

RESEARCH ARTICLE



Improving mixing and renewal in drinking water storage tanks: lessons learnt and practical measures

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ABSTRACT

This paper investigates the effect of different types of structural and operational measures on water mixing and renewal time in circular and rectangular cross-section water storage tanks, aiming at a better understanding of the flow dynamics to find practicable solutions to improve their design, rehabilitation and operation. An experimental programme, including traditional tracer and dye tracer tests, was carried out in small-scale tanks for different configurations and operating conditions. Two tanks were tested with and without interior structures, with the inlet/outlet pipes at different locations and for constant and variable water level. The main findings are that: i) the most effective measure is operating with fill-and-draw cycles, however, for tanks operated nearly full structural measures are recommendable; ii) reducing the inlet pipe diameter and installing nozzles near the tank bottom improve the mixing conditions; iii) the use of baffles is recommendable when the inlet and outlet pipes are very close.

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Water storage tanks; water mixing; water renewal; short-circuiting; improvement measures

1. Introduction

Extensive research has been developed and knowledge attained on design methods for ensuring a better water mixing and renewal inside water storage tanks to comply with water quality standards. These design practices have been widely used and have been validated by field sampling and monitoring. Some studies are limited in their application and tested conditions, but they have given some insights on how to improve the quality of the stored water. It is indisputable that water mixing and ageing are two related phenomena that affect the water quality in storage tanks (Grayman et al. 2004). Although a completely mixed flow is preferred for its minimal potential loss of disinfectant, it is usually not achievable for a full-scale tank because of the (large) size and the limited mixing energy provided by the inflow momentum (Xavier and Janzen 2017). In the case of nonuniform mixing, water may stay for a long time in stagnant zones, which can result in diminished disinfectant (e.g. chlorine) concentration and in higher bacterial regrowth (Fisher et al. 2009; Ho et al. 2016; Rauen, Angeloudis, and Falconer 2012).

Water storage tanks are generally operated without active mixing devices, such as turbines or impellers (specially in Portugal), being the inflow momentum the only driving force for mixing the new water with the stored one. The number, the position and the orientation of the inlet pipes are important factors that affect the degree of mixing inside the tank. About 92% of the tanks in Portugal have only one water inlet pipe and 5% have two inlet pipes (Monteiro et al. 2021).

According to Grayman et al. (2004) and Lemke and Deboer (2013), the inlet pipes must be located to avoid stagnation

zones inside the tank and to promote the water mixing. The water jet formed by the inflow should have sufficient momentum to promote the effective circulation and mixture of the stored water. However, this very much depends on the diameter of the inlet pipe as well as on the flow rate value and time-variation. Turbulent flows lead to a better mixing than laminar flows (Grayman et al. 2004).

The effect of the location and orientation of the inlet pipes on mixing effectiveness has been widely studied (Grayman et al. 2004; Okita and Oyama 1963; Tian and Roberts 2008; Zhang et al. 2014). Okita and Oyama (1963) show that jets are useful for mixing water within the confined geometry of storage tanks, particularly when a vertical jet reaches the water surface or when a horizontal jet reaches the opposite wall and the water reverses its direction. Grayman et al. (2004) experimentally analysed several inlet pipe configurations for a constant water level tank, observing that some may cause poor mixing inside the circular cross-section tanks: the tangential inlet can lead to swirling flow, which may result in a stagnation area in centre of the tank; the inlet pipe extension and directed at the wall and to deflectors do not allow jet to completely develop, which may result in incomplete mixing or long mixing times; and large diameter inlet pipes may lead to low inlet velocity and momentum, which increase mixing times.

Tian and Roberts (2008) studied the influence of different inlet configurations on mixing conditions during the tank filling cycle. The authors carried out experiments in which the quantity, location and direction of the jet were combined. The results show that a higher number of jets is usually better when considering the same direction, and that horizontal jets lead to better mixing conditions than vertical or 45° angle jets.

Both Grayman et al. (2004) and Tian and Roberts 2008a studied the influence of submerged jets on the mixing conditions; however, it is not clear how the influence of a jet would be on the surface of the stored water volume, nor on the resulting flow patterns. The location of the outlet pipe relative to the inlet pipe can also affect the water mixing inside the tank. The existing of an outlet close to the inlet pipe can promote the short-circuiting phenomena, leading to newer water leaving the tank before the water already stored. This dynamics in the flow can lead to the formation of older water pockets inside the tank that have more difficulty getting out. Studies conducted considering both inflow and outflow simultaneously and considering filling and drawing cycles are scarce (Pinheiro et al. 2021; Zhang et al.,2013).

According to Mahmood et al. (2005) the inlet pipe diameter and orientation must be carefully designed to provide adequate mixing within the filling period. Zhang et al. (2013) applied computational fluid dynamics (CFD) to simulate the flow patterns and water age distributions of a rectangular cross-section tank considering a fill-and-draw cycle condition. The results show that, depending on the cycle phase, the flow pattern changes. This means that it is important to consider both the outlet pipe position and the inlet. The same study only considers the submerged inflow. Due to the large dimensions of the tanks and limited mixing energy provided by the inflow momentum, completely mixed flows are usually not achievable and stagnation zones tend to be formed (Grayman et al. 1996). One way to provide a higher inlet momentum is by increasing the flow rate, whenever possible. Other way is by decreasing the inlet pipe diameter. The reduction of the diameter allows to increase the flow velocity and, consequently, the jet momentum. Based on the experiments of small-scale cylindrical tanks, other researchers (Rossman and Grayman 1999; Tian and Roberts 2008) reported that the mixing time was inversely proportional to the square root of inflow momentum. Reducing the nozzle cross-sectional area by half will (theoretically) reduce the mixing time by up to 50%.

