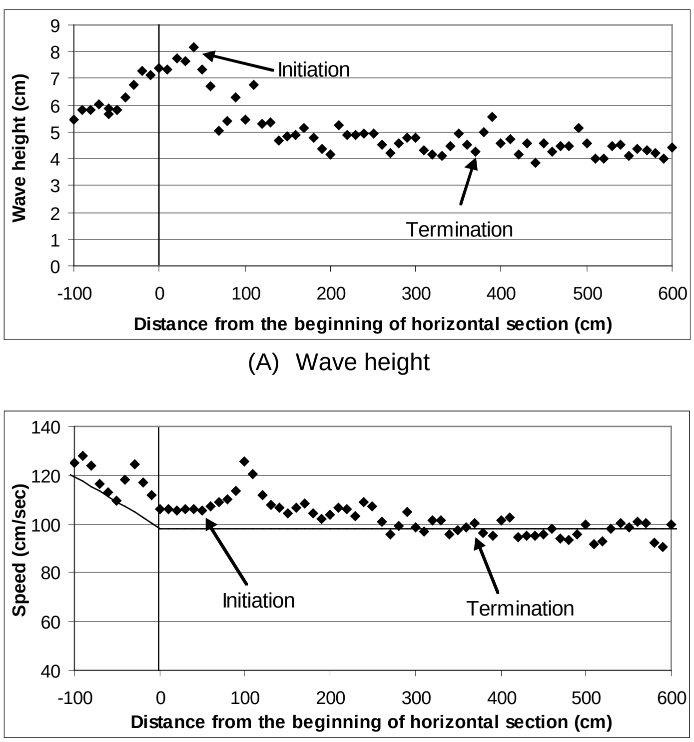
FIGURE 1 : Wave celerity evolution over a bar-trough profiled beach (1/20 for lee side of bar), T=1.5sec, H=8cm (From Okamoto et al. [3])

ACCELERATED BORE PROPAGATION SPEED

Okamoto et al. [3] focused on the wave celerity at the termination location and after the termination of wave breaking. Therefore, the complete structure of bore propagation speed during the wave breaking is still unknown. However, it is clear that the bore propagation speed evolution can be divided in two parts; the acceleration stage in the outer surfzone and the deceleration stage in the inner surfzone.

The wave celerity at the wave breaking initiation can be easily found by using existing wave celerity formulae because it is still governed by the potential flow theory. The KD86 given by Eqs. 3, 4 and 5 provides very accurate results for this region as Catalan and Haller [7] discussed. So, if the accelerated bore propagation speed appearing at the border between the outer and the inner surfzone can be estimated, we can know all three boundary values for two sections. Then, construction of the bore propagation speed model would be possible.

As shown in Fig. 1, the maximum bore propagation speed appears slightly after the initiation of wave breaking. This location roughly corresponds with where the formation of the roller is completed. Fig. 2 shows that the wave height decay has been almost completed when the bore propagation speed becomes the maximum (around x=100cm).



(A) Wave height

(b) Wave CELERITY and bore propagation speed

FIGURE 2 : Comparison of wave height evolution and wave celerity evolution on a horizontal bottom (T=2.5sec, H=3cm) (Okamoto and Fortes [13])

This is not very good for existing formulae. As explained, they provide higher value than the linear wave theory because of the nonlinear effects, which is basically given by the term corresponding to the wave height. Thus, when the wave height gets smaller, the wave celerity becomes closer to the linear wave celerity. However, as shown in Fig. 2, the bore propagation speed becomes much faster with the wave height close to that of the post-breaking wave, which is smaller than the initiation wave height. This result illustrates that, different from non-breaking waves, the wave height does not give strong influence on the bore propagation speed.

Okamoto and Fortes [13] evaluated the roller kinetic energy based on the experimental data. The basic concept follows the roller model: the roller is the separated water mass from the wave motion. The propagation of the roller was considered not to be a part of the potential flow motion but to be the simple moving object. It was a very simplified model but successfully illustrated the evolution of the roller kinetic energy during the wave breaking event. The same concept is employed here to derive the formula for the maximum bore propagation speed appearing at the border of outer and inner surfzone.

The roller is considered as a water mass propagating with the bore propagation speed, but the rotation and the turbulence are not considered here. Therefore, the energy of the roller is simply described as;

$$E = \frac{1}{2} mV^2 = \frac{1}{2} \rho AC^2 \tag{6}$$

where m is the mass of the roller per unit width, V is the speed of the moving object, ρ is the density of water, and A is the roller area shown in Fig. 3.

