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New bed shear stress estimator for net sand transport rate predictions under non-linear waves

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ABSTRACT

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The estimation of the bed shear stress is a crucial step in many sediment transport formulae since, when bedload is dominant, sediment transport can be parameterized in terms of the bed shear stress. In this work, the performance of a new bed shear stress estimator for nonlinear waves is analysed. The effects of velocity and acceleration skewness are incorporated in the time-varying bed shear stress using two parameters: the index of skewness or nonlinearity, r, and the waveform parameter, ϕ . The results are compared with two different data sets obtained in the Aberdeen Oscillatory Flow Tunnel, under accelerated-skewed oscillatory flows. When applied to a typical bedload formula, the new expression considerably improves the prediction of the net sand transport rates. A sensitivity analysis for the phase-lead between the bed shear stress and the outer flow indicates that the new expression provides the best trends between predicted and measured transport rates when compared to other parameterizations. This work provides further insights in the correct prediction of sediment transport modelling, accounting for the effect of nonlinear wave shapes.

ADDITIONAL INDEX WORDS: bed shear stress, sand transport, non-linear waves, oscillatory flow

INTRODUCTION

The knowledge and description of the hydrodynamics and sediment transport of sands in the nearshore zone is crucial to beach evolution predictions. As waves travel and shoal towards a beach, nonlinear effects start playing a fundamental role in wave motion and subsequent nearshore transport of sediments. In the shoaling zone the wave becomes steeper and of shorter-duration at the crest and flatter and of longer-duration at the trough. In the surf zone a rapid change in velocity during the forward-leaning steep wave front gives rise to high fluid accelerations, while at the gentle sloping rear face of the wave the accelerations are much smaller. These asymmetries of the wave shape are reflected in the near-bed orbital flow which consequently results in non-zero net sediment transport, as for example shown by the experimental studies of Ribberink and Al-Salem (1994), Watanabe and Sato (2004), Van der A *et al.* (2010) and Silva *et al.* (in press).

In the literature, several approaches can be found, relating the frictional force that the fluid exerts on the bed to the mass of sediment moved. In particular, quasi-steady sand transport formulae relate the time-dependent transport directly to the time-dependent bed shear stress or orbital velocity near the sea bed, just above the wave boundary layer. This approach holds as long as bedload is the dominant mode of transport, as is the case in the sheet flow plane-bed regime (Hsu and Hanes, 2004), or when unsteady effects resulting from the phase lag between the sediments concentration and the flow velocity are weak.

Nevertheless, even when these assumptions are not fulfilled (e.g., rippled beds or fine sand sheet flow), the accurate prediction of bed shear stress is still essential to sediment transport formulae because it drives the mobilization of sediments.

In this paper a new instantaneous bed shear stress parameterization for arbitrary nonlinear oscillatory flows is presented. The effects of velocity and acceleration skewness are incorporated in the time-varying bed shear stress using two parameters recently proposed by Abreu *et al.* (2010) who extended the work of Drake and Calantoni (2001): the index of skewness or nonlinearity, *r*, and a waveform parameter, ϕ , corresponding to the biphase (Elgar and Guza, 1985). The new formulation extends the work of Nielsen (1992, 2002), Nielsen and Callaghan (2003) and Terrile *et al.* (2009) and shows that, beside acceleration effects, the shape of the wave described through *r* and ϕ needs to be considered in the instantaneous shear stress estimations.

The performance of the new formulation is tested using two different data sets. First we compare the predictions of bed shear stress with bed shear stress measurements by Van der A *et al.* (subm.) in the Aberdeen Oscillatory Flow Tunnel for acceleration-skewed flows over fixed rough beds. Secondly, we incorporate the new bed shear stress predictor in the quasi-steady bed load formulation of Nielsen (2006) and compare the transport rate predictions with net transport rate measurements of Van der A *et al.* (2010) obtained in the same facility.



