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Sliding stability analyses of a rock slope using deterministic, semi-probabilistic and probabilistic methods

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Sliding stability analyses of a rock slope using deterministic, semi-probabilistic and probabilistic methods

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Abstract. Stability analyses of geotechnical structures in rock are traditionally performed using deterministic methods. In Europe, Eurocode7, introduced in the beginning of the 21st century, adopts limit state design and semi-probabilistic methods, using partial factors for the design of geotechnical structures. Meanwhile, reliability-based design, using probabilistic methods, is becoming more common in practical cases. The paper considers an intentionally simple case study—the analysis of a slope in a rock mass with one discontinuity, considered in a discrete way, forming a rock block to be stabilised by anchors—to compare the results obtained with the different methods. The objective is to calculate the force applied by the anchors so that the ultimate limit states of sliding of the rock block is verified. Deterministic-based design optimization considering both the traditional global safety factor approach and the partial factor approach following the Eurocode 7 are first applied. A reliability-based design optimization procedure—which takes geometrical and mechanical properties of the discontinuities as random variables—is then used, and the results are compared to the former ones. A discussion is presented concerning the consistency of the obtained results.

1. Introduction
Stability analyses of geotechnical structures in rock are traditionally performed with deterministic methods, using global factors of safety (FoS). The FoS values are mainly based on experience and differ from case to case, depending on the data uncertainty and on the acceptable risk of failure [1].

In Europe, the structural Eurocodes (EN 199x) for design of buildings and civil engineering works were introduced in the beginning of the 21st century and brought structural reliability concepts to the design of geotechnical structures. EN 1990 (Eurocode: Basis of structural design) [2] is the main Eurocode, with which all other Eurocodes comply, and EN 1997 (Eurocode 7: Geotechnical design) [3] deals with the design of geotechnical structures. The suite of Eurocodes is currently undergoing a process of evolution to a second generation and a major feature of the revision of Eurocode 7 is to consider soil and rock on an equal basis. Hence, it also applies to the design of geotechnical structures in rock masses. Eurocode 7 adopts limit state design using a semi-probabilistic approach, where uncertainties are considered implicitly by the specification of characteristic values of random variables (actions, effect of actions, resistances, and ground properties) and partial factors applied to those variables.

The inclusion of rock masses in the current revision of the Eurocode 7 raises the problem of calibrating the values of the partial factors affecting the ground properties, since values used for soils may, or may not, be adequate for rock masses and for rock discontinuities. In the latest draft of the revised Eurocode 7, issued in April 2021 [4], when dealing with rock, only partial factors for the "shear strength of rock" are considered, and their values are the same as for the "effective shear strength" of soils. Although there is no mention to it, they should apply indiscriminately to rock masses, rock

discontinuities and rock matrix. Validity of these values of the partial factors should be investigated, so that geotechnical structures designed with them display the reliability levels prescribed in the Eurocodes for probabilistic analyses and do not differ from the inherent reliability levels achieved with the current deterministic design methods. Another issue that also deserves attention when applying Eurocode 7 to rock masses is the consideration of the geometrical uncertainties. The semi-probabilistic approach using partial factors applies to actions and to ground strength, but cannot be applied to geometrical properties, such as those related to the rock discontinuities, namely their orientation, and thus they are considered in a purely deterministic way.

Reliability-based design using probabilistic methods is becoming more common in practical cases due to the increasing availability of specific software and to the continuously improved computational capabilities that designers have at hand. These methods are proper alternatives when the semi-probabilistic approach fails to produce structures with the required reliability level, owing to the uncertainties involved. These situations are mainly caused by the complexity of the partial safety factor calibration process for a wide range of structures and variables covered by the code. Besides, probabilistic methods can easily consider the uncertainty of other variables, such as geometrical properties, which are of relevance in rock masses. These methods are also essential for calibration of the partial factors used in semi-probabilistic methods, and this is particularly important at the present stage of the revision of Eurocode 7 for its application to geotechnical structures in rock masses.

This paper presents an intentionally simple case study, where deterministic, semi-probabilistic and reliability-based approaches are used, and where the uncertainties in the mechanical properties and in the geometry are considered. The results obtained are compared, and their consistency is discussed.

2. The rock slope case study

The geotechnical structure selected as case study of this analysis is represented in figure 1. Being a two-dimensional example, a 1 m slice will be considered. A 25 m high slope is excavated in a good quality rock mass. The slope face dips at 85º and a persistent rock discontinuity outcrops at the top horizontal surface, with a trace parallel to the slope crest, at a distance of 8 m, dipping towards the slope at an angle ψ.

