

Steel bars complying with Eurocode 2 used in reinforced concrete in Portugal. Variation of their yield strength with the rebar diameter

Varões de aço conformes com o Eurocódigo 2 utilizados em betão armado em Portugal. Variação da sua tensão de cedência com o diâmetro do varão

António Manuel Baptista

Abstract

The yield strength of the steel rebars is one of their essential characteristics considered in the safety checking of reinforced concrete structures. It may present substantial variations, due to the influence of different factors.

This paper presents a statistical study of the yield strength R_e of A500 steel rebars used in Portugal, from three different ductility classes according to the Eurocode 2. The statistical distributions of R_e are based on a very large collection of more than 30 000 test results, obtained from 1994 to 2019. These rebars were produced by several dozens of different manufacturers, mostly from foreign countries.

The detailed data presented here, about the main statistical parameters of the R_e distributions and their variation with the rebars grade and diameter, for instance, may be useful for different studies, such as structural refurbishment projects to evaluate the safety of existing structures, in their actual state or after their restoration.

Resumo

A tensão de cedência dos varões de aço é uma das suas características essenciais consideradas na verificação da segurança de estruturas de betão armado. O seu valor pode apresentar variações importantes, devido à influência de vários fatores.

Este artigo apresenta um estudo estatístico da tensão de cedência R_e de varões A500 utilizados em Portugal, de três classes de ductilidade diferentes previstas no Eurocódigo 2. As distribuições estatísticas de R_e baseiam-se num conjunto de mais de 30 000 resultados de ensaios, obtidos entre 1994 e 2019. Estes varões foram produzidos por várias dúzias de fabricantes, maioritariamente de países estrangeiros.

A informação detalhada sobre os principais parâmetros das distribuições estatísticas de R_e e sua variação com o tipo e diâmetro dos varões p.ex., podem ser úteis em diversos tipos de estudos, nomeadamente em projetos de reabilitação estrutural para avaliar a segurança de estruturas existentes, no seu estado atual ou após a sua reabilitação.

Keywords: Reinforced concrete / Eurocode 2 / Steel rebars / Yield strength / Diameter / Statistical analysis

Palavras-chave: Betão armado / Eurocódigo 2 / Varões de aço / Tensão de cedência / Diâmetro / Análise estatística

António Manuel Baptista

Laboratório Nacional de Engenharia Civil
Lisboa, Portugal
ambaptista@lnec.pt

Aviso legal

Os conteúdos incluídos na Revista Portuguesa de Engenharia de Estruturas são da exclusiva responsabilidade dos seus autores.

Legal notice

The contents included in the Portuguese Journal of Structural Engineering are the sole responsibility of the authors.

BAPTISTA, A. – Steel bars complying with Eurocode 2 used in reinforced concrete in Portugal. Variation of their yield strength with the rebar diameter **Revista Portuguesa de Engenharia de Estruturas**. Ed. LNEC. Série III. n.º 21. ISSN 2183-8488. (março 2023) 49-64.

1 Introduction

The European Standard EN 1992-1-1:2004 (Eurocode 2, Part 1-1) [1] describes the principles and requirements for safety, serviceability and durability of reinforced and prestressed concrete structures, together with specific provisions for buildings. The Eurocode 2 (EC2) applies to ribbed and weldable reinforcement. The application rules for design and detailing in EC2 are valid for a specified characteristic yield strength range, from 400 N/mm² to 600 N/mm².

The Annex C of Eurocode 2 establishes the properties of reinforcement suitable for use with this Eurocode. This Annex C specifies three different ductility classes (A, B and C), but it does not define any steel grades.

The requirements for the steel reinforcing bars (rebars) to be used in reinforced concrete structures in Portugal are established in the LNEC Specifications ([2], [3], [4]) which contemplate two different strength classes (A400 and A500) and three different ductility classes (ER, NR and NR SD). These three ductility classes correspond to the three ductility classes specified in the Annex C of Eurocode 2 [1]: A (ER), B (NR) and C (NR SD).

The yield strength R_e of the rebars may present substantial variations, which depend on the grade of the steel rebars, on their origin, on the manufacturing processes, on the sampling procedures, on the diameter of the rebars and on the test methods used in its assessment, for instance.

The statistical distribution of the rebars yield strength has been studied by several authors from different regions of the world, such as North America [5], Africa [6], Middle East [7], [8], Asia [9] and Oceania [10]. However, in some cases, these studies were based in relatively small samples of test results, which might not be representative of the global distributions of the yield strength for all the steel rebars used in those regions.

The National Laboratory for Civil Engineering (LNEC), located in Lisbon, has ensured, along the last three decades, the technical support for the periodic control of the manufacture of the steel rebars used as reinforcements in concrete structures built in Portugal. Thus, LNEC has collected a very large number of experimental results, which provide representative samples of different characteristics of these steel rebars. These results have been used to support statistical studies about these characteristics, namely about the rebars yield strength [11], [12], [13].

The steel bars used in a specific construction often come from a single manufacturer and from a limited number of production batches. For this reason, a detailed statistical study on the variation of the rebars yield strength distributions with those factors may help a better assessment of the range of the real values of the yield strength used in a particular construction or in some reinforced concrete members, for instance.

Besides its application on building refurbishment studies, the information on these statistical distributions of the steel rebars yield strength may also be useful for the reliability assessment of existing or new constructions by means of probabilistic methods. It may also provide an advantageous tool for theoretical studies, such as the calibration of partial safety factors for these steel rebars.

This paper presents a statistical analysis of the yield strength of A500 steel bars, based on a very large collection of more than 30 000 yield strength values, obtained by the National Laboratory for Civil Engineering (LNEC) between the years of 1994 and 2019, in the frame of Certification Follow-up Audits of the steel rebars manufacturing. These rebars have been produced by several dozens of different manufacturers, most of them from foreign countries.

The differences between the statistical distributions of the steel rebars yield strength, and between their parameters (average, standard deviation and 5% characteristic values), are put in evidence for steel rebars from the three different ductility classes mentioned above (ER, NR and NR SD), with different diameters, made by different manufacturers.

2 Variation of the A500 rebars yield strength with their ductility class

As mentioned before, the Annex C of Eurocode 2 specifies three different ductility classes (A, B and C), which correspond to the ER, NR and NR SD ductility classes, respectively, defined in the LNEC Specifications [2], [3] and [4].

The statistical distributions of the steel rebars yield strength values, R_e , gathered by LNEC between 1994 and 2019, are quite different from one ductility class to another, as shown in Figure 1.

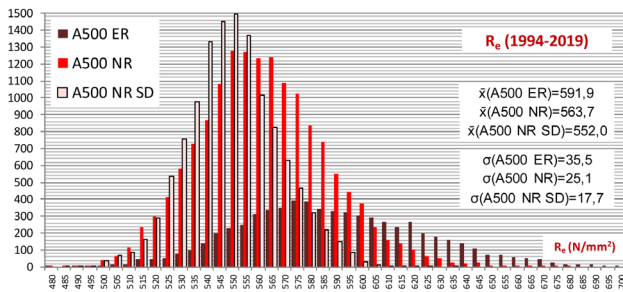


Figure 1 Histogram of the yield strength values R_e (N/mm²) obtained by LNEC and the manufacturers

The R_e distribution of the A500 ER cold-rolled rebars presents the highest average value \bar{x} and standard deviation value σ . On the other hand, the R_e distribution of the A500 NR SD special ductility hot-rolled rebars presents the smallest average value \bar{x} and standard deviation value σ , which indicates a better quality control of their manufacturing.

The R_e distribution of the A500 NR high ductility hot-rolled rebars is closer to the one of the A500 NR SD rebars. Although its average value \bar{x} and standard deviation value σ are larger than those of the A500 NR SD rebars, the 5% characteristic values $x_{5\%}$ of these two distributions are practically the same (522,1 N/mm² or 522,6 N/mm², respectively); both these $x_{5\%}$ values comply with the minimum limit of 500 N/mm² established for these two grades [2], [4].

The 5% characteristic value $x_{5\%}$ of the A500 ER rebars is 2% larger than those mentioned before; therefore, it also complies with its minimum limit (500 N/mm²) [3].

The experimental results presented in Figure 1 are supposed to follow normal distributions. This hypothesis is supported by the very large size of the results samples taken in account in these histograms (more than 15 000 or 12 000 results, in the cases of the A500 NR and A500 NR SD rebars, respectively).

According to this assumption, the 5% characteristic values $x_{5\%}$ have been evaluated, for each R_e distribution, with a 90% confidence level, by means of equation (1), where \bar{x} and σ represent, respectively, the average and standard deviation of the set of R_e values taken in account for that distribution, and $k_{5\%,n}$ is a coefficient that depends on the dimension n of this set of R_e values:

$$x_{5\%} = \bar{x} - k_{5\%,n} \sigma \quad (1)$$

Due to the large values of n used in this study, the \bar{x} and σ values are supposed to be very close to the average and standard deviation of the R_e distributions of the whole production of these rebars, during the same period of time, by the same manufacturers.

In order to access the reliability of equation (1), for the evaluation of the $x_{5\%}$ values, these characteristic values have been evaluated directly by means of the cumulated frequency of the observed R_e values from each sample. The $x_{5\%}^*$ value obtained by this procedure corresponds to the R_e value for which 0,05 n values of R_e from the sample in question are less or equal than $x_{5\%}^*$. The comparison between these $x_{5\%}^*$ values and the $x_{5\%}$ values obtained by means of equation (1) is presented in Table 1.

