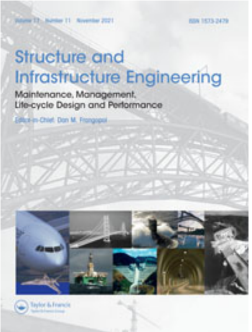

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À priori uplift pressure model for concrete dam foundations based on piezometric monitoring data

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ABSTRACT
The uplift pressures change the stress state of the rock mass foundation in a coupled hydromechanical process and may compromise the dam stability. Consequently, dam safety regulations specify the maximum values of the hydraulic conductivity and uplift pressures that can usually only be fulfilled after the execution of seepage and uplift control measures. The design of waterproofing and drainage systems is still based on an equivalent continuum approach, even though rock mass foundations are discontinuous media. The analysis of the piezometric monitoring data of several Portuguese large concrete dam foundations reveals that uplift pressures are site-specific and may even vary considerably across a given site, which can be critical for the safety assessment. From the data gathered in several dams, a probabilistic model that can be seen as an *a priori* prediction model for uplift pressures is proposed. Considering the difficulty in classifying the geologic foundation conditions beneath each piezometer in existing dams, it is assumed that the data come from a mixture of two beta distributions, representing foundations with regular and unfavorable geologic conditions. This model is a significant improvement over available approaches and will be instrumental for the assessment of existing dams and the design of new dams.

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Beta distribution; concrete dams; monitoring; piezometric data; probabilistic model; reliability analysis; uncertainty modelling; uplift pressures

1. Introduction

The gradient in the hydrostatic pressure installed between upstream and downstream faces of dams results in water seepage, under pressure, mainly through higher permeability zones such as cracks, lift joints, the dam-foundation interface and the rock mass foundation discontinuities. The permeability of the dam-foundation system and the resulting uplift pressures were firstly recognised in the design of Vyrnwy dam (Powys, Wales, UK), in 1882–1891. To collect seeping water and reduce uplift (or water-seeping) pressures, a drainage system consisting of a set of rock drains connected to a horizontal gallery with an outlet in the downstream face (Thomas, 1976) was built. However, the consideration of the effects of uplift pressures on the stability of gravity dams remained overlooked until the failure of Bouzey dam (France), in 1895. The stability of Bouzey dam was compromised due to higher unexpected uplift pressures installed in a horizontal crack caused by a very poor bond between mortar and masonry that could not withstand the tensile stresses developed with full reservoir (Smith, 1994). In fact, the elementary concepts applied at that time to gravity dam design, derived from the reasoned application of mathematical theory to structural engineering (Rankine, 1872; Sazilly, 1853), took into account only the reservoir pressure and the dam height in stability calculations. To account for the effects of uplift pressures, Lévy (1895) stated that the compressive stress must be equal or higher than the water pressure at each point of the upstream face. Additionally, a triangular uplift distribution (Figure 1(a)), varying from the reservoir water pressure at the upstream face to the tailwater pressure at the downstream face, should be adopted, in stability calculations, for any horizontal joint.

The failure of Bouzey dam and the subsequent controversy over the dam design principles, promptly clarified by Lévy (1895), were followed by a period of theoretical investigations on the advent of uplift pressures, their intensity and acting area and, mainly, the uplift control measures. Focus was put on the study of water seepage through the rock mass foundations since, with proper construction techniques, either in masonry or subsequent concrete dams, adequate waterproofing conditions could be ensured for the upstream face of the dam. Although discontinuities govern the mechanical and hydraulic behaviour of rock mass foundations, those studies were advantageously based on the assumption that an impermeable dam is founded on a porous and continuum media through which water flows in steady-state conditions. Analytical solutions (Weaver, 1932) for determining pressures were deduced by solving the Laplace's equation obtained from the Darcy's law (Darcy, 1856) which admits a laminar flow, such as can be adequately considered for the foundation of concrete dams (Wittke, 1990). The obtained pressure distribution, at the dam-foundation interface, is approximately linear (Figure

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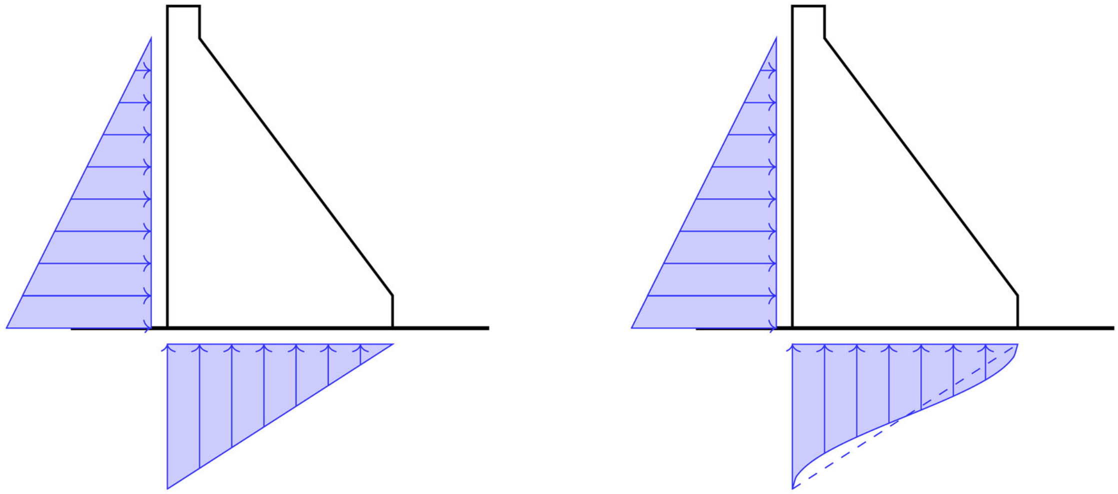


(a) Linear distribution (Lévy, 1895). (b) Theoretical distribution (Weaver, 1932).

Figure 1. Uplift pressure distribution at the dam-foundation interface.

1(b)), such as suggested by Lévy (1895) based on stability principles.

The efficiency of a cut-off wall (Weaver, 1932) and a drainage trench (Brahtz, 1936), aiming to reduce or even eliminate the uplift pressures, were also studied. Analytical solutions for the pressure distribution at the dam-foundation interface are illustrated in Figure 2(a) and 2(b), respectively. In practice, however, waterproofing curtains, composed by a single row of grout holes, proved to be ineffective in reducing the uplift pressures as soon as typical uplift observations were published (Keener, 1950; TVA, 1952). Proving that uplift reduction was mainly attained by the drainage system, Casagrande (1961) presented the analytical solution of a theoretical problem that is still considered nowadays for the design of a drainage system composed by a set of evenly-spaced boreholes (Mascarenhas, 2005).