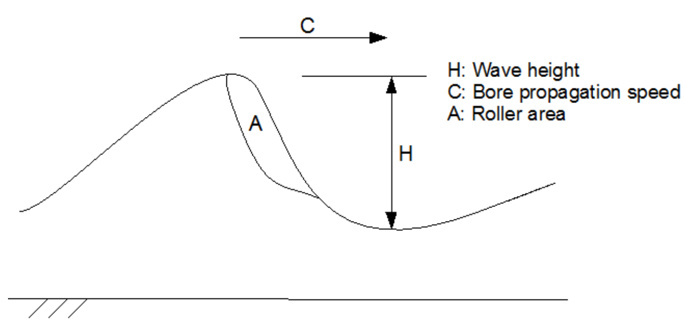


FIGURE 3 : Definition sketch of wave breaking roller (redrawn form Duncan [14])

Svendsen [1] re-analyzed the data obtained by Duncan [14] and got a relation between the wave height, H, and the roller area, A, as;

$$A = 0.9H^2 \tag{7}$$

Substituting Eq. 7 into Eq. 6 provides the roller kinetic energy as a function of the wave height and the bore propagation speed. However, notice that Eq. 7 is given as the universal number here. Duncan [14] only provided data for the fully developed breaking wave, and it seems that Svendsen [1] considered that it is valid for all the stage of wave breaking event.

As a matter of fact, the constant ratio between the roller area and the wave height seems to be valid, or at least, it provides a reasonable approximation when wave breaking at the shoreline is considered, because both items continuously decrease in the inner surfzone and vanish at the end of wave breaking. Not only at the termination (*H=d=0*), the constant ratio sounds very reasonable but also for the inner surfzone situated on positive slopes. Many wave breaking indices approach to some constant numbers while the breaking wave propagates on positive slopes. Dally et al. [15] examined the relative wave height (*H/d*) on plane slope beaches with several different slope angles and showed that the relative wave height asymptotically approaches to a constant value although such value varied with the slope angle setting. Dally et al. [15] called it "stabilized" state. Thus, it is reasonable to think that the ratio between the roller and the wave height also becomes "stabilized" on positive slopes.

However, in the case of offshore wave breaking, the ratio between the roller area and the wave height should not be a constant. The roller disappears at the termination of wave breaking, while the wave keeps certain wave height and continues to propagate even after the wave breaking. On horizontal bottom, as shown in Fig. 2, the wave height does not change so much after certain point (around x=150cm in this case). On the other hand, it is easily understood that the roller keeps decreasing its size towards zero and does not have constant size in this region. Therefore, a coefficient to adjust the roller size regarding to the stage of the breaking event has to be added. Then, Eq. 6 becomes as;

$$E = 0.45\alpha\rho H^2 C^2 \tag{8}$$

where α is the coefficient to adjust the roller size. Since the roller size evolution has not been examined with experimental data yet, a very simple assumption is employed here: there is no roller at the initiation and the termination of wave breaking so that α is zero at both locations, and α is one at the location where the bore propagation speed becomes the maximum.

Since the wave energy based on the potential flow theory is described by the wave height and the wave length, it is obvious that the wave decay reflects the energy dissipation from the wave. However, it is also obvious that all of the energy is not completely dissipated by this. Some of the energy exerted from the wave motion is transferred into the roller kinetic energy and this energy accelerates the bore propagation speed much faster than the theoretical estimation based on the potential flow theory, as discussed previously.

First, it is considered the situation in which the exerted energy due to the wave breaking is completely transferred into the roller kinetic energy. Under this condition, the energy in the system is conserved. Therefore, the conservation of energy is given as;

$$\frac{1}{8}\rho g H^2 L + 0.45\alpha\rho H^2 C^2 = const. \tag{9}$$

where L is the wave length.

At the wave breaking initiation location, the roller has not been formed yet, so the roller kinetic energy term becomes zero. Taking the initiation location of the wave breaking and the location where the maximum bore propagation speed appears, Eq. 9 becomes as;

$$\frac{1}{8}\rho g H_b^2 L_b = \frac{1}{8}\rho g H_m^2 L_m + 0.45\rho H_m^2 C_m^2 \tag{10}$$

where subscripts b and m denote the locations where the wave breaking initiates and where the maximum bore propagation speed appears, respectively.