Table 1 shows that the estimation of the 5% characteristic values $x_{5\%}$ by means of equation (1) gives a conservative approach of the $x_{5\%}^*$ values obtained directly from the cumulated frequencies of the R_e values from each sample. This conclusion sustains the validity of the characteristic values presented along the following sections of this paper, which have been evaluated by means of equation (1).

Table 1 Comparison between the $x_{5\%}^*$ and the $x_{5\%}$ values

A500 ER welded fabric rebars		A500 ER rebars		A500 NR rebars		A500 NR SD rebars	
$x_{5\%}$ (N/mm ²)	$x_{5\%}^*$ (N/mm ²)	$x_{5\%}$ (N/mm ²)	$x_{5\%}^*$ (N/mm ²)	$x_{5\%}$ (N/mm ²)	$x_{5\%}^*$ (N/mm ²)	$x_{5\%}$ (N/mm ²)	$x_{5\%}^*$ (N/mm ²)
535,8	541,9	527,3	533,0	522,1	524,6	522,6	524,4

3 Influence of the source of the experimental data

The R_e test results presented in Figure 1 have been obtained by the manufacturers, from their own checking tests, and by LNEC, from the external supervision tests, in the frame of the periodic Certification Follow-up Audits of the production of the steel rebars.

Since the testing laboratories, from the manufacturers and from LNEC, have different technical operators, using different testing equipment and different testing procedures (within the boundaries of the same testing standard), the influence of the source of the test results on the dispersion of the yield strength values, presented in Figure 1, may be put in question.

In order to clarify this topic, Figure 2 shows a comparison between the statistical distributions of the test results obtained by the manufacturers and those obtained by LNEC, for the yield strength of the A500 ER rebars. It should be mentioned that these cold-rolled rebars do not usually present a yield plateau; hence, the yield strength values indicated in Figure 2 correspond to the 0,2% proof stress, $R_{ep0,2\%}$ of these rebars.

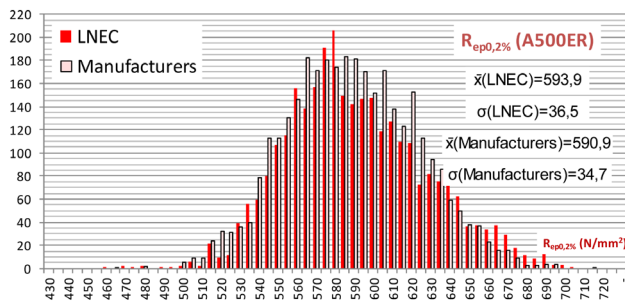


Figure 2 Histogram of the $R_{ep0,2\%}$ values obtained by LNEC or the manufacturers for A500 ER rebars

The examination of Figure 2 shows that, globally, the statistical distributions of the test results obtained by the manufacturers or by LNEC are very close to each other, which justifies their global analysis, as presented in Figure 1. The relative difference between the average values of these two distributions is only 0,5%, and their 5% characteristic values are almost the same.

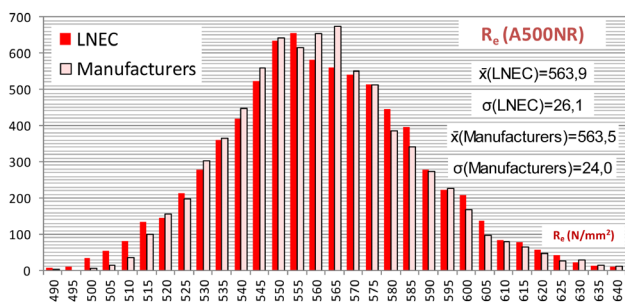


Figure 3 Histogram of the R_e values obtained by LNEC or the manufacturers for A500 NR rebars

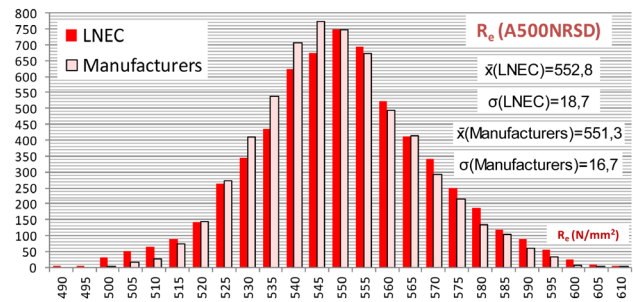


Figure 4 Histogram of the R_e values obtained by LNEC or the manufacturers for A500 NR SD rebars

Figure 3 and Figure 4 present the same comparison, between the statistical distributions of the test results obtained by the manufacturers or by LNEC, for the A500 NR and A500 NR SD rebars, respectively.

Once again, the two statistical distributions presented in each one of these Figures are quite close to each other and their average and 5% characteristic values are practically the same.

Based on these conclusions, it is possible to assume that the source of these experimental data (from the manufacturers or from LNEC) does not affect their global analysis, which allows duplicating the size of the samples analysed in the next chapter.

4 Examples of yield strength distributions for different rebars diameters

As mentioned before, the R_e test results presented in Figure 1 have been obtained by the manufacturers and by LNEC, in the frame of the periodic Certification Follow-up Audits of the production of the steel rebars. The statistical distributions shown in this Figure 1 include the R_e test results obtained for all the rebars diameters controlled in the frame of these Certification Follow-up Audits.

Nevertheless, there are indications that the manufacturing procedures may affect the R_e values, depending on the rebars diameters. This influence has already been studied by other authors [5], [8], [10]; however, the effect of the rebar diameter on the yield strength was not easily explained and it was not possible to establish a clear relation between these two variables.

Hence, in order to study that influence, the main aim of this work consisted in the assessment of the variation of the statistical distributions of the R_e test results, and their main parameters, with the rebars diameters.

Figure 5 shows the statistical distributions of the R_e ($R_{ep0,2\%}$) test results, obtained by the manufacturers and LNEC, for the yield strength of the A500 ER rebars with diameters of 8 mm, 10 mm and 12 mm.

First of all, it may be observed that some of these distributions are quite different from each other and from the global distribution presented in Figure 1, for the cold-rolled A500 ER rebars. Yet, the relative differences between their average values are small (less than

2%) and their standard deviations are reasonably close. Therefore, their 5% characteristic values are quite similar (their relative differences are also smaller than 2%).

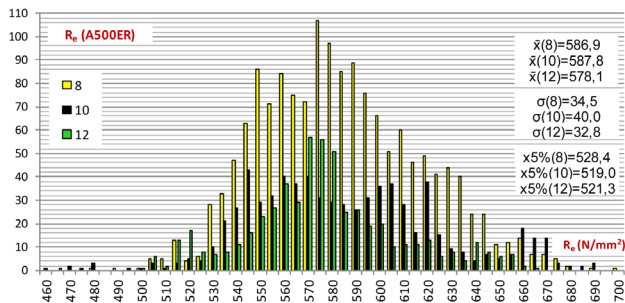


Figure 5 Histograms of the yield strength values R_e ($R_{ep0,2\%}$) of A500 ER rebars with diameters of 8 mm, 10 mm and 12 mm, obtained by LNEC and the manufacturers

The fact that the histograms shown in Figure 5 present a multimodal shape, not as close to a normal distribution as in the case of the global distributions presented in Figure 2, may be related to the smaller size of the corresponding samples of R_e test results and to the consequent lack of diversity of these samples.

These histograms seem to result from the accumulation of different partial distributions that may be associated to different origins (manufacturers) of the rebars. The study of the combined influence of the rebar diameters and their origin, on the main parameters of the statistical distributions of the yield strength values of each type of rebars, will be presented later in this paper.

Figure 6 and Figure 7 show the statistical distributions of the R_e test results obtained for the same set of diameters (8 mm, 10 mm and 12 mm) as before, in the case of the A500 NR and A500 NR SD rebars, respectively.

These figures show that, as regards these ductility classes, the individual R_e statistical distributions are much more similar to each other and to the global distributions shown in Figure 1, for the A500 NR and A500 NR SD rebars. In each case, their standard deviations are very similar.

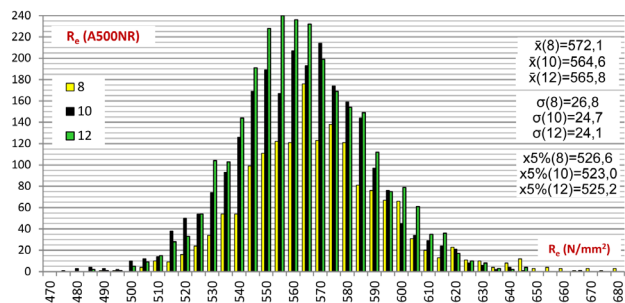


Figure 6 Histograms of the yield strength values R_e of A500 NR rebars with diameters of 8 mm, 10 mm and 12 mm, obtained by LNEC and the manufacturers

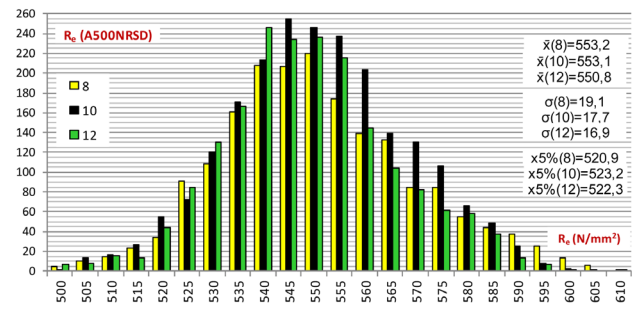


Figure 7 Histograms of the yield strength values R_e of A500 NR SD rebars with diameters of 8 mm, 10 mm and 12 mm, obtained by LNEC and the manufacturers

Therefore, the relative differences between their 5% characteristic values are very small (less than 0,5%, for both the A500 NR and A500 NR SD rebars).