Current regulations specify the requirements of stability, deformation and low permeability that rock mass foundations shall fulfil, which are usually only accomplished after treatment works. Regarding their hydraulic behaviour, the waterproofing and drainage curtains are designed to meet the specifications of hydraulic conductivity and maximum uplift pressures, respectively. In the design phase, when no site information on the uplift pressure is available, a bi-linear pressure distribution (Figure 3(a)) is usually considered, characterised by the average pressure head at the drainage line H_d, given by

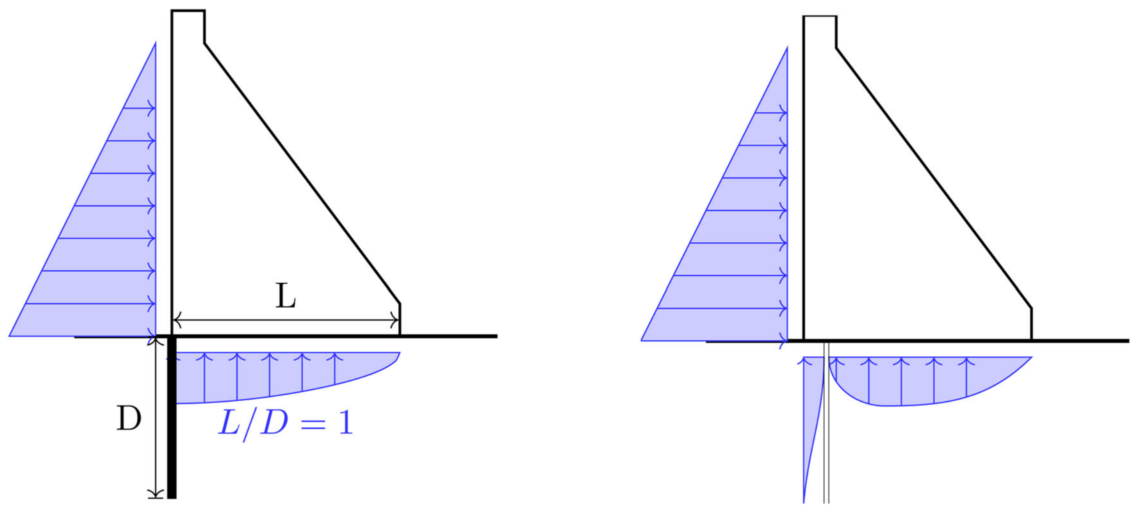
$$H_d = k_u \cdot \frac{L - L_d}{L} \cdot (H_r - H_t) \tag{1}$$

where H_r and H_t are the reservoir and tailwater pressure heads, respectively, and k_u is an uplift factor, inversely proportional to the drainage effectiveness. However, under extreme loading conditions, a crack may open from the dam heel, along which the reservoir pressure is established, compromising the efficiency of the drainage system if it propagates beyond the drainage line (Figure 3(b)).

The different specifications found in dam safety regulations worldwide, roughly ranging the uplift factor k_u between 0.25 and 0.60 (RSB, 2018; Ruggeri, 2004a; USACE, 1995; USBR, 1976), are probably justified by the conservatism that national authorities impose on the assumptions regarding the theoretical drainage effectiveness. In fact, as the hydromechanical behaviour of rock mass foundations depends on the spatial variation of their properties, its simplification by means of equivalent continuum idealizations may result in erroneous predictions regarding the uplift pressure distribution under concrete dams. Furthermore, uplift pressures in rock mass foundations may exhibit high spatial variations (Ruggeri, 2004b) since they are not only influenced by the geologic features, such as the geologic structure, the rock type and the joint pattern, but also by the joint permeability, mainly characterised by the filling, roughness and specially the joint aperture (EPRI, 1992). In that sense, a distinction between regular and unfavourable geologic conditions regarding the hydromechanical behaviour of the rock mass foundation is proposed in the French guidelines (CFBR, 2012) which recommend the uplift lowering coefficient (equivalent to the uplift factor k_u) presented in Table 1.

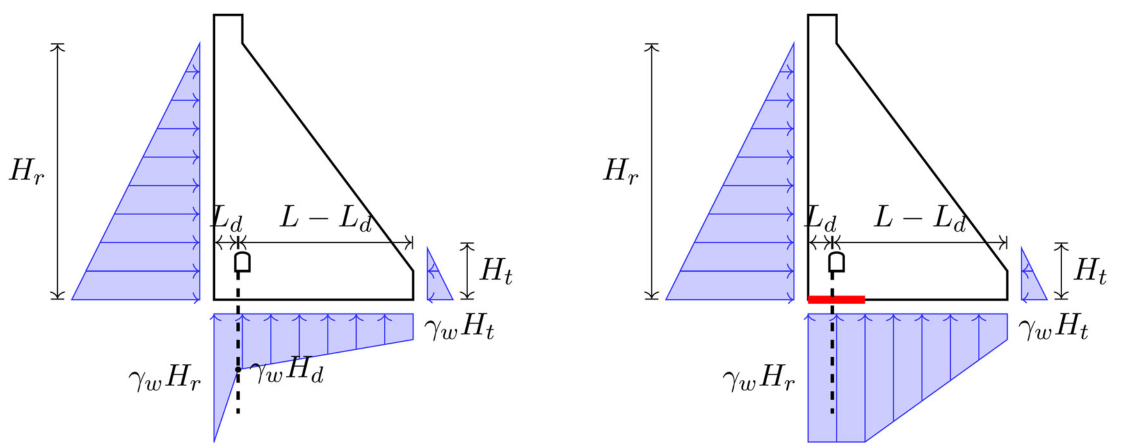
Loads from the dam construction, reservoir filling and operation cause variations in the stress state of rock masses in a coupled hydromechanical process (Farinha, 2010; Rutqvist & Stephansson, 2003). Changes over time in the drainage effectiveness due to the reservoir water level variations (Grenoble, Harris, Meisenheimer & Morris, 1995; Ruggeri, 2004b) and cyclic thermal variations (Guiducini & Andrade, 1988; Kalkani, 1992) are then expected. Also a time-dependent variation is possible due to drain clogging.

In the context of reliability analysis, increasingly applied to the safety analysis of concrete dams (Altarejos, 2009; Westberg, 2010), the recognised uncertainties shall be taken into account by probabilistic models that combine both data available (objective information) and physical arguments, experience and judgement (subjective information). On one



(a) Cut-off wall (Weaver, 1932). (b) Drainage trench (Brahtz, 1936).

Figure 2. Effects of uplift control measures on the pressure distribution at the dam-foundation interface.



(a) With an operational drainage system. (b) With a crack propagated beyond the drainage system.

Figure 3. Regulatory uplift pressure distribution.

Table 1. Recommended values for the uplift reduction coefficient (CFBR, 2012).

Foundation conditions	k_u
Regular geology	$0.33 - 0.50$
Unfavourable geology	>0.50
No drainage	1.00

hand, the objective information available, either from the scientific literature or gathered data, shall be treated from a technical perspective in order to attain the most suitable models. On the other hand, when such information is lacking, engineering judgement is the main tool to define conservative probabilistic models. For the uncertainty quantification of the uplift pressures, Altarejos (2009), using a limited number of piezometric recordings, derived a probabilistic model to account for the variability of the drainage efficiency in a specific dam. Westberg (2009a,b), intending to derive a generic probabilistic model for the utilisation in reliability analysis, followed a geostatistical approach. The model properties, however, are highly dependent on the statistical descriptors of the spatial variation and correlation of the foundation permeability whose selection requires extensive field tests to obtain a realistic representation of the foundation hydraulic properties.