Making the wave length the product of the wave celerity by the wave period and solving Eq. 10 for C_m provides the maximum bore propagation speed as follows;

$$C_m = \frac{1}{2}\sqrt{\left(\frac{gT}{3.6}\right)^2 + 1.1gT\left(\frac{H_b}{H_m}\right)^2 C_b} - \frac{gT}{7.2} \tag{11}$$

where T is the wave period.

Of course, this provides too much for the accelerated bore propagation speed because all the energy is transferred into the roller kinetic energy. So, these values are compared with the experimental data to find how much of the exerted energy is transferred into the roller. If the transfer coefficient can be determined, the maximum bore propagation speed can be estimated.

WAVE TANK EXPERIMENTS

Wave tank experiments were conducted at National Laboratory of Civil Engineering (LNEC), Lisbon, Portugal. The wave tank is 32m long from the wave maker to the end of wave tank. The bottom configuration includes a horizontal section in the middle so that no water depth change occurs in the inner surfzone. As a result, shoring effects and other non-linear transformations can be excluded in this experiment. The bottom was made out of concrete so there is no permeability in the bottom. Horsehair sheets were installed in the shore section to reduce the reflected energy. Preliminary test results show that only few percent of energy reflects back to the tank. The coordinate system was set the beginning of the horizontal section to be zero, with shoreward to be positive as shown in Fig. 4.

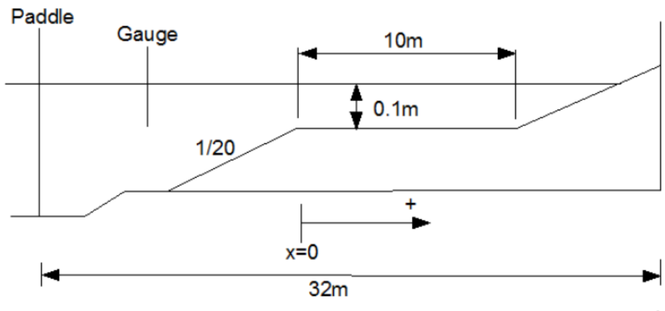


FIGURE 4 : Wave tank settings of LNEC wave tank

The input wave conditions were chosen to make the wave breaking to initiate in the front slope and to terminate within the horizontal section. Four wave periods (T=1.1, 1.5, 2.0, and 2.5sec) and four wave heights (H=8, 10, 15, and 20cm) were chosen. A case with T=1.1sec and H=20cm was discarded because it breaks in front of the wave maker due to the wave steepness violation. One special case, T=2.5sec and H=3cm was also tested. The input wave height of this case is so small that the initiation of wave breaking occurs in the horizontal section. As a result, the whole wave breaking event occurs within the horizontal section. Note that the input wave condition here is the condition at the toe of the front slope of the bar, not in front of the wave maker, because the original bottom configuration of wave tank is not horizontal as shown in Fig. 4.

Free surface elevation was measured to calculate the wave height and the wave celerity/the bore propagation speed. Eight resistant-type wave gauges were used in this experiment. Two kinds of gauges with different probe length were used in this experiment because of the very shallow water depth at the horizontal section. A wave gauge connected with the wave generation system was also installed at the toe of the front slope to check the input wave conditions since the original bottom configuration of the tank is not flat.

Those measuring gauges were placed 20cm apart from each other and the set of wave gauges was moved as a group. Wave gauges were placed at both odd (x=10, 30, 50,…) and even (x=0, 20, 40,…) series. It makes the final resolution of the data set 10cm. The measured area covers from 500cm before the beginning of horizontal section (x=-500cm) to almost the end of the horizontal section (x=940cm).

To reduce the influence of natural oscillation of the tank, the input wave amplitude was amplified gradually at the beginning, and the recording was started three minutes after the beginning of the operation. The duration of each record is 2.5 minutes and the sampling frequency is 100Hz.

The initiation location of the wave breaking was determined by the eye-observation since the wave tank has only partial glass walls near the end of horizontal section. In fact, it is not a very accurate measurement but it is good enough to find it in 10cm increment scale. The smaller waves (H=8 and 10cm) initiate the wave breaking very close to the horizontal section and most of the inner surfzone is located in the horizontal section. The larger waves (H=15 and 20cm) initiate the wave breaking enough far from the horizontal section that the maximum bore propagation speed appears in the front face. The situation of these cases is similar to the wave breaking occurred on a plane slope in terms of the roller formation.