Figure 8 and Figure 9 show the statistical distributions of the R_e test results obtained for the A500 NR and A500 NR SD rebars with diameters of 16 mm, 20 mm and 25 mm.

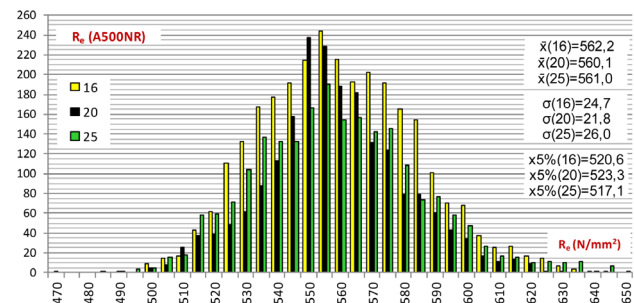


Figure 8 Histograms of the yield strength values R_e of A500 NR rebars with diameters of 16 mm, 20 mm and 25 mm, obtained by LNEC and the manufacturers

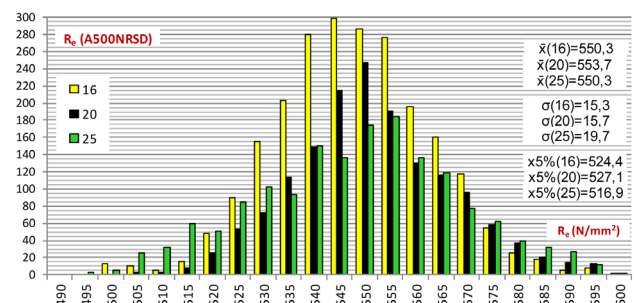


Figure 9 Histograms of the yield strength values R_e of A500 NR SD rebars with diameters of 16 mm, 20 mm and 25 mm, obtained by LNEC and the manufacturers

Once again, the individual R_e statistical distributions are quite similar to each other and to the global distributions shown in Figure 1. The standard deviations of these distributions are a bit larger in the case

of the rebars with the largest diameter (25 mm) than in the case of the other two diameters. Yet, the relative differences between the 5% characteristic values of these distributions are still very small (around 1% for A500 NR rebars and less than 0,5% for A500 NR SD rebars).

Thus, the R_e statistical distributions presented in these figures, for these particular diameters, show that they are not significantly affected by the rebars diameters, especially in the case of the hot rolled A500 NR and A500 NR SD steel rebars. As it concerns the cold rolled A500 ER rebars, these distributions presented some significant differences, from each other and from the global distribution, which may be explained by the reasons mentioned before.

5 Statistical main parameters of the yield strength for different diameters

5.1 Potential applications of the yield strength statistical parameters

The yield strength of the steel rebars is one of the main mechanical characteristics considered in the safety checking of reinforced concrete structures. The safety checking procedure is often based on a comparison between the design values of the acting stresses and of the yield strength of the steel rebars. The design value of the yield strength is evaluated after the division of its characteristic value by a partial safety factor. This characteristic value of the yield strength should be based on only the reinforcement used in a particular structure [1].

However, in practice, that characteristic value is not known and it is taken as the nominal value $R_{e,nom}$ of the yield strength given in the rebars product standard, which is based on the long-term quality level of production. In the particular case of the A500 steel rebars, the nominal value of the yield strength is $R_{e,nom} = 500 \text{ N/mm}^2$ ([2], [3], [4]). This value is supposed to be equal or smaller than the R_e characteristic value of the rebars used in the structure.

In the lack of information about this R_e characteristic value, it may be interesting to have some detailed data about its variation with the rebars steel grade, diameters and year of manufacturing. This information may be used on structural refurbishment studies, to evaluate the safety of existing structures, in their actual state or after their restoration.

It may also be useful to have some information about other statistical parameters of the yield strength distributions, such as their average values and standard deviations, when using probabilistic methods on the reliability assessment of existing or new constructions.

The statistical distributions of the R_e test results presented in the previous chapter have been obtained by the manufacturers and by LNEC, altogether. The charts presented in this chapter give detached data about the statistical parameters of the distributions of the R_e test results obtained by LNEC or by the manufacturers, in the frame of the periodic Certification Follow-up Audits of the production of the steel rebars. These charts allow a comparison between the variations of the yield strength with the rebars diameter, depending on the source of the test results used on the R_e evaluation.

5.2 Variation of the yield strength average values with the rebars diameters

Figure 10 shows the average values \bar{x} of the R_e test results, obtained by LNEC or by the manufacturers for cold-rolled A500 ER rebars with diameters \varnothing from 5,0 mm to 16 mm.

In the case of the LNEC's results, the maximum value of \bar{x} is 631,8 N/mm^2 , for $\varnothing = 5,5 \text{ mm}$, and the minimum value of \bar{x} is 581,7 N/mm^2 , for $\varnothing = 12 \text{ mm}$; so the maximum difference of \bar{x} is $\Delta\bar{x} = 50,0 \text{ N/mm}^2$.

In the case of the manufacturers' results, the maximum value of \bar{x} is 635,3 N/mm^2 , for $\varnothing = 5,5 \text{ mm}$, and the minimum value of \bar{x} is 547,7 N/mm^2 , for $\varnothing = 11 \text{ mm}$; their maximum difference is $\Delta\bar{x} = 87,5 \text{ N/mm}^2$.

Nevertheless, it should be stated that the sample size of the R_e test results available for the diameters of 5,5 mm, 11,0 mm and 14,0 mm is much smaller than in the case of the other diameters. If these diameters were excluded, then the maximum differences of \bar{x} would be $\Delta\bar{x} = 36,1 \text{ N/mm}^2$ and $\Delta\bar{x} = 48,3 \text{ N/mm}^2$, for the LNEC's and manufacturers' results, respectively.

This fact puts in evidence the influence of the small size of test results samples, which may lead either to an overestimation or to an underestimation of their statistical parameters. This may be due to the influence of the variations in the production processes, from one manufacturer to another or from one production batch to another, which is not entirely taken into account in the case of the smaller samples.

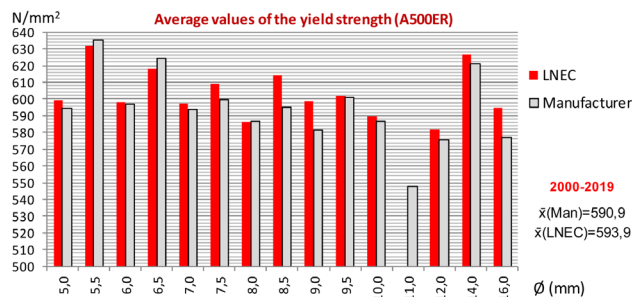


Figure 10 Average values of the yield strength R_e (N/mm^2) of A500 ER rebars with diameters from 5,0 mm to 16 mm, obtained by LNEC or the manufacturers

Figure 11 and Figure 12 show the average values \bar{x} of the R_e test results, obtained by LNEC or by the manufacturers for hot-rolled A500 NR and A500 NR SD rebars with diameters \varnothing from 6,0 mm to 40 mm. It should be mentioned that the sample sizes of the results available for the diameters of 6,0 mm, 14 mm and 40 mm are much smaller than those of the other diameters, for both these types of steel rebars.

As it concerns the LNEC's results obtained for the A500 NR rebars, the maximum value of \bar{x} is 574,1 N/mm^2 (for $\varnothing = 8,0 \text{ mm}$) and the minimum value of \bar{x} is 554,7 N/mm^2 (for $\varnothing = 40 \text{ mm}$); so the maximum difference of \bar{x} is $\Delta\bar{x} = 19,3 \text{ N/mm}^2$. As regards the manufacturers' results, the maximum value of \bar{x} is 570,1 N/mm^2 (for $\varnothing = 8,0 \text{ mm}$), and the minimum value of \bar{x} is 544,0 N/mm^2 , (for $\varnothing = 6,0 \text{ mm}$); their maximum difference is $\Delta\bar{x} = 26,1 \text{ N/mm}^2$.

In the case of the A500 NR SD rebars, the maximum value of \bar{x} for the results obtained by LNEC is 583,3 N/mm² (for $\varnothing = 14$ mm) and the minimum value of \bar{x} is 529,6 N/mm² (for $\varnothing = 40$ mm); so the maximum difference of \bar{x} is $\Delta\bar{x} = 53,6$ N/mm². The maximum value of \bar{x} for the results obtained by the manufacturers is 584,6 N/mm² (for $\varnothing = 14$ mm), and the minimum value of \bar{x} is 535,8 N/mm² (for $\varnothing = 40$ mm); their maximum difference is $\Delta\bar{x} = 48,8$ N/mm².