Some water utilities have started to develop solutions in which the inlet pipe is extended and has several horizontal and/or vertical flexible nozzles. These nozzles allow to optimize the jet velocity at all flow rates unlike fixed diameter pipes. The combination of an elaborate passive system of pipes inside the tank and multiple nozzles can be something to consider given the resulting effectiveness when well designed, however, it can entail high construction costs. These multiport systems can mix the water up to 50% faster than a single inlet pipe, because the inlet flow momentum is distributed through the tank instead of being concentrated in the inlet area (Duer 2011). Properly designed passive mixing systems require knowledge of jetinduced mixing characteristics as a function of tank shape, flow rate, turnover time, jet momentum and water density differences to determine the proper location, elevation, spacing, discharge angle and jet velocity of inlet and outlet pipes to achieve complete mixing (Duer 2011).

For tanks that have extremely low turnover, one way to mix water is to use an active mixing system that can operate 24 hours/day, if necessary. The application of active mixing devices, such as electric submersible mechanic mixers, solar powered mechanical mixers and air bubblers (Zhang et al., 2013b), are not common measures, given the high costs of the equipment and of operation and maintenance. However, these solutions can be effective, when properly sized, designed and tested. Some are used for disinfectant injection.

The application of structures, like tank inner walls (baffles), is often used to direct the water flow inside the tank. These structures are common in contact tanks in water treatment plants, since they promote plug-flow regimes needed to assure the long and precise contact time between the water and the disinfectant (e.g. ozone or chlorine in primary disinfection) (Angeloudis, Stoesser, and Falconer 2014; Nasyrlayev and Demirel 2022; Rauen, Angeloudis, and Falconer 2012; Taylor, Carlston, and Venayagamoorthy 2015; Wang, Falconer, and R 1998). The use of baffles in water storage tanks is also common in Portugal, in particular in large storage tanks with very close inlet and outlet pipes. In circular cross-section tanks, the use of baffles allows to separate the tank in two half circles and the gap between these two parts tends to be between 25% and 50% of the tank diameter (Figure 1(a)). However, the use of baffles is more frequent in rectangular cross-section tanks, in particular, for larger tank volumes (Figure 1(b)).

In water storage tanks, the inflow contains a residual concentration of a disinfectant, usually chlorine in Portugal,

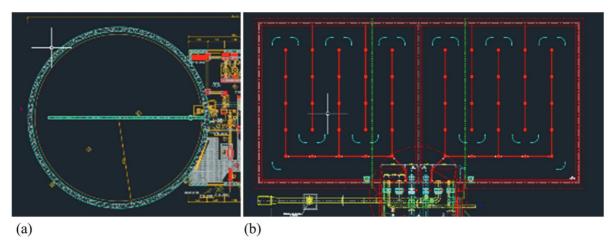


Figure 1. Examples of water storage tanks with baffles in Portugal with: a) circular cross-section, b) rectangular cross-section.



necessary to prevent the deterioration of the stored water. To avoid the loss of this disinfectant residual, regimes close to complete mixing operation are preferable (i.e. the water enters the tank and is automatically mixed with the existing water). In a plug-flow operation, like the one promoted in a tank with baffles, the disinfectant concentration is higher at inlet region and lower at the exit region (depending on the operation, this may be a problem). Some authors recommend not using baffles in water storage tanks since they lead to poorer mixing conditions (Grayman et al. 2004; Zhang et al. 2012; Gualtieri, 2009). However, the application of baffles may not result on worse results in terms of the time needed to replace the water that is in the tank compared to fully open tanks.

Operating tanks with time-variable water levels has shown to be an important operational mechanism to minimize the water age inside storage tanks. The required amount of turnover time depends on the system size and characteristics, being a common turnover goal 3-5 days or 20-33% of the daily fluctuation. However, tanks can have a significant localized increase in water age when they short-circuit and are not completely mixed, even if they are fluctuated 20-33%. Often, the increased water age and all associated water quality problems are specifically attributed to the inlet and outlet pipes (Calabrò and Viviani 2006; Duer 2003; Ghorpade, Sinha, and Kalbar 2021). The influence of a fill-and-draw mode is analysed in Pinheiro et al. (2021), and it should be highlighted that, for water level variations of 50% and 80%, the necessary time to renew the water inside the tank is practically independent of the tank configuration.

Tracer tests are typically used to understand the mixing phenomena in storage tanks both in laboratory and field conditions (Grayman et al. 2004; Rossman and Grayman 1999). However, these testing techniques are limited to one single or to very few sampling points that are clearly insufficient to describe the three-dimensionality nature of water dynamics inside these facilities. Also, computational fluid dynamics (CFD) has become a feasible and reliable method to complement tracer tests and to investigate alternative designs and operating measures to mitigate mixing problems including modelled water velocity profiles, temperature or tracer concentration (Boulos et al. 1996; Duer 2003; Montoya-Pachongo et al. 2016; Nordblom and Bergdahl 2004; Van der Walt 2002; J. M. Zhang et al. 2013), mixing characteristics (Alizadeh Fard, Baruah, and Barkdoll 2021; Meroney and Colorado 2009) and inflow jet conditions (Martins and Covas 2022).

However, little is known about the effect of various structural and operational conditions on the flow dynamics inside the tanks and only for specific conditions. Also, there is no quantification of the effects of structural and operational measures on tanks with the characteristics similar to those existing in Portugal (Monteiro et al. 2021).

This paper presents an extensive experimental study to analyse the effect of different structural measures (i.e. changes in the characteristics of the inlet pipe through the number of nozzles, the size and orientation; use of baffles with different dimensions) and of operational changes (variation in storage volume; flow rates) on water mixing and renewal. This paper investigates the effect of different types of measures (structural and operational) on water mixing in circular and rectangular cross-section tanks. Three structural improvement measures are analysed, namely, the inlet pipe diameter reduction; the use of multiple nozzles; and the use of baffles with different lengths, as well as the operational improvement measure of operating the tank with variable water level. Tests are carried out for the inlet pipe located above the surface of the water volume. This is based on the typical characteristics of existing inlet pipes, as well as of tank filling and emptying cycles as a result of consumption patterns at downstream networks.