In this work, data gathered from the piezometric monitoring of the Portuguese large concrete dam stock is used to derive a probabilistic model characterising the uncertainty on uplift pressures. This strategy had already proved useful in quantifying the uncertainty on the reservoir water level (Pereira, Batista & Neves, 2018), in which the monitoring data was used to derive probabilistic models. The large set of data, including an extensive variety of foundation properties, shall yet be divided into subgroups of similar characteristics in order to better study and distinguish the variability within a population and between populations (JCSS, 2001). Given the lack of information that could be used to classify the geologic foundation conditions beneath each piezometer in existing dams, it is assumed that two major groups exist, such as distinguished in the French guidelines (CFBR, 2012). Considering the uplift factor k_u as a random variable limited to the unit interval, the underlying probabilistic descriptors of each group can be estimated through the

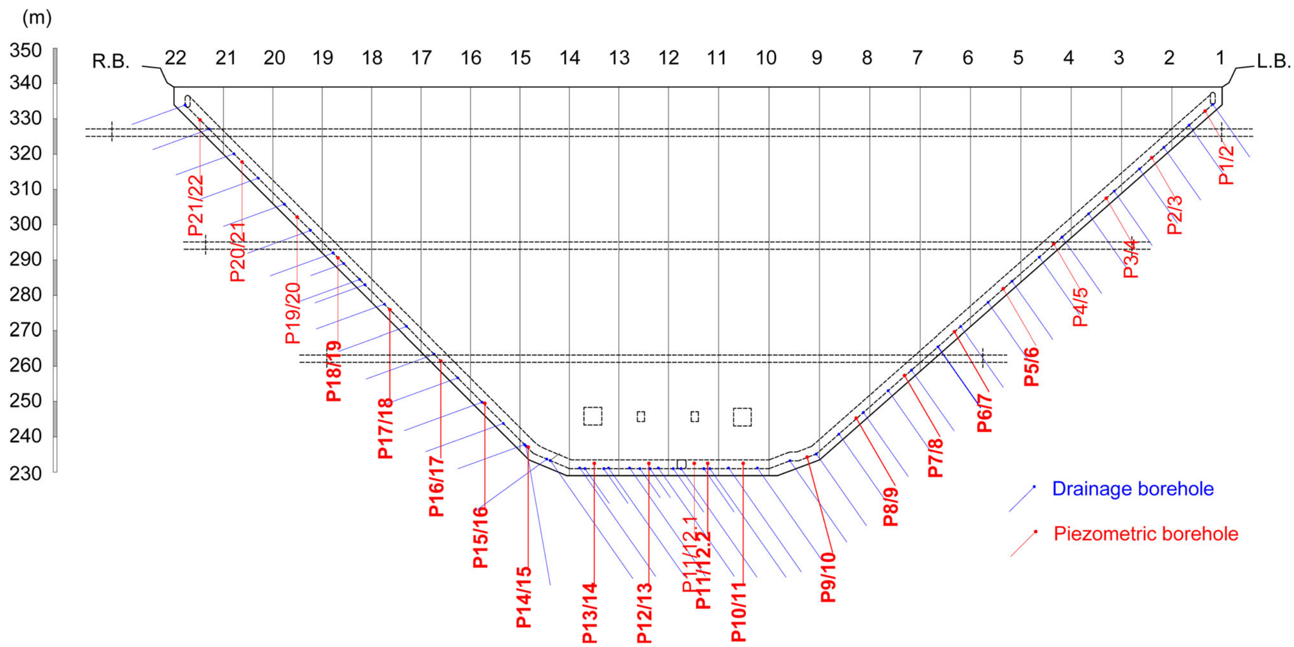


Figure 4. Alto Lindoso dam's piezometric system.

Table 2. Main characteristics of the dams considered for the uncertainty modelling of the uplift factor k_u.

Dam	$H(m)$	Year	N_{pz}
Alqueva	96.00	2003	19
Alto Lindoso	110.00	1991	12
Alto Rabagão	94.10	1964	5
Bouçoais-Sonim	43.00	2004	2
Cabril	132.00	1954	7
Castelo do Bode	115.00	1951	4
Ferradosa	33.40	2005	2
Fronhas	62.00	1985	9
Pedrógão	43.00	2006	12
Penha Garcia	25.00	1979	8
Pretarouca	28.50	2007	1
Raiva	36.00	1981	11
Rebordelo	35.50	2005	1
Varosa	76.00	1976	18
Vilarinho das Furnas	94.00	1972	9
Total			120

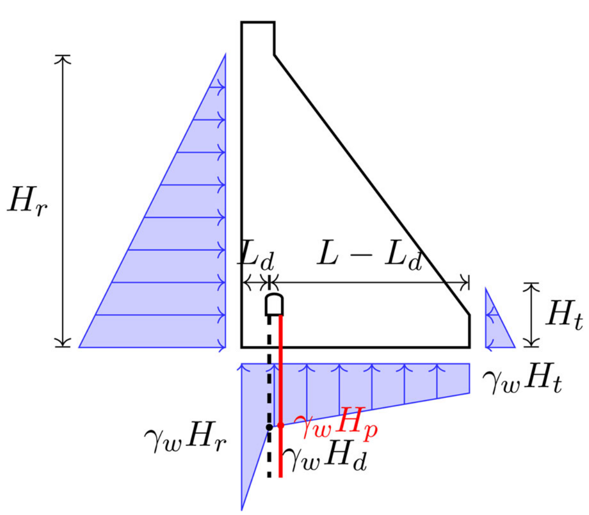


Figure 5. Extrapolation of the piezometric recordings to the drainage line.

maximum likelihood estimation (MLE) method which maximises a likelihood function so that, under the assumed probabilistic model, the observations are most probable. The obtained probability distribution can be seen as an *a priori* prediction model and can later be updated, for a specific case, as soon as new data are available. The strong monitoring basis of this approach is a significant improvement over other alternatives for characterising the uncertainty on uplift pressures in large dams and will be instrumental for the assessment of existing dams and the design of new dams.

2. Monitoring data analysis

During the dam operation period, a monitoring system provides useful information to assess the dam condition and to predict its future behaviour. Uplift pressures are measured using piezometers, usually installed in sealed boreholes, drilled downstream from both waterproofing and drainage systems. Since the piezometers are distributed along the dam-foundation interface, the piezometric recordings provide only localised information. Multi-chamber piezometers, crossing different major discontinuities and allowing the measurement of the water pressures in specific sections, are usually not used in Portugal. Instead, single-chamber piezometers, providing therefore only the average pressure from the water inflow and outflow over their length, are often used. Furthermore, more than one piezometer, forming an upstream-downstream network, is rarely installed within a specific cross-section. Figure 4 shows, as an example, the piezometric system of the Alto Lindoso dam.