Wave heights were calculated with the zero up-crossing method to couple the wave crest and the trough in front of it. In order to calculate the wave celerity and the bore propagation speed from the two wave gauge data obtained at fixed locations, it is necessary to estimate the time that requires a wave to pass between two wave gauges. In this work, the cross-correlation method was employed to estimate the time lag between the two wave gauges. First, the time series was divided into short portions with the length of one wave period. The time lag to the next wave gauge for each wave was determined with the corresponding section of the record at the next wave gauge by finding the highest correlation coefficient value. Those time lags for each single wave were gathered and the averaged time lag was calculated. Then, the wave celerity was calculated by dividing the gauge distance (20cm) by the time lag.

The wave celerity data obtained here was checked with the calculation result given by KD86 (Eq. 3) for the region before the wave breaking initiation. The data showed very good agreement with KD86, so it was concluded that this calculation scheme provides enough accuracy for the wave celerity and the bore propagation speed.

Fig. 5 and 6 displays an example of the evolution of wave height and the wave celerity/bore propagation speed. As shown in these figures, the maximum bore propagation speed appears after most of the wave decay has completed. In this case, the wave height at the initiation of wave breaking is about 15.5cm and the stabilized wave height is about 4cm. The wave height at the location where the maximum bore propagation appears is about 7cm. So, nearly 75% of wave decay has already completed by this point.

In this case, Fig. 6 displays that the bore propagation speed becomes back to the linear wave celerity. But as shown in Fig. 1, this does not mean the wave celerity after the wave breaking termination follows the linear wave theory. If the bottom slope is negative so that the water depth is deeper than this example, the discrepancy in the propagation speed between the main frequency component and higher harmonics becomes larger and the in-situ wave celerity becomes slower than the linear wave theory due to the influence of higher harmonics.

Fig. 6 also displays that the bore propagation speed in the inner surfzone (after the maximum speed appears) continuously decreases although the water depth is constant in this region. Fig. 5 shows that the wave height in this region has already stabilized mostly, and as a matter of fact, it is almost completely stabilized after the wave enters into the horizontal section. This result displays that the local water depth and non-linear effects given by the finite amplitude of the wave do not give strong influence on the bore propagation in the inner surfzone. That is why existing formulae cannot solve this kind of situation.

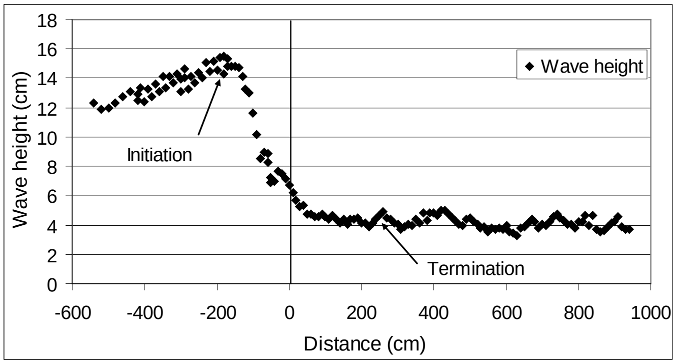


FIGURE 5 : Wave height evolution of T=1.5sec and H=10cm

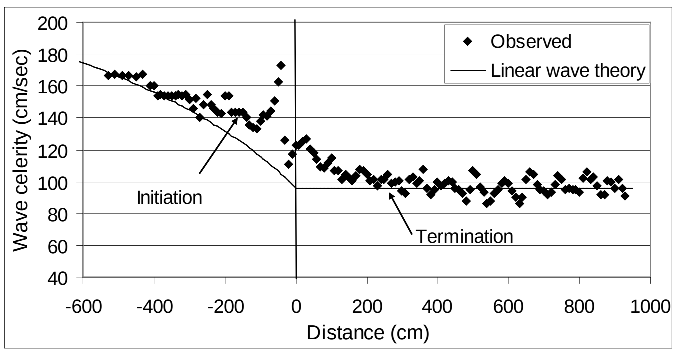


FIGURE 6 : Wave celerity evolution of T=1.5sec and H=10cm

RESULTS

The wave height data at the initiation location and where the maximum bore propagation speed appears, and the wave celerity at the initiation location were substituted into Eq. 11 to calculate the maximum bore propagation speed under the assumption of full energy transfer. The calculated results were compared with the observed maximum bore propagation speed. Table 1 displays the calculated maximum bore propagation speed given by Eq. 11 and the observed maximum bore propagation speed. In Table 1, the case label indicates the wave period and the input wave height. For example, T11H08 means that the input wave conditions are T=1.1sec and H=8cm.

