Empirical equations for calculating the rate of liquid flow through GCL-geomembrane composite liners

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ABSTRACT: This paper presents equations for the evaluation of advective flow rates though composite liners involving GCLs. Advective flow is due to the existence of defects in the geomembrane, and depends on the contact condition between the geomembrane and the GCL. In this paper the geomembrane–geosynthetic clay liner contact condition is quantitatively defined, based on experimental data. Accordingly, empirical equations are presented for circular defects having diameters in two different ranges: 2 to 20 mm and 100 to 600 mm. The validity of the empirical equation obtained for the smallest range of diameters is compared with experimental results and with an existing empirical equation. The empirical equations developed in this paper are then combined in a simple analytical solution, leading to semi-empirical equations that allow one to predict flow rates for narrow and wide defects, taking account of the flow that takes place at both ends of these defects of finite length. A parametric study shows, through a correlation factor, the importance of the flow at both ends of defects of finite length, mainly for narrow defects.

KEYWORDS: Geosynthetics, Composite liners, Geomembrane, Geosynthetic Clay liner, Defects, Empirical equation, Flow rate

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1. INTRODUCTION

Modern landfills are generally designed to protect the environment against contaminants by using a composite liner. Unfortunately, despite all precautions regarding manufacturing, transportation, handling, storage and installation, defects in the geomembrane (GM) seem to be unavoidable, as shown, for example, by Nosko and Touze-Foltz (2000), Rollin *et al.* (2002) and Needham *et al.* (2004). Defects in the geomembrane represent preferential advective flow paths for leachate migration. Their impact can be minimised by proper design of the landfill liner. It is thus of primary importance to predict the flow rate through composite liners due to the existence of defects in the geomembrane.

Tools currently available for predicting flow rates through composite liners for situations where there exists an interface between geomembrane and soil liner include analytical solutions (Rowe 1998; Touze-Foltz *et al.* 1999; Foose *et al.* 2001) and empirical equations. Analytical solutions are accurate but complex, especially when it is necessary to solve them for circular defects in the geomembrane. Simple tools such as empirical equations (e.g. Giroud and Bonaparte 1989; Giroud *et al.* 1989, 1998; Giroud *et al.* 1992; Giroud 1997; Foose *et al.* 2001; GSE 2001; Touze-Foltz *et al.* 2002; Touze-Foltz and Giroud 2003, 2005; Chai *et al.* 2005; Giroud and Touze-Foltz 2005) are thus preferred by design engineers for evaluating flow rates.

Although the empirical equations have been developed for different contact conditions, such as poor, good and excellent, a geomembrane–geosynthetic clay liner (GCL) contact condition, which will be denoted GM–GCL contact condition in the following, has never been defined, and most of the existing equations were developed for composite liners consisting of a geomembrane and a soil liner. The definition of excellent contact condition proposed by Touze-Foltz and Giroud (2003) can, according to these authors, be used for composite liners consisting of a geomembrane and a GCL overlying a compacted clay liner (CCL) for soil liner equivalent hydraulic conductivities in the range 10^{-10} to 10^{-8} m/s. However, the validity of this assumption has never been experimentally studied. Therefore existing equations need to be extended or validated for composite liners involving GCLs. To the

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authors' knowledge, only Foose *et al.* (2001) have proposed some empirical equations for composite liners involving GCLs, but, even those were applicable to one only type of defect, i.e. narrow defects of infinite length. Furthermore, the empirical equation proposed by GSE (2001) consists of an adaptation of the empirical equations proposed by Giroud (1997) for circular defects, and is applicable for a particular type of composite liner involving geomembrane-supported GCLs. Geomembrane-supported GCLs will not be studied in this paper.

As a result, the first goal of this paper is to propose a relationship to define GM–GCL contact conditions based on a set of experimental data given by Barroso (2005). This relationship is presented in Section 2 of this paper.

The second goal of this paper is to develop empirical equations for circular defects for GM–GCL composite liners, in case the GCL lies on an underlying soil layer.

Relevant parameters that govern the flow through composite liners due to defects in the geomembrane are presented in Section 3. The development of empirical equations for circular defect diameters in the ranges 2 to 20 mm and 100 to 600 mm is presented in Section 4. These empirical equations can be used either to evaluate flow rates linked to the existence of circular defects in flat areas of geomembranes, or to evaluate flow at both ends of defects of finite length. Indeed, in the case of defects of finite length, the analysis is performed assuming the plan view of the defect is a long rectangle with a half-circle at each end. The diameter of the half-circle considered is equal to the width of the rectangle. Consequently, for wrinkle widths in the range 100 to 600 mm, the diameter of the half-circle considered also varies between 100 and 600 mm, which justifies the development of an empirical equation for large circular defects.

The empirical equations developed for circular defects were combined with a simple existing analytical solution given for infinitely long defects, in order to predict flow rates for defects of finite length, according to the rationale used by Giroud and Touze-Foltz (2005). Semi-analytical solutions are therefore obtained. These equations are presented in Section 5.

Section 6 discusses both the accuracy of the empirical equations developed in this paper and the validity of a previously published empirical equation developed for geomembrane-supported GCLs. This validation will in part be based on a comparison between experimental results presented by Barroso (2005) and calculations performed thanks to empirical equations presented in this paper and existing analytical solutions. Section 6 also discusses the importance of flow at the ends of defects of finite length. Finally, Section 7 summarises the main conclusions obtained in this paper.

