MATHEMATICAL MODEL FOR A 400 mm EXTENSOMETER: CONSTRUCTION, VALIDATION AND PARAMETERIZATION

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ABSTRACT

This paper describes the work in progress on the construction, validation and parameterization of a mathematical model for a 400 mm base length Extensometer for measurement of structural deformations. Previous work (Oliveira, 2011) showed the instrument has high linearity, low hysteresis and good overall performance, thus setting the need to further enhance the existing model and to develop practical experiments to validate it and certify the instrument's accuracy is within the desired value of 1 μm. Some preliminary results are disclosed and discussed.

INTRODUCTION

The 400 mm base length Extensometer is a device intended to measure deformations on civil engineering structures. Although it can be applied to structures of different types of material, it is aimed at concrete structures, hence the 400 mm base length. This instrument is meant to be a high accuracy, portable, robust and easy handling solution for measurements in particular points in space and in time. Some of its requisites are: ±200 μm range; accuracy within 1 μm; easy handling and installation; reliability; good frequency response up to the fundamental frequency of structures (below 10 Hz); temperature independence; and affordability. Its working principle can be described as follows: (a) one end of the instrument is attached to a frame, (b) the other end is attached to a rod protruding from this frame, (c) within the frame four pre-stressed steel strips link the rod to the frame, (d) the geometry of this linkage is such that as the ends of the instrument move closer or apart, thus forcing the rod into or out of the frame, two steel strips are further stressed while the other two are relaxed, (e) each one of these steel strips has a strain gauge glued on it thus forming a complete Wheatstone bridge. Further details can be found on Oliveira (2011).

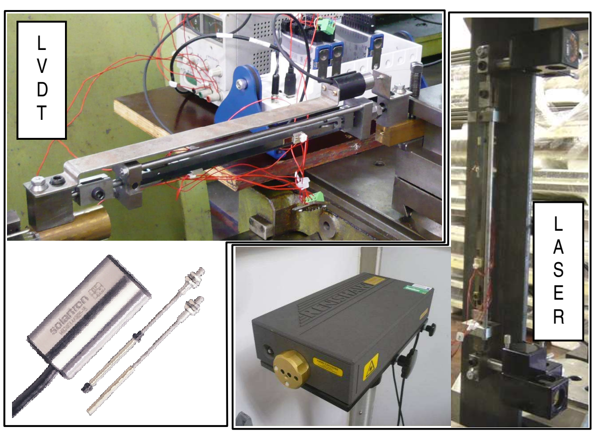

Fig. 1 – 400mm Extensometer being tested against an LVDT and a LASER Interferometer.

A simplified mathematical model of the instrument resulted on a relationship between the output of the Wheatstone bridge and the deformation imposed (Table 1). One of this model's parameters is an equivalent cross section of the instrument (A_d), which can only be determined through parameterization. Comparison of instrument's performance to an LVDT's (Fig. 2) revealed the need to use higher quality traceable standards.

RESULTS AND CONCLUSIONS

Preliminary temperature experiments revealed an unexpected small but consistent temperature dependence of the extensometer. A detailed mathematical model of the instrument was built to accommodate for temperature and internal friction effects (Table 1). In addition, unlike the previous model, the detailed model allows for the calculation of the equivalent cross section of the instrument (A_d).

Table 1 – Mathematical model of the Extensometer

Simplified Model
$\frac{\Delta U_{out}}{U_{in}} = \frac{S_g}{L_s + 2 \cdot A_s \cdot L_d / A_d} \Delta L$
Detailed Model
$\frac{\Delta U_{out}}{U_{in}} = \frac{S_g \cdot \left[\Delta L - \alpha \Delta T L_d + f \right]}{L_s + 4 \cdot A_s \cdot L_d / A_d}$
$A_d = L_d \cdot \left(\frac{E}{K} - \frac{L_s}{4A_s} \right)^{-1}$
U_{out} – brige output U_{in} – brige input S_g – gauge factor L_s – strip length A_s – strip cross section L_d – extensometer length A_d – extensometer cross section ΔL – imposed displacement α – thermal coefficient of expansion ΔT – temperature difference f – friction term E – Young's modulus of steel K – extensometer rigidity

A quasistatic experiment was then carried out with the use of a traceable LASER interferometer, the extensometer being attached to a steel bar and its deformation being imposed by a hydraulic press (Fig. 1). As displayed in Fig. 2 the shape and the magnitude of the extensometer's measurements residuals, obtained from present (LASER) and previous (LVDT) experiments, clearly indicate a dependence of the results on the standards used and further experiments' conditions. As shown in Table 2 values for hysteresis and linearity error are now more compatible with the pre-defined requisite of a 1 μm accuracy. This experiment also allowed for a first evaluation of the instrument's frequency response: at an imposed displacement frequency of 5 Hz there wasn't any significant signal loss.

Table 2 – Linearity and hysteresis

[%FS]	LVDT	LASER
Linearity error	0,97	0,59
Hysteresis	1,06	0,33

Mechanical devices are being built in order to perform (1) temperature experiments with a high quality traceable climatic chamber, and thus shed some light on the temperature (in)dependence of the instrument as well as determine its overall thermal coefficient of expansion (α); and (2) stationary experiments which will allow an evaluation of the significance of the friction term (f), the global validation and parameterization of the model, the evaluation of potential creep in the instrument and its sensitivity. Ultimately it is desired that these experiments along with the validated model will sustain a calibration procedure of the instrument.

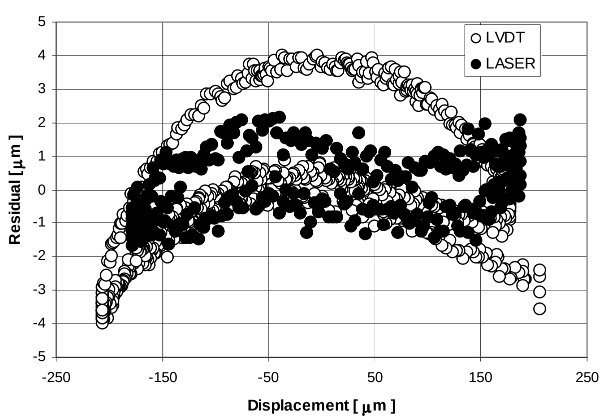

Fig. 2 – Residuals for LVDT and LASER experiments.

REFERENCES

Oliveira F, Morais P, Freitas A. 400 mm base Extensometer for measurement of structural deformations. Proceedings of the 4th International Conference on Experimental Vibration Analysis for Civil Engineering Structures (EVACES), 2011, p. 735-742.