The piezometric data are often collected manually, even though, recently, automated data acquisition systems, which increase the frequency of data collection, have been tested. Most data available was then collected weekly, biweekly or even monthly. For the uncertainty quantification of the uplift pressures, data gathered from 120 piezometers installed in 15 Portuguese large gravity and thick arch concrete dams is used. Only the piezometers installed in higher dam monoliths are considered, since spurious uplift

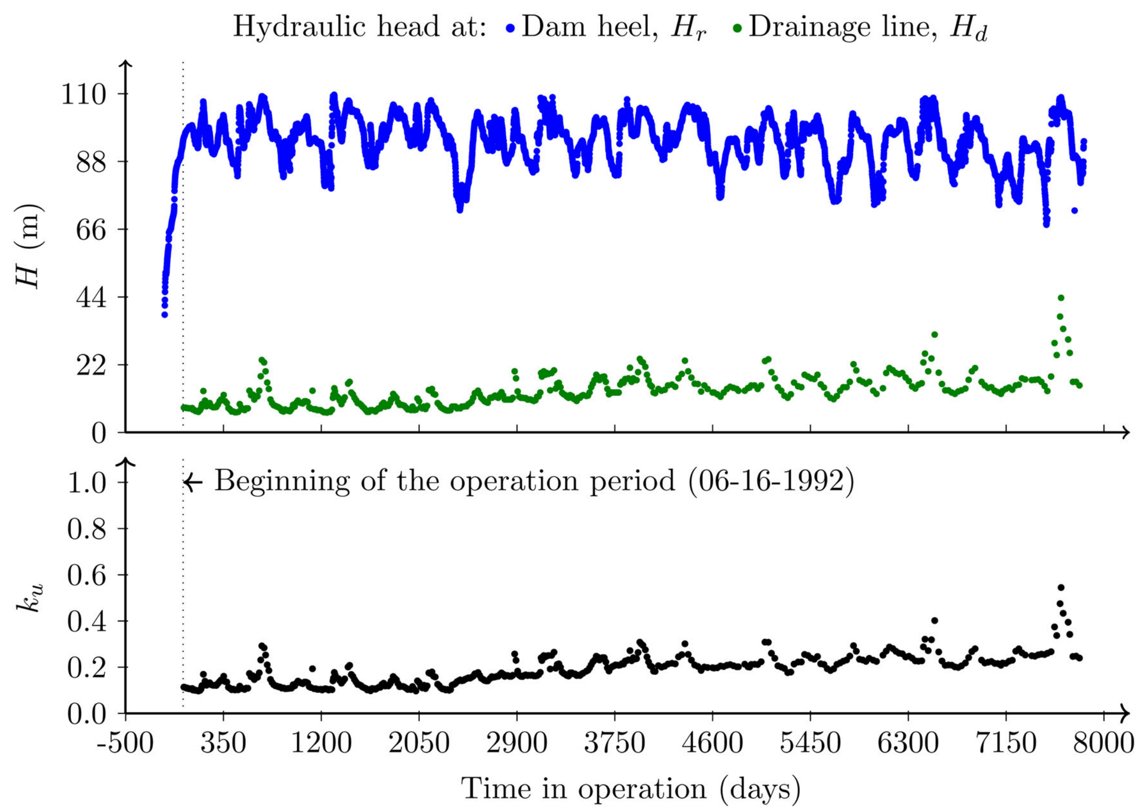


Figure 6. Uplift pressure history, interpolated to the drainage line from the recordings of the piezometer 'P10/11' of Alto Lindoso dam.

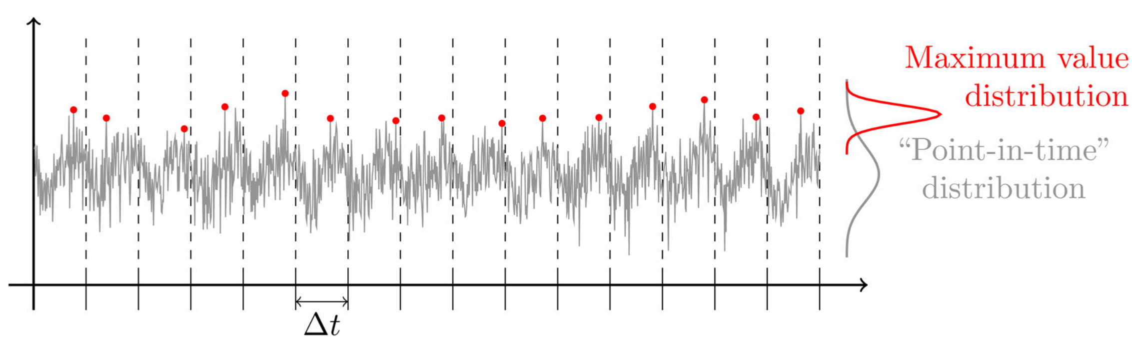


Figure 7. Stochastic process and the associated 'point-in-time' and maximum value distributions.

pressures, exceeding the reservoir water level, may be observed near the abutments (EPRI, 1992). Table 2 presents the relevant characteristics of the dams considered for the uncertainty modelling of the uplift factor k_u, namely the maximum height (H), the year of completion and the number of piezometers (N_{pz}) considered.

Piezometers, which are located downstream from the drainage line (Figure 5), are usually not aligned with drains, in the upstream-downstream direction, being installed in an intermediate zone. Yet, considering that there is no relevant spatial variation of the foundation properties along the dam-axis direction, the recorded pressures can be interpolated to the drainage line allowing their use for the uncertainty modelling of the uplift factor k_u, by,

$$H_d = (H_p - H_d) \cdot \frac{L - L_d}{L - L_p} + H_t \tag{2}$$

where H_d and H_p are the hydraulic heads at the drainage and piezometric lines, respectively, and L_d and L_p are the corresponding distance, from the dam heel. In practice, however, the non-negligible spatial variation of the foundation properties due to the discontinuous nature of the rock mass foundations justifies the variability of the recorded pressures and is indirectly taken into consideration on the uncertainty quantification of the uplift factor k_u.

As an example, Figure 6 shows the pressure head history of Alto Lindoso dam, interpolated to the drainage line from the recordings of the piezometer 'P10/11' using the Equation (2), and the corresponding history of the uplift factor k_u obtained by inverting Equation (1).

Frequently, as illustrated in Figure 6, the uplift factor k_u presents a non-linear relation with the reservoir water level and an increase over time probably due to loss of drainage effectiveness. It also shows variability, when comparing the pressure history obtained in different piezometers, due to the variation of the foundation hydromechanical properties. In a probabilistic perspective, this evidence is compatible with the representation of uplift pressures as trend stationary stochastic processes, i.e., mathematical objects that characterise a collection of random variables whose unconditional joint probability distribution changes over time according to an underlying trend function (Coleman, 1974).

3. Uncertainty characterisation

By definition, structural safety is a time-dependent problem since both actions and the structural capacity vary with time. In reliability analysis, when the uncertain nature of the problem is explicitly taken into account, time-varying actions are modelled by stochastic processes. However, the structural safety problem is often simplified by analysing the most demanding conditions that can occur during specific time intervals. When a structure is subjected to multiple simultaneous time-varying actions, a load combination, that represents the foreseeable load conditions, must then be derived, considering the theory of stochastic load combinations, in order to proceed with a simplified time-independent formulation. Since it is not expected that each action reaches its maximum value at the same moment in time, the Turkstra's rule (Turkstra, 1970) or the Ferry Borges-Castanheta (FBC) rule (Ferry Borges & Castanheta, 1971) are often invoked to derive the foreseeable maximum combined effect of actions. These load combination rules consider either the arbitrary 'point-in-time' distribution of each action, which characterise the associated uncertainty at each moment t, or the distribution of maximum values that shall occur during reference time periods Δt. Figure 7 illustrates a stochastic process and the associated 'point-in-time' and maximum value distributions.

