Effect of topography on the distribution of *in situ* stresses due to gravity and tectonic loadings at Paradela site (Portugal)

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Abstract: A methodology is proposed for dealing with the unbalanced stresses that arise from numerical models due to topography effects when horizontal stresses are applied at the vertical boundaries. The proposed methodology is used for understanding the role of topography on the distribution of *in situ* stresses due to both gravity and tectonic loadings. In particular, the influence of Poisson's ratio on the orientation of principal stresses is illustrated for the case where only gravity loading is introduced. This approach is applied to the Paradela site, in Portugal, where a large underground repowering scheme is underway.

Theme: Geological site characterization.

Keywords: Topography, tectonic and gravitational stresses, numerical model, boundary conditions.

1 INTRODUCTION

Paradela site refers to the location of the re-powering scheme of Paradela dam located on the Cávado River in the North of Portugal. Figure 1 shows the location of Paradela site and the orientations of the regional maximum horizontal compressive stresses taken from the World Stress Map (Heidbach et al., 2008). The re-powering scheme includes a new 10 km long hydraulic circuit and a powerhouse complex placed about halfway in the circuit and comprising a new powerhouse cavern, a valves chamber and a large surge chamber with several adits located 500 m bellow ground surface (Fig. 2). The water intake is placed at Paradela dam (bottom right corner of the figure), which is a concrete face rockfill dam built in the 50's. The circuit will run approximately along the Cávado River exiting near the confluence of the Rabagão River (a left bank tributary). It will cross the flank of Serra do Gerês that corresponds essentially to an outcrop of the Gerês granite, which is a post-tectonic biotite granite with calcite plagioclase of medium size with porphyritic trends.

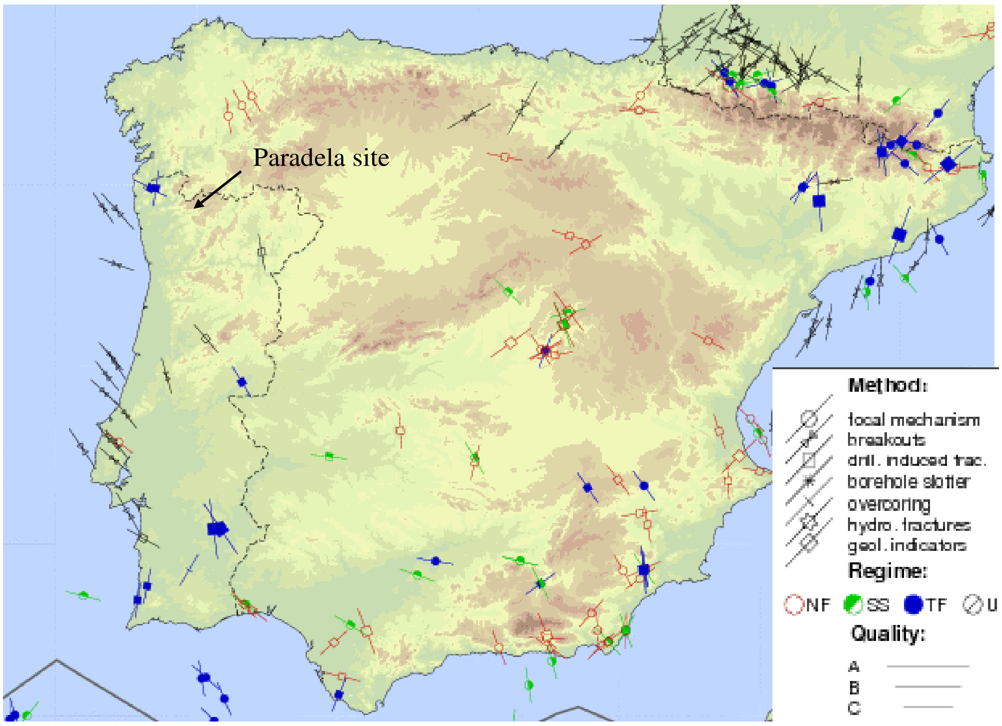

Figure 1. Location of Paradela site and orientations of the regional tectonic maximum horizontal compressive stress from the World Stress Map (Heidbach et al., 2008).

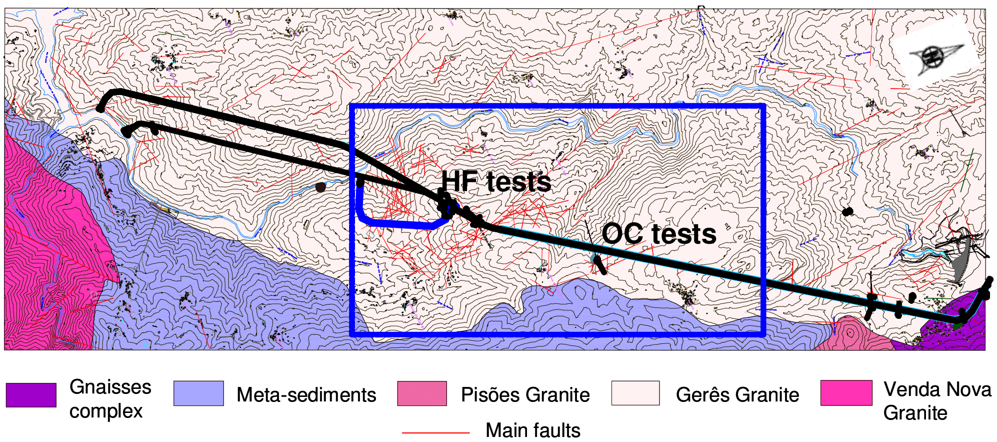

Figure 2. Top view of the hydropower circuit at Paradela (adapted from a drawing provided by EDP).

The design of Paradela re-powering scheme requires a sound understanding of the regional stress field, since the state of stress is frequently the main load to be considered in the design of underground works in rock masses. The determination of rock stresses is a great challenge, due to its spatial variability and the many factors that influence it.