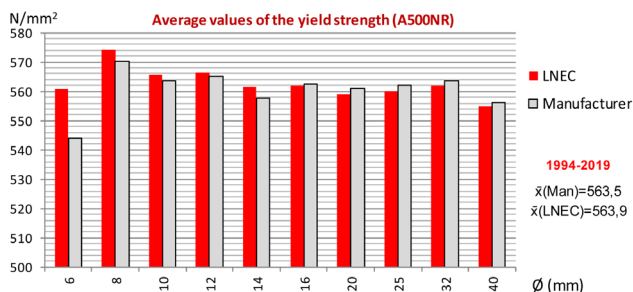


Figure 11 Average values of the yield strength R_e (N/mm²) of A500 NR rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

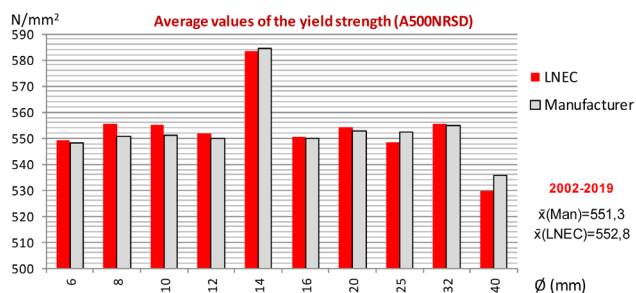


Figure 12 Average values of the yield strength R_e (N/mm²) of A500 NR SD rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

Once again, the above extreme values of \bar{x} may have been affected by the smaller size of the test results. If the 6,0 mm, 14 mm and 40 mm diameters were excluded, then the maximum differences for the A500 NR rebars would be only $\Delta\bar{x} = 14,9$ N/mm² and $\Delta\bar{x} = 9,0$ N/mm², in the case of the LNEC's and manufacturers results respectively. As for the A500 NR SD rebars, the maximum differences would be only $\Delta\bar{x} = 7,3$ N/mm² and $\Delta\bar{x} = 4,9$ N/mm², for the LNEC's and manufacturers results respectively.

Table 2 Variation of the yield strength average values \bar{x} with the rebars diameters

Rebar Diameter (mm)	A500 ER		A500 NR		A500 NR SD	
	\bar{x}_{LNEC} (N/mm ²)	$\bar{x}_{Manuf} - \bar{x}_{LNEC}$ (N/mm ²)	\bar{x}_{LNEC} (N/mm ²)	$\bar{x}_{Manuf} - \bar{x}_{LNEC}$ (N/mm ²)	\bar{x}_{LNEC} (N/mm ²)	$\bar{x}_{Manuf} - \bar{x}_{LNEC}$ (N/mm ²)
5,0	599,3	-5,0	-	-	-	-
5,5	631,8	3,5	-	-	-	-
6,0	597,9	-0,9	560,7	-16,7	549,4	-1,3
6,5	617,8	6,5	-	-	-	-
7,0	597,5	-3,5	-	-	-	-
7,5	609,1	-9,5	-	-	-	-
8,0	586,5	0,4	574,1	-3,9	555,6	-4,9
8,5	614,4	-19,4	-	-	-	-
9,0	598,9	-17,7	-	-	-	-
9,5	601,7	-0,8	-	-	-	-
10	589,3	-2,9	565,8	-2,3	555,0	-3,8
12	581,7	-5,8	566,5	-1,5	551,8	-1,8
14	626,8	-5,8	561,7	-3,9	583,3	1,3
16	594,8	-18,1	561,9	0,8	550,7	-0,9
20	-	-	559,1	2,0	554,3	-1,3
25	-	-	560,0	2,2	548,3	4,1
32	-	-	562,0	1,7	555,6	-0,9
40	-	-	554,7	1,5	529,6	6,1
Max. Δx_{\varnothing}	50,0	87,5	19,3	26,1	53,6	48,8

The above results show that the variations of the yield strength average values with the rebars diameters are much larger in the case of the cold-rolled A500 ER rebars than in the case of the hot-rolled A500 NR and A500 NR SD rebars. In the last case, the A500 NR SD special ductility hot-rolled rebars have the smallest variations of the yield strength average values with the rebars diameters.

Table 2 presents a list of the average values \bar{x}_{LNEC} of the R_e test results obtained by LNEC for each diameter, as well as the difference $\bar{x}_{Manuf} - \bar{x}_{LNEC}$ between the average values of the R_e test results obtained separately by the manufacturers and by LNEC for the same diameter.

Table 2 shows that the maximum difference $\Delta\bar{x}_{\emptyset}$ between the extreme average values of the R_e test results obtained for each diameter \emptyset , may be much larger than the difference $\bar{x}_{Manuf} - \bar{x}_{LNEC}$ for a single diameter, which puts in evidence the influence of the diameter on the average values \bar{x} of the yield strength, when compared to the influence of the source of these test results.

5.3 Variation of the yield strength standard deviation values with the rebars diameters

Figure 13 shows the standard deviation values σ of the R_e test results, obtained by LNEC or by the manufacturers, for cold-rolled A500 ER rebars with diameters \emptyset from 5,0 mm to 16 mm.

As it concerns the LNEC's results, the maximum difference of σ is $\Delta\sigma = 48,6 \text{ N/mm}^2$ and, in the case of the manufacturers' results, it is $\Delta\sigma = 40,0 \text{ N/mm}^2$.

However, if the R_e test results regarding the 5,5 mm, 11,0 mm and 14,0 mm diameters are discarded, due to the smaller size of their test results samples, as mentioned before in 5.2, then the maximum differences of σ would be $\Delta\sigma = 35,2 \text{ N/mm}^2$ and $\Delta\sigma = 24,3 \text{ N/mm}^2$, for the LNEC's and manufacturers' results, respectively.

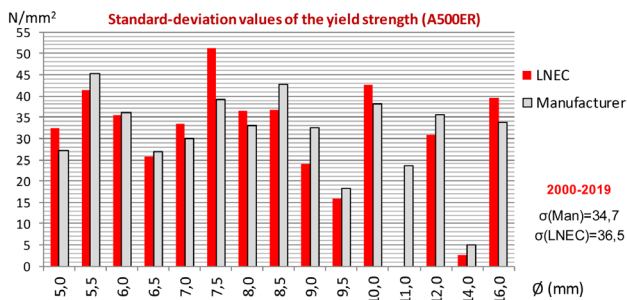


Figure 13 Standard-deviation values of the yield strength R_e (N/mm^2) of A500 ER rebars with diameters from 5,0 mm to 16 mm, obtained by LNEC or the manufacturers

Figure 14 and Figure 15 show the standard deviation values σ of the R_e test results, obtained by LNEC or by the manufacturers, for hot-rolled A500 NR and A500 NR SD rebars with diameters \emptyset from 6,0 mm to 40 mm.

As it concerns the A500 NR rebars, the maximum difference of σ is $\Delta\sigma = 8,1 \text{ N/mm}^2$, for the LNEC's results, and $\Delta\sigma = 15,4 \text{ N/mm}^2$ for the manufacturers' results.

In respect of the A500 NR SD rebars, the maximum difference of σ is $\Delta\sigma = 12,7 \text{ N/mm}^2$, for the LNEC's results, and $\Delta\sigma = 14,2 \text{ N/mm}^2$ for the manufacturers' results.

If the R_e test results regarding the 6,0 mm, 14 mm and 40 mm diameters are discarded, due to the smaller size of their test results samples, as mentioned before in 5.2, then the maximum differences of σ would be $\Delta\sigma = 5,9 \text{ N/mm}^2$ and $\Delta\sigma = 6,2 \text{ N/mm}^2$, for the LNEC's and manufacturers' results, respectively, in the case of the A500 NR rebars. In relation to the A500 NR SD rebars, the maximum differences would be only $\Delta\sigma = 5,2 \text{ N/mm}^2$ and $\Delta\sigma = 4,6 \text{ N/mm}^2$, for the LNEC's and manufacturers results, respectively.

The above results show that the variations of the yield strength standard deviation values with the rebars diameters are much larger in the case of the cold-rolled A500 ER rebars than in the case of the hot-rolled A500 NR and A500 NR SD rebars. Furthermore, these $\Delta\sigma$ variations with the rebars diameters may also depend on the source of the test results, from LNEC or from the manufacturers.

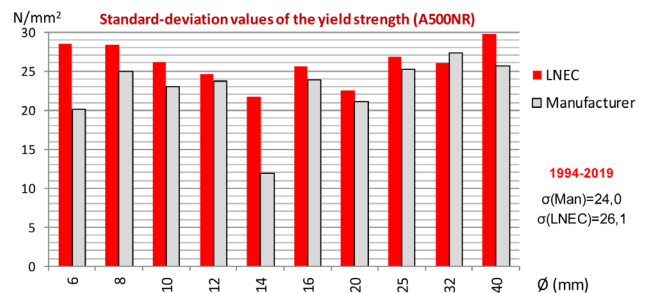


Figure 14 Standard deviation values of the yield strength R_e (N/mm^2) of A500 NR rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

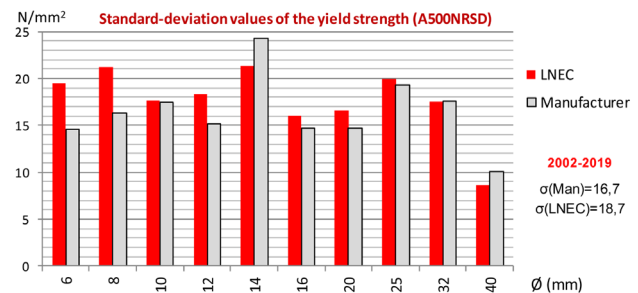


Figure 15 Standard deviation values of the yield strength R_e (N/mm^2) of A500 NR SD rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

Table 3 presents a list of the standard deviation values σ_{LNEC} of the R_e test results obtained by LNEC for each diameter, as well as the difference $\sigma_{Manuf} - \sigma_{LNEC}$ between the standard deviation values of the R_e test results obtained separately by the manufacturers and by LNEC for the same diameter.