Figure 1. Domain of solutions (β , *R*) for Equation (1) (shaded area) with specific values of ϕ and $0 \le r < 1$ and for Drake and Calantoni's (2001) (dashed line) (reproduction of Abreu *et al.*, 2010)

FORMULATION

Recently Abreu *et al.* (2010) proposed the following free-stream horizontal velocity:

$$u(t) = U_w f \frac{\left[\sin(\omega t) + \frac{r\sin\phi}{1 + \sqrt{1 - r^2}}\right]}{\left[1 - r\cos(\omega t + \phi)\right]}.$$
 (1)

to describe nonlinear motions. Here, $U_{\rm w}$ represents the amplitude of the orbital velocity, $U_{\rm w} = (u_{\rm max}-u_{\rm min})/2$, $\omega = 2\pi/T$ is the angular frequency, and the dimensionless factor $f = \sqrt{1 - r^2}$ allows the velocity amplitude to be equal to U_{w} . Furthermore, r is an index of skewness or nonlinearity, and ϕ a waveform parameter. A purely acceleration-skewed flow (i.e. sawtooth wave) is obtained for $\phi =$ 0 and a pure velocity-skewed flow for $\phi = -\pi/2$. Between these two extreme values the orbital flow contains both velocity and acceleration skewness. Figure 1 shows the application range of Equation (1), between $0 \le r < 1$ and $-\pi/2 \le \phi \le 0$, in terms of the more common velocity and acceleration skewness coefficients, R $(=u_{\max}/(u_{\max}-u_{\min}))$ and β $(=a_{\max}/(a_{\max}-a_{\min}))$, where *a* is acceleration. Figure 1 further illustrates that Drake and Calantoni's (2001) solution for the free stream velocity (dashed line) can be assumed as a particular solution of Equation (1) with r = 0.8 (see Abreu *et al.*, 2010, for further details).

The time-varying bed shear stress near the bottom follows:

$$\tau_{b}(t) = \rho u_{*}(t) |u_{*}(t)|.$$
 (2)

with the corresponding shear velocity:

$$u_{*}(t) = \sqrt{\frac{f_{w}}{2}} \left(\cos(\varphi_{\tau}) u(t) + \frac{\sin(\varphi_{\tau})}{\omega} \left[\frac{\partial u(t)}{\partial t} - S(t,\phi,r) \right] \right), (3)$$



Figure 2. Measured and predicted free-stream velocity for the experimental condition S706015m of Van der A *et al.* (2010) and the corresponding predicted bed shear stress using the parameterizations of Nielsen (1992, 2002), Terrile *et al.* (2009) and Equation (3) with $\varphi_{\tau} = 51^{\circ}$.

where f_w represents the wave friction factor, ρ is the fluid density, φ_{τ} is the phase-lead between the shear stress and the corresponding free stream velocity. This expression, apart from the usual drag force component acting in sediment particles (when $\varphi_{\tau} =0$) also takes into account the pressure gradient/free stream acceleration. The last term in the brackets, $S(t,\phi,r)$, adjusts the fluid acceleration effect as suggested by Nielsen (1992, 2002), Nielsen and Callaghan (2003). According to Equation (1) this term is written as the following function that depends on the shape of the orbital motion:

$$S(t,\phi,r) = \omega \cdot f \cdot U_{w} \cdot \frac{r[-(-1+f)\cos\phi - 2r\cos(\omega t) + (1+f)\cos(2\omega t + \phi)]}{2(1+f)[-1+r\cos(\omega t)]^{2}}$$
⁽⁴⁾

Hence, the effects of velocity and acceleration skewness are incorporated in the time-varying bed shear stress (Equations 2-4) using the two above parameters r and ϕ .

As an example, Figure 2 shows in the upper panel the instantaneous free stream velocity using Eq. (1) and, in the lower panel, the corresponding bed shear stress estimations for the experimental condition S706015m presented in Van der A *et al.* (2010). For comparison, we have added the results of τ_b obtained with the original parameterizations of Nielsen (1992, 2002) and Terrile *et al.* (2009).