Overcoring tests (OC), hydraulic fracture tests (HF), and hydraulic tests in pre-existing fractures (HTPF) were carried out in order to characterize *in situ* stresses at the locations of the new hydraulic circuit and powerhouse. Overcoring tests were performed in two parallel 60 m deep vertical boreholes (PD1 and PD2), 150 m apart. They were drilled 160 m bellow ground surface inside an existing adit located approximately 4550 m downstream from the water intake and 1850 m upstream from the future powerhouse cavern (Fig. 2). The adit is located 620 m above sea level and 170 m above the future hydraulic circuit. HF and HTPF were carried out in two 500 m deep vertical boreholes (PD19 and PD23), 100 m apart, located 1600 m downstream from the PD1 and PD2 boreholes (Fig. 2). The boreholes were drilled at an elevation of 730 m above sea level. Data from both measurement techniques is being analyzed separately in order to determine stresses in the rock mass at both locations.

A three dimensional model, including the locations where overcoring and hydraulic fracturing tests were carried out, has been developed for the interpretation of the tests results. Since topography plays an important role on the distribution of *in situ* stresses at Paradela site, a sensitivity analysis regarding the dimensions of the area to be considered in the numerical model was undertaken. This analysis concluded that the region delimited by the window shown in Figure 2 is large enough for obtaining reliable estimates of the stress field associated with gravity and tectonic loadings at the locations of the *in situ* tests. The detailed elevation of the simulated area together with the position of the test boreholes is shown in Figure 3. The figure shows that the maximum difference in elevation is about 600 m.

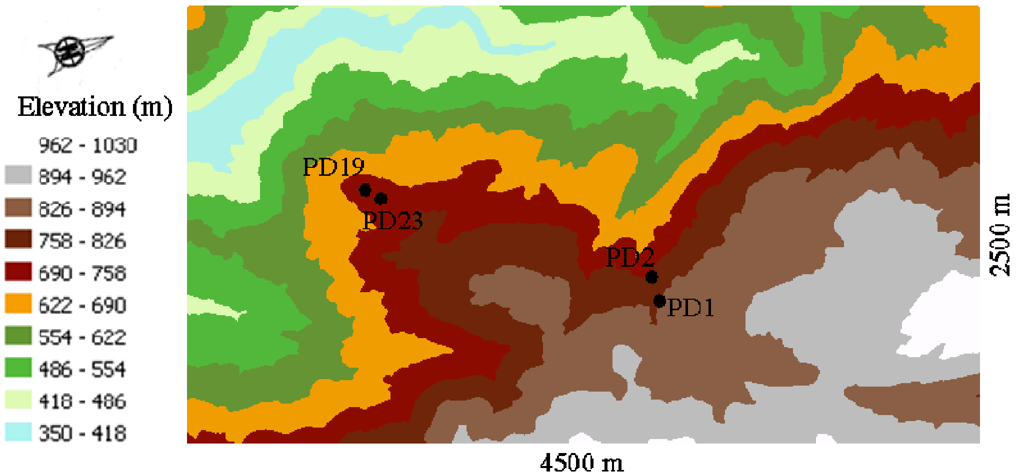

Figure 3. Elevation of the region considered in the three dimensional numerical model.

2 CONSIDERATION OF THE TECTONIC STRESSES IN NUMERICAL MODELS

Numerical modelling is a useful tool for estimating the influence of various factors on the stress field, such as the constitutive models for the rock mass constituents, the loading history, the geological structures, or the role of heterogeneity and of topography (Hart, 2003). However, without stress measurements data for validating the numerical models, such modelling is only of academic interest. Numerical models and stress measurements data must always be combined in order to contribute to a better understanding of the stress field at a given site.

It is common practice to divide the stress field in the earth's crust into vertical and horizontal components. The vertical component is generally equal to the weight of the overlying material and is of gravitational origin. For this component of the gravity loading it is also common practice to assume that lateral confinement results from preventing the normal displacements at the boundaries of numerical models.

For the tectonic loading it is usually assumed that the tectonic stresses act entirely upon the vertical boundaries of the models.

To estimate the tectonic boundary stresses to be considered in the numerical models from stress measurements data the following methodology is commonly used. Unit normal and shear tractions are applied to the model boundaries and the response is computed at the location of the stress measurements. For analysis purposes, it is assumed that the tectonic components of the far field stress tensor can be considered as constant plus a vertical gradient with depth. Then, an optimization procedure is used to compute the proportions of each tectonic stress component that is required, in addition to the gravitational stresses, to reproduce the state of stress at the location of the stress measurements (Li et al. 2009, McKinnon 2001, Tonei et al. 2001).

The main drawback of this technique is that the unit stress components can be applied only at the boundaries of models that simulate rock masses at a scale large enough so that topography effect is negligible when compared with the length assigned to the vertical direction of the model. Otherwise, without assuming certain artificial boundary conditions, such as prescribed displacement boundary conditions or prescribed stress boundaries, unbalanced stresses occur due to topography effects. Furthermore, in this approach the influence of topography on the distribution of the horizontal stresses at the lateral boundaries of the model is neglected.

Let us consider the following example of a unit and uniform compressive stress distribution S_{xx} applied at the lateral boundaries of the numerical model (Fig. 4a). Due to the topography effect, an unbalanced stress state ($F_{res} \neq 0$) occurs when this loading condition is applied.

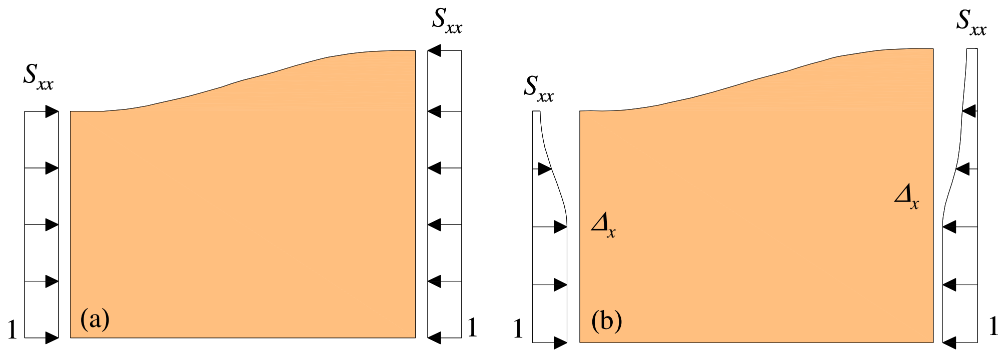

Figure 4. (a) Unbalanced stress state due to the effect of topography when a uniform stress S_{xx} distribution is applied at the boundaries of the numerical model (b) Uniform normal displacement Δ_x applied at the boundaries to induce a horizontal unit stress S_{xx} at the bottom boundary of the model.