This paper presents: i) innovative analyses of several improvement measures of water mixing and renewal in smallscale storage tank; ii) the assessment of measures effectiveness through three hydraulic indexes; and iii) the identification of the most adequate measures for different circular and rectangular cross-section tanks.

2. Experimental research

Small-scale water storage tanks were used to analyse the effects of different types of measures (structural and operational) on water mixing and renewal conditions. Most common circular and rectangular cross-section tanks configurations were tested in concrete tanks.

2.1. Experimental tests

The experimental tests were carried out in a laboratory facility designed to develop tracer tests (Figure 2). Water mixing and renewal improvement measures were analysed using dye tracers' tests (Rhodamine WT 20% at a 1:50 000 dilution) and conductivity tracer tests (sodium chloride). Tracer tests with NaCl have been carried out for a more quantitative analysis and dye tracer tests for a more qualitative analysis. A step input tracer tests were used and consists of the injection of a solution of NaCl (0.05 gL⁻¹) at the inlet of the tank and its monitoring at the exit of the tank. The small-scale tanks were gravity fed with deionized water or with a tracer solution from two secondary tanks, and the outflow was controlled by three peristaltic pumps in parallel. The inflow rate was regulated using a control valve located immediately upstream of the smallscale tank. Experiments were carried out for two different operating conditions: steady-state flow with a constant water level, repeated for three different flow rates and variable water level conditions (fill-and-draw cycles) with different operational levels and flow rates.

Two small-scale tanks with different cross-section made in acrylic were tested. The circular cross-section small-scale tanks used had a diameter of 392 mm and a maximum water-depthto-tank-diameter ratio of 0.15. The tank dimensions were determined by down-scaling sizes and shapes of drinking water storage tanks common in Portugal (Monteiro et al. 2021). Three different inlet/outlet pipe configurations were tested (internal diameter = 4 mm) (Figure 3). In all tested configurations, an inlet pipe was located 14 mm above the maximum water level (70 mm). Circular A has an outflow pipe at the opposite side of the tank, aligned with the inlet pipe, and located 4 mm above the bottom. In circular B, the outlet pipe is near the inlet pipe, with a circumference arc length of 30 mm and 4 mm above the bottom. In circular C, the inlet/outlet pipes

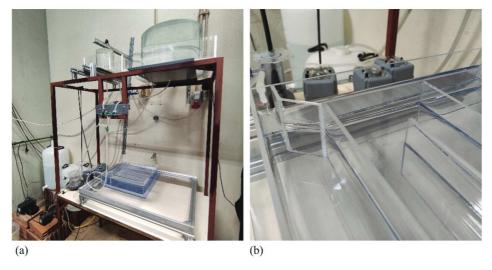


Figure 2. Tracer tests facility pictures: a) general view of the installation, b) small-scale tank.

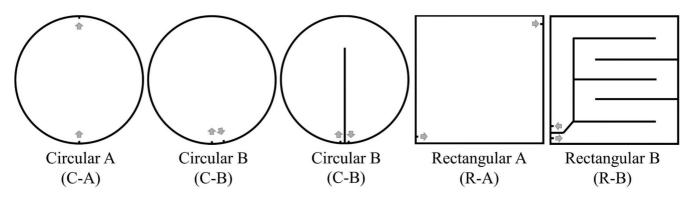


Figure 3. Circular and rectangular cross-section small-scale tanks tested.

location was the same as in circular B, but an acrylic baffle of 75% of the tank diameter and 4 mm of thickness was included. Both rectangular cross-section tanks have a square crosssection (350 \times 350 mm²), allowing a maximum water depth of 70 mm (corresponding to a maximum depth–width ratio of 0.2) (Figure 3). The inlet and outlet pipes have an internal diameter of 4 mm. In both configurations, the inlet pipe is 14 mm above the maximum water level (70 mm) and the outlet pipe at 4 mm above the bottom of the tank. Rectangular A is a tank without baffles with pipes in two opposite corners (Figure 3). Rectangular B has internal baffles that force the flow through a series of 90° and 180° turns (Figure 3). The resulting width of each channel (because of the baffles) is 55 mm; the total length of the water path is approximately 2.2 m. For structural simplification, the small-scale tanks have an inlet pipe perpendicular to the tank wall and above the maximum water height, and the outlet pipe lies above the tank floor.

The methodology used for data acquisition, treatment and analysis was similar to that presented in Pinheiro et al. (2021). Conductivity step tracer experiments were carried out according to two tank operating modes: constant water level and variable water level allowing the quantification of mixing (Morrill index, Mo), short-circuiting (t_{10}) and turnover (t_{95}) characteristics that were used to assess each tested improvement measure.

Tracer tests in small-scale tanks allow to calculate water mixing and renewal indexes extracted from cumulative distribution function, F(t), and residence time distribution function, E(t), and are used to assess water short-circuiting and mixing inside the tank. In the current work, the short-circuiting index used was time t_{10} , which refers to the time needed for 10% of the injected tracer particles to leave the tank. The Morrill index was used to assess mixing conditions, which is defined as the ratio between t_{90} and t_{10} , that is the ratio between the time needed for 90% and 10% of the injected tracer particles to leave the tank, respectively (Teixeira and Siqueira 2008). The Morrill index is approximately 22 for a Continuous Stirred Tank Reactor (CSTR) and 1 for a Plug Flow Reactor (PFR). The time for total water renewal (turnover time), t_{95} , proposed in Pinheiro et al. (2021) and corresponding to F(t) = 0.95, was also estimated.

To compare the experimental data, F(t) and E(t) were normalized. The normalized short-circuiting index and turnover time can be defined as the ratio between t_{10} or t_{95} and τ , respectively, being τ the theoretical residence time given by the ratio between the volume of fluid inside the tank, V, and the flow rate, O.