The example condition corresponds to a pure accelerationskewed oscillatory flow (sawtooth wave) with degree of acceleration skewness $\beta = 0.70$, wave period of T = 6s and rootmean-square velocity of $u_{\rm rms} \approx 0.93$ m/s. This flow can be estimated with Equation (1) using $U_w=1.34$ m/s, r = 0.40 and $\phi =$ 0. For the phase-lead parameter φ_{τ} we have considered 51° as proposed by Nielsen (2006). Nielsen (2006) found this value by applying his shear stress approach to a quasi-steady Meyer-Peter and Müller type bedload formula and optimizing the net transport predictions to Watanabe and Sato's (2004) measurements for acceleration-skewed flows.

From Figure 2 it can be seen that Terrile *et al.*'s (2009) formulation leads to two distinct peaks in the positive direction. That is due to the fact that those authors used Drake and Calantoni's (2001) free stream velocity expression to obtain their



Figure 3. (a) Measured and predicted free-stream velocity; (b) measured and predicted bed shear stress using $\varphi_{\tau} = 51^{\circ}$; (c) idem using $\varphi_{\tau} = 26^{\circ}$ Flow condition: T = 7s, $U_{w} = 1.1$ m/s, r = 0.451, $\phi = 0.161$, $k_{s} = 13.8$ mm.

parameterization. Such solution correspond to r = 0.8 in Equation (1). Thus we obtain Terrile *et al.* (2009) parameterization by fixing r = 0.80 in Equation (3). That means that their solution is not prepared for r = 0.40 as the example presented. Similarly, Nielsen's (1992, 2002) parameterization results in the particular case of Equation (3) with r = 0 because it leads to S = 0 in Equation (4). This corresponds to the sinusoidal case from where Nielsen developed his formulation.

BED SHEAR STRESS

In a recent experimental study in the Aberdeen Oscillatory Flow Tunnel (AOFT), Van der A *et al.* (subm.) measured the instantaneous velocity profiles in the boundary layer for a range of acceleration-skewed oscillatory flows over fixed rough beds. The test conditions had flow periods of T = 5s and 7s, with $U_w = 0.9$ or 1.1m/s, respectively. The bed roughness consisted of sand (with Nikuradse roughness $k_s = 1.1$ mm) or gravel ($k_s = 13.8$ mm) glued to the fixed bed. The free-stream oscillatory flow was dominated by acceleration skewness. Bed shear stress was estimated for large parts of the oscillatory flow cycle by applying the so-called "lawof-the-wall" to the measured instantaneous velocity profiles. For the present comparison we focus on the results from the gravel bed experiments, for which all the flow conditions were in the rough turbulent regime.

Figure 3 shows an example comparison of measured bed shear stress with the predicted bed shear stress using the new predictor (Equation (3)) and using Nielsen's (2006) method (i.e., S = 0). Figure 3a compares the measured and predicted free-stream velocity time-series, the latter is used as input to both bed shear stress predictors. Bed shear stress is shown for two scenario's; Figure 3b shows the results using Nielsen's (2006) optimized phase-lead value of $\varphi_{\tau} = 51^{\circ}$, while in Figure 3c the results are shown using the measured (first harmonic) phase-lead, which is approximately $\varphi_{\tau} = 26^{\circ}$ for this condition. It is shown that using $\varphi_{\tau} = 51^{\circ}$ Nielsen's (2006) approach largely overestimates the

Table 1: Ratio of maximum positive and maximum negative bed shear stress $|\tau_{bmax}/\tau_{bmin}|$. Test conditions were dominated by acceleration skewness (i.e. wave form parameter $\phi \approx 0$)

		$\varphi_{\tau} = 51^{\circ}$		φ_{τ} meas.	
Test conditions	meas.	N06	Eq. (3)	N06	Eq. (3)
$T=5s, U_w=0.9m/s, r=0.22$	1.0	1.4	1.4	1.0	1.2
$T=5s, U_w=0.9m/s, r=0.25$	1.3	1.4	1.4	1.0	1.2
$T=5s, U_w=0.9m/s, r=0.62$	1.7	3.4	1.7	1.5	1.8
$T=7$ s, $U_w=1.1$ m/s, $r=0.16$	1.0	1.3	1.3	1.0	1.1
$T=7$ s, $U_w=1.1$ m/s, $r=0.31$	1.2	1.9	1.7	1.2	1.4
$T=5s, U_w=1.1m/s, r=0.45$	1.8	2.9	2.4	1.5	1.8