In this paper a new numerical technique for dealing with the unbalanced stress state problem in order to estimate the role of topography when both tectonic and gravity forces are active is presented. It consists in applying a uniform normal displacement Δ_x at the lateral boundaries of the numerical model that yields a unit horizontal stress at the elements of the bottom boundary of the model (Fig. 4b). For this method to be successful, it is necessary that the stresses at the bottom boundary of the model are not influenced by topography which may be achieved by extending the model in the vertical direction. In this approach, non-uniform distributions are generated at the lateral boundaries of the model. These stress distributions are balanced ($F_{res} = 0$) and take into account the effect of topography.

An equivalent procedure would be to apply unit stress components in all elements of the model and preventing boundary displacements. However, for those models that aim to simulate the poor quality of the rock mass near the surface by decreasing the local elastic modulus, this loading condition is not effective, since it leads to stress distributions that are unaffected by the variability of the elastic modulus.

3 APPLICATION TO PARADELA SITE TOPOGRAPHY

3.1 Numerical model

A three dimensional model has been developed for Paradela site using the software FLAC 3D (Itasca, 2009). The model includes the location of the overcoring and hydraulic fracturing tests (Fig. 5) and the terrain topography of the region shown in Figure 3. Since the data analysis is not completed yet, directions of the principal stresses are not known. The *x* axis of the numerical model is oriented to N30ºE, so that it is approximately aligned with the direction of the regional (tectonic) minimum principal stresses suggested by the World Stress Map (Heidbach et al., 2008). The introduction of tectonic shear stresses S_{xy} enables to simulate other principal stress orientations, yet using the same grid.

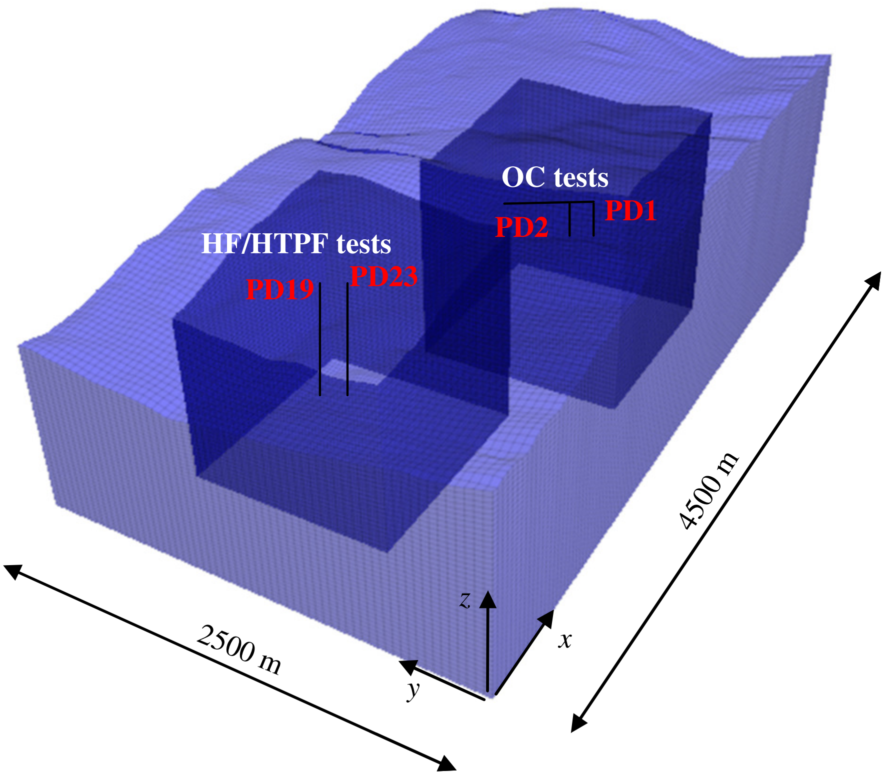

Figure 5. Three dimensional numerical model for Paradela site.

In the dark blue region of Figure 5, the mesh is composed by 25 m side cubes, and in the light blue region by 50 m side cubes. A large vertical extension has been considered so that topography effects do not influence the stress distribution at the bottom of the model. Hence, the bottom boundary of the model has been placed 4000 m bellow sea level.

In this analysis, a linear elastic, isotropic behavior for the rock mass has been assumed. Results from laboratory tests conducted on the cores extracted from Paradela provided average values of 50 GPa for the Young's modulus, 0.25 for the Poisson's ratio, and 2700 kg/m^3 for the rock mass density.

3.2 Influence of topography on the distribution of *in situ* stresses due to tectonic and gravity loadings

The model has been used for assessing the variation of magnitude and orientation of the principal stresses along the boreholes where tests have been carried out.

First, the influence of topography on the distribution of the stress tensor components due to unit tectonic S_{yy} and S_{xy} components at the location of the boreholes was analyzed. Figure 6 shows the variation with depth of all stress tensor components at the location of the boreholes. They have been obtained for a unit horizontal stress component S_{yy}. Figure 7 shows the variation with depth of the stress tensor components at the same locations for a unit tectonic shear stress S_{xy} component introduced in the numerical model. These figures clearly show the effect of topography, since stress profiles are quite different at the locations of overcoring and hydraulic fracturing tests. The stresses displayed in both figures are compared with those that would be the result of the same unit stress loading considering a horizontal ground surface.

Figure 6 shows that the stresses σ_{yy} vary 45% along a 1000 m depth range at the location of the overcoring tests. At the location of boreholes PD19 and PD23 variations of 80% and 60% along a 1000 m depth range are exhibited, respectively. Variations of the horizontal stresses σ_{xx} and vertical stresses σ_{zz} that reach maximums of 10% and 5%, respectively, are also seen. The shear stresses σ_{xy}, σ_{yz} and σ_{xz} exhibit variations that reach a maximum of 10%, 15% and 5%, respectively. This analysis also concludes that the effect of topography at 1000 m depth is higher at the location of overcoring data.

Figure 7 shows that the horizontal stresses σ_{xx} exhibit a 20% variation along a 1000 m depth range at the location of boreholes PD1 and PD19 and a 40% variation at the location of boreholes PD2 and PD23. The horizontal stresses σ_{yy} exhibit a variation that reach an average of 40% at the location of overcoring data. The shear stresses σ_{xy} also displays a variation along the same depth range that can reach maximums of 20% and 60 % at the location of overcoring and hydraulic fracturing data, respectively. From 500 m bellow ground surface, the influence of topography on this stress tensor component is negligible at the two locations. The shear stresses σ_{xz} exhibits a variation that reaches a maximum of 20% at the location of the borehole PD2.

