MEASUREMENT UNCERTAINTIES IN REGRESSION ANALYSIS WITH SCARCITY OF DATA

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Abstract

The evaluation of measurement uncertainties, in certain fields of science, faces the problem of scarcity of data. This is certainly the case in testing with geological soils in civil engineering, where tests can last for several days or weeks and where the same sample is not available for further testing, being destroyed during the experiment.

In this particular study attention will be paid to compression triaxial tests made on geological soils. The purpose of the testing is to fit a straight line to a series of, usually, three points derived from the experimental data. The coordinate axes represent the average effective normal stress and the shear stress resulting from different consolidation stresses applied to the soil sample. The straight line is then used to determine two parameters that characterize the soil: cohesion and angle of resistance stress, directly from the intersection with the ordinate and the slope of the line, respectively.

The analysis will be centred on problems linked to the construction of a straight line with the data usually available from these experiments. Having only three points and no replicates, the uncertainties associated with each point must come from the uncertainty evaluation of their own model functions, which are non-linear ratio models. But if there are results from two experiments at each consolidation stress point, should a rectangular distribution be assumed for each point, especially if the corresponding uncertainty (taken as the standard deviation of the rectangular distribution) is larger than the one calculated from their function models? Since a regression analysis using least squares is typically based on an underlying Gaussian assumption for each point, what is the magnitude of the approximation entailed by this new assumption? And finally, having three replicates at each point, instead of just two, should a t-distribution be used in this instance?

All these questions will be analysed, and a Monte Carlo method [1] will be used to underline the poor approximation of the GUM [2] in certain ratio models, and the regression analysis will take into account the uncertainty in both coordinate axes [3]. A Monte Carlo method [4] is also an appropriate approach to discuss the approximation made by assuming a rectangular distribution associated with the points of the regression. Finally, an extension to multivariate regression analysis will be attempted.

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The impact of this work can be considerable and go beyond the strict area of measurement science, since the findings related to errors and approximations made when working with few data is potentially relevant to a variety of fields in sciences. More specifically, the area of destructive testing is a very broad field, with expensive experiments and scarcity of data that can adequately be represented by the examples studied in this paper.

References

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