Table 3 shows that the maximum difference $\Delta\sigma_{\emptyset}$ between the extreme standard deviation values of the R_e test results obtained for

Table 3 Variation of the yield strength standard deviation values σ with the rebars diameters

Rebar Diameter (mm)	A500 ER		A500 NR		A500 NR SD	
	σ_{LNEC} (N/mm ²)	$\sigma_{Manuf} - \sigma_{LNEC}$ (N/mm ²)	σ_{LNEC} (N/mm ²)	$\sigma_{Manuf} - \sigma_{LNEC}$ (N/mm ²)	σ_{LNEC} (N/mm ²)	$\sigma_{Manuf} - \sigma_{LNEC}$ (N/mm ²)
5,0	32,3	-5,0	-	-	-	-
5,5	41,3	3,9	-	-	-	-
6,0	35,5	0,7	28,4	-8,3	19,5	-4,9
6,5	25,8	1,1	-	-	-	-
7,0	33,5	-3,6	-	-	-	-
7,5	51,2	-11,9	-	-	-	-
8,0	36,5	-3,5	28,4	-3,4	21,2	-4,9
8,5	36,6	6,0	-	-	-	-
9,0	23,9	8,5	-	-	-	-
9,5	15,9	2,4	-	-	-	-
10	42,6	-4,5	26,2	-3,2	17,6	-0,1
12	30,8	4,7	24,6	-0,9	18,4	-3,2
14	2,6	2,6	21,7	-9,8	21,3	3,0
16	39,4	-5,6	25,6	-1,7	16,0	-1,3
20	-	-	22,5	-1,4	16,6	-1,9
25	-	-	26,8	-1,6	20,0	-0,7
32	-	-	26,0	1,3	17,5	0,1
40	-	-	29,8	-4,2	8,6	1,5
Max. $\Delta\sigma_{\phi}$	48,6	40,0	8,1	15,4	12,7	14,2

each diameter ϕ , is usually larger than the difference $\sigma_{Manuf} - \sigma_{LNEC}$ for a single diameter, which puts in evidence the influence of the diameter on the standard deviation values σ of the yield strength, when compared to the influence of the source of these test results.

5.4 Variation of the yield strength 5% characteristic values with the rebars diameters

Figure 16 shows the 5% characteristic values $x_{5\%}$ of the R_e test results, obtained by LNEC or by the manufacturers for cold-rolled A500 ER rebars with diameters ϕ from 5,0 mm to 16 mm.

As it concerns the LNEC's results, the maximum value of $x_{5\%}$ is 620,1 N/mm², in the case of $\phi = 14$ mm, and the minimum value of $x_{5\%}$ is 504,4 N/mm², in the case of $\phi = 7,5$ mm; so the maximum difference of $x_{5\%}$ is $\Delta x_{5\%} = 115,8$ N/mm².

In relation to the manufacturers' results, the maximum value of $x_{5\%}$ is 607,7 N/mm², in the case of $\phi = 14$ mm, and the minimum value of $x_{5\%}$ is 495,4 N/mm², in the case of $\phi = 11$ mm; their maximum difference is $\Delta x_{5\%} = 112,4$ N/mm².

However, as mentioned before, the sample size of the R_e test results available for the diameters of 5,5 mm, 11,0 mm and 14,0 mm is much smaller than in the case of the other diameters. If these diameters were excluded, then the maximum differences of $x_{5\%}$ would

be $\Delta x_{5\%} = 59,8$ N/mm² and $\Delta x_{5\%} = 62,0$ N/mm², for the LNEC's and manufacturers' results, respectively.

Once again, this fact puts in evidence the influence of the small size of test results samples, which may lead either to an overestimation or to an underestimation of their statistical parameters.

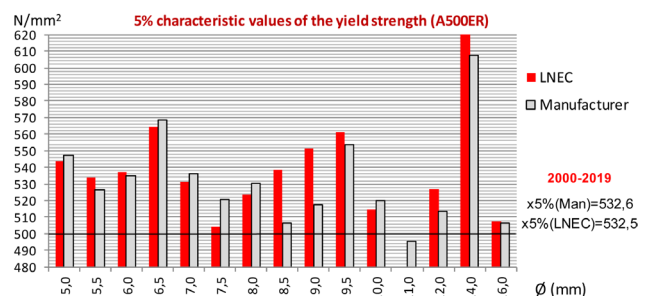


Figure 16 5% characteristic values of the yield strength R_e (N/mm²) of A500 ER rebars with diameters from 5,0 mm to 16 mm, obtained by LNEC or the manufacturers

Furthermore, it is also relevant the fact that the variation of the yield strength 5% characteristic values with the rebars diameters is much larger than the one of the yield strength average values

(see Figure 10). This fact is due to the influence of the variation of the standard deviation values of the yield strength with the rebars diameters (see Figure 13).

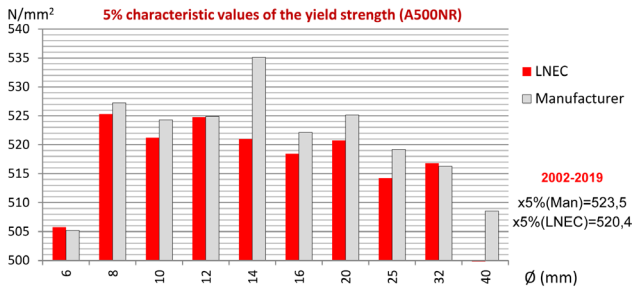


Figure 17 5% characteristic values of the yield strength R_e (N/mm²) of A500 NR rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

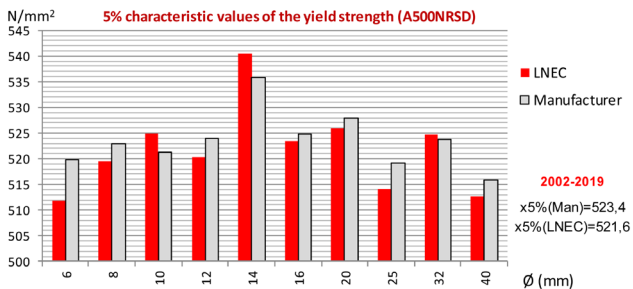


Figure 18 5% characteristic values of the yield strength R_e (N/mm²) of A500 NR SD rebars with diameters from 6,0 mm to 40 mm, obtained by LNEC or the manufacturers

Figure 17 and Figure 18 show the 5% characteristic values $x_{5\%}$ of the R_e test results, obtained by LNEC or by the manufacturers for hot-rolled A500 NR and A500 NR SD rebars with diameters \varnothing from 6,0 mm to 40 mm.

As it concerns the LNEC's results obtained for the A500 NR rebars, the maximum value of $x_{5\%}$ is 525,3 N/mm² (for $\varnothing = 8$ mm), and the minimum value of $x_{5\%}$ is 499,2 N/mm² (for $\varnothing = 40$ mm); so the maximum difference of $x_{5\%}$ is 26,2 N/mm². As regards the manufacturers' results, the maximum value of $x_{5\%}$ is 535,1 N/mm² (for $\varnothing = 14$ mm), and the minimum value of $x_{5\%}$ is 505,2 N/mm² (for $\varnothing = 6,0$ mm); their maximum difference is $\Delta x_{5\%} = 29,9$ N/mm².

In the case of the LNEC's results obtained for the A500 NR SD rebars, the maximum value of $x_{5\%}$ is 540,5 N/mm² (for $\varnothing = 14$ mm), and the minimum value of $x_{5\%}$ is 511,8 N/mm² (for $\varnothing = 6,0$ mm); so the maximum difference of $x_{5\%}$ is 28,7 N/mm². In relation to the manufacturers' results, the maximum value of $x_{5\%}$ is 535,8 N/mm² (for $\varnothing = 14$ mm), and the minimum value of $x_{5\%}$ is 515,9 N/mm², (for $\varnothing = 40$ mm); their maximum difference is $\Delta x_{5\%} = 19,8$ N/mm².

Once again, the above extreme values of $x_{5\%}$ may have been affected by the smaller size of the test results. If the 6,0 mm, 14 mm and 40 mm diameters were excluded, then the maximum differences

would be only $\Delta x_{5\%} = 11,1$ N/mm² and $\Delta x_{5\%} = 10,9$ N/mm² for the LNEC's and manufacturers' results, respectively, in the case of the A500 NR rebars. As for the A500 NR SD rebars, the maximum differences would be only $\Delta x_{5\%} = 11,9$ N/mm² and $\Delta x_{5\%} = 8,6$ N/mm² for the LNEC's and manufacturers results, respectively.

The above results show that the variations of the yield strength 5% characteristic values with the rebars diameters are also much larger in the case of the cold-rolled A500 ER rebars than in the case of the hot-rolled A500 NR and A500 NR SD rebars.

Table 4 presents a list of the 5% characteristic values $x_{5\%,LNEC}$ of the R_e test results obtained by LNEC for each diameter, as well as the difference $x_{5\%,Manuf} - x_{5\%,LNEC}$ between the 5% characteristic values of the R_e test results obtained separately by the manufacturers and by LNEC for the same diameter.

Table 4 shows that the maximum difference $\Delta x_{5\%,\varnothing}$ between the extreme 5% characteristic values of the R_e test results obtained for each diameter \varnothing , is much larger than the difference $x_{5\%,Manuf} - x_{5\%,LNEC}$ for a single diameter, which puts in evidence the influence of the diameter on the 5% characteristic values $x_{5\%}$ of the yield strength, when compared to the influence of the source of these test results.

6 Influence of the origin of the rebars for different diameters

6.1 Yield strength average values of rebars from different manufacturers

It has been mentioned before that there are indications that the manufacturing procedures may affect the R_e values, depending on the rebars diameters.