Dye experiments were carried out to identify the predominant flow patterns and the most important low-velocity regions (almost stagnation areas) for each tank



configuration. The application of a dye tracer allowed a better understanding of the mixing phenomenon that occurs inside the tank. For each configuration tested in dye experiments, the top view was recorded by a GoPro camera.

The complete experimental facility and experimental procedure description are presented in Pinheiro et al. (2021).

2.2. Tested structural measures

The effect of several types of structural measures on water mixing and renewal conditions in circular and rectangular cross-section tanks is analysed, namely: i) the inlet pipe diameter effect (two different diameters were analysed, 2 and 4 mm, in the circular cross-section tank); ii) the use of multiple nozzles effect (three nozzles were tested in each circular crosssection tanks and three nozzles in one rectangular cross-section tank); and iii) the effect of baffles with different lengths (0%, 50% and 75% of the circular cross-section tank diameter).

2.2.1. Reduction of the inlet pipe diameter

Two inlet pipe diameters were tested and compared: the initially tested 4 mm pipe (as described in Pinheiro et al. (2021) and a 2 mm pipe, corresponding to a 50% reduction in diameter. Tested flow rate values are grouped in three categories since it was very difficult to experimentally control such low flows rates: Q1 ranging from 5.0 to 5.4 Lh^{-1} ; Q2 from 7.2 to 7.4 Lh^{-1} ; and Q3 from 9.1 to 9.4 Lh^{-1} .

2.2.2. Use of baffle with different lengths

The effect of baffles with different lengths (50% and 75% of the diameter of the circular cross-section tank) was analysed and compared with a tank without baffles. The circular cross-section tank with an inner wall with a total length of 75% of the tank diameter baffle corresponds to the one studied in Pinheiro et al. (2021) (corresponding to C-A). The second case is a baffle of 50% of the tank diameter. Both cases are compared with the no baffle circular cross-section tank configuration (C-B). Tested flow rates were grouped in four categories: Q1 with 5.3 Lh⁻¹; Q2 with 7.3 Lh⁻¹; Q3 with 9.2 Lh⁻¹; and Q4 with 11.8 Lh⁻¹.

2.2.3. Use of multiple nozzles

Three different nozzle solutions were analysed for the tanks type: Circular A, Circular B and Rectangular A, referred as C-Ai, C-Bi and R-Ai, respectively (being i the identificatory of the nozzle type). The main objectives of these structures are: i) to increase the jet momentum by reducing the diameter of each nozzle (1 mm) with respect to the original inlet pipe diameter (4 mm); and ii) to guide the flow to multiple directions. For the three tank configurations, the inflow pipe with multiple nozzles is submerged, as opposed to the initial conditions analysed in Pinheiro et al. (2021) in which the pipe was above the water level.

For configuration Circular A (C-A), two solutions were analysed with nozzles with an angle of 45° (C-A1 and C-A2) and one solution with nozzles parallel to the bottom of the tank (C-A3) (Figure 4). Since the inlet pipe was submerged and to prevent the formation of a jet directly into the outlet pipe, the nozzles were only directed towards the sides of the tank (Figure 5(a)). According to Grayman et al. (2004), this inlet orientation can promote a poor mixing condition inside the tank, however the authors' experimental conditions did not include the influence of an outflow or nozzles with the proposed orientations. In Circular B (C-B), the objective is to locate the inlet pipe as far as possible from the outlet pipe, to reduce the short-circuiting effect, which is particularly relevant in this configuration. For this purpose, three solutions C - B1, C - B2 and C - B3 were developed (Figure 4) and located 3 cm from the opposite wall of the tank and aligned with the outlet pipe (Figure 5(b)). The nozzle directed towards the tank wall was intended to avoid creating low velocities in that zone of the tank. For Rectangular A (R-A), the effect of three nozzles on mixing was analysed at three locations (Figure 5(c)). The solution R-A1 was located near the tank inlet, R-A2 was located at 1/4 of the diagonal of the tank cross-section and R-A3 was in the middle of the tank (Figure 5(c)).

2.3. Tested operational measures

The tested operational measures aim to simulate the most common operating schemes of storage tanks in water supply systems, with successive fill-and-draw cycles, as a result of daily demand variation at the downstream networks. In

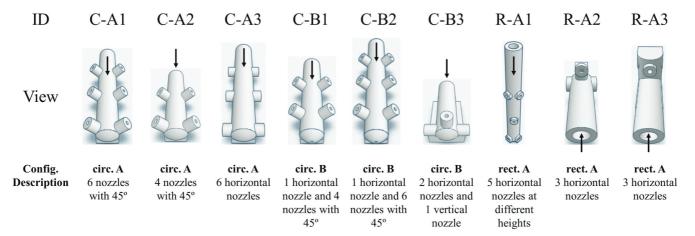


Figure 4. Inlet pipe configurations tested in the circular and rectangular cross-section tanks.

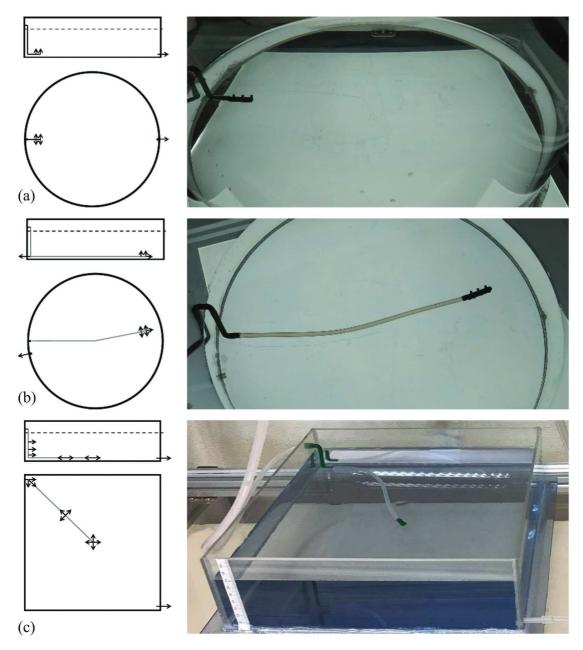


Figure 5. Circular and rectangular cross-section tanks configurations with the location of the nozzles: a) C-A, b) C-B, c) R-A.