2. DISCUSSION OF CONTACT CONDITIONS

2.1. General concerns

Among others factors, advective flow (also called 'leakage', and herein simply referred to as 'flow') through a composite liner due to a defect in the geomembrane component of the composite liner depends on the features of the interface between the two components of the composite liner, the geomembrane and the underlying soil liner. The features of the interface are often defined in terms of contact conditions, expressed by a contact factor. The definition of contact conditions was first done in qualitative terms, such as perfect contact (Giroud and Bonaparte 1989), excellent contact conditions (Giroud and Bonaparte 1989; Touze-Foltz and Giroud 2003), good and poor contact (Giroud 1997), and perfect and imperfect contact (Foose *et al.* 2001; Chai *et al.* 2005). The case where there is perfect contact between the two components of the composite liner is not considered in this paper. Only the more complex, and generally more realistic, case described by Brown *et al.* (1987) and Giroud and Bonaparte (1989), where there is an interface between the two components of the composite liner, is considered herein.

Qualitative definitions of contact conditions are subjective. This may lead to different interpretations of a given field case. To overcome this limitation, Rowe (1998) proposed quantitative definitions linking the soil liner hydraulic conductivity to the interface transmissivity for poor and good contact conditions. These quantitative definitions were extended by Touze-Foltz and Giroud (2003) for excellent contact conditions. Furthermore, Touze-Foltz and Giroud (2003) proposed a simplified form of the quantitative definitions given by Rowe (1998) that gives the three parallel lines represented in Figure 1 for poor, good and excellent contact conditions. Defined contact conditions can thus be quantified based on relationships between the hydraulic conductivity of the soil liner and the interface transmissivity.

A drawback of the existing contact conditions is that they were defined for composite liners consisting of a geomembrane and a soil liner, and are valid for soil liner hydraulic conductivities in the range 10^{-10} to 10^{-8} m/s. Consequently, there is no specific GM–GCL contact condition existing at the moment.

2.2. Definition of a GM–GCL contact condition based on experimental data

2.2.1. *Experimental results*

Composite liners consisting of a geomembrane, with a 3 mm-diameter circular defect, a GCL and a CCL were studied in tests at three scales by Barroso (2005), and the flow rate in the interface between the geomembrane and the GCL was measured. Three different geotextile-supported GCLs were used. GCL-1 and GCL-2 were needle-punched, with granular and powdered natural sodium bentonite respectively. GCL-3 was adhesive bond plus semi-needle-punched, with granular sodium bentonite. Details of the features of these products can be found in Barroso (2005). Small-scale tests were performed using a 0.2 m-diameter cell. An intermediate-scale test (IST) was conducted using a 1 m-diameter cell, and a large-scale test (LST) was performed in a square 2.2 m wide test facility. The purpose of the small-scale tests was to

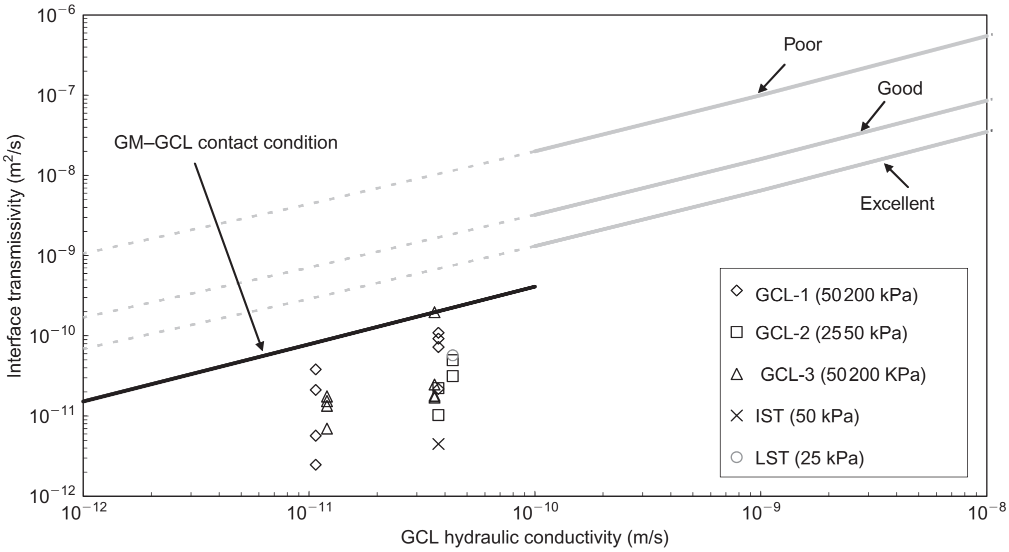

Figure 1. Interface transmissivity against GCL hydraulic conductivity

examine the influence of prehydration of the GCLs, of the increase in confining stress in the 25 to 200 kPa range, and of the hydraulic head on flow rates through composite liners due to circular defects in the geomembrane, whereas the main goal of the intermediate and large-scale tests was to check the feasibility of an extrapolation of results obtained on small-scale tests to field conditions. Only one confining stress was evaluated in the intermediate- (50 kPa) and large-scale tests (25 kPa).

2.2.2. *Interpretation of experimental results in terms of interface transmissivity*

Experimental results obtained by Barroso (2005) were interpreted in terms of hydraulic transmissivity, based on the knowledge of the hydraulic parameters of the GCL and the underlying soil liner, and the size of the testing devices.