In order to verify this eventuality, further studies have been carried out to assess the influence of the origin of the steel rebars on the variation of the main statistical parameters of the yield strength distributions for rebars with different diameters.

Figure 19 shows the average values \bar{x} of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for cold-rolled A500 ER rebars with diameters \varnothing from 5,0 mm to 12 mm, made by five different manufacturers, designated as F039, F040, F041, F043 and F059.

The maximum value of \bar{x} is 667,3 N/mm² ($\varnothing = 6,0$ mm) and the minimum value of \bar{x} is 555,1 N/mm² ($\varnothing = 10$ mm); so the global maximum difference of \bar{x} is $\Delta \bar{x} = 112,2$ N/mm².

For a specific diameter, the maximum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta \bar{x} = 77,6$ N/mm² ($\varnothing = 6,0$ mm), for the manufacturers F040 and F059.

The minimum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta \bar{x} = 4,4$ N/mm² ($\varnothing = 7,5$ mm), for the manufacturers F041 and F043. Yet, for this diameter $\varnothing = 7,5$ mm, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above

Table 4 Variation of the yield strength 5% characteristic values $x_{5\%}$ with the rebars diameters

Rebar Diameter (mm)	A500 ER		A500 NR		A500 NR SD	
	$x_{5\% \text{ LNEC}}$ (N/mm ²)	$x_{5\% \text{ Manuf}} - x_{5\% \text{ LNEC}}$ (N/mm ²)	$x_{5\% \text{ LNEC}}$ (N/mm ²)	$x_{5\% \text{ Manuf}} - x_{5\% \text{ LNEC}}$ (N/mm ²)	$x_{5\% \text{ LNEC}}$ (N/mm ²)	$x_{5\% \text{ Manuf}} - x_{5\% \text{ LNEC}}$ (N/mm ²)
5,0	543,6	3,7	–	–	–	–
5,5	534,2	-7,5	–	–	–	–
6,0	536,9	-1,9	505,7	-0,6	511,8	8,1
6,5	564,2	4,2	–	–	–	–
7,0	531,7	4,5	–	–	–	–
7,5	504,4	16,2	–	–	–	–
8,0	523,8	6,4	525,3	1,9	519,4	3,4
8,5	538,3	-31,9	–	–	–	–
9,0	551,4	-33,8	–	–	–	–
9,5	560,8	-6,9	–	–	–	–
10	514,5	5,3	521,2	3,1	524,9	-3,7
12	527,1	-13,8	524,7	0,1	520,4	3,6
14	620,1	-12,4	521,0	14,1	540,5	-4,7
16	507,7	-1,3	518,4	3,7	523,4	1,4
20	–	–	520,7	4,4	525,9	1,9
25	–	–	514,2	5,0	514,0	5,2
32	–	–	516,8	-0,5	524,7	-1,0
40	–	–	499,2	9,4	512,7	3,2
Max. $\Delta x_{5\%,\varnothing}$	115,8	112,4	26,2	29,9	28,7	19,8

are considered, the minimum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\bar{x} = 64,1 \text{ N/mm}^2$ ($\varnothing = 5,0 \text{ mm}$), for the manufacturers F040 and F059.

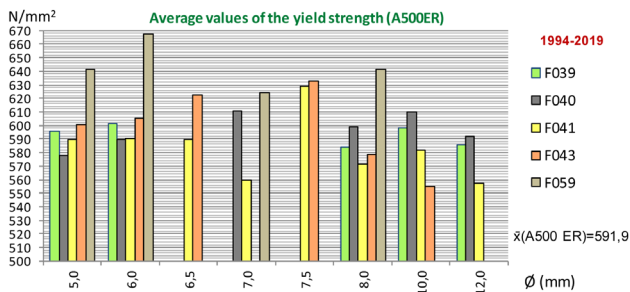


Figure 19 Average values of the yield strength R_e (N/mm²) of A500 ER rebars with diameters from 5,0 mm to 12 mm, made by the manufacturers F039, F040, F041, F043 and F059

Figure 20 shows the average values \bar{x} of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR rebars with diameters \varnothing from 6,0 mm to 32 mm, made by five different manufacturers, designated as F010, F021, F028, F029 and F030.

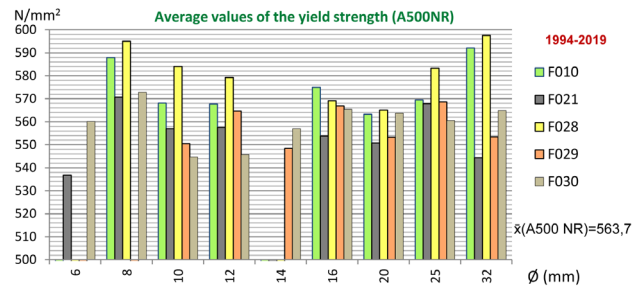


Figure 20 Average values of the yield strength R_e (N/mm²) of A500 NR rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

The maximum value of \bar{x} is 597,6 N/mm² ($\varnothing = 32 \text{ mm}$) and the minimum value of \bar{x} is 536,8 N/mm² ($\varnothing = 6,0 \text{ mm}$); so the global maximum difference of \bar{x} is $\Delta\bar{x} = 60,7 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\bar{x} = 53,2 \text{ N/mm}^2$ ($\varnothing = 32 \text{ mm}$), for the manufacturers F021 and F028.

The minimum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is

$\Delta\bar{x} = 8,5 \text{ N/mm}^2$ ($\varnothing = 14 \text{ mm}$), for the manufacturers F029 and F030. Yet, for this diameter $\varnothing = 14 \text{ mm}$, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\bar{x} = 14,3 \text{ N/mm}^2$ ($\varnothing = 20 \text{ mm}$), for the manufacturers F021 and F028.

Figure 21 shows the average values \bar{x} of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR SD rebars with diameters \varnothing from 6,0 mm to 32 mm, made by five different manufacturers, designated as F010, F021, F028, F029 and F030.

The maximum value of \bar{x} is $574,3 \text{ N/mm}^2$ ($\varnothing = 12 \text{ mm}$) and the minimum value of \bar{x} is $533,7 \text{ N/mm}^2$ ($\varnothing = 6,0 \text{ mm}$); so the global maximum difference of \bar{x} is $\Delta\bar{x} = 40,6 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\bar{x} = 31,5 \text{ N/mm}^2$ ($\varnothing = 12 \text{ mm}$), for the manufacturers F010 and F028.

The minimum difference between the average values \bar{x} of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\bar{x} = 10,3 \text{ N/mm}^2$ ($\varnothing = 8,0 \text{ mm}$), for the manufacturers F021 and F028.

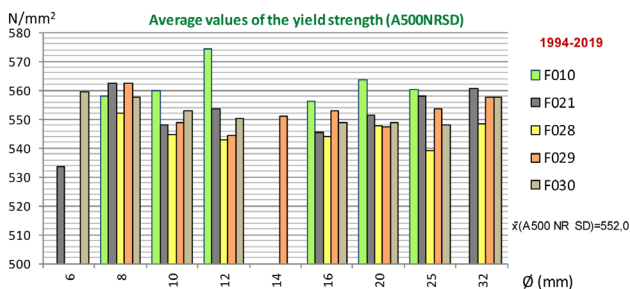


Figure 21 Average values of the yield strength R_e (N/mm^2) of A500 NR SD rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

6.2 Yield strength standard deviation values of rebars from different manufacturers

Figure 22 shows the standard deviation values σ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for cold-rolled A500 ER rebars with diameters \varnothing from 5,0 mm to 12 mm, made by the same five different manufacturers mentioned in 6.1, designated as F039, F040, F041, F043 and F059.

The maximum value of σ is $52,1 \text{ N/mm}^2$ ($\varnothing = 10 \text{ mm}$), and the minimum value of σ is $4,4 \text{ N/mm}^2$ ($\varnothing = 7,0 \text{ mm}$); so the global maximum difference of σ is $\Delta\sigma = 47,7 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the standard deviation values σ of each set of yield strength values,

corresponding to a single manufacturer, is $\Delta\sigma = 45,8 \text{ N/mm}^2$ ($\varnothing = 10 \text{ mm}$), for the manufacturers F039 and F043.

The minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 0,9 \text{ N/mm}^2$ ($\varnothing = 6,5 \text{ mm}$), for the manufacturers F041 and F043. Yet, for this diameter $\varnothing = 6,5 \text{ mm}$, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 14,2 \text{ N/mm}^2$ ($\varnothing = 8,0 \text{ mm}$), for the manufacturers F040 and F041.

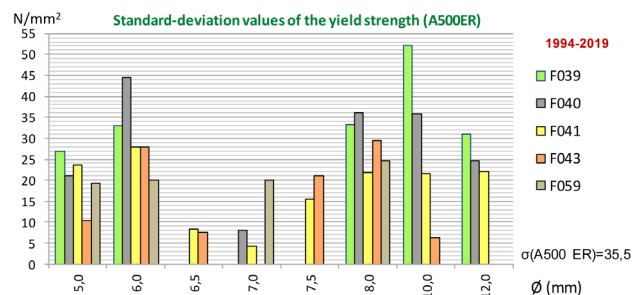


Figure 22 Standard deviation values of the yield strength R_e (N/mm^2) of A500 ER rebars with diameters from 5,0 mm to 12 mm, made by the manufacturers F039, F040, F041, F043 and F059

Figure 23 shows the standard deviation values σ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR rebars with diameters \varnothing from 6,0 mm to 32 mm, made by the same five manufacturers mentioned in 6.1, designated as F010, F021, F028, F029 and F030.