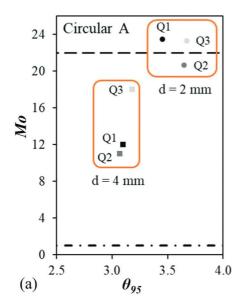
this mode, a variable outflow pattern with four steps is used, as a proxy of the daily demand cycle of a water distribution system.

In this mode, a constant inflow rate is applied until a predefined water level is reached (filling period). Tank filling is resumed when a set minimum water level is attained. During this fill-and-draw mode, the water is continuously drawn out through the outlet pipe at variable flow rates, according to the consumption pattern. Three minimum water levels are tested (20%, 50% and 80% of the maximum water level), which correspond to volume variations of 80%, 50% and 20%, respectively. The experimental procedure description is presented in Pinheiro et al. (2021).

3. Main results

Structural and operational changes on the two most common water storage tank configurations with circular and rectangular cross-section tanks in Portugal are tested for improving water mixing and renewal conditions. Three structural measures are analysed, namely: the inlet pipe diameter reduction; the use of multiple nozzles; and the use of baffles with different lengths. One operational improvement measure which corresponds to operating the tank with variable water level, that is with fill-and-draw cycles with several amplitudes of the volume variation (20%, 50% and 80%) is analysed.





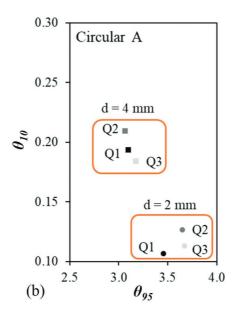


Figure 6. Hydraulic indexes for configuration C-A (Circular A) in steady-state conditions with different inlet pipe diameter: a) Morrill index vs normalized turnover time, b) normalized short-circuiting index vs normalized turnover time. The dashed line corresponds to the Morrill index value for a CSTR and the dotted line corresponds to a PFR

3.1. Effects of the inlet pipe diameter

The momentum of the inlet jet is used to stir the water inside the tank. The inlet pipe orientation and dimension with respect to the tank size and geometry determine the effectiveness of the resulting water mixing. Reducing the diameter of the inlet pipe is a passive method used to promote water mixing since it allows to increase the flow velocity and the kinetic energy of the inflow.

Water mixing in the circular cross-section tank with the inlet and outlet pipes located at opposite side walls (C-A) when reducing the inlet pipe diameter has shown improvements in mixing indicators, for all tested flow rates (Figure 6(a)). Morrill index values approach those of a perfectly mixed tank, namely 22. Also, C-A leads to lower θ_{10} values when the diameter is reduced, which means that 10% of the injected tracer reaches the outlet pipe faster and shows that turnover time increases (Figure 6(b)). According to these results, reducing the inlet pipe diameter for steady-state operation (for constant water level) and, consequently, increasing the flow velocity may not have conclusive results. The fact is that in terms of water mixing, this reduction of diameter is beneficial, but in terms of the time taken to renew stored water it is not.

3.2. Effects of inlet pipe nozzles

The installation of oriented inlet pipes with several nozzles is tested for tank C-A, C-B and R-A to study whether the use of this type of structure improves the water mixing and renewal conditions in existing tanks. This solution consists of extending the pipe from the inlet located above the water level along the wall until the bottom of the tank and then along the tank floor. For configuration C-A, the multiple nozzles are located near the entrance of the tank and with the nozzles with inclinations of 0 and 45°. In C-B, the diffusers are intended to reduce the bypass caused by the proximity of the inlet and the outlet, so the

diffuser was placed at the other end in line with the outlet. In R-A, three locations were tested (Vaz 2021), namely near the entrance, 1/4 and 1/2 of the diagonal of the tank.

In the C-B tank, the use of the diffusers leads to significant improvements in the mixing indexes (Figure 7(a)) and also minimizes the short-circuiting effect, especially the C-B1 diffuser. The three diffusers allow to achieve lower turnover times, being C-B3 the one with the best results (Figure 7(a), (b)). In R-A tank, the three diffusers significantly improve the tank mixing with the diffusers R-A2 and R-A3 having very similar results (Figure 7(c)). In terms of turnover time, all analysed diffusers show slight improvements, especially the R – A1 diffuser (Figure 7(c,d)). The use of diffusers makes it possible to slightly reduce the short-circuiting effect (Figure 7(d)).

3.3. Effects of baffle structures

Building baffles in tanks can direct water through regions of a tank that would otherwise have poor turnover conditions, thus eliminating short-circuiting flow. Optimal configurations can be difficult to determine. Generally, in Portugal, the circular cross-section tanks have a baffle wall, which can have between 50% and 75% of the tank diameter. Circular C (C-C) with two baffles sizes was tested (50% and 75% of diameter) to understand the effect of the size of the baffle on the water mixing and renewal conditions. Figure 8 depicts the evolution of the dye tracer at three instants after the beginning of the injection (1, 5 and 10 min) for the two baffled circular cross-section tanks. The existence of a baffle separating the inlet and outlet pipes causes longer flow paths inside the tanks. These baffles promote recirculation in the first half of the tank, thus delaying the dye from reaching the second half and outlet pipe.