Critical stability conditions of concrete dams can be reached not only when the water level is high but also under other unusual loading conditions, such as, for instance, during seismic events. Thus, both maximum value and 'point-in-time' distributions of the uplift factor k_u shall be considered when the water actions (including the water level) are the dominant load or otherwise, respectively. The consideration of data observed during the entire life of the structure not only allows a more robust model but also makes the distributions proposed independent of the water level, which is useful given the generalisation intended.

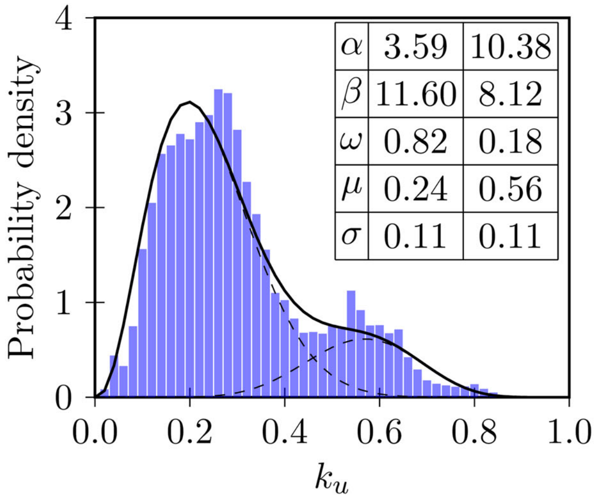
As a trend stationary stochastic process, the properties of the arbitrary 'point-in-time' distribution of the uplift factor k_u vary over time according to an underlying trend function that characterise its time evolution. This underlying function is also site-specific since, as mentioned, it depends on both the local hydromechanical properties and the unpredictability of loss of drainage effectiveness. A conservative way to overcome the difficulty in removing the underlying trend function is to analysed the gathered data as if it came from a strict-sense stationary process. In this case, the properties of the arbitrary 'point-in-time' distribution would not change over time and each piezometric recording is considered, after interpolation to the drainage line, as an observation of the uplift factor k_u. In this way, the obtained distribution characterises both the inherent variability and the time effect. The histogram of the uplift factor k_u, obtained by interpolation of the piezometric data gathered during normal operation periods in several dams, is shown in Figure 8(a).

The maximum value distribution of the uplift factor k_u during an interval Δt is obtained in a similar way. In fact, stationarity ensures that each interval Δt provide equally valid information regarding the statistical properties of the entire process. Therefore, the maximum recorded pressure at each interval Δt is considered as one observation of a population that characterises the maximum uplift pressures in such intervals. In the particular case of concrete dams, the load combinations commonly referred in dam safety regulations (RSB, 2018; Ruggeri, 2004a; USACE, 1995; USBR, 1976), which characterise the foreseeable and most conditioning loading conditions that the structure shall withstand, are derived from the occurrence of the maximum design earthquake and the design flood. For the time-invariant reliability analysis, considering these or other load combinations, the probability distribution that models the uncertainty on the maximum values of the uplift pressures would then depend on the duration of the leading action. Figure 8(b–f) shows the histogram of the maximum uplift pressures, defined by an uplift factor k_u, for intervals of two weeks, one month, three month, six month and one year, respectively. In fact, since, in most cases, the piezometric data is collected at most once a week, and may even be collected only once a month, for shorter time intervals the only recorded pressure is considered as a maximum value, which may distort the results. This effect naturally vanishes for larger time intervals.

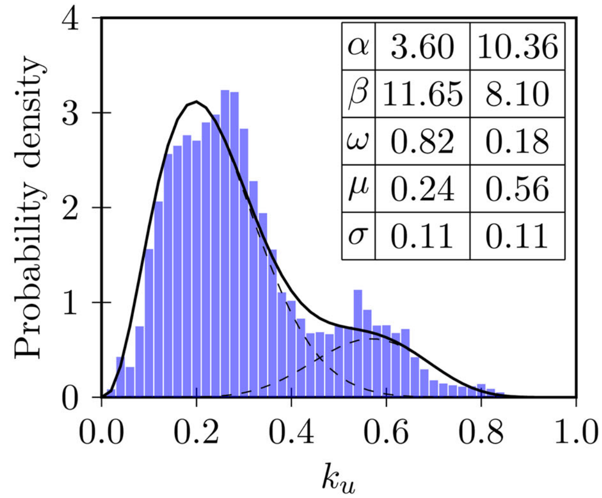
4. Parametric inference

The hydromechanical behaviour of rock mass foundations determines the order of magnitude of the uplift pressures installed. Its prediction, either in the design/feasibility or operation stages, requires both geologic investigations, to identify the major joint patterns and to evaluate the hydraulic conductivity, and numerical modelling. This strategy allows the classification of the rock mass foundation regarding its hydromechanical behaviour, defining qualitatively the expected uplift pressures. The first operation period, when uplift pressures are monitored by piezometers installed along the dam-foundation interface, is the first real test to the assumptions derived from the geologic investigations.

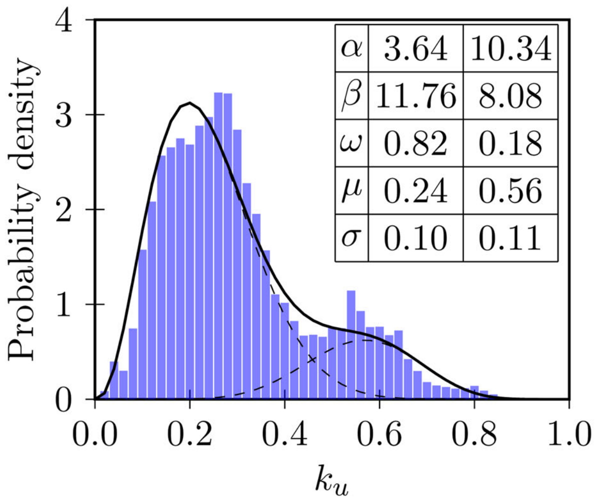
The role of the geologic structure and joint permeability on the hydromechanical behaviour of rock mass foundations justifies not only the high spatial variations possibly exhibited (Ruggeri, 2004b) but also its categorisation according to the favourableness for an efficient uplift control. The analysis of the histograms presented in Figure 8 leads to the identification of some peaks (local maxima). Considering