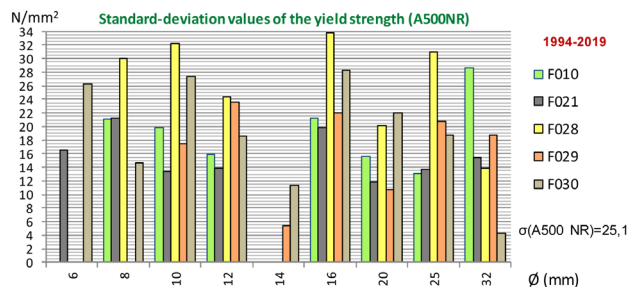


Figure 23 Standard deviation values of the yield strength R_e (N/mm^2) of A500 NR rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

The maximum value of σ is $33,8 \text{ N/mm}^2$ ($\varnothing = 16 \text{ mm}$), and the minimum value of σ is $4,3 \text{ N/mm}^2$ ($\varnothing = 32 \text{ mm}$); so the global maximum difference of σ is $\Delta\sigma = 29,5 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 24,3 \text{ N/mm}^2$ ($\varnothing = 32 \text{ mm}$), for the manufacturers F010 and F030.

The minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 9,8 \text{ N/mm}^2$ ($\varnothing = 6,0 \text{ mm}$), for the manufacturers F021 and F030. Yet, for this diameter $\varnothing = 6,0 \text{ mm}$, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 10,6 \text{ N/mm}^2$ ($\varnothing = 12 \text{ mm}$), for the manufacturers F021 and F028.

Figure 24 shows the standard deviation values σ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR SD rebars with diameters \varnothing from 6,0 mm to 32 mm, made by the same five manufacturers mentioned in 6.1, designated as F010, F021, F028, F029 and F030.

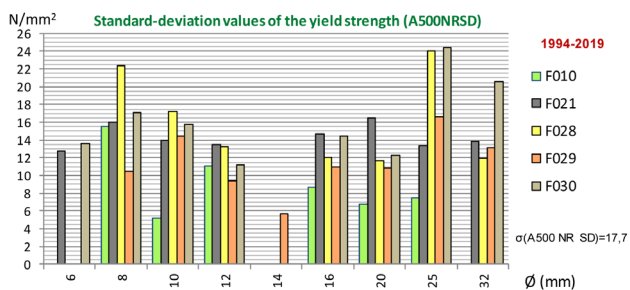


Figure 24 Standard deviation values of the yield strength R_e (N/mm^2) of A500 NR SD rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

The maximum value of σ is $24,4 \text{ N/mm}^2$ ($\varnothing = 25 \text{ mm}$), and the minimum value of σ is $5,2 \text{ N/mm}^2$ ($\varnothing = 10 \text{ mm}$); so the global maximum difference of σ is $\Delta\sigma = 19,2 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 16,9 \text{ N/mm}^2$ ($\varnothing = 25 \text{ mm}$), for the manufacturers F010 and F030.

The minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 0,9 \text{ N/mm}^2$ ($\varnothing = 6,0 \text{ mm}$), for the manufacturers F021 and F030. Yet, for this diameter $\varnothing = 6,0 \text{ mm}$, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the standard deviation values σ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta\sigma = 4,1 \text{ N/mm}^2$ ($\varnothing = 12 \text{ mm}$), for the manufacturers F021 and F029.

6.3 Yield strength 5% characteristic values of rebars from different manufacturers

Figure 25 shows the 5% characteristic values $x_{5\%}$ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for cold-rolled A500 ER rebars with diameters \varnothing from 5,0 mm to

12 mm, made by five different manufacturers, designated as F039, F040, F041, F043 and F059.

The maximum value of $x_{5\%}$ is $629,3 \text{ N/mm}^2$ ($\varnothing = 6,0 \text{ mm}$), and the minimum value of $x_{5\%}$ is $500,1 \text{ N/mm}^2$ ($\varnothing = 10 \text{ mm}$); so the global maximum difference of $x_{5\%}$ is $\Delta x_{5\%} = 129,3 \text{ N/mm}^2$.

For a specific diameter, the maximum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 119,6 \text{ N/mm}^2$ ($\varnothing = 6,0 \text{ mm}$), for the manufacturers F040 and F059.

The minimum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 6,5 \text{ N/mm}^2$ ($\varnothing = 7,5 \text{ mm}$), for the manufacturers F041 and F043. Yet, for this diameter $\varnothing = 7,5 \text{ mm}$, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 67,4 \text{ N/mm}^2$ ($\varnothing = 8,0 \text{ mm}$), for the manufacturers F039 and F059.

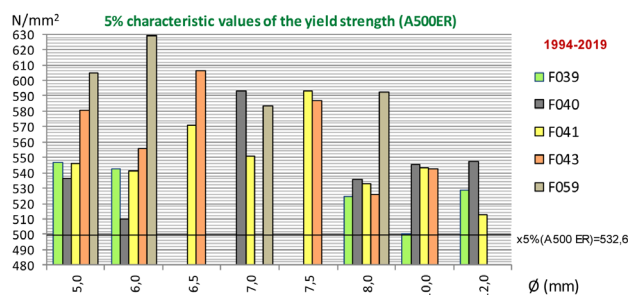


Figure 25 5% characteristic values of the yield strength R_e (N/mm^2) of A500 ER rebars with diameters from 5,0 mm to 12 mm, made by the manufacturers F039, F040, F041, F043 and F059

Figure 26 shows the 5% characteristic values $x_{5\%}$ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR rebars with diameters \varnothing from 6,0 mm to 32 mm, made by five different manufacturers, designated as F010, F021, F028, F029 and F030.

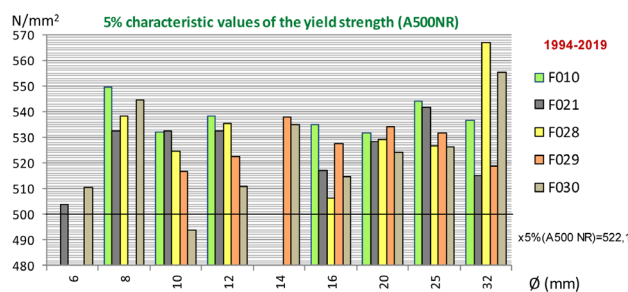


Figure 26 5% characteristic values of the yield strength R_e (N/mm^2) of A500 NR rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

The maximum value of $x_{5\%}$ is 566,9 N/mm² ($\varnothing = 32$ mm) and the minimum value of $x_{5\%}$ is 493,9 N/mm² ($\varnothing = 10$ mm); so the global maximum difference of $x_{5\%}$ is $\Delta x_{5\%} = 73,1$ N/mm².

For a specific diameter, the maximum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 51,8$ N/mm² ($\varnothing = 32$ mm), for the manufacturers F021 and F028.

The minimum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 2,7$ N/mm² ($\varnothing = 14$ mm), for the manufacturers F029 and F030. Yet, for this diameter $\varnothing = 14$ mm, there are R_e test results merely from these two manufacturers; when only the diameters with R_e test results from the five manufacturers mentioned above are considered, the minimum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 9,9$ N/mm² ($\varnothing = 20$ mm), for the manufacturers F029 and F030.

Figure 27 shows the 5% characteristic values $x_{5\%}$ of the R_e test results, obtained by LNEC and by the manufacturers, altogether, for hot-rolled A500 NR SD rebars with diameters \varnothing from 6,0 mm to 32 mm, made by five different manufacturers, designated as F010, F021, F028, F029 and F030.

The maximum value of $x_{5\%}$ is 549,9 N/mm² ($\varnothing = 12$ mm), and the minimum value of $x_{5\%}$ is 496,2 N/mm² ($\varnothing = 25$ mm); so the global maximum difference of $x_{5\%}$ is $\Delta x_{5\%} = 53,8$ N/mm².

For a specific diameter, the maximum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 49,9$ N/mm² ($\varnothing = 25$ mm), for the manufacturers F010 and F028.

The minimum difference between the 5% characteristic values $x_{5\%}$ of each set of yield strength values, corresponding to a single manufacturer, is $\Delta x_{5\%} = 16,8$ N/mm² ($\varnothing = 32$ mm), for the manufacturers F021 and F030.

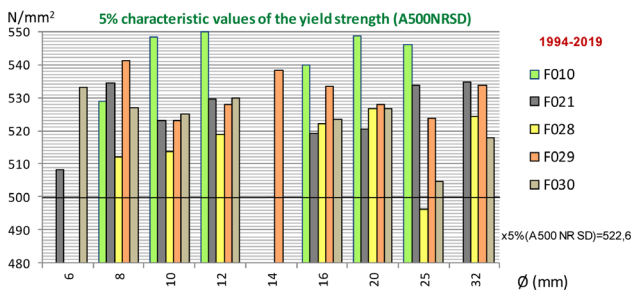


Figure 27 5% characteristic values of the yield strength R_e (N/mm²) of A500 NR SD rebars with diameters from 6,0 mm to 32 mm, made by the manufacturers F010, F021, F028, F029 and F030

Table 5 presents a list of the maximum variations, for each rebar diameter, of the yield strength average values \bar{x} , standard deviation values σ and 5% characteristic values $x_{5\%}$. The values of \bar{x} , σ and $x_{5\%}$ have been calculated separately for each set of test results, obtained

by LNEC and each manufacturer, altogether, for all the rebars with a same diameter, made by the manufacturer in question, between the years of 1994 and 2019.