Next, the influence of the tectonic loading S_{yy} component on the results obtained for the gravity loading *G* was analyzed. Three values were considered for the S_{yy} tectonic component: S_{yy1}, S_{yy2} and S_{yy3} equal to 1, 5 and 10 MPa, respectively. After data processing, if the directions of the principal stresses do not coincide with the directions of the model axis, a shear tectonic stress S_{xy} component may be introduced to better fit the data.

Figures 8 and 10 show the orientation of the principal stresses directions with depth at the locations of boreholes PD1 and PD19, respectively, obtained for the gravity loading *G* and for the combined effect of the gravity and tectonic S_{yy1}, S_{yy2} and S_{yy3} loadings . The orientations of the principal stresses are shown in a lower hemisphere stereographic projection in which the direction of the *x* axis of the numerical model is shown. Figures 9 and 11 show the magnitudes of the principal stresses with depth at the location of boreholes PD1 and PD19, respectively.

The results for the magnitude and orientation of the principal stresses at the location of the boreholes PD2 and PD23 are not presented in the paper since they are very similar to the results obtained at the location of the boreholes PD1 and PD19, respectively.

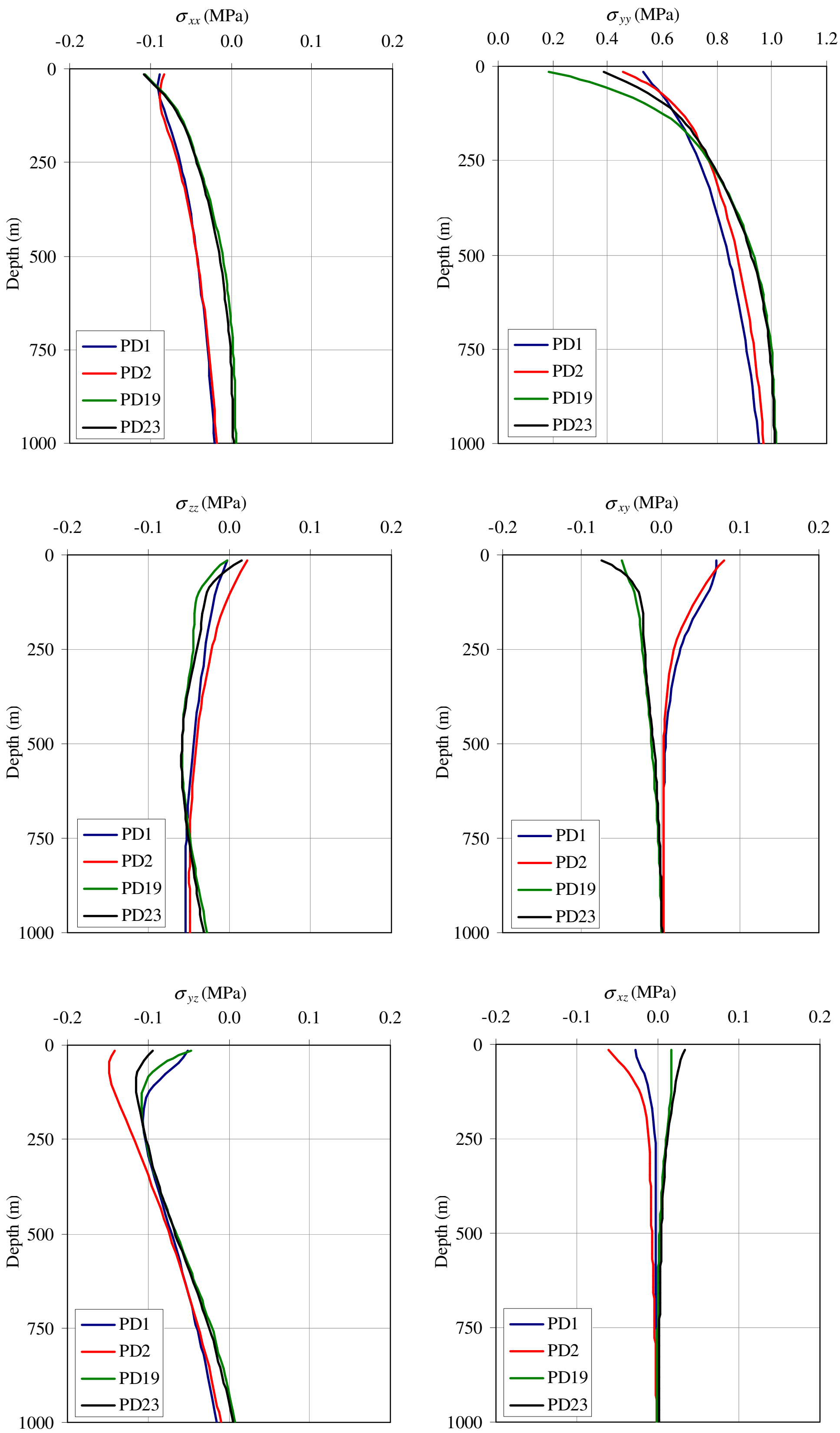

Figure 6. Variation of the stress tensor components for a unit tectonic stress S_{yy}.