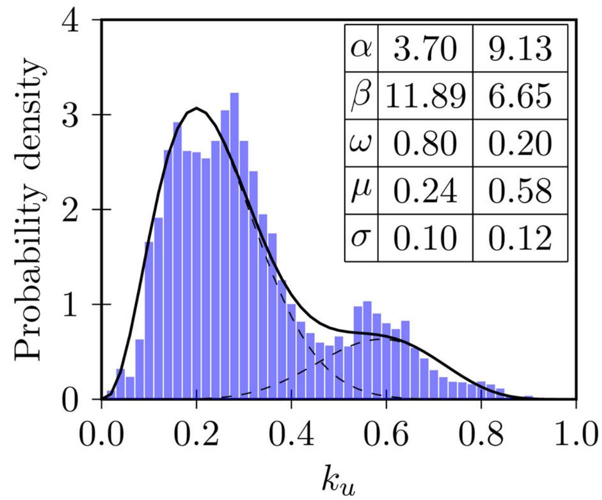
(a) Arbitrary "point-in-time" values ($\Delta t \to 0$)



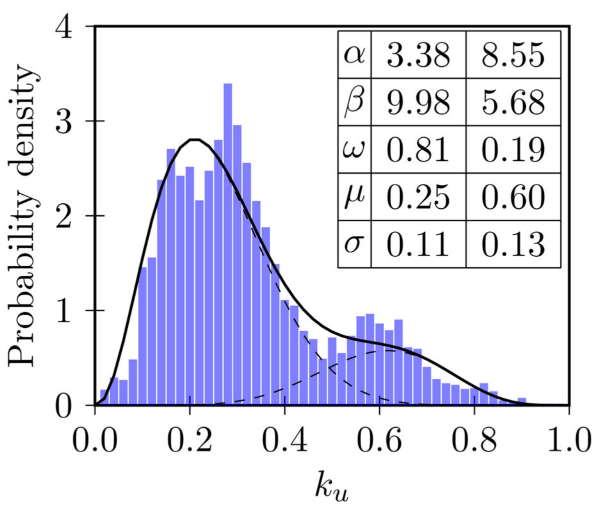
(b) Maximum values for intervals of two weeks ($\Delta t = 15$ days)



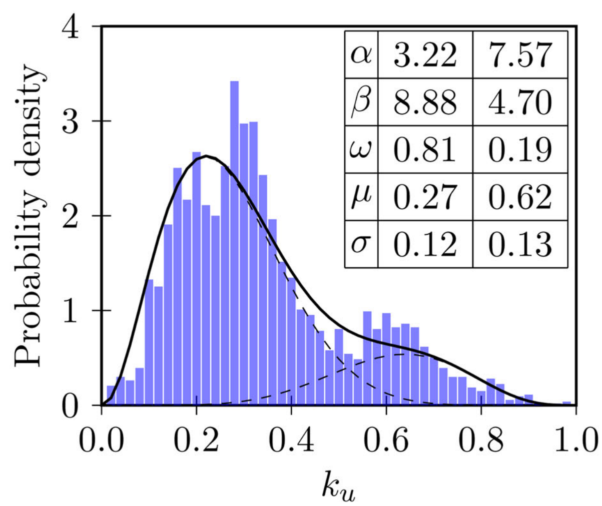
(c) Maximum values for intervals of one month ($\Delta t = 30$ days)



(d) Maximum values for intervals of three months ($\Delta t = 90$ days)



(e) Maximum values for intervals of six months ($\Delta t = 180$ days)



(f) Maximum values for intervals of one year ($\Delta t = 365$ days)

Figure 8. Mixture of two beta distributions adjusted to the uplift factor k_u.

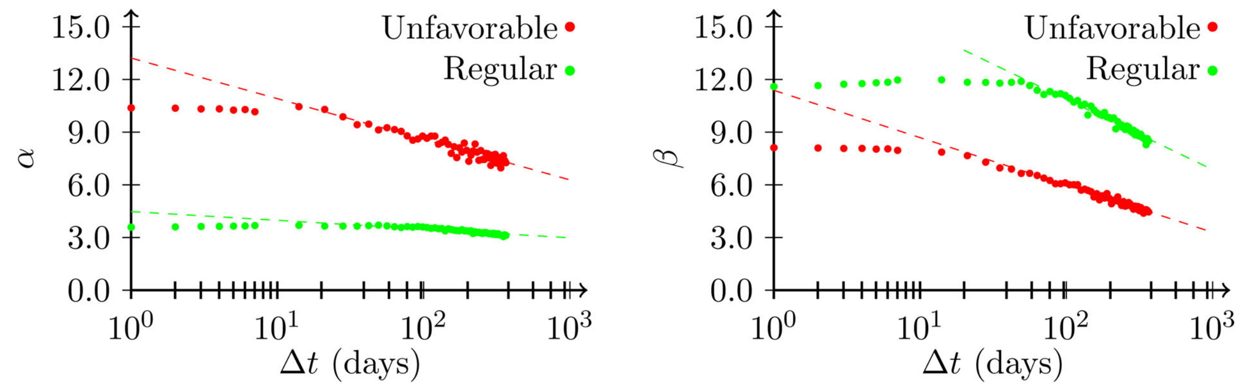


Figure 9. Beta distribution shape parameters for the uncertainty characterisation of the uplift factor k_u.

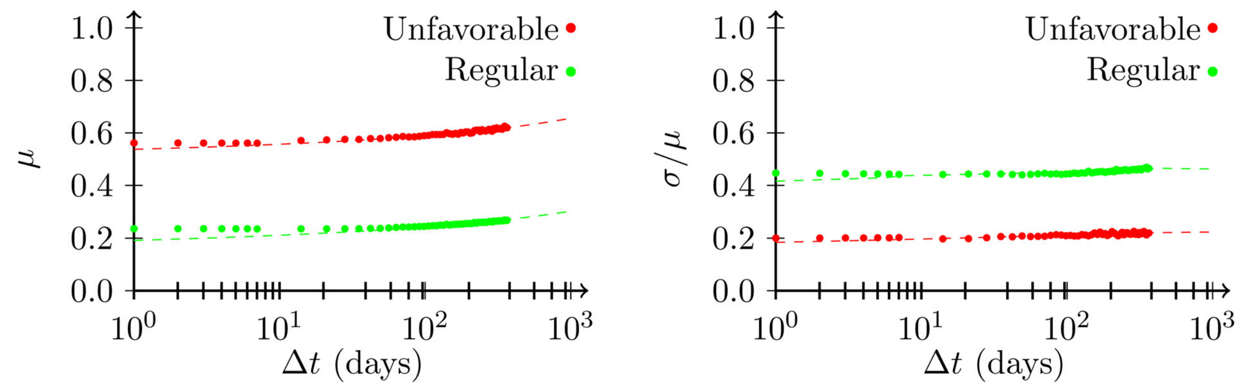


Figure 10. Beta distribution mean value and coefficient of variation for the uncertainty characterisation of the uplift factor k_u.

Table 3. Relevant statistics of the 'point-in-time' and annual maximum value distributions of the uplift factor k_u.