As the slope is excavated, an anchor will be placed 6 m from the initial rock mass surface, dipping 10º downwards, to prevent sliding of the rock block that may be formed by the discontinuity. Finally, after full excavation of the slope and rock anchoring is completed, a permanent vertical distributed load with a representative value $G_{rep} = 200$ kN/m² will be applied on the top horizontal surface of the slope.
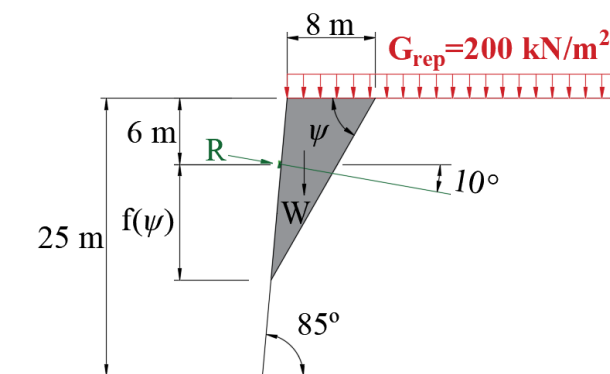

Figure 1. Schematic representation of the rock slope main geometrical data and loading.

The objective of this case study is to calculate the force to be applied by the anchor to prevent failure by sliding of the rock block. Firstly, deterministic-based design optimization is used, considering both the traditional global factor of safety approach and the partial factor approach following the Eurocode 7. Then, a reliability-based design optimization procedure is applied, taking geometrical and mechanical properties of the discontinuity as random variables described by appropriate probability functions.

The shear strength of the rock discontinuity was derived from pull tests and Schmidt hammer rebound values carried out on 18 rock discontinuity specimens and pull tests of saw cut rock surfaces using the procedure proposed by Barton and Choubey [5]. The following values of the parameters of the Barton non-linear shear strength criterion were calculated: 8.6 for the joint roughness coefficient (JRC), 79 MPa

for the joint wall compressive strength (JCS) and 25.8° for the residual friction angle (ϕ_r) [6]. Subsequently, the Barton envelope was linearized to obtain equivalent Mohr-Coulomb shear strength parameters using a linear regression for the range of normal stresses foreseen in the problem under analysis, which was taken between 0.15 and 0.25 MPa. The values calculated in this way for the apparent cohesion (c) and the friction angle (ϕ) were 28.8 kPa and 44.1°, respectively.

For the probabilistic characterization of the apparent cohesion and of the friction coefficient (tan ϕ), it was assumed that they follow lognormal distributions, with the mean values given above. Indicative values of their coefficients of variation were taken from table A.2 of [4]. Since this table gives ranges for the values of the coefficient of variation, the mid values of the ranges were used, which are $V_c = 40\%$ for the apparent cohesion and $V_{\tan \phi} = 10\%$ for the friction coefficient.

For the geometrical properties, values of the dip of the rock discontinuity (ψ) between 40° and 70° were considered in the analyses. For each analysis, the dip is given with its range of variation ($\varDelta$): $\psi \pm \varDelta$. Values of $\varDelta = 2.5°$ and $\varDelta = 5.0°$ were considered. For the probabilistic characterization of the dip angle, a Fisher distribution [7] centred at ψ was used. The values the Fisher constant (k), which is a concentration parameter, were chosen so that there is a 95% probability the dip values fall in the range $\psi \pm \varDelta$.

For the variability of the rock mass weight density (γ) a normal distribution was adopted with a mean value of 25 kN/m^3 and indicative values of the coefficient of variation (V_γ) given in table A.2 of [4]. The lower, mid, and upper values of the range given for were used which are 5%, 7.5% and 10%.

3. Deterministic, semi-probabilistic and reliability-based approaches

The stability of rock blocks is a common safety issue in rock engineering projects, and an area where probabilistic approaches are more frequent. The safety assessment of rock blocks can be found in rock slopes, created by surface excavations for open pit mining and rail or motorways, and in rock faces in structurally controlled underground excavations, such as caverns, for hydropower plants or waste disposal, and tunnels for water supply or transportation networks.

The reason why probabilistic studies related to the stability analysis of rock blocks have been carried out since long [8-12] is the fact that the required critical parameters are few and not difficult to model [13]. Moreover, no elaborate methods or equations are required for the evaluation of safety, as simple limit equilibrium analyses can be used to determine the failure probability or the reliability index [14].