In terms of mixing, the tank with a 50% diameter baffle gives better results especially for lower flow rates (Mo), whereas the tank with a 75% baffle is closer to a PFR (Mo = 1) (Figure 9). As

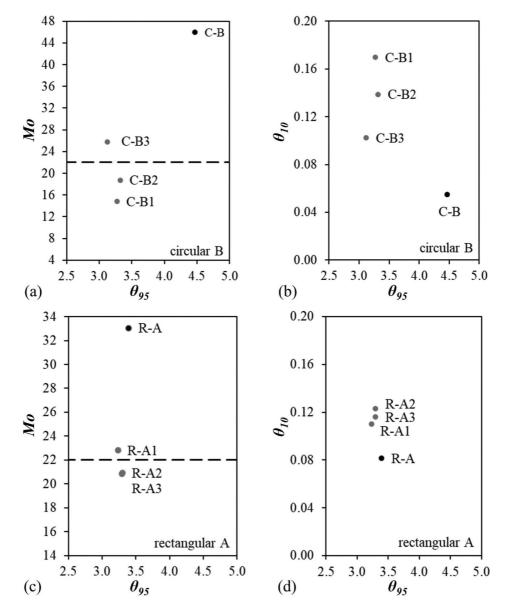


Figure 7. Hydraulic indexes for configuration C-B and R-A in steady-state conditions with different inlet pipe nozzles: a and c) Morrill index vs normalized turnover time, b and d) normalized short-circuiting index vs normalized turnover time. The upper dashed line corresponds to the Morrill index value for a CSTR.

expected, the short-circuiting effect is smaller for the larger baffle since the flow has a longer path. In terms of renewal times, the results are considerably better for the 75% baffle regardless of flow rate.

Effects of operational measures

Tracer tests were carried out in the circular and rectangular cross-section tanks operated at variable water levels (Pinheiro et al. 2021). Three levels of variation in tank volumes were analysed (20%, 50% and 80%) and an outflow pattern was used to simulate the real daily water consumption and storage tanks operating conditions.

The short-circuiting index θ_{10} does not significantly vary with the volume exchange and confirms the strongest short-circuiting effect in C-B (Figure 10). These results suggest that operating the tanks at variable water levels can intensify existing short-circuiting effects, compared with what is observed in

steady-state tests. The Morrill index increases for all configurations operated at variable water levels, in comparison with the steady-state conditions. Such increase in Mo is due to much higher θ_{90} and lower θ_{10} values when the tanks operate at variable water level. Though these conditions are different from the steady-state ones, it seems reasonable to assume that a θ_{90}/θ_{10} ratio of 22, as in a CSTR (ideal reactor), would be the target value for Mo of a homogeneously mixed tank. For all configurations, the high Mo values decrease as the percentage of exchange volume increases. This suggests that the deviation from the ideal mixing condition is higher when the water level variation is small and that increasing the exchange volume in each cycle enhances mixing.

When water volume variation within one cycle is high (80%), all storage tank configurations present similar turnover times suggesting a small effect of the tank configuration on water renewal (Figure 10). However, when volume variation

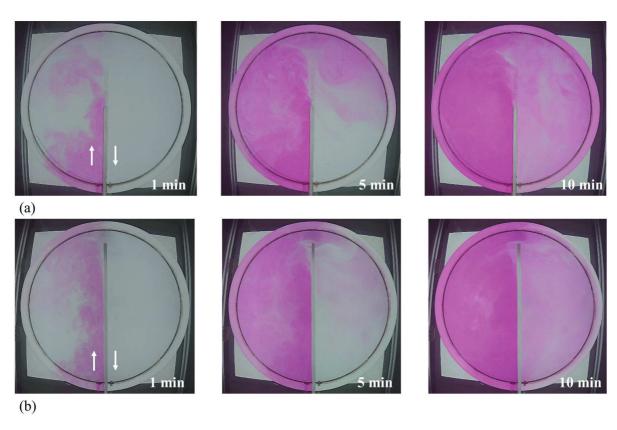


Figure 8. Rhodamine dispersion within the two baffled configurations at 1, 5, and 10 min after injection (constant water level, flow rate Q3): a) baffle 50% tank diameter, b) baffle 75% tank diameter (C-C, (Pinheiro et al. 2021)).

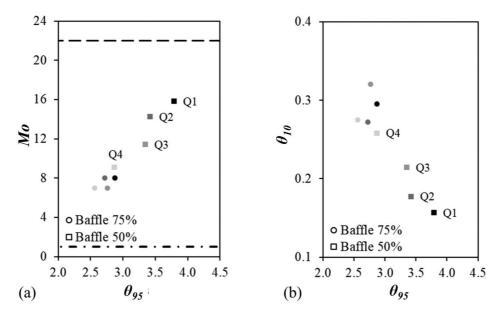


Figure 9. Hydraulic indexes for configurations with baffle with 50% and 75% of the tank diameter in steady-state conditions: a) Morrill index vs normalized turnover time, b) normalized short-circuiting index vs normalized turnover time. The dashed line corresponds to the Morrill index value for a CSTR and the dotted line corresponds to a PFR.

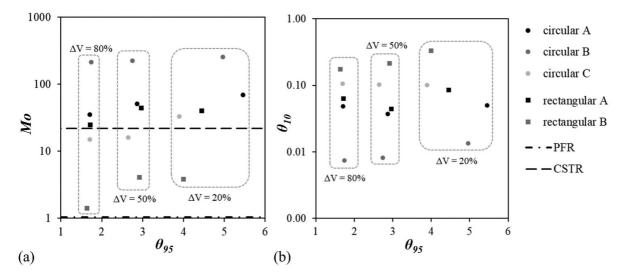


Figure 10. Hydraulic indexes for circular and rectangular cross-section tanks in variable water level condition: a) normalized short-circuiting index vs normalized turnover time, b) Morrill index vs normalized turnover time. The dashed line corresponds to the Morrill index value for a CSTR and the dotted line corresponds to a PFR.

within one cycle is low (20%), which is frequent in daily operation of storage tanks of water supply systems, the turnover time is significantly different for all tanks configurations. For the smaller volume variations (20% to 50%), C-C and R-B promote faster water replacement compared to the other three configurations and C-A presents the highest turnover times. That is the typical tank C-A, which has the potential to achieve good mixing levels, has also to potential to have the longest water retention times, due to the low mixing levels, if inadequately operate.