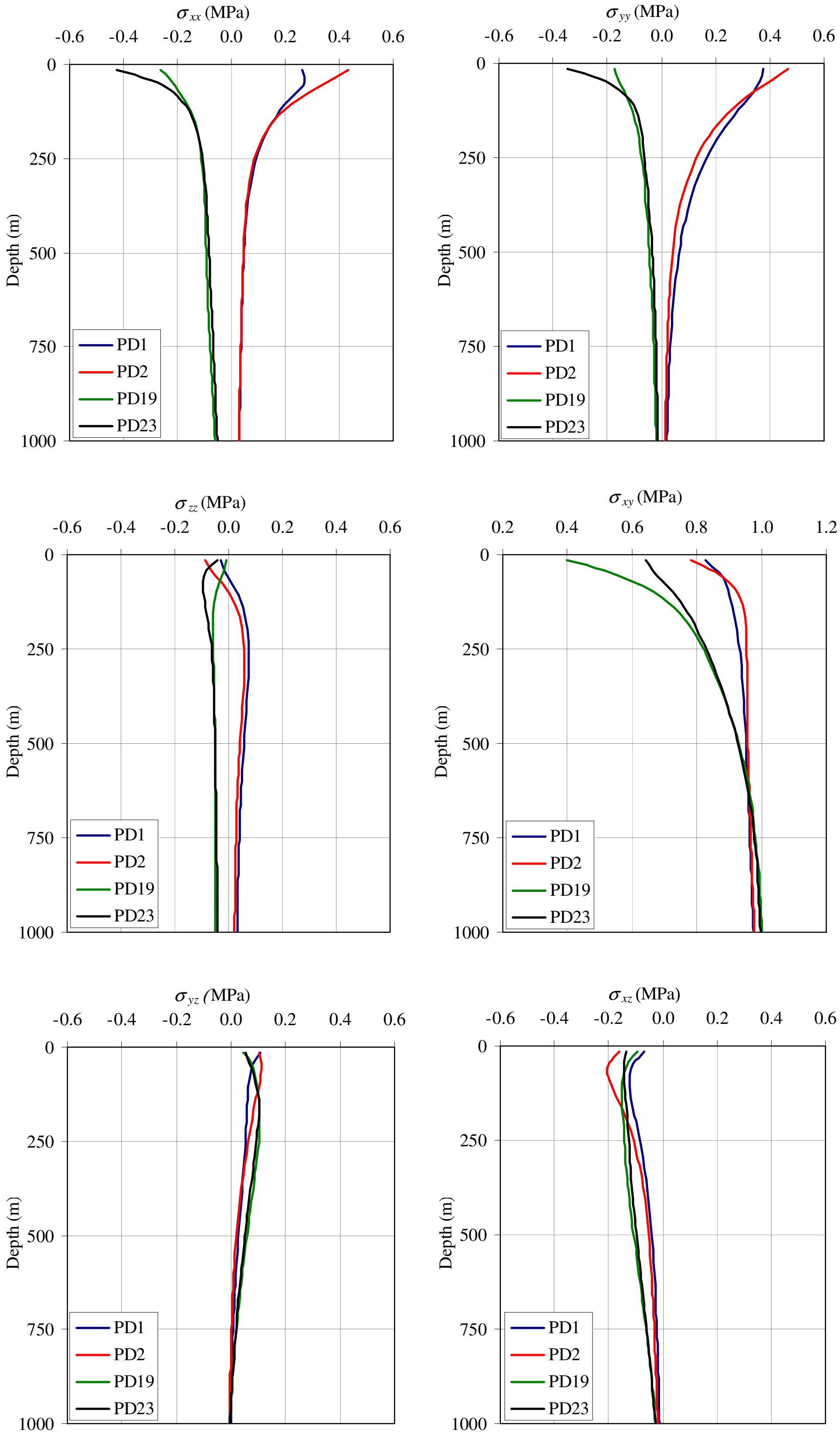

Figure 7. Variation of the stress tensor components for a unit shear stress S_{xy}.

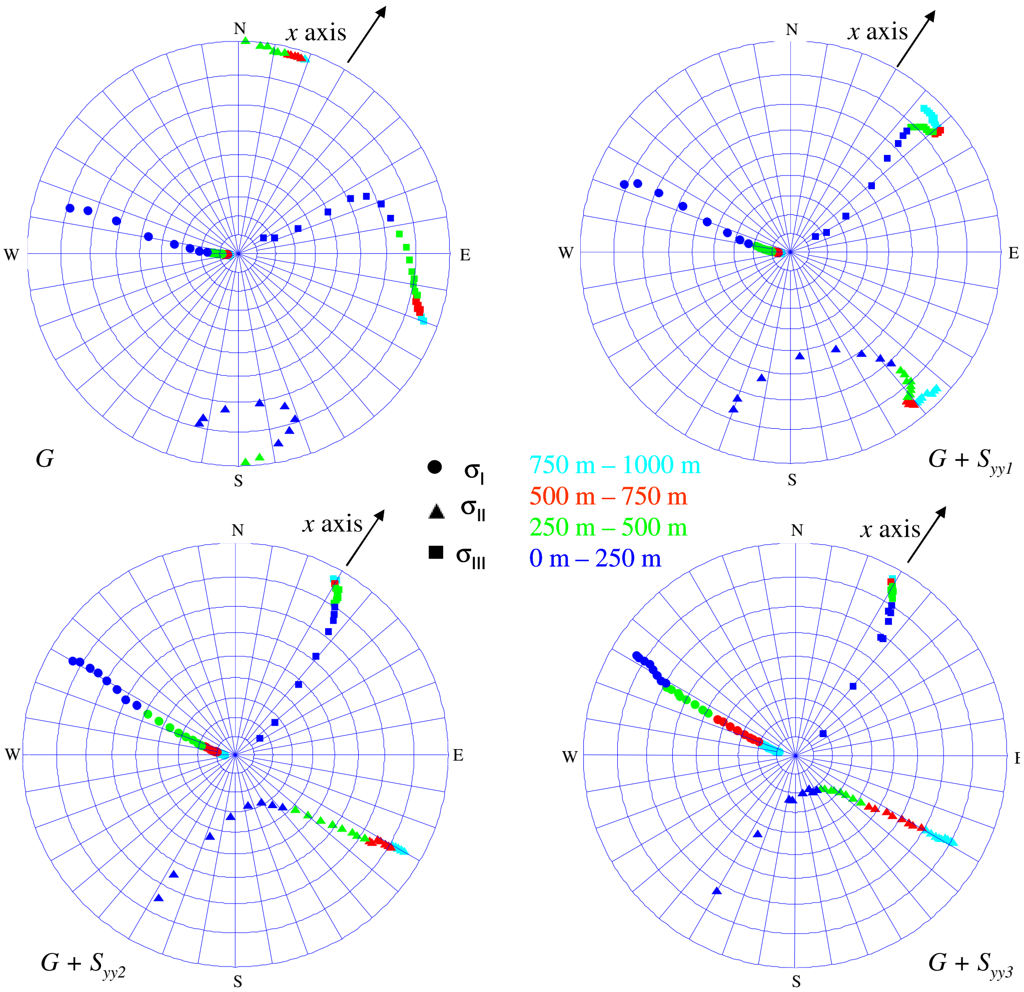

Figure 8. Orientation of the principal stresses obtained at the location of the borehole PD1 for the combined effect of gravity *G* and tectonic S_{yy} loadings.