maximum shear stress, while much better agreement is obtained with the new method which includes the additional term to adjust the acceleration contribution. On the other hand, both predicted maxima in the negative direction agree quite well with the measurements. As a first approach, for sediment transport rate predictions, the ratio of the shear stress maxima, $|\tau_{bmax}/\tau_{bmin}|$, can be considered as an important parameter. The measured and predicted values of this ratio are listed in Table 1 for the whole set of experimental tests. It is shown that using $\varphi_{\tau} = 51^{\circ}$ both methods generally overestimate the measured ratio. Figure 3c shows that using the measured phase-lead values significantly improves the predictions of bed shear stress maxima, except in the negative direction for Nielsen (2006) which now overestimates the measurements somewhat. The last two columns in Table 1 show that both methods present an overall much better agreement when the measured phase-lead is considered (for the measurements 26° $\leq \varphi_{\tau} \leq 30^{\circ}$). However, there is, not a predictor that is superior to the other: Nielsen (2006) represents better the lower ratio of $|\tau_{bmax}\!/\!\tau_{bmin}|$ and the new method tends to represent better the high measured ratios.

NET SAND TRANSPORT RATE

The performance of the new formulation to predict sediment transport rates is tested against the net transport rates measured by Van der A *et al.* (2010) in the AOFT. The experiments involved acceleration-skewed flows and mobile beds of three different median grain sizes ($d_{50} = 0.15$, 0.27 and 0.46 mm). We note that these oscillatory flows also contained a small degree of velocity skewness, which was taken into account in the new formulation through *r* and ϕ .

To estimate the sediment transport rates, q_s , we followed Nielsen's (2006) formulation, which is a modified version of the Meyer-Peter Müller (1948) bedload type formula:

$$q_{\rm s}(t) = 12\sqrt{(s-1)gd_{50}^{3}} \left[\theta(t) - \theta_{\rm cr}\right]\sqrt{\theta(t)} \frac{u_{*}}{|u_{*}|} \text{ for } \theta > \theta_{\rm cr} \quad (5)$$

The instantaneous Shields parameter $\theta(t)$ is defined by $\theta(t) = \tau(t)/(\rho(s-1)gd_{50})$, where $s = \rho_s/\rho$ is the ratio between sediment and water densities and $\theta_{\rm cr}$ is the critical value of θ , at the threshold of motion. A typical value of $\theta_{\rm cr} = 0.05$ is assumed for the present conditions.

For the wave friction coefficient f_w we considered the nominal grain roughness friction factor, $f_{2.5}$, proposed by Nielsen (1992):



Figure 4. Measured and predicted net transport rates based on (a) Nielsen's (2006) bed shear stress approach; (b) new method to compute bed shear stress. $\varphi_{\tau} = 51^{\circ}$.

$$f_{2.5} = \exp\left[5.5\left(\frac{2.5d_{50}}{A}\right)^{0.2} - 6.3\right]$$
(6)

where A represents the near bed semi-excursion.

The new bed shear stress predictor requires the introduction of φ_{τ} . For the net sediment transport rates computations we again adopted the phase difference of 51°, as a reference. Since the present dataset equally concerns sheet flow conditions under acceleration-skewed flow, it seems reasonable to evaluate the performance of the new model with $\varphi_{\tau} = 51^{\circ}$.

Figure 4 shows the results of the predicted q_s using Nielsen's bed shear stress model (top panel) and the new bed shear stress predictor (lower panel). The solid lines correspond to a 1:1 match between predicted and measured, whereas the dashed lines refer over- and under-predictions by a factor of 2. These results clearly illustrate that the introduction of $S(t,\phi,r)$ given by Equation (4) in Equation (3) results in more accurate estimates of the net transport rates but also, perhaps more importantly, the trends in the transport rates are much better predicted. There is, however, some disagreement in the magnitude of the transport rates; the medium and coarse sands are generally within a factor of about 2, while the fine sands are under-estimated by a higher factor. It is not entirely surprising that the finer sand are under-estimated since it was shown by Van der A *et al.* (2010) that these transport rates are subject to significant phase lag effects, which are of course not

captured in the present quasi-steady approach. We furthermore note that, to allow for comparison with Nielsen (2006), in the present approach the roughness is taken proportional to the grain size, but it can also be linked to the mobile bed roughness (e.g. Gonzalez-Rodriguez and Madsen, 2007), which will affect the magnitude of the predicted transport rates. Much better agreement between predicted and measured transport rates can also be obtained if φ_{τ} is calibrated separately for the different sand sizes, as suggested by Van der A *et al.* (2010).