4. Discussion of improvement measures

Different types of improvement measures on water mixing and renewal in circular and rectangular cross-section tanks were experimentally testes. Three structural measures were tested: the reduction in the diameter of the inlet pipe, the inclusion of baffles and the use of diffusers to guide the flow. Three hydraulic indexes (Morrill index, short-circuiting index, turnover time) are used to assess the short-circuiting and the water mixing, being relevant results summarised in Tables 1 and 2.

Discussion presented herein will account for two aspects: mixing and turnover time. Some tanks may have better mixing conditions than others, but that have much higher times to renew the stored water. There is a tendency, in literature, to give more importance to mixing and hardly any studies are carried on water renewal that can be a relevant issue, depending on the disinfection conditions of the water.

The comparison of different improvement measures for tank C-A, considering the operation close to steady-state condition (i.e. with the tank almost full), is presented in Table 1. The effect of each measure was classified using a colour-scale for a better understanding of the results.

Decreasing the inlet pipe diameter in C-A is a simple way to increase the mixing efficiency inside the tank, regardless of the flow rate in which the tank is operated (see close to 22 values of Morrill index results in Table 1). Likewise, the use of diffusers C – A2 and C - A3 are good options to increase the mixing conditions. However, since the turnover time is also important, it should be noted that reducing the diameter will increase the renewal time by 11% to 19%, while the diffuser C – A2 increases it by 6%. The short-circuiting effect seems to increase by any of the analysed measures, since the inlet flow jet tends to reach faster (given the higher velocities due to the inlet diameter reduction) or to be located closer (nozzles) to the outlet pipe; thus, both reducing the diameter and using diffusers increase the short-circuiting effect. Overall, considering a balance between the three hydraulic indexes, the diffuser C - A3 demonstrates to be the best compromise solution, since both indexes Mo and θ_{95} have improved (Mo = 22 is similar to ideal mixing conditions and the renewal time has a 2% reduction), despite the increase of the short-circuiting effect that is quite similar in all measures analysed.

The comparison of different improvement measures for tank C-B is also presented in Table 1. In C-B, a simple solution to increase water mixing efficiency is to reduce the diameter of the inlet pipe or to use diffusers, like those analysed herein. The construction of baffles with 50% or 75% of the tank diameter will only enhance the plug flow effect and move away from the ideal mixing conditions of a CSTR. In terms of turnover time, any of the solutions demonstrates improvements ranging from 12% to 42%. Overall, considering the balance between the hydraulic indexes, the reduction of the diameter of the inlet pipe and the installation of diffusers are the best compromise solutions to improve water mixing and reduce renewal times.

The comparison of different measures for C-C is presented in Table 1. The mixing conditions in tanks with a baffle with 75%

Table 1. Structural improvement measures tested for C-A, C-B, C-C and R-A with the corresponding hydraulic indexes.

	Inlet pipe	Flow rate	Morrill index		Short-circuiting index			Turnover time		
	(mm)	(-)	Mo (-)	$\theta_{I\theta}(-)$	Variation		$\theta_{95}(-)$	Variation	
	4	Q1	12		0.19			3.10		
Circular A (C-A) initial conditions	4	Q2	11		0.21			3.07		
	4	Q3	18		0.18			3.18		
	4	Q4	15		0.16			3.10		
Inlet diameter reduction	2	Q1	23	•	0.11	- 45 %	•	3.45	+ 11 %	•
	2	Q2	21	•	0.13	- 39 %	•	3.65	+ 19 %	•
	2	Q3	23	•	0.11	- 39 %	•	3.67	+ 15 %	•
Diffusers A	A1 6×1	Q3	18	•	0.13	- 30 %	•	3.18	0 %	•
	4×1	Q3	20	•	0.12	- 36 %	•	3.36	+6%	•
	6×1	Q3	22	•	0.10	- 44 %	•	3.10	-2%	•
Circular B (C-B) initial conditions	4	Q1	27		0.10			4.29		
	4	Q2	36		0.08			4.72		
	1	Q3	46		0.06			4.78		
	4	Q4	70		0.04			4.13		
Inlet diameter reduction	2	Q1	24	•	0.11	+ 10 %	•	3.12	- 27 %	•
	2	Q2	19		0.13	+ 63 %	•	3.15	- 33 %	•
	2	Q3	18		0.13	+ 117 %		3.13	- 34 %	•
Diffusers B2		Q3	15	•	0.17	+ 183 %	•	3.27	- 32 %	•
	-	Q3	19	•	0.14	+ 133 %	•	3.32	- 31 %	•
		Q3	26		0.10	+ 67 %		3.12	- 35 %	
Baffle 50%	4	Q1	16	•	0.16	+ 60 %	•	3.79	- 12 %	•
	4	Q2	14	•	0.18	+ 125 %	•	3.42	- 28 %	•
	4	Q3	11	•	0.10	+ 250 %		3.35	- 30 %	
	4	04	9	•	0.26	+ 550 %	•	2.87	- 31 %	•
Baffle 75% circular C	4	Q1	8	•	0.30	+ 200 %	•	2.87	- 33 %	•
	4	Q2	8		0.30	+ 238 %		2.72	- 42 %	•
	4	Q2 Q3	7	•	0.32	+ 433 %	•	2.76	- 42 % - 42 %	•
	4	Q4	7	•	0.32	+ 575 %		2.56	- 38 %	
Circular C (C-C) initial conditions	4	Q1	8	_	0.30	1 373 70		2.87	30 70	_
	4	Q1 Q2	8		0.30			2.72		
	1	Q2 Q3	7		0.27			2.72		
	4	Q3 Q4	7		0.32			2.76		
Inlet diameter reduction	2	Q4 Q1	27	•	0.27	- 67 %	•	3.53	+ 23 %	_
	2	Q1 Q2	14	•	0.10	- 67 % - 30 %		4.08	+ 23 % + 50 %	
	2	Q2 Q3	12		0.19	- 30 % - 8 %		4.44	+ 61 %	
Baffle 50%	4	Q3 Q1	16	•	0.29	- 47 %	·	3.79	+ 32 %	•
	4		14	•	0.18	- 47 % - 35 %		3.42	+ 32 % + 26 %	•
	4	Q2 Q3	11	•	0.18	- 33 % - 33 %		3.42	+ 26 %	•
	4	Q3 Q4	9		0.21	- 33 % - 6 %		2.87	+ 21 % + 12 %	
D 4 1 4	4	_ `	33	•		- 0 %	•		+ 12 %	•
Rectangular A initial condition		Q2 Q4	26		0.08			3.40 3.13		
A1 A2		Q4 Q2	23	•	0.09	+ 35 %		3.13	- 5 %	•
	-	`	23 20		0.11	+ 35 % + 31 %	•	3.24		-
	5×1	Q4		•			•		-1 %	•
Diffusers		Q2	21	•	0.12	+ 51 %	•	3.30	- 3 % - 4 %	•
	3×1	Q4	18	•	0.14	+ 51 %	•	3.26	+ 4 %	•
1	4×1	Q2	21	•	0.12	+ 42 %	•	3.29	-3%	•
	4×1	Q4	20	•	0.12	+ 29 %	•	3.13	0 %	•