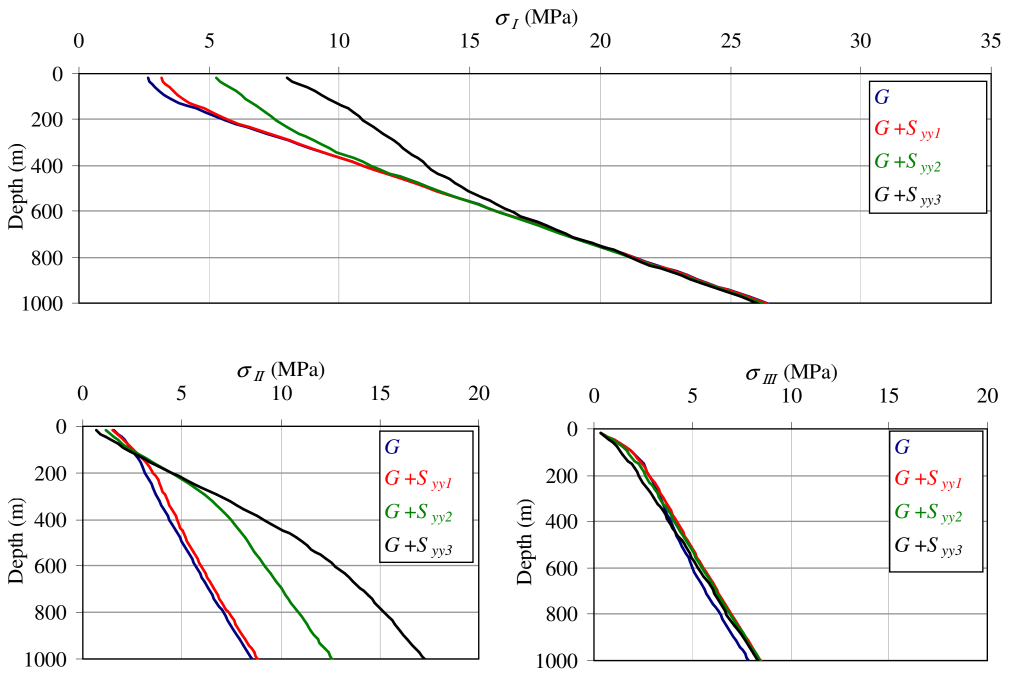

Figure 9. Magnitudes of the principal stresses obtained at the location of the borehole PD1 for the combined effect of gravity *G* and tectonic S_{yy} loadings.

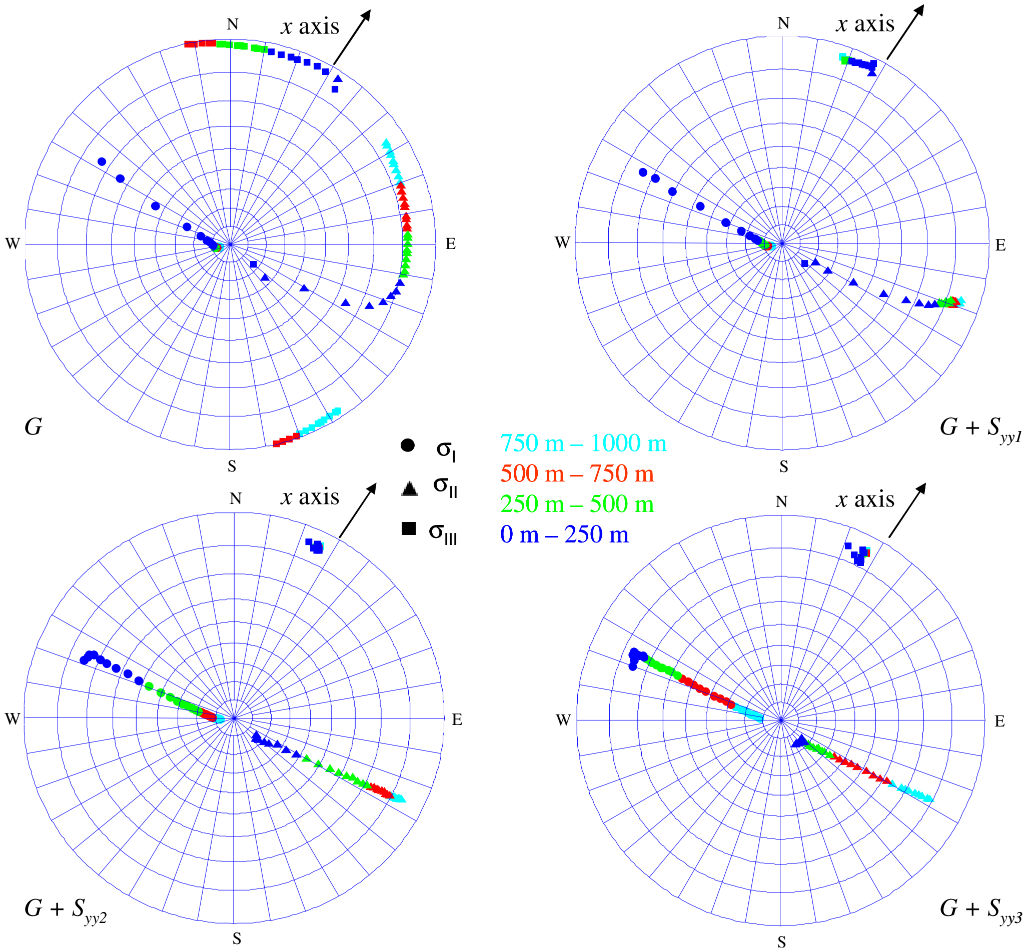

Figure 10. Orientation of the principal stresses obtained at the location of the borehole PD19 for the combined effect of gravity G and tectonic S_{yy} loadings.