Since the energy between the wave motion and the roller is conserved in Eq. 11, calculated results must be much bigger than the observed maximum bore propagation speed. As shown in Table 1, the calculated maximum bore propagation speed is about twice as big as the observed value in most of the cases. This means that about half of the energy exerted from the wave is transferred into the roller. The rest of the energy is dissipated in the outer surfzone through the formation of the roller.

The case with T=1.1sec and H=15cm gives higher ratio than the others. The result shows that about three quarter of energy is transferred into the roller. Comparison of the wave heights between the initiation location and the location where the maximum bore propagation speed appears reveals that the wave height difference between two locations in this case is smaller than the other cases (Table 2). These things are probably related but the reason is currently unknown.

The case of T=2.5sec and H=3cm also gives higher ratio than the other cases. As explained, the wave breaking initiation occurs after the wave enters the horizontal section. So, the bottom slope angle at the initiation location of this case is different from the other cases. Therefore, there might be some dependency on the slope angle.

TABLE 1 : Comparison between the calculated maximum bore propagation speed and the observed maximum bore propagation speed

Case	Calculated(m/s)	Observed(m/s)	Ratio
T11H08	2.74	1.3	0.47
T11H10	2.50	1.3	0.52
T11H15	2.10	1.6	0.76
T15H08	3.15	1.4	0.44
T15H10	3.51	1.6	0.45
T15H15	3.20	1.7	0.54
T15H20	3.45	1.9	0.55
T20H08	3.41	1.8	0.53
T20H10	3.35	1.7	0.49
T20H15	3.99	2.0	0.50
T20H20	4.55	2.2	0.48
T25H03	1.94	1.2	0.62
T25H08	3.03	1.6	0.53
T25H10	3.77	2.0	0.53
T25H15	4.18	2.0	0.48
T25H20	4.60	2.5	0.54

Besides these two cases, the ratio between the calculated values and the observed values are fairly similar. For the range of values considered here, input wave conditions do not affect so much on the result. This data set also contains both spilling and plunging breakers. So, the breaker type does not affect so much on the energy transfer ratio either.

Thus, it is determined that the energy transfer rate is 0.5 and the maximum bore propagation speed can be calculated by using the following equation;

$$C_m = \frac{1}{4}\sqrt{\left(\frac{gT}{3.6}\right)^2 + 1.1gT\left(\frac{H_b}{H_m}\right)^2 C_b} - \frac{gT}{14.4} \qquad (12)$$

Eq. 12 must provide reasonable estimation of the maximum bore propagation speed. But this is not very convenient since it includes a term related to the location where the maximum bore propagation speed appears. If this term was eliminated, we could predict the maximum bore propagation speed only from the information at the initiation location of the wave breaking.

The ratio between the wave height at the initiation location and the location where the maximum bore propagation speed appears was calculated. Table 2 shows the wave heights at the two locations and the ratio between them.

TABLE 2 : Wave height at the initiation location and the location where the maximum bore propagation speed appears

Case	H_b (cm)	H_m (cm)	H_b/ H_m
T11H08	11	5	2.20
T11H10	14	7	2.00
T11H15	16	10	1.60
T15H08	13	6	2.17
T15H10	15	7	2.14
T15H15	19	10	1.90
T15H20	25	13	1.92
T20H08	15	8	1.88
T20H10	17	9	1.89
T20H15	23	11	2.09
T20H20	26	12	2.17
T25H03	9	6	1.50
T25H08	16	9	1.78
T25H10	19	9	2.11
T25H15	22	11	2.00
T25H20	28	14	2.00

As shown in Table 2, the ratio between the wave height at the initiation location and at the location where the maximum bore propagation speed appears is about 2 in most of the cases. Two cases (T11H15 and T25H03) give smaller values than the others as mentioned before. The average value of all cases is 1.96. If these two cases are excluded, the average value becomes 2.01.

It should be careful to treat this ratio to be 2 because of the scatter in data. The wave height ratio is squared in Eq. 12, so the scatter becomes greater than what is shown in Table 2. If this scatter is assumed to be negligible, applying this ratio to Eq. 12 removes the wave height terms, and the maximum bore propagation speed can be estimated only with the wave celerity at the initiation location.

$$C_m = \frac{1}{4}\sqrt{\left(\frac{gT}{3.6}\right)^2 + 4.4gTC_b} - \frac{gT}{14.4} \qquad (13)$$

This is a rough estimation as mentioned. But as long as concerning waves in the laboratory scale, this should provide the maximum bore propagation speed with certain level of accuracy.

DISCUSSION

The formula to calculate the maximum bore propagation speed was constructed from the energy conservation between the wave motion and the roller kinetic energy and the adjustment was made by using the data obtained from the wave tank experiment.