	'Point-in-time' distribution		Annual maximum value distribution	
Foundation conditions	Regular	Unfavourable	Regular	Unfavourable
Computed from inferred statistics	$\alpha = 3.59$	$\alpha = 10.38$	$\alpha = 3.12$	$\alpha = 7.26$
	$\beta = 11.60$	$\beta = 8.12$	$\beta = 8.50$	$\beta = 4.46$
	$\mu = 0.24$	$\mu = 0.56$	$\mu = 0.27$	$\mu = 0.62$
	$\sigma = 0.11$	$\sigma = 0.11$	$\sigma = 0.12$	$\sigma = 0.14$
	$\sigma/\mu = 45\%$	$\sigma/\mu = 20\%$	$\sigma/\mu = 46\%$	$\sigma/\mu = 22\%$
	$k_{95} = 0.43$	$k_{95} = 0.74$	$k_{95} = 0.50$	$k_{95} = 0.83$
	$k_{98} = 0.48$	$k_{98} = 0.78$	$k_{98} = 0.56$	$k_{98} = 0.87$
Estimated from logarithmic trend functions	$\alpha = 4.47$	$\alpha = 13.22$	$\alpha = 3.20$	$\alpha = 7.29$
	$\beta = 18.87$	$\beta = 11.38$	$\beta = 8.62$	$\beta = 4.48$
	$\boldsymbol{\mu} = 0.19$	$\boldsymbol{\mu} = 0.54$	$\mu = 0.27$	$\boldsymbol{\mu} = 0.62$
	$\sigma = 0.08$	$\sigma = 0.10$	$\sigma = 0.12$	$\sigma = 0.14$
	$\boldsymbol{\sigma/\sigma} = 42\%$	$\boldsymbol{\sigma/\sigma} = 18\%$	$\boldsymbol{\sigma/\sigma} = 46\%$	$\boldsymbol{\sigma/\sigma} = 22\%$
	$k_{95} = 0.34$	$k_{95} = 0.70$	$k_{95} = 0.50$	$k_{95} = 0.83$
	$k_{98} = 0.38$	$k_{98} = 0.73$	$k_{98} = 0.56$	$k_{98} = 0.87$

that the available data covers a wide variety of hydromechanical properties, these peaks can be associated with different categories of rock mass foundations with similar behaviour. This encourages the performance of a multi-modal statistical analysis, considering that multiple sets of values, coming from different populations, are mixed in a single sample. If a link between those maxima and the hydromechanical properties can be established, it is possible to associate expected values of uplift pressures with each category. In the French guidelines (CFBR, 2012), this strategy led to a distinction between rock mass foundations that present regular and unfavourable geologic conditions, to which determined values of an uplift lowering coefficient (Table 1) were proposed. Although these guidelines omit what are and how to obtain the indicators associated with such classification, which could be used to split the data into two sets and analyse it separately, a bi-modal statistical analysis of the data allow to derive the statistical properties (distribution parameters) for each set.

In the context of a multi-modal statistical analysis, parametric inference consists in estimating the parameters of a mixture of probability distributions that expectedly characterises the data set. Those probability distributions shall be from the same family since they model different categories of the same random variable. Although the number of potential distribution models is very large, most random variables with application in the civil engineering field, such as external loads and material properties, can be modelled by a small set of probability distributions either because they have desirable mathematical characteristics or because they adjust particularly well to reality (Forbes, Evans, Hastings & Peacock, 2000). When dealing with stochastic processes, their maximum values over a specific time interval are usually well characterised by the set of generalised extreme value distributions. However, since the uplift factor k_u can only take values in the unit interval, both its 'point-in-time' and maximum values shall be modelled by beta distributions, which have been applied to model random variables limited to finite length intervals. Therefore, considering that the sample $\hat{\boldsymbol{x}}$ is composed by n independent occurrences of a mixture of two beta-distributed populations, each one related to a category of foundation geologic conditions, the best estimators of the beta distribution parameters are those that maximise the likelihood function L, given by,

$$L(\boldsymbol{\Theta}, \boldsymbol{\Omega}|\hat{\boldsymbol{x}}) = \prod_{i=1}^{n} \sum_{j=1}^{2} \omega_j \cdot f_X(\hat{x}_i|\alpha_j, \beta_j) \tag{3}$$

subjected to,

$$0 \leq \omega_j \leq 1, \text{ and} \sum_{j=1}^{2} \omega_j = 1 \tag{4}$$

where $\boldsymbol{\Theta} = \{\alpha_1, \beta_1, \alpha_2, \beta_2\}$ and $\boldsymbol{\Omega} = \{\omega_1, \omega_2\}$ are the unknown beta distribution parameters and weights, respectively, of the variables in the mixture. f_X is the beta density function, given by,

$$f_X(x|\alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \tag{5}$$

where B is the beta function and α and β are two positive shape parameters related to the beta distribution mean value (first moment) and variance (second central-moment), respectively, by,

$$\mu = \frac{\alpha}{(\alpha+\beta)} \tag{6}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{7}$$

The analytical maximisation of the likelihood function is generally not as straightforward as in the context of usual uni-modal variables, since it requires solving a system of $k \times n$ non-linear equations (Casaca & Pereira, 2017), where k is the number of variables in the mixture. Alternatively, numerical maximisation, grid search methods or expectation-maximisation algorithms shall be considered. In this work, the likelihood function of a mixture of two beta variables (Equation (3)) was numerically maximised for both the arbitrary 'point-in-time' and the maximum value distribution, considering, in the last case, increasingly larger interval durations (Δt). The shapes of the adjusted mixture distribution, for some cases, are also presented in Figure 8(a–f). The estimated parameters of the beta mixture, for each case, are also shown.

The identification of modes is an essential step for a justified characterisation of the uncertainty related to a phenomenon. When several modes are identified, the presence of groups/categories with distinct characteristics is suggested. By observation of the histograms illustrated in Figure 8, two well-defined modes (for k_u at around 0.30 and 0.60) are clearly distinguished. However, if the sample has a substantial size, other small humps may also be relevant, indicating the presence of subgroups. In this case, a rigorous analysis may identify modes of k_u at around 0.20, 0.30, 0.60 and 0.80. Special attention should yet be payed to the fact that virtual modes may be observed if the sample size is not large enough. These virtual modes are caused by statistical uncertainty due to the limited information available. Nonetheless, a retrospective study must be carried out in order to not only justify the number of categories that should be distinguished but also to determine the local foundation characteristics that may be used to classify the rock mass foundations, before the dam construction, according to the favourableness for an efficient uplift control.

For now, given the distinction made in the French guidelines (CFBR, 2012), it was possible to proceed with a bi-modal statistical analysis considering the piezometric data gathered during normal operation periods in several Portuguese large concrete dams. Both for the instantaneous and interval maximum values, the mixtures of two beta distributions adjust well the available data. Expectedly, as the reference time interval increases, the distributions move towards the right tail.

5. Result analysis and model proposal

In order to study the influence of the low frequency of data collection into the 'point-in-time' and maximum value distributions, especially for shorter time periods, the parametric inference procedure was repeated for increasingly larger reference time intervals: from one day (daily maximum values) to one year (annual maximum values). The evolution of the maximum value distribution properties shall be characterised by monotonic continuous functions, since, as the reference time interval Δt is reduced, the maximum value distribution shall gradually tend to the 'point-in-time' distribution. This is due to the logical convergence between maximum and minimum values occurring in an infinitesimal time period. Furthermore, higher maximum values of uplift pressures shall be more likely to occur in larger time intervals. Figure 9 shows the beta distribution shape parameters for the arbitrary 'point-in-time' ($\Delta t \rightarrow 0$) and the maximum value ($\Delta t > 0$) distributions, considering the first and seconds distributions associated with regular and unfavourable geologic conditions, respectively, such as designated in the French guidelines (CFBR, 2012).