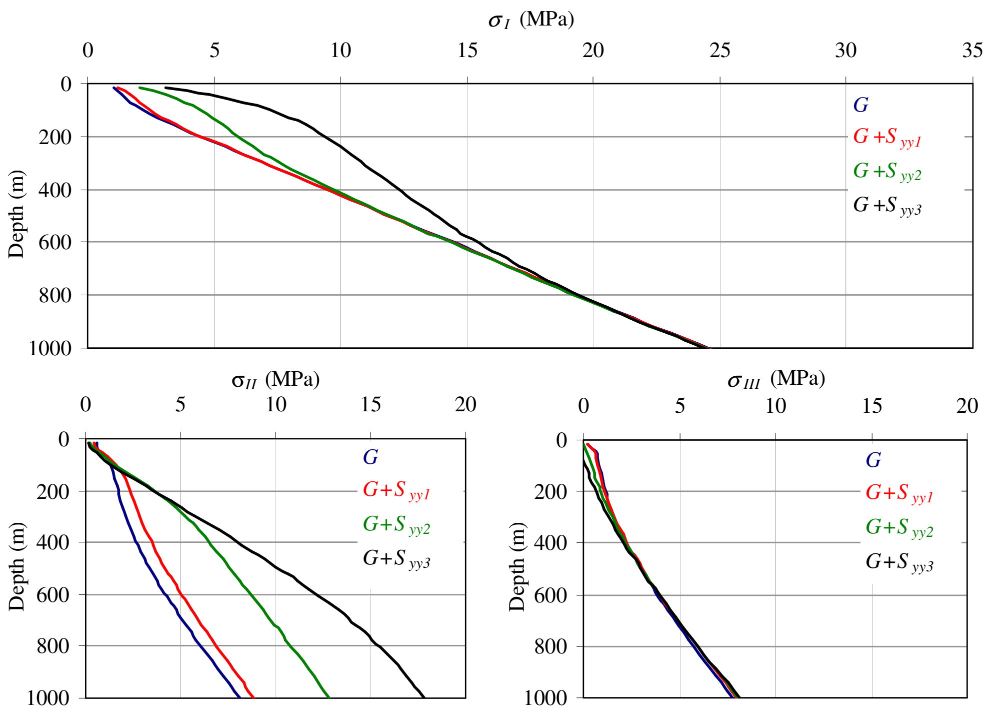

Figure 11. Magnitudes of the principal stresses obtained at the location of the borehole PD19 for the combined effect of gravity G and tectonic S_{yy} loadings.

For the gravity loading, at the location of the borehole PD1 the direction of the minimum principal stress (σ_{III}) is approximately constant (N 65º E) down to 200 m bellow the surface. From this depth down to 1000 m this direction rotates about 45º.

For the combined effect of gravity *G* loading and a unit horizontal stress applied in the *y* axis direction of the model, the minimum principal stress is orientated N 40ºE at 1000 m depth and rotates about 20º till the ground surface. As the tectonic loading increases, the rotation of the minimum and intermediate principal stress directions with depth decreases and these components get aligned with the *x* and *y* axis of the model, respectively. The direction of the maximum principal stress (σ_I) is not influenced as much by the tectonic loading as the direction of the minimum principal stress, since it rotates between 5º and 50º when a tectonic loading S_{yy} is applied. This principal stress component is practically aligned with the *y* axis of the numerical model.

At the location of borehole PD19, the minimum principal stress (σ_{III}) remains sub-horizontal with depth for the considered loading conditions. For the gravity loading its direction rotates about 70º with depth. For the combined effect of gravity and tectonic loadings the direction of this principal stress component only rotates about 10º and is practically aligned with the *x* axis of the model. The maximum principal stress (σ_I) is sub-vertical for depths greater than 200 m bellow the surface when the gravity loading alone is applied. However, its inclination varies substantially with depth when the tectonic loading is introduced in the model. The direction of this principal stress component is practically constant with depth for all loading conditions and is practically aligned with the *y* axis of the model.

The magnitude of the principal stresses at the location of boreholes PD1 and PD19 is similar. The magnitude of the minimum principal stress is practically unaffected by the tectonic loading. The magnitude of the maximum principal stress obtained for the gravity loading at the locations of the boreholes PD1 and PD19 is not influenced in a range from 600 m and 800 m, respectively, when a tectonic loading is applied. In this way, for greater depths the magnitude of the maximum principal stress is controlled by the gravity loading.

Several steps are being taken to integrate the three dimensional numerical model with the results from *in situ* stress measurements in order to estimate the regional stress field at Paradela site. The main goal consists in fitting the results from stress measurements with those computed with the numerical model at the same locations by adjusting the horizontal stress components that must be applied to the model boundaries. In this analysis, it is assumed that the rock mass has a linear, elastic and isotropic behaviour and the tectonic stress field can be described as a linear combination of the stress tensors calculated for unit tectonic S_{xx}, S_{yy} and S_{xy} components applied at the boundaries of the numerical model.

However, if the rock mass does not exhibit a linearly elastic behaviour such an analysis is not correct. A viscoelastic model taking into account the influence of time on the distribution of *in situ* stresses may be considered by adjusting the values of the elastic parameters of the rock mass.

As a matter of fact, the values of the elastic parameters obtained from tests of the cores extracted from the rock mass are unrealistic when the aim is the simulation of the behaviour of the rock mass at a large scale. To understand the influence of the elastic parameters in the stress field, different values of the Poisson's ratio (0.30 and 0.35) were used in the numerical model, since changing the Young's modulus does not induce variations in the results.

A comparison between the principal stress directions obtained with Poisson's ratios equal to 0.25 and to 0.35 has been conducted. Results obtained along the axis of borehole PD19 for the gravity loading show that they differ by about 25º (Fig. 12). Increases of the Poisson's ratio of 0.05 and 0.10 yield increases of about 35% and

75%, respectively, for the magnitude of the intermediate and minimum principal stresses at 500 m depth bellow ground surface. The magnitude of the maximum principal stress is not influenced by the Poisson's ratio because this stress component is sub-vertical. (Fig. 13).