Rather than predicting the exact magnitude of the transport rates, which can be achieved in various ways and it's not the main aim of this study, we instead focus on predicting the correct trends in the transport rates. This is illustrated in Figure 5 which shows the value of the squared correlation coefficient, r^2 , between the measured and predicted transport rates for each sediment size as a function of φ_{τ} . The two extreme values of φ_{τ} , 0° and 90°, correspond to drag dominated sediment transport and to pressure gradient dominated scenarios like plug flows (Sleath, 1999).

These results show that for any φ_{τ} other than 0, the new formulation improves the correlation compared to Nielsen's (2006) original approach. Note that the absolute differences in r^2 for the different sand sizes results from the different number of conditions. The optimal correlations are found with $\varphi_{\tau} = 57^{\circ}$, 73° and 42° for $d_{50} = 0.15$, 0.27 and 0.46 mm, respectively. These values are significantly larger than the phase-lead values resulting from the bed shear stress comparisons for fixed beds in the previous section (within the range of 20° - 30°). The reason for these large φ_{τ} values, most even in excess of the 45° for laminar flows, is not entirely clear. We do note however, that such values not necessarily disagree with measurements for mobile bed sheet flows. For example, Guard and Nielsen (2008) reported phaseleads that asymptotically increase to 90° when we look further into the sheet flow layer. They show that the magnitude and phase lead of the total shear stress depends on the chosen elevation within the sheet flow layer which makes it difficult to define a particular phase lead under mobile sand beds.

CONCLUSIONS

The performance of a new bed shear stress estimator for net sand transport rate predictions under non-linear waves is investigated. The parameterization extends the work of Nielsen (1992, 2002), Nielsen and Callaghan (2003) and Terrile et al. (2009) and shows that, beside acceleration effects, the shape of the wave, described through two parameters (r,ϕ) recently proposed in Abreu et al. (2010), can be considered in the instantaneous shear stress computations, bringing in more physics. These parameters characterize the regular nonlinear orbital motions through an index of skewness or nonlinearity, r, and a waveform parameter, ϕ , which reflect the degree of velocity and acceleration skewness of the orbital motion.

The new expression together with Nielsen's (2006) original approach are first compared to bed shear stress measurements for acceleration-skewed flows over fixed rough beds. It is shown that using the phase-lead between the shear stress and the corresponding free stream velocity $\varphi_{\tau} = 51^{\circ}$, as recommended by Nielsen (2006), the new formulation gives a better agreement, but both methods generally overestimate the measured ratio of $|\tau_{bmax}/\tau_{bmin}|$. Using the measured phase-lead values, the predictions of bed shear stress maxima are significantly improved for both formulations. However, based solely on the ratio of $|\tau_{bmax}/\tau_{bmin}|$ there is not a predictor that is overall superior to the other.

Secondly, the new bed shear stress method is incorporated in a quasi-steady bed load formula (Nielsen, 2006) and its performance is tested against the measured net transport rates of Van der A *et*



Figure 5. Correlation between measured and predicted transport rates using Nielsen's (2006) bed shear stress approach (N06) and the new approach (Eq. (3)), as a function of the phase-lead φ_{τ} .

al. (2010) under sheet flow conditions. Using the default value $\varphi_{\tau} = 51^{\circ}$, the new bed shear stress approach leads to better estimates of the net transport rates when compared to Nielsen's original approach. A sensitivity analysis for φ_{τ} also illustrates that the new approach always results in better correlations with measurements. Maximum correlation is obtained for φ_{τ} between 42° and 73°: these values are much larger than the optimum values found for the fixed bed measurements, in the range of 20°-30°.