Transmissivity values were back calculated from flow rates obtained at steady state using an analytical solution proposed by Touze-Foltz *et al.* (1999) for a hydraulic head equal to zero at a distance corresponding to the radius of the testing device. Indeed, Barroso (2005) systematically observed a flow rate at the outlet of the transmissivity cell in her experiments. The following equation was used accordingly.

$$Q = \pi r_0^2 k_s \frac{h_w + H_s}{H_s} - 2\pi r_0 \theta \alpha \left[A I_1(\alpha r_0) - B K_1(\alpha r_0) \right] \quad (1)$$

where r_0 is the circular defect radius; k_s is the hydraulic conductivity of the soil component of the composite liner; h_w is the hydraulic head on top of the geomembrane; H_s is the thickness of the soil component of the composite liner; θ is the interface transmissivity; I_1 and K_1 are modified Bessel functions of the first order; and α, A and B are parameters given by the following equations.

$$\alpha = \sqrt{\frac{k_s}{\theta H_s}} \quad (2)$$

$$A = -\frac{h_w K_0(\alpha R) + H_s\left[K_0(\alpha R) - K_0(\alpha r_0)\right]}{K_0(\alpha r_0) I_0(\alpha R) - K_0(\alpha R) I_0(\alpha r_0)} \quad (3)$$

$$B = \frac{h_w I_0(\alpha R) + H_s\left[I_0(\alpha R) - I_0(\alpha r_0)\right]}{K_0(\alpha r_0) I_0(\alpha R) - K_0(\alpha R) I_0(\alpha r_0)} \quad (4)$$

where K_0 and I_0 are modified Bessel functions of zero order; and R is the radius of the wetted area, which was in most tests the cell radius. If the wetted radius is unknown, which was the case for IST and LST in which no flow at the outlet of experimental test devices was detected during the 6 month tests, it can be determined by solving the following equation for R (zero hydraulic head at radius R).

$$A I_0(\alpha R) + B K_0(\alpha R) - H_s = 0 \quad (5)$$

As the soil liner is a combination of a GCL and an underlying soil liner in the case studied in this paper, k_s is the equivalent hydraulic conductivity calculated according to the following equation (Rowe 1998):

$$k_s = \frac{H_{GCL} + H_f}{H_{GCL}/k_{GCL} + H_f/k_f} \quad (6)$$

where k_s is the equivalent hydraulic conductivity; k_f is the hydraulic conductivity of the foundation layer (CCL); k_{GCL} is the hydraulic conductivity of the GCL; H_f is the thickness of the underlying soil; and H_{GCL} is the thickness of the GCL. H_s is the total thickness of the soil liner (GCL + CCL), given by

$$H_s = H_{GCL} + H_f \quad (7)$$

Results obtained in terms of transmissivity as a function of hydraulic conductivity of the GCL are plotted in Figure 1. GCL hydraulic conductivities were measured by Barroso (2005) based on flow rate (ASTM D 5887) and on thickness measurements. These measurements were carried out considering two confining stresses: 50 and 200 kPa.

Poor, good and excellent contact conditions are valid for soil liner hydraulic conductivities in the range 10^{-10} to 10^{-8} m/s, and cannot strictly speaking be used for lower hydraulic conductivities. It is tempting to extrapolate them to hydraulic conductivities in the range 10^{-12} to 10^{-10} m/s anyway, which is what was done in Figure 1. The results obtained show that the relationships obtained for poor, good, and even excellent contact conditions overestimate the experimental results obtained.

2.2.3. *GM–GCL contact condition*
Based on the aforementioned comparison of existing contact conditions and experimental data, it is clear that a definition of a GM–GCL contact condition is necessary to quantify more precisely the interface transmissivity for composite liners involving GCLs.

In an attempt to obtain a relationship consistent with the existing relationships linking the hydraulic conductivity of the soil liner, including a GCL and a soil layer in the present case, and the interface transmissivity, it is assumed that the GM–GCL contact condition can be represented by a straight line in log–log scale parallel to the straight lines representing poor, good and excellent contact conditions in Figure 1 defined by Touze-Foltz and Giroud (2003) and passing through the highest value of transmissivity obtained by Barroso (2005) (black solid line in Figure 1). This contact condition will thus represent an upper bound for greater loads applied on the composite bottom liner. From a mathematical point of view, the GCL contact condition can be represented by the following expression, which is consistent with previous formulations of poor, good and excellent contact conditions given by Touze-Foltz and Giroud (2003):

$$\log \theta = -2.2322 + 0.7155 \log k_{\mathrm{GCL}} \quad (8)$$

where θ is the interface transmissivity, and k_{GCL} is the hydraulic conductivity of the GCL component of the composite liner. This equation can only be used with the following units: θ (m^2/s) and k_{GCL} (m/s).

The definition of the GM–GCL contact condition in quantitative terms is a step forward for performing more accurate evaluations of the flow rate, as the interface transmissivity is an input parameter in the analytical solutions used to perform these evaluations. This is also very important in the context of the present work. Indeed, according to the methodology proposed by Touze-Foltz and Giroud (2003), the analytical solutions will be used to assist in the development of empirical equations for the case of circular defects to predict flow rates through composite liners consisting of a geomembrane, a GCL and an underlying soil liner.