The low frequency of data collection seems to affect the computed statistical properties of the probability distribution that characterise the maximum values of uplift pressures in shorter time intervals. In fact, the statistical inference procedure for time intervals below 10 days, for the α-shape parameter, and below 45 days, for the β-shape parameter, leads to apparently distorted results since a gradual change in the distribution parameters were expected. Ignoring the information affected by the low frequency of data collection, logarithmic functions were then adjusted to the distribution parameters of both regular and unfavourable groups, considering only the results inferred for larger time intervals. Afterwards, these functions were used to estimate the 'point-in-time' distribution parameters by back-extrapolating to the origin. Figure 10 shows the mean value and coefficient of variation for the arbitrary 'point-in-time' ($\Delta t \rightarrow 0$) and the maximum value ($\Delta t > 0$) distributions of both groups. Their estimation using the logarithmic functions adjusted to the distribution parameters, given in terms of the time interval, is compared to the values computed directly from the inferred statistics.

If pressures were recorded few times within a specific time interval, the associated maximum value could be underestimated since the true maximum may not be recorded. This could affect even larger time intervals, depending on the data collection frequency. However, for shorter time intervals, such as observed in Figure 10, this assumption is not necessarily valid. In fact, considering the logarithmic descreasing tendency of the shape parameters for larger time intervals (Figure 9), the mean value and the coefficient of variation of the maximum value distributions are overestimated when using the shape parameters computed directly from the inferred statistics. The lack of continuous data collection may cause it since only time intervals in which pressures were recorded are considered, which may skew the results. The consideration of the logarithmic function to define the shape parameters in terms of the time interval aims to correct that. Thus, as the reference time interval increases, a logical distribution transformation, gradually moving towards the right tail, is achieved.

Careful should be placed in extrapolating the distribution properties beyond the range of time intervals considered since beta distributions, for shape parameters lower than one, become U-shaped which have no application in this case. For that purpose, the rule used to transform extreme value distributions to other time intervals can still be used, i.e.,

$$F_{i_2}(x) = \left(F_{i_1}(x)\right)^{i_2/i_1} \tag{8}$$

where $F_i(x)$ is the cumulative probability distribution of the maximum value observed in a time interval i and i_1 and i_2 are the original and target time intervals, respectively.

For the uncertainty quantification of the uplift factor k_u, the 'point-in-time' and annual maximum value distributions, characterised by the shape parameters estimated from the adjusted logarithmic functions, are proposed. The relevant statistics are synthesised in Table 3. For comparison purposes, the values computed directly from inferred statistics are shown. Furthermore, characteristic values corresponding to 95% and 98% of probability of non-exceedance (k_{95} and k_{98}, respectively), such as usually assumed in semi-probabilistic safety analysis, are also presented.

Both categories of foundation conditions are well distinguished with the statistical properties proposed. On one hand, for regular geologic foundation conditions, mean values of 0.19 (coefficient of variation of 42%) or 0.27 (coefficient of variation of 46%), regarding instantaneous or annual maximum values, respectively, are proposed. Although it is not explicitly stated, the safety regulations (CFBR, 2012; RSB, 2018; Ruggeri, 2004a) shall address to instantaneous uplift pressures, since the corresponding characteristic values are within the range of values recommended.

On the other hand, for unfavourable foundation conditions, mean values of 0.54 (coefficient of variation of 20%) or 0.62 (coefficient of variation of 22%), regarding instantaneous or annual maximum values, respectively, are proposed. The corresponding characteristic values are higher than the worst cases admitted in safety regulations (Ruggeri, 2004a), even though the French guidelines (CFBR, 2012) recommend values above 0.50. The weights ω ($\omega \simeq 0.80$) obtained in the parametric inference procedure (Figure 8) indicate the frequency of occurrence of regular geologic conditions in the set of rock masses that presented adequate hydromechanical properties to accommodate concrete dam foundations. Accordingly, unfavourable geologic conditions occur only in occasional situations.

The significant standard deviation obtained (around 0.10 for instantaneous values and above it for maximum values) alerts for the fact that the probabilistic nature of the uplift pressures shall be considered in safety analysis.

6. Conclusions

Since uplift pressures installed in rock mass foundations may compromise the stability of gravity dams, the regulatory specifications regarding the requirements that rock mass foundations shall fulfil are usually only attained after the execution of uplift control measures. The design of drainage systems, to meet the specifications of maximum uplift pressures, is still based on an idealised problem with known solution (Casagrande, 1961), even though, in practice, distinct field conditions are generically found.

In the context of reliability analysis, the uncertainty on the uplift pressure quantification, considering the uplift factor k_u as a random variable limited to the unit interval, shall be properly taken into account by probabilistic models that represent realistically the variety of field conditions that are expected to exist in the foundation of concrete dams. Therefore, the information obtained from the piezometric monitoring of several Portuguese large concrete dams was treated from a technical perspective in order to attain the most suitable models, combining physical arguments, experience and judgement. Since the data revealed not only the dependency on the local hydromechanical properties of the rock mass foundation but also a time effect, the uplift pressures were considered, through the uplift factor k_u, as stochastic processes.

Regarding the influence of the hydromechanical behaviour of the rock mass foundation, two categories were identified and advantageously associated with the situations distinguished in the French guidelines (CFBR, 2012) describing the uplift pressures on rock mass foundations with regular and unfavourable geologic conditions. To overcome the time effect in defining priorly the uplift pressures, due also to the unpredictability of loss of drainage effectiveness, the fundamentals of the theory of stochastic load combinations were revised in order to proceed with a simplified time-independent formulation. In that way, the arbitrary 'point-in-time' distribution and maximum value distribution, associated with each category, could be derived considering the data observed during the entire life of the structure which ensured the generalisation intended.

Probabilistic models characterised by beta distributions are here proposed for the uplift factor k_u. For regular geologic foundation conditions, mean values of 0.19 (coefficient of variation of 42%) or 0.27 (coefficient of variation of 46%), regarding instantaneous or annual maximum values, respectively, are recommended. The corresponding characteristic values are within the range of values used in safety regulations (CFBR, 2012; RSB, 2018; Ruggeri, 2004a). On the other hand, for unfavourable foundation conditions, mean values of 0.54 (coefficient of variation of 20%) or 0.62 (coefficient of variation of 22%), regarding instantaneous or annual maximum values, respectively, are recommended. The corresponding characteristic values are higher than the admitted in safety regulations (Ruggeri, 2004a), even though the French guidelines (CFBR, 2012) recommend values above 0.50. However, these cases shall occur only in occasional situations since the weights obtained in the parametric inference procedure were around 20% (Figure 8). This distinction emphasises the importance of a correct classification since different geometric properties would be required to fulfil the stability requirements. Furthermore, the significant standard deviations obtained alerts for the fact that the probabilistic nature of the uplift pressures shall definitely be considered in safety analysis and the probabilistic models proposed in this paper can then be used for that purpose.

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