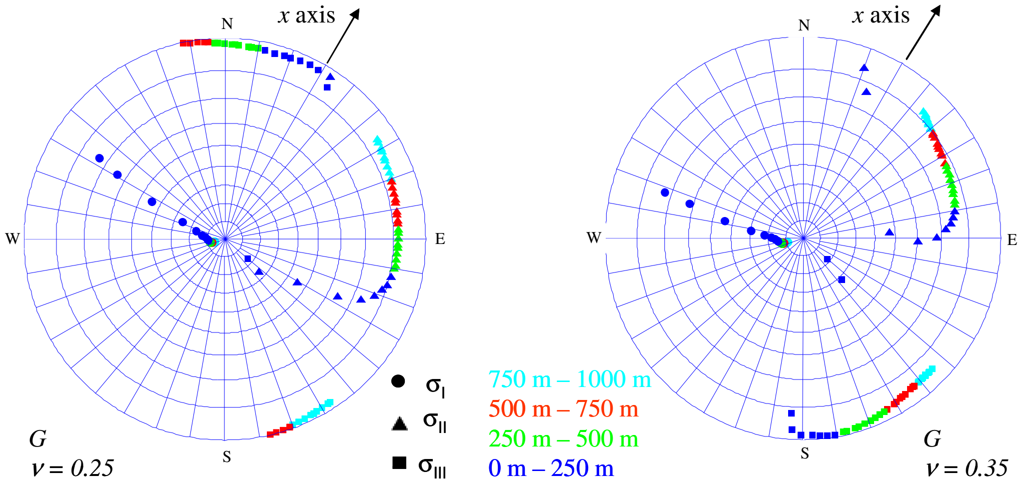

Figure 12. Orientations of the principal stresses obtained at the location of the borehole PD19 for the gravity loading with the values of 0.25 and 0.35 for the Poisson's ratio.

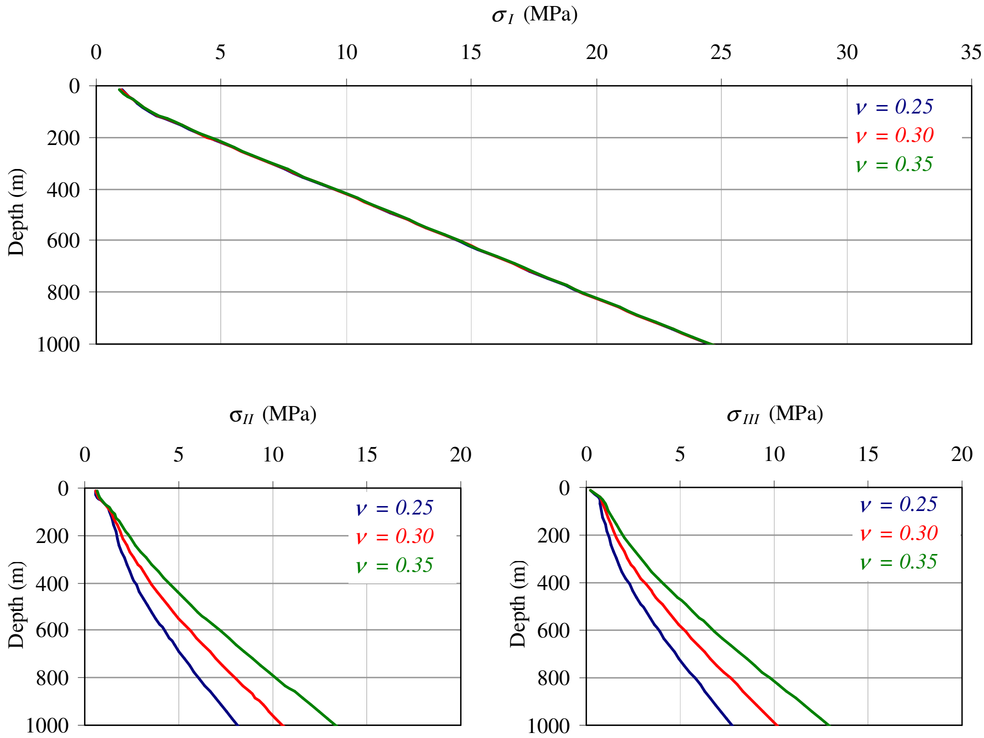

Figure 13. Comparison of the magnitudes of the principal stresses obtained at the location of the borehole PD19 for the gravity loading, with the values of 0.25, 0.30 and 0.35 for the Poisson's ratio.

4 CONCLUSIONS

The design of Paradela hydroelectric re-powering scheme required a set of overcoring and hydraulic fracturing *in situ* tests in order to characterize the stress field at the location of the new hydraulic circuit and powerhouse.

A three dimensional numerical model including the location of the *in situ* tests has been developed. A methodology for dealing with the unbalanced stresses due to effect of topography when horizontal stresses are applied at the boundaries of numerical models has been developed. This technique was used to understand the role of topography on the distribution of *in situ* stresses resulting from gravity G and tectonic loadings S_{yy} at the location of the tests. A tectonic shear stress component S_{xy} enables to consider different directions of the principal stresses, yet using the same grid l.

The analysis concludes that due to gravity loading the influence of topography on the distribution of *in situ* stresses is significantly different at the two test locations. At the location of the hydraulic fracturing tests the minimum principal stress remains sub-horizontal as depth increases, while at the location of the overcoring tests it is sub-horizontal only down to a 500 meter depth. The orientation of the minimum principal stress changes significantly when tectonic stresses S_{yy} are considered. As the magnitude of this tectonic component increases, rotation of the minimum principal stress with depth decreases. The analysis also concludes that the stresses are controlled by gravity effects at the location of the overcoring tests, while in the boreholes where HF and HTPF tests have been conducted, stresses may also be influenced by tectonic stresses.

Assuming a linear and elastic behaviour for the rock mass, the tectonic stress tensor at any given point may be described as a linear combination of the stress tensors calculated for horizontal unit far field stress components (two normal and one shear components). In this way, six model parameters are required for describing the tectonic far field stress (three components at a given depth plus their vertical gradients) and three model parameters for describing the rock mass properties (elastic modulus, Poisson's ratio and density), accounting a total of nine model parameters.

In order to consider that the rock mass may not be linearly-elastic, visco-elasticity may be assumed for taking into account some relaxation in the rock mass. Indeed, the elastic properties of the rock mass vary with time and the values obtained from the extracted cores might not be realistic for a simulation of its behaviour at a large time scale. Visco-elastic effects can be considered in the model by performing an elastic analysis but with relaxed elastic properties.

A first trial of relaxation was performed by investigating the influence of Poisson's ratio on the magnitude and orientation of the principal stresses at the location of borehole PD19. This analysis concludes that it may not be simple to separate visco-elastic effects from tectonic effects, when only stress measurements at isolated locations are considered. However, the availability of vertical stress profiles provides data that may enable separating both effects more efficiently.

5 ACKNOWLEDGEMENTS

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