3. RANGE OF PARAMETERS ADOPTED

The flow through composite liners due to defects in the geomembrane is related to the type and size of the defects, the hydraulic head above the geomembrane, the hydraulic conductivity and thickness of the GCL and soil layer. Consequently, all these parameters are described in the following paragraphs.

3.1. Type and size of the defects

The types of defect considered in this study are the same as those considered in the series of papers by Giroud and Touze-Foltz (2005) and Touze-Foltz and Giroud (2003, 2005):

- circular defects located in a flat area of the geomembrane (e.g. punctures);
- defects of infinite length located in a flat area of the geomembrane, such as defective seams and long cuts or tears, also called ‘narrow defects’; and
- defects of any shape located on wrinkles in the geomembrane resulting in damaged wrinkles, also called ‘wide defects’.

According to the rationale developed by Giroud and Touze-Foltz (2005), narrow and wide defects can be subdivided into defects of finite length and defects of infinite length. Giroud and Touze-Foltz (2005) have shown that, in the case of defects of finite length, the fraction of the flow rate due to flow at the end of the defect is generally large. Therefore they do not recommend treating defects of finite length as though they had an infinite length. Accordingly, the impact of ends of two-dimensional defects will be considered in equations that will be presented in Section 5.

For the sake of consistency with previous papers by Giroud and Touze-Foltz (2005) and Touze-Foltz and Giroud (2003, 2005) dealing with the development of empirical equations, the same range of parameter values was adopted wherever possible in this paper. Accordingly, the range of defect widths is 2 to 20 mm for narrow defects (tears, cuts and defective seams) and 100 to 600 mm for wide defects (damaged wrinkles), and these dimensions correspond to the range of diameters of circular defects for which empirical equations were developed.

3.2. Hydraulic conductivity and thickness of GCL

The hydraulic conductivity of GCLs depends on confining stress (Estornell and Daniel 1992; Petrov *et al.* 1997; Ruhl and Daniel 1997). Data compiled by Bouazza (2002) and Barroso (2005) indicate that it can vary between 1×10^{-12} and 1×10^{-10} m/s when water is used as permeant. Therefore this range of hydraulic conductivity was adopted in this paper.

GCL thicknesses in the range 6×10^{-3} to 14×10^{-3} m are considered in this paper. This range covers the GCL thicknesses that may be expected in landfills as a result of the coupling effect between confining stress and swelling, according to the results obtained by Lake and Rowe (2000) during the performance of constant stress swell tests.

3.3. Hydraulic head, soil liner hydraulic conductivity and thickness and contact condition

For the sake of consistency with previous papers by Giroud and Touze-Foltz (2005) and Touze-Foltz and Giroud (2003, 2005) dealing with the development of empirical equations, the following range of parameters was adopted in this paper for the hydraulic head, soil thickness and soil hydraulic conductivities, respectively.

- The liquid head on top of the geomembrane is between 0.03 m and 3 m.
- The thickness of the soil component of the composite liner is between 0.3 m and 5 m.
- The hydraulic conductivity of the soil component of the composite liner ranges between 1×10^{-10} m/s and 1×10^{-8} m/s.

The contact condition considered herein is the GM–GCL contact condition defined in Section 2.2.3 by Equation 8.

4. EMPIRICAL EQUATIONS FOR CIRCULAR DEFECTS

4.1. Methodology

In the case of circular defects the methodology used in this paper derives mainly from the methodology adopted by Touze-Foltz and Giroud (2003). It consists in selecting a simple mathematical expression for the empirical equations and selecting values for the unknowns of the empirical equations such that flow rates calculated using the empirical equations are as close as possible to flow rates rigorously calculated using existing analytical solutions.

4.2. Form of the mathematical expression for the empirical equation for circular defects

In order to be consistent with the approach used by Touze-Foltz and Giroud (2003, 2005), the same form of empirical equation, given by Equation 9, was adopted:

$$Q = C_c h_w^{\chi} a^{\xi} k_s^{\kappa} \left[1 + \lambda \left(\frac{h_w}{H_s} \right)^{\mu} \right] \quad (9)$$

where Q is the rate of flow through a composite liner due to a circular defect in the geomembrane component of the composite liner; C_c is the contact condition factor; h_w is the hydraulic head on top of the geomembrane; a is the circular defect area; λ is a factor; H_s is the equivalent thickness of the soil liner (GCL+CCL); and χ, ξ, κ and μ are exponents. Equation 9 can only be used with SI units as follows: Q (m^3/s), h_w (m), a (m^2), k_s (m/s), and H_s (m); dimension of C_c is variable; χ, ξ, κ, λ and μ are dimensionless.

4.3. Determination of the unknowns of the empirical equations

By adopting the same procedure as that used by Touze-Foltz and Giroud (2003), the values of the unknown exponents and factors of Equation 9, i.e. χ, ξ, κ, C_c, λ and μ, are determined by comparing the values of Q calculated using Equation 9 with the values of Q calculated using the analytical solution expressed by Equation 1.

This general methodology gives a range of values for each exponent and factor. As a result, a value of exponent or factor located within the given range is selected. In this paper, the selected value was the mean of values obtained.

Determination of the unknowns of the empirical equations is performed in three steps:

(1) determination of the exponents χ, ξ and κ;
(2) determination of the contact factor (C_c); and
(3) determination of the factor λ and exponent μ.