Table 5 shows that the maximum variations $\Delta \bar{x}$, $\Delta \sigma$ and $\Delta x_{5\%}$ between the extreme values of these parameters, within the full set of manufacturers that made rebars with the diameter in question, are always larger in the case of the cold rolled A500 ER rebars, for any diameter. As it concerns the hot rolled A500 rebars, the maximum variations of those parameters are almost always larger for the A500 NR rebars than for the special ductility A500 NR SD rebars.

Furthermore, the maximum variations $\Delta \bar{x}$, $\Delta \sigma$ and $\Delta x_{5\%}$ between two different manufacturers, may present large differences from one diameter to another, as indicated in the last two lines of Table 5; nevertheless, it was not possible to correlate them with the diameter value.

Yet, the results obtained in the frame of this work (see Figure 25 to Figure 27, for instance) have shown that the 5% characteristic values $x_{5\%}$ of the rebars yield strength usually comply with its nominal minimum limit of 500 N/mm² established for these three grades (A500 ER, A500 NR and A500 NR SD) [2], [3], [4], as assumed in the safety verification of the reinforced concrete structural members, no matter the rebar diameter and its manufacturer.

7 Conclusions

The yield strength of the steel rebars is one of the main mechanical characteristics considered in the safety checking of reinforced concrete structures.

The yield strength of the rebars may present substantial variations, which depend on the grade of the steel rebars, on their origin, on the manufacturing processes, on the sampling procedures, on the diameter of the rebars and on the test methods used in its assessment, for instance.

The steel bars used in a specific construction often come from a single manufacturer and from a limited number of production batches. For this reason, a detailed statistical study on the variation of the rebars yield strength distributions with those factors may help a better assessment of the range of the real values of the yield strength used in a particular construction or in some reinforced concrete members, for instance.

This paper presents a statistical study of the yield strength of A500 steel bars from three different ductility classes (A500 ER, A500 NR and A500 NR SD), which correspond to the ductility classes (A, B and C) defined in the Annex C of Eurocode 2 [1]. The statistical distributions of the yield strength of these rebars, used in Portugal for reinforced concrete structures, were built from a very large collection of more than 30 000 yield strength values, obtained by the National Laboratory for Civil Engineering (LNEC) between the years of 1994 and 2019, in the frame of Certification Follow-up Audits of these steel rebars manufacturing. These rebars have been produced by several dozens of different manufacturers, most of them from foreign countries.

Table 5 Variations, for each diameter, of the yield strength average values \bar{x} , standard deviation values σ and 5% characteristic values $x_{5\%}$ of steel rebars from different manufacturers

Rebar Diameter (mm)	A500 ER			A500 NR			A500 NR SD		
	$\Delta\bar{x}$ (N/mm ²)	$\Delta\sigma$ (N/mm ²)	$\Delta x_{5\%}$ (N/mm ²)	$\Delta\bar{x}$ (N/mm ²)	$\Delta\sigma$ (N/mm ²)	$\Delta x_{5\%}$ (N/mm ²)	$\Delta\bar{x}$ (N/mm ²)	$\Delta\sigma$ (N/mm ²)	$\Delta x_{5\%}$ (N/mm ²)
5,0	81,0	22,8	70,9	–	–	–	–	–	–
5,5	–	–	–	–	–	–	–	–	–
6,0	118,4	30,7	119,6	23,3	9,8	6,8	25,8	4,1	25,0
6,5	60,9	0,9	61,7	–	–	–	–	–	–
7,0	64,0	15,7	43,9	–	–	–	–	–	–
7,5	55,5	32,2	112,1	–	–	–	–	–	–
8,0	90,7	44,7	128,9	76,7	35,1	56,2	31,6	11,9	29,2
8,5	4,9	40,0	83,4	–	–	–	–	–	–
9,0	49,0	17,5	81,3	–	–	–	–	–	–
9,5	–	–	–	–	–	–	–	–	–
10	102,6	45,8	100,1	43,4	21,3	62,4	42,4	16,7	52,1
12	90,3	35,1	57,9	66,8	19,9	70,4	34,4	12,2	47,2
14	–	–	–	64,1	8,5	65,2	44,1	7,9	30,4
16	–	–	–	56,9	28,0	72,2	32,2	10,6	32,7
20	–	–	–	88,2	29,9	90,5	21,1	11,9	28,4
25	–	–	–	98,9	28,8	106,3	47,5	18,4	79,0
32	–	–	–	67,5	32,2	76,6	54,3	13,5	57,4
40	–	–	–	16,7	28,2	49,1	11,9	10,9	13,4
Max. Dif.	118,4	45,8	128,9	98,9	35,1	106,3	54,3	18,4	79,0
Min. Dif.	4,9	0,9	43,9	16,7	8,5	6,8	11,9	4,1	13,4

The global statistical distributions of the steel rebars yield strength values are quite different, from the A500 ER cold-rolled rebars to the A500 NR high ductility and the A500 NR SD special ductility hot-rolled rebars. Yet, their 5% characteristic values $x_{5\%}$ are very similar, and they all comply with the corresponding specified minimum limit of 500 N/mm².

The yield strength test results have been obtained by the manufacturers, from their own checking tests, and by LNEC, from the external supervision tests, in the frame of the periodic Certification Follow-up Audits of the production of the steel rebars. In spite of the different technical operators, testing equipment and testing procedures, the source of the experimental data does not affect their global analysis, as shown by means of their relative comparison.

Yet, a detailed comparative study has been carried out on the variations of the average, standard deviation and 5% characteristic values of the yield strength distributions obtained for each particular diameter of the rebars.

The results of this comparative study have shown that the statistical parameters mentioned above may, eventually, present very

significant variations, from one diameter to another, especially in the case of the cold-rolled A500 ER rebars.

It was also shown that the size of the available test results samples may have a significant impact, since it may lead either to an overestimation or to an underestimation of those statistical parameters.

Finally, some further studies have been carried out to assess the influence of the origin of the steel rebars on the variations of the main statistical parameters of the yield strength values for rebars with different diameters. It was found that the magnitude of these variations, from one manufacturer to another, may also present large differences, depending on the diameter in question.

In the lack of information about the real distribution of the yield strength of the steel rebars used in a specific construction, the detailed data about the statistical parameters presented in this paper and their variation with the rebars steel grade and diameter may be used on structural refurbishment studies, to evaluate the safety of existing structures, in their actual state or after their restoration.

These data may also be useful when using probabilistic methods on the reliability assessment of existing or new constructions.

Acknowledgment

The author wishes to thank the collaboration of Mr. João Filipe (MSc) in the gathering of the test results analyzed in the frame of this work.

Referências

- [1] EN 1992-1-1:2004 (E) – “Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings”, European Committee for Standardization (CEN), 2004, 225 p.
- [2] Especificação LNEC E 450:2017 – “Varões de aço A500 NR para armaduras de betão armado. Características, ensaios e marcação”, Laboratório Nacional de Engenharia Civil, maio 2017.
- [3] Especificação LNEC E 456:2021 – “Varões de aço A500 ER para armaduras de betão armado. Características, ensaios e marcação”, Laboratório Nacional de Engenharia Civil, outubro 2021.
- [4] Especificação LNEC E 460:2017 – “Varões de aço A500 NR de ductilidade especial para armaduras de betão armado. Características, ensaios e marcação”, Laboratório Nacional de Engenharia Civil, janeiro 2017.
- [5] Mirza, Sher Ali; MacGregor, James G. – “Variability of mechanical properties of reinforcing bars”, *Journal of the Structural Division, ASCE*, Vol 105, ST5, May, pp.921-937, 1979.
- [6] Jibrin, Mohammed Umar – “Characterisation of reinforcing steel bars in the Nigerian construction industry”, PhD Dissertation, Department of Civil Engineering, Faculty of Engineering, Ahmadu Bello University, Zaria – Nigeria, August 2012, 205 p.
- [7] Djavanroodi, F.; Salman, A. – “Variability of mechanical properties and weight for reinforcing bar produced in Saudi Arabia”, *International Conference on Materials Sciences and Nanomaterials, IOP Conf. Series: Materials Science and Engineering 230 (2017) 012002*, doi:10.1088/1757-899X/230/1/012002.
- [8] Firat, Fatih K. – “Mechanical properties of reinforcing steel in R/C: uncertainty analysis and proposal of a new material factor”, *Arab J Sci Eng (2016) 41:4019–4028*, doi:10.1007/s13369-016-2077-7.
- [9] Bandara, C.S.; Jayasinghe, J.A.S.C.; Dissanayake, P. B. R. – “Variation of mechanical properties and load carrying capacities of reinforcing steel bars used in Sri Lanka”, *CIDA Journal*, Vol. XV, December 2017, p. 40-48, doi: 10.13140/RG.2.2.30542.97600.
- [10] Lim, Wai Tat – “Statistical analysis of reinforcing steel properties”, MSc Dissertation in Mechanical Engineering, University of Canterbury, Christchurch, New Zealand, June 1991, 139 p.
- [11] Pipa, M. – “Ductilidade de elementos de betão armado sujeitos a ações cíclicas. Influência das características mecânicas das armaduras. Tese de Doutoramento em Engenharia Civil”, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1993, 333 p.
- [12] Baptista, A. M.; Filipe, J. – “Tensão de cedência dos aços de varões para betão armado. Análise da sua distribuição estatística nas duas últimas décadas”, *Encontro Nacional BETÃO ESTRUTURAL - BE2016, FCTUC*, 2016, 10 p.
- [13] Baptista, A. M.; Filipe, J. – “Tensão de cedência de aços de varões A500 para betão armado. Influência da origem dos varões na sua distribuição estatística”, *Reabilitar & Betão Estrutural 2020, LNEC*, 10 p.