The result shows that about half of the energy exerted due to the wave decay in the outer surfzone is transferred into the roller. It is converted to the roller kinetic energy and accelerates the bore propagation speed much faster than the wave celerity estimated by the potential flow theory. The other half of the energy is actually dissipated in the outer surfzone. Even if the half of the energy is carried into the inner surfzone, the intensity of the dissipated energy is much bigger in the outer surfzone than in the inner surfzone because of the difference in the spatial scale between two regions.

The model description is a very simplified one with no rotation or turbulence. However, the comparison between the calculated maximum bore propagation speed based on the full energy transfer assumption and the observed data provides a fairly stable relation between the exerted energy due to the wave decay and the acceleration of bore propagation speed among the tested cases. So, the model successfully demonstrates that the bore propagation speed cannot be calculated from the local wave height.

The agreement between the model and the observation confirms that the energy transfer into the roller makes the acceleration of bore propagation speed. This explains why the bore propagation speed structure is so different between the positive slope cases and the horizontal or negative slope cases and existing models can only applicable for the wave breaking on shoreline.

In the case of horizontal bottom and negative slopes, the broken wave does not decay so much after it enters into the inner surfzone. This means that there is no energy dissipation from the wave motion but only the dissipation from the roller is occurring in the inner surfzone. This is probably because no further wave shoaling occurs in these cases so that the wave height or the shape of the wave does not exceed the breaking criteria. Consequently, the formation of the roller in the outer surfzone is almost the only chance to transfer or feed the energy to the roller. That is why the bore propagation speed continuously decreases while the wave height is kept at a certain size, which cannot be explained by the non-linear effect of the wave motion.

On the other hand, in the case of positive slope, e.g. on plane slope beaches, the broken wave keeps shoaling up due to the decreasing of the water depth while the roller dissipates the energy in the inner surfzone. Therefore, the wave geometry exceeds the breaking criteria. So, the wave keeps exerting the energy through the wave decay and feeds the energy to the roller. That is why the wave height itself keeps decreasing on positive slopes. And because of the continuous energy feeding from the wave motion, the bore propagation speed is kept accelerated.

This probably makes the relative size of the roller to the wave height to be constant as Svendsen [1] considered. The meaning of "breaking" and the energy feeding in the inner surfzone are different from the ones due to the roller formulation in the outer surfzone. It should be much smaller and milder than the one in the outer surfzone. Thus, the feeding energy and the energy dissipation from the roller becomes to some sort of balanced state.

This is probably another factor to let the bore propagation speed near the wave breaking termination faster on the positive slopes and slower on the negative slopes, besides the effect of the higher harmonics. And therefore, existing models and formulae based on the observation on positive slopes cannot be utilized for wave breaking events in which the wave breaking termination occurs at the finite water depth.

SUMMARY

A new formula to calculate the maximum bore propagation speed is constructed in this paper. The model is based on the concept of the roller model given by Svendsen [1] but it is a very simplified one. Comparison between the full energy transfer model and the observed bore propagation speed determined the energy transfer coefficient to be about 0.5.

The model successfully illustrates the independence of the bore propagation speed from the local wave height, and the fact that energy transfer from the wave motion to the roller accelerates the bore propagation speed. This provides an important point for the difference in the bore propagation speed structure between the positive slope cases and the negative slope cases. The continuous shoaling on the positive slope keeps feeding the energy to the roller and it accelerates the bore propagation speed even in the inner surfzone. Existing formulae can handle only this type of wave breaking.

The result of this paper is only related to the bore propagation speed at a point. In order to construct the complete bore propagation speed model for the breaking wave, it is necessary to establish the energy dissipation model of the roller and the feeding energy model in the inner surfzone.

NOMENCLATURE

A: the roller area
C: Wave celerity/Bore propagation speed
C_b: Wave celerity at the initiation location
C_m: Maximum bore propagation speed
d: Mean water level
g: The acceleration of gravity
H: Wave height
H_b: Wave height at the initiation location
H_m: Wave height at the location where the maximum bore propagation speed
h: Total water depth from the free surface to the bottom
h_1: Total water depth at the wave trough
h_2: Total water depth at the wave crest
k: Wave number
L: Wave length
L_b: Wave length at the initiation location
L_m: Wave length at the location where the maximum bore propagation speed
m: mass of the roller per unit width
T: Wave period
V: Speed of moving object
α: Coefficient for adjusting the roller area
ρ: Density of water

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