These steps are detailed in the paper by Touze-Foltz and Giroud (2003), and will not be repeated here for the sake of brevity.

Calculations for more than 32,000 cases were performed to develop theses empirical equations. These 32,000 cases cover the entire range of values of the parameters listed in Section 3.

4.4. Equations obtained for circular defects

The following equation was obtained for circular defects having diameters in the 2 to 20 mm range.

$$Q = 2 \times 10^{-4} h_w^{0.87} a^{0.07} k_s^{0.64} \left[1 + 0.31 \left(\frac{h_w}{H_s} \right)^{0.79} \right] \quad (10)$$

The following equation was obtained for circular defects having diameters in the 100 to 600 mm range.

$$Q = 0.116 a^{0.4} h_w^{0.54} k_s^{0.82} \left[1 - 0.22 \left(\frac{h_w}{H_s} \right)^{-0.35} \right] \quad (11)$$

In these equations Q is the flow rate; h_w is the hydraulic head on top of geomembrane; a is the circular defect area; k_s is the equivalent hydraulic conductivity of the soil liner (GCL + CCL); and H_s is the total thickness of the soil liner. These equations must be used with the following units: Q (m^3/s); h_w (m); a (m^2); k_s (m/s); and H_s (m).

4.5. Modification of the Touze-Foltz and Giroud (2003, 2005) equations for circular defects

The exponents and coefficients obtained in Equations 10 and 11 are different from those obtained by Touze-Foltz and Giroud (2003), who attempted to develop empirical equations exhibiting similar exponents and gradients for all types of contact conditions (poor, good and excellent), for a given type of geomembrane defect.

Accordingly, a second set of empirical equation was elaborated, by simply modifying the value of the contact factor assuming that similar exponents and hydraulic gradient expressions as for the good, poor and excellent contact conditions could be adopted, as in papers from Touze-Foltz and Giroud (2003, 2005) The following equations were thus obtained respectively for circular defect diameters in the range 2 to 20 mm and in the range 100 to 600 mm.

$$Q = 2.4 \times 10^{-3} a^{0.1} h_w^{0.90} k_s^{0.74} \left[1 + 0.1 \left(\frac{h_w}{H_s} \right)^{0.95} \right] \quad (12)$$

$$Q = 0.078 a^{0.18} h_w^{0.84} k_s^{0.77} \left[1 - 0.1 \left(\frac{h_w}{H_s} \right)^{0.027} \right] \quad (13)$$

where Q is the flow rate; h_w is the hydraulic head on top of geomembrane; a is the circular defect area; k_s is the equivalent hydraulic conductivity of the soil liner (GCL + CCL); and H_s is the total liner thickness. These equations must be used with the following units: Q (m^3/s); h_w (m); a (m^2); k_s (m/s); and H_s (m).

The issue is to know for both range of diameters which empirical equation gives the closest results to the analytical solution, i.e. Equation 10 or Equation 12 in the case of circular defects diameters in the range 2 to 20 mm and Equation 11 or Equation 13 in the case of circular defect diameters in the range 100 to 600 mm. To answer this question, more than 8,000 calculations were performed for the range of parameters presented in Section 3. The percentage of the number of cases studied corresponding to the number of calculations performed is plotted against the relative difference between the flow rates calculated using the analytical solution and Equations 10 and 12 in Figure 2. As can be seen in this figure, for circular defect diameters ranging from 2 to 20 mm the flow rates rigorously calculated using the analytical solution given by Equation 1 and the approximate flow rates calculated using Equations 10 and 12 are almost identical.

This result suggests that changing the contact factor is sufficient to correctly represent the GM–GCL contact condition. This tends to show that, for circular defect diameters in the 2 to 20 mm range, both Equations 10 and 12 can be used to predict the flow rate. Taking into account that practitioners are more familiar with exponents and gradient expressions from Equation 12, it is thus recommended that this equation be used.

In the case of large circular defects, Figure 3 tends to show that a simple change in the contact factor (Equation 13) does not lead to the least difference from the analytical solution, suggesting that Equation 11 should be preferred to Equation 13.

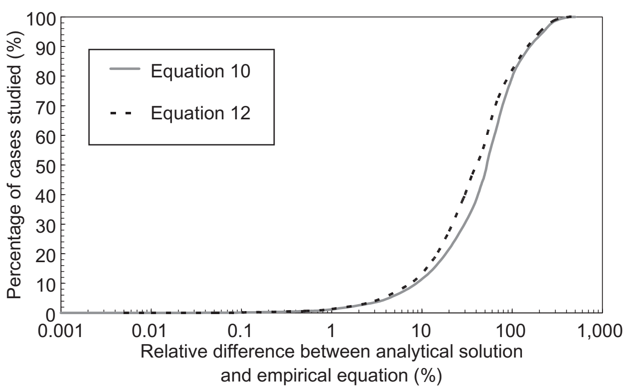

Figure 2. Relative difference between analytical solution and empirical equations developed in this paper for circular defect diameters in the 2 to 20 mm range for Equations 10 and 12