Existing rock slopes have an intrinsic safety level, which may or may not meet performance requirements. In a deterministic approach, the safety level is measured by the FoS, generically given by the quotient between stabilizing and destabilizing effects of actions. Considering that constant nominal values of loads and resistance are usually adopted, the inherent uncertainty of the problem is handled by imposing values of FoS greater than one. Although there are no broadly accepted safety criteria, the target FoS for rock slopes is often set subjectively, being the range from 1.3 to 1.5 [1] related to short-term and long-term stability [15], respectively, often taken as a "rule of thumb".

The semi-probabilistic approach followed in the Eurocodes adopts the limit state design rationale (or underlying principle) and the partial factor method. Limit states are defined as boundary conditions beyond which a structure does not fulfil performance requirements. The probabilistic background is accounted by defining performance requirements as target values for the probability of failure or the reliability index. The Eurocodes, intending to harmonize the design practice across Europe, use representative values and partial factors to deal with several sources of uncertainty in a way that reliability performance requirements can be deterministically verified. Thus, safety criteria are generically translated into a problem of verifying that load effects E_d on a specific limit state are lower than the corresponding resistance R_d, i.e.:

$$E_d \leq R_d \tag{1}$$

For the verification of ultimate limit states using the partial factor method, conceptually different approaches can be followed. For the stability of slopes, Eurocode 7 indicates the adoption of the material factor approach (MFA), where E_d and R_d are respectively calculated as:

$$E_d = E\{\textstyle\sum F_{rep} \cdot \gamma_F; a_d; X_{rep}\} \tag{2}$$

$$R_d = \left\{ X_{rep}/\gamma_M \, ; a_d ; \Sigma F_{Ed} \right\} \qquad (3)$$

where γ_F are the partial factors applied to actions for design case DC3 (table A.1.8 in [2]), which are distinguished between permanent (γ_F=1.0) and unfavourable or destabilizing variable (γ_F=1.3) actions; and γ_M are the set "M2" of partial factors applied to ground properties (table 4.7 in [4]). For rock structures, to allow non-linear failure envelopes to be used, only the total shear strength is factored (γ_M=1.25K_m).

Eurocodes also differentiate structures according to the consequences of their structural failure, categorizing them as lower (CC1), medium (CC2) and higher (CC3) consequence classes, which are used to establish the consequence factor K_m applied for the ground properties (K_m = 0.9 for CC1, K_m = 1.0 for CC2 and K_m = 1.1 for CC3). Using these partial and consequence factors, the resulting structures supposedly share the same reliability level ($p_f = 10^{-3}$ for CC1, $p_f = 10^{-4}$ for CC2 and $p_f = 10^{-5}$ for CC3). However, the calibration of partial factors is subject to a trade-off between optimality and simplicity, resulting in scatter of practical reliability indices around target values. Structural optimization, taking explicitly inherent uncertainties into account, can only be fully accomplished following probabilistic approaches.

In the probabilistic approach, performance functions $G(x)$, which describe the functional relations between random variables x, are used to define limit states ($G(x)$=0). The safety levels measured by the probability of failure obtained by the integration of the joint probability density function f_x over the failure domain, i.e.:

$$p_f = P(G(x) < 0) = \int_{G(x)<0} f_x(x)dx \qquad (4)$$

Designing a structure to meet performance requirements is a rational optimization process, which depends on the selection of design control variables θ (type of material, geometry, reinforcement elements, etc.), considering all its possible constraints, in order to minimize an objective function $f(\theta)$ often associated with overall costs.

Although conceptually probabilistic, a deterministic design optimization (DDO) process is undertaken to design a structure according to the semi-probabilistic approach, just as the deterministic approach, since safety criteria (Eq. 1) are deterministic. On the other hand, to explicitly consider the inherent uncertainties of the problem, a reliability-based design optimization (RBDO) is followed. In the RBDO, the solution of the minimization problem is subjected to a reliability constraint imposing that the obtained reliability level meets its target value.

In this case study, the effective anchor force is the only design variable considered since the slope geometry and material is previously defined and unchangeable. For the DDO process, the solution is exactly known by solving the deterministic safety criteria. For the RBDO process, the solution of the corresponding inverse reliability problem can be obtained using an extended version of the Rackwicz-Fiessler algorithm [16]. The effective anchor force is iteratively searched such that a first-order reliability (FORM) estimate of the probability of failure meets its target value. From this analysis, also sensitivity factors α, representing the influence of each random variable, can be extracted.

4. Calculations and analysis

4.1. General
Stability analyses via semi-probabilistic and probabilistic approaches require realistic descriptions of the ground model, external and internal loads, and ground properties. The former uses the concept of characteristic or representative values, as in Eurocode 7 representative values can be characteristic values, if they obtained by statistical analysis, or nominal values fixed on a non-statistical basis). The latter explicitly describe random variables as probability distributions. Guidance to properly assign characteristic values and probability distributions to random variables can be found in [4].