Note: • positive effect; • identical effect; • negative effect

of the tank diameter show improvements when the inlet diameter is reduced (from 4 to 2 mm) and, also, when the baffle size is reduced to 50%. However, these structural measures will increase by 12% to 61% the time required for water renewal. Thus, these two measures should be avoided in tanks with C-C.

The comparison of different diffusors for R-A are presented in Table 1. The mixing conditions in tanks with these diffusors show improvements approaching an ideal reactor. These diffusors will reduce by 29% to 51% the short-circuiting effect. However, the improvements in terms of renewal time are negligible, with higher flows showing worse results compared to lower flows.

The comparison of different fill-and-draw cycles for the two tank configurations (circular and rectangular cross-section) is

Table 2. Hydraulic indexes for the circular and rectangular cross-section tanks for variable water level.

Configuration	Volume variation	Morrill index		Short-circuiting index			Turnover time			
	(%)	Mo (-)		(min)	Variation		<i>t95</i> (min)	Variation		
	20 %	69		5.6			610			
C-A	50 %	51	•	5.0	- 11 %	•	386	−37 %	•	
	80 %	35	•	6.5	+ 16 %	•	230	− 62 %	•	
	20 %	254		1.5			555			
С-В	50 %	223	•	1.1	− 27 %	•	370	− 33 %	•	
	80 %	215	•	1.0	− 33 %	•	235	− 58 %	•	
C-C	20 %	33		11.3			436			
	50 %	16	•	13.9	+ 23 %	•	357	− 18 %	•	
	80 %	15	•	14.3	+ 27 %	•	229	-47%	•	
	20 %	40		9.5			498			
R-A	50 %	44	•	5.9	- 38 %	•	400	- 20 %	•	
	80 %	25	•	8.5	- 11 %	•	231	- 54 %	•	
	20 %	4		37.0			447			
R-B	50 %	4	•	28.7	− 22 %	•	393	- 12 %	•	
	80 %	1	•	23.5	- 36 %	•	220	- 51 %	•	

Note: • positive effect; • identical effect; • negative effect

presented in Table 2. Each fill-and-draw cycle is characterized by a volume variation of 20%, 50% and 80% of the total water volume. The comparison is made with the smallest volume variation (20%). The mixing conditions in the three circular cross-section tanks show improvements when the volume variation is higher, especially in C-A and C-C. The short-circuiting effect is increased in C-B due to the proximity of the inlet and outlet pipes. Regarding the water renewal time, there is a significant improvement for the three configurations with improvements between 18% and 62%. For the three circular configurations, higher inflow rates (Q3 and Q4) and higher water volume variations in each cycle can result in good mixing levels in such tanks. Thus, from the operational point of view, larger variations in water level are better for improving water mixing and renewal than smaller and more frequent water level variations (Table 2).

5. Conclusions

The effect of different types of measures on water mixing in circular and rectangular cross-section tanks was investigated. Several structural improvement measures were analysed: i) the inlet pipe diameter effect; ii) the use of multiple nozzles effect; iii) the effect of baffles with different lengths; and iv) one operational improvement measure which corresponds to operating the tank with variable water level.

The reduction of the inlet pipe diameter is an efficient structural measure to improve mixing in the three circular cross-section tanks, however only in C-B this measure can contribute positively to lower short-circuiting effect and lower turnover time as the inlet moves away from the outlet pipe.

The use of multiple nozzles directed to the horizontal or making a 45° angle with the horizontal located at different areas of the tank (both circular and rectangular cross-section tanks) near the entrance or near the bottom far from the inlet pipe is another structural measure that contributes to improve mixing. The use of this type of structures contributes significantly to lower turnover times especially on C-B and has little significance in the C-A and R-A configurations.

The use of baffles aims to direct the water flow inside the tank and promote the plug-flow phenomena that consequently worsen the water mixing conditions. However, these structures can reduce the short-circuiting effect and reduce the renewal time. The use of higher-size baffles (i.e. 75% of the diameter, instead of 50%) is more efficient for reducing the turnover time in tanks in close inlet/outlet pipes, however it impairs the mixing conditions.

When operating tanks with fill-and-draw cycles, the larger the water volume variation is, the better the mixing conditions and lower is the renewal time and the shortcircuiting effect. Thus, operating tanks with fill-and-draw cycles is preferable to operating the tanks with quasiconstant water level or with minor but frequent variations of the water level, which is the current practice of many water utilities to increase the global reliability of the distribution system in case of supply disruption upstream the storage tank.

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Data availability statement

The data that support the findings of this study are available from the corresponding author, Pinheiro, A., upon reasonable request.

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