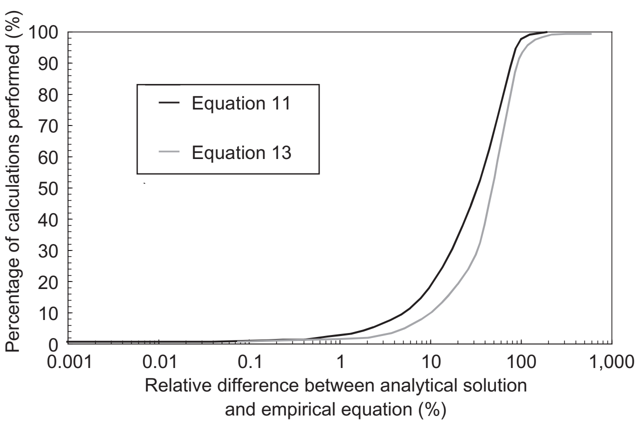

Figure 3. Relative difference between analytical solution and empirical equations developed in this study for circular defect diameters in the 100 to 600 mm range for Equations 11 and 13

5. EQUATIONS FOR DEFECTS OF FINITE LENGTH

5.1. Assumptions

In order to develop semi-empirical equations for the case of defects of finite length (narrow and wide defects) the methodology used derives mainly from the methodology adopted by Giroud and Touze-Foltz (2005). The following assumptions are made.

- A defect of finite length L and width b has the shape of a long rectangle with a half-circle at each end.
- Flow through the rectangular portion of the defects, i.e. L–b and width b, is the same as in the case of a defect of infinite length; in other words, the analysis is two-dimensional.
- Flow through the semi-circular portions of the defect is the same as in the case of a circular defect.

Based on these assumptions, from a practical point of view this type of defect can be seen as the sum of a two-dimensional defect and a circular defect.

The form of the analytical solution proposed in the paper by Giroud and Touze-Foltz (2005) for defects of finite length is given by

$$Q_L = b k_s \left(1 + \frac{h_w}{H_s} \right) + 2 \sqrt{k_s \theta h_w \left(2 + \frac{h_w}{H_s} \right)} \quad (14)$$

where Q_L is the flow rate per unit length; and b is the width of defect of infinite length. Equation 14 can be used with any set of coherent units. The basic SI units are: Q_L (m^2/s); k_s (m/s); b (m); h_w (m); and H_s (m).

Equation 14 will be combined with the empirical equations developed in this paper for circular defects, i.e. Equations 11 and 12 depending on the defect width.

5.2. Equations obtained for narrow defects of finite length

For narrow defects, Equation 12, where the area of the circular defect, a, is replaced by the diameter, d, is coupled with Equation 14 to derive the following equation.

$$Q_T = (L-b)\left[bk_s\left(1+\frac{h_w}{H_s}\right)+2\sqrt{k_s\theta h_w\left(2+\frac{h_w}{H_s}\right)}\right] + 2.3\times10^{-3}d^{0.2}h_w^{0.9}k_s^{0.74}\left[1+0.1\left(\frac{h_w}{H_s}\right)^{0.95}\right] \quad (15)$$

The first term on the right-hand side of Equation 15 quantifies the flow rate into the soil liner (GCL + CCL) of the composite liner located directly under the rectangular portion of the geomembrane defect, whereas the second term quantifies the flow rate at the ends of the defect.

5.3. Equations obtained for wide defects of finite length

For wide defects, Equation 14 is coupled with Equation 11, and the area of the circular defect, *a*, is replaced by the diameter, *d*, in Equation 11 to derive the following equation.

$$Q_T = (L-b)\left[bk_s\left(1+\frac{h_w}{H_s}\right)\right]+2\sqrt{k_s\theta h_w\left(2+\frac{h_w}{H_s}\right)} + 0.111d^{0.8}k_s^{0.82}h_w^{0.54}\left[1-0.22\left(\frac{h_w}{H_s}\right)^{-0.35}\right] \quad (16)$$

The first term on the right-hand side of Equation 16 quantifies the flow rate into the GCL plus CCL of the composite liner located directly under the rectangular portion of the geomembrane defect, whereas the second term quantifies the flow rate at the ends of the defect.

6. DISCUSSION

6.1. Comparison of empirical equation for circular defect diameters in the 2 to 20 mm range with existing equations

GSE (2001) proposed a contact condition factor for the case of geomembrane-supported GCLs equal to 0.01. As a result, in the case of geomembrane-supported GCLs the flow rate can be evaluated thanks to the following equation.

$$Q = 0.01a^{0.1}h_w^{0.9}k_{GCL}^{0.74}\left[1+0.1\left(\frac{h_w}{H_{GCL}}\right)^{0.95}\right] \quad (17)$$

where Q is the flow rate; h_w is the hydraulic head on top of geomembrane; a is the circular defect area; k_{GCL} is the equivalent hydraulic conductivity of the GCL; and H_{GCL} is the thickness of the GCL. These equations must be used with the following units: Q (m^3/s); h_w (m); a (m^2); k_{GCL} (m/s); and H_{GCL} (m).

Equation 17 is considered valid for $h_w < 3$ m and defect diameters between 0.5 and 25 mm.

The accuracy of Equations 12 and 17 was studied by comparison with the analytical solution presented in Equation 1.

No comparison was undertaken with the equation proposed by Touze-Foltz and Giroud (2003) for excellent contact conditions, as the range of hydraulic conductivities for which this empirical equation is valid (10^{-10} to 10^{-8} m/s) is not identical to the range of equivalent hydraulic conductivity for which empirical equations were developed in this paper.