Future work is aimed at improving the magnitude of the transport rate predictions by applying a mobile bed roughness and/or formulating different φ_{τ} for the different sand sizes. The new formulation provides further insights in sediment transport predictions, accounting for the effect of nonlinear wave shapes, and can be useful in several engineering applications, in particular for morphological models.

LITERATURE CITED

- Abreu, T.; Silva, P.A.; Sancho, F. and Temperville, A., 2010. Analytical approximate wave form for asymmetric waves. *Coastal Engineering*, 57(7), 656-667.
- Drake, T.G. and Calantoni, J., 2001. Discrete particle model for sheet flow sediment transport in the nearshore. *Journal Geophysical Research*, 106(C9), 19,859–19,868.
- Elgar, S. and Guza, R.T., 1985. Observations of bispectra of shoaling surface gravity waves. *Journal of Fluid Mechanics*, 161, 425-448.
- Gonzalez-Rodriguez, D. and Madsen, O.S., 2007. Seabed shear stress and bedload transport due to asymmetric and skewed waves. *Coastal Engineering*, 54(12), 914-929.
- Guard, P. and Nielsen, P., 2008. Unsteady flow effects on bed shear stress and sheet flow sediment transport. *Proceedings* 31st International Conference on Coastal Engineering (Hamburg, Germany, World Scientific), 1521-1532.

- Hsu, T.J. and Hanes, D.M., 2004. Effects of wave shape on sheet flow sediment transport. *Journal of Geophysical Research*, 109 (C05025). doi:10.1029/2003JC002075.
- Meyer-Peter, E. and Müller, R., 1948. Formulas for bed-load transport. *In: Report from the 2nd Meeting of the International Association for Hydraulic Structures Research* (Stockholm, Sweden, IAHR), 39-64.
- Nielsen, P. 1992. Coastal bottom boundary layers and sediment transport. World Scientific, 324p.
- Nielsen, P. 2002. Shear stress and sediment transport calculations for swash zone modelling. *Coastal Engineering*, 45(1), 53–60.
- Nielsen, P. 2006. Sheet flow sediment transport under waves with acceleration skewness and boundary layer streaming. *Coastal Engineering*, 53(9), 749–758.
- Nielsen, P. and Callaghan, D.P., 2003. Shear Stress and Sediment Transport Calculations for Sheet Flow under Waves. *Coastal Engineering*, 47(3), 347-354.
- Terrile, E.; Reniers, A.J.H.M. and Stive, M.J.F., 2009. Acceleration and skewness effects on the instantaneous bedshear stresses in shoaling waves. *Journal of Waterway, Port, Coastal and Ocean Engineering*, 135(5), 228-234.
- Ribberink, J.S. and Al-Salem, A.A., 1994. Sediment transport in oscillatory boundary layers in cases of rippled beds and sheet flow. *Geophysical Research*, 99 (C6), 12707–12727.
- Silva, P.A.; Abreu, T.; Van der A, D.A.; Sancho, F.; Ruessink, B.G.; Van der Werf, J.J., and Ribberink, J.S., in press. Sediment tranport in non-linear skewed oscillatory flows: the Transkew experiments. *Journal of Hydraulic Research*.
- Sleath, J.F.A., 1999. Conditions for plug formation in oscillatory flow. *Continental Shelf Research*, 19(13), 1643-1664.
- Van der A, D.A.; O'Donoghue, T.; Davies, A.G., and Ribberink, J.S., (submitted). Experimental study of the turbulent boundary layer in acceleration-skewed oscillatory flow. *Journal of Fluid Mechanics.*
- Van der A, D.A.; O'Donoghue, T., and Ribberink, J.S., 2010. Measurements of sheet flow transport in acceleration-skewed oscillatory flow and comparison with practical formulations. *Coastal Engineering*, 57(3), 331-342.
- Watanabe, A. and Sato, S., 2004. A sheet-flow transport rate formula for asymmetric forward-leaning waves and currents. *Proceedings 29th International Conference on Coastal Engineering* (Lisbon, Portugal, World Scientific), 1703–1714.

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