Figure 4 presents the percentage of cases studied (more than 8,000) for parameters in the range defined in Section 3 (i.e. percentage of cases studied) with the relative difference between the flow rate calculated using Equation 12 and Equation 17. As can be seen, Equation 12 gives the closest results to the analytical solution as compared with Equation 17. Therefore evaluations based on the empirical equation previously published in the literature to evaluate flow rates for geomembrane supported GCLs are not suitable for GM–GCL composite liners.

6.2. Comparison with experimental results from Barroso (2005)

The accuracy of Equation 12 is also studied by comparing the flow rates predicted for circular defects with the flow rates obtained in the intermediate scale test (1 m-diameter test cell) and in the large-scale test (4.84 m^2 test pad) by Barroso (2005). Those tests were selected as no flow was noticed at the outlet of test cells at steady state. Consequently, the results obtained in these tests are supposed to correspond to what would be observed at field scale, and accordingly to be comparable to evaluations given by empirical equations. Comparisons performed are presented in Table 1.

Table 1 shows that the relative difference between the flow rates measured in tests and the flow rate calculated using Equation 12 is much less than the relative difference between flow rates measured in tests and flow rates calculated using Equation 17. For the large-scale test this difference is about 15%. This result is all the more logical as the largest value of transmissivity experimentally

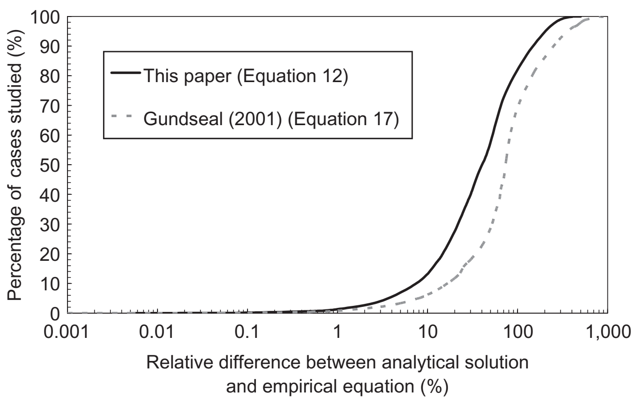

Figure 4. Relative difference between analytical solution and empirical equations for circular defect diameters in the 2 to 20 mm range for Equation 12 (this paper) and Equation 17 (GSE 2001)

Table 1. Comparison between flow rates calculated using empirical equations for circular defects and those obtained in tests

Laboratory tests (Barroso 2005)	Empirical equation	Flow rate: measured (m^3/s)	Flow rate: calculated (m^3/s)	Relative difference (%)
Intermediate-scale test	Equation 12	2.7×10^{-12}	2.1×10^{-11}	682
	Equation 17		8.9×10^{-11}	3,157
Large-scale test	Equation 12	2.5×10^{-11}	2.2×10^{-11}	15
	Equation 17		8.9×10^{-11}	255

obtained, and on which the GM–GCL contact condition is based, corresponds to the large-scale test experimental result where the confining pressure was equal to 25 kPa. Nor should it be a surprise that the experimental flow rate obtained is overestimated for larger confining pressures applied on the composite liner, as is the case in the intermediate-scale test in which the confining pressure was equal to 50 kPa. Therefore the empirical equation developed represents an upper bound for the experimental results obtained by Barroso (2005). Further developments of the kind of those proposed by Chai *et al.* (2005) could be undertaken were more experimental data to be collected to evaluate the influence of confining stress on leakage rate.

6.3. Influence of flow at the ends of defects of finite length

This analysis was undertaken according to a recent work by Giroud and Touze-Foltz (2005), which has shown that, in the case of defects of finite length, for GM–CCL composite liners a large fraction of the flow takes place at the ends of the defects. Thus, based on the methodology presented by Giroud and Touze-Foltz (2005) to compensate for the error made by neglecting the flow at the two ends of the defect, a parametric study was conducted in order to evaluate the factor λ_{2D} for the case of the GM–GCL contact condition, which can be obtained through the following approximate equation (Giroud and Touze-Foltz 2005).

$$Q_{2D} \approx \lambda_{2D} L \left[b k_s \left(1 + \frac{h_w}{H_s} \right) + 2\sqrt{k_s \theta h_w} \left(2 + \frac{h_w}{H_s} \right) \right] \tag{18}$$

where Q_{2D} is the approximate value of rate of flow through defects of finite length in geomembrane obtained from Equations 15 or 16 depending on defect width, assuming that the hydraulic head on top of the geomembrane is small compared with the thickness of the soil component of the composite liner.

An example of values of λ_{2D} obtained is presented in Figure 5 for narrow defects and Figure 6 for wide defects. It appears that in many cases the factor λ_{2D} is large. This means that a large fraction of the flow takes place at the ends of defects of finite length. This result is especially true for narrow defects. Therefore, in most cases, it is safer to calculate the flow rate using Equations 15 and 16, depending on defect width, rather than do a two-dimensional calculation using Equation 14.

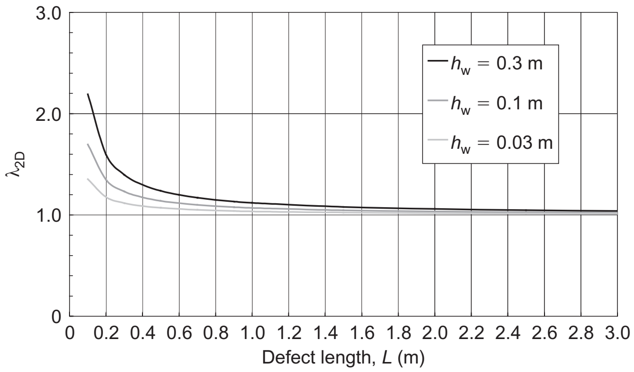

Figure 5. Correlation factor, λ_{2D}, for a 2 mm-wide defect, a GCL soil liner hydraulic conductivity equal to 2×10^{-11} m/s, and 9 mm thick, a soil liner hydraulic conductivity equal to 10^{-9} m/s, a liquid head on top of the geomembrane smaller than the total thickness of the soil liner (GCL + CCL), and GM–GCL contact condition

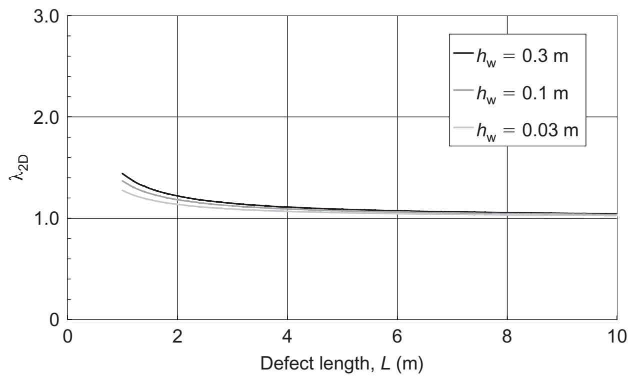

Figure 6. Correlation factor, λ_{2D}, for a 100 mm-wide defect, a GCL soil liner hydraulic conductivity equal to 2×10^{-11} m/s[s], and 9 mm thick, a soil liner hydraulic conductivity equal to 10^{-9} m/s, a liquid head on top of the geomembrane smaller than the total thickness of the soil liner (GCL + CCL), and GM–GCL contact conditions

7. CONCLUSIONS

This paper focuses on the development of empirical and semi-empirical equations for calculating the rate of liquid flow through circular defects (small and large) and finite-length defects (narrow and wide) in the geomembrane component of composite liners involving GCLs.

As the rate of liquid flow depends on the contact conditions at the interface between the two components of the composite liner (the geomembrane and the GCL), a new contact condition, herein termed the 'GM–GCL contact condition', was first defined in quantitative terms considering the experimental data reported by Barroso (2005).

Based on the GM–GCL contact condition, the rate of flow through composite liners involving GCLs could thus be rigorously calculated using the analytical solutions proposed by Touze-Foltz *et al.* (1999), which were used to assist in the development of empirical equations for the case of circular defects. Two equations are presented, one for defect diameters ranging from 2 to 20 mm and the other for defect diameters ranging from 100 to 600 mm. The methodology used to develop these equations was that proposed by Touze-Foltz and Giroud (2003).

Also, the need to consider the flow at the ends of the finite length defects (narrow and wide defects) was analysed for composite liners involving GCLs through a parametric study. This analysis was undertaken according to a recent work by Giroud and Touze-Foltz (2005), which has shown that for this type of defect a large fraction of the flow takes place at the ends. Thus, based on the methodology presented by Giroud and Touze-Foltz (2005), the empirical equations previously developed for circular defects were combined with simple analytical solutions to take into account the flow at both ends of the defects. Two semi-empirical equations were obtained in this way for narrow and wide defects, respectively.

The results of the parametric study showed that, in most cases, it is safer to take into account the flow rate at the ends of the defect, rather than perform a two-dimensional calculation.

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NOTATIONS

Basic SI units are given in parentheses.

- A constant (m)
- a area of circular defect in geomembrane (m^2)
- B constant (m)
- b width of geomembrane defect (m)
- C_c contact condition factor (dimension is variable)
- d diameter of circular defect (m)
- H_f thickness of soil layer (m)
- H_{GCL} thickness of GCL (m)
- H_s total soil liner thickness (m)
- h_w liquid head on top of geomembrane (m)
- I_m modified Bessel function of *m*th order (dimensionless)
- K_m modified Bessel function of *m*th order (dimensionless)
- k_f hydraulic conductivity of soil layer (m/s)
- k_{GCL} hydraulic conductivity of GCL (m/s)
- k_s equivalent hydraulic conductivity (m/s)
- L defect length (m)
- Q rate of flow through circular defect in geomembrane component of composite liner (m^3/s)
- Q_L rate of flow per unit length through two-dimensional defect in geomembrane component of a composite liner (m^2/s)
- Q_{2D} approximate value of rate of flow through defects of finite length in geomembrane obtained from two-dimensional calculation (m^3/s)
- Q_T total rate of flow through defect in geomembrane component of composite liner including flow at both ends (m^3/s)
- R radius of wetted area (m)
- r_0 radius of defect in geomembrane (m)
- α $\sqrt{k_s/H_s\theta}$ (m^{-1})
- θ interface transmissivity (m^2/s)
- κ exponent of soil layer hydraulic conductivity in empirical equation (dimensionless)
- λ factor in hydraulic gradient expression (dimensionless)
- λ_{2D} correction factor used to compensate for error made by neglecting flow at two ends of defect (dimensionless)
- μ exponent in hydraulic gradient expression (dimensionless)
- ξ exponent of defect area or width in empirical equation (dimensionless)
- χ exponent of hydraulic head in empirical equation (dimensionless)

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