The case study presented in this paper, intending to test the alternatives to address the geotechnical problem, ignores the possible variability of the external load, taking it as a permanent load with constant value. The analyses performed compare the solution of the problem (force applied by the anchor) following three different approaches: deterministic, such that global factors of safety from 1.0 to 2.0,

with a step of 0.1, are achieved; semi-probabilistic, such that the safety criterion (Eq. 1) is verified for structures included in consequence classes CC1, CC2 or CC3; and probabilistic, such that the target probability of failure for each consequence class is reached. For the deterministic and semi-probabilistic approaches, a DDO process is carried out, whereas, for the probabilistic approach, a RBDO process is implemented. At first, sensitivity analyses for every source of uncertainty are performed, by taking them as random variables while keeping the remaining variables constant at the respective mean value. At the end, a fully probabilistic analysis is performed by modelling all relevant sources of uncertainty as random variables.

4.2. Sensitivity analysis of the variability of the ground properties
In accordance with Eurocode 7, the characteristic value of a ground property shall be the value affecting the occurrence of the limit state. It corresponds to an average value of the ground property in the volume involved in the limit state, when the occurrence of the limit state in study is insensitive to the spatial variability of the ground property in the volume of the ground involved in the limit state. The sliding stability analysis of a rock slope along a large plane is generally insensitive to the spatial variability of the strength properties. In this case, estimates of their average values in the volume involved shall be considered as their characteristic values.

As indicated in Annex A of [4], the estimate of the mean value of a ground property to be used as the characteristic value (X_k) depends on the number of the sample derived values (n) used for the evaluation of the mean (X_{mean}), and can be obtained, when the coefficient of variation (V_x) is known or assumed, by the following equation:

$$X_k = X_{mean} \cdot \left(1 - N_{95} V_x / n^{1/2}\right) \tag{5}$$

where $N_{95} = 1.645$ represents the 95% confidence level for the normal distribution. The use of this procedure, considering the test results on 18 discontinuity samples and the mean values and coefficients of variation given in section 2, results in the following characteristic values: $c_k = 24.3$ kPa, $\phi_k = 43.0°$.

It should be borne in mind that the coefficients of variation values used so far account for the inherent variability of the ground properties but do not consider the effect of their spatial variability along the discontinuity. The total coefficient of variation can be obtained by combining the effects of the inherent and spatial variabilities, which can be done multiplying V_x by a variance reduction factor (Γ), which depends on the relation between the sliding surface length (L) and the effective autocorrelation distance or fluctuation scale (δ) of the ground property, given, for instance [17], as:

$$\Gamma^2 = \begin{cases} \frac{\delta}{L} \cdot \left(1 - \frac{\delta}{3L}\right), & \frac{L}{\delta} > 1 \\ 1 - \frac{\delta}{3L}, & \frac{L}{\delta} \leq 1 \end{cases} \tag{6}$$

The three graphs of figure 2 present the results of the calculations for three values of the fluctuation scale and thus of the variance reduction factor: $\Gamma = 1$ ($\delta = 0$), $\Gamma = 0.50$ (for $L/\delta = 3.63$) and $\Gamma = 0.25$ (for $L/\delta = 15.66$). The mean value of the discontinuity dip angle (ψ) is represented in the horizontal axes and the vertical axes represent the calculated anchor forces (R) required to stabilize the rock block. The 11 thin grey curves are the results of the deterministic calculations for FoS = 1.0, 1.1, 1.2, …, 2.0, respectively. The black continuous, dashed and dotted lines are the results of the semi-probabilistic calculations for the three consequence classes CC1, CC2 and CC3. The green, blue and red lines are the results of the probabilistic calculations for the probabilities of failure (p_f) of 10^{-3}, 10^{-4} and 10^{-5}, assigned in Eurocodes to consequence classes CC1, CC2 and CC3, respectively.

Being the Mohr-Coulomb failure criterion described by a linear envelope, the solutions of the probabilistic and deterministic problems are parallel. Different partial factors for the apparent cohesion and the friction coefficient would have distorted the semi-probabilistic solution. The solutions for the semi-probabilistic problem, in this simple case, with partial factors of 1.25 K_m applied on the characteristic values of the two strength parameters, resulted in equivalent global factors of safety (FoS) of 1.20, 1.33 and 1.46 for the consequence classes CC1, CC2 and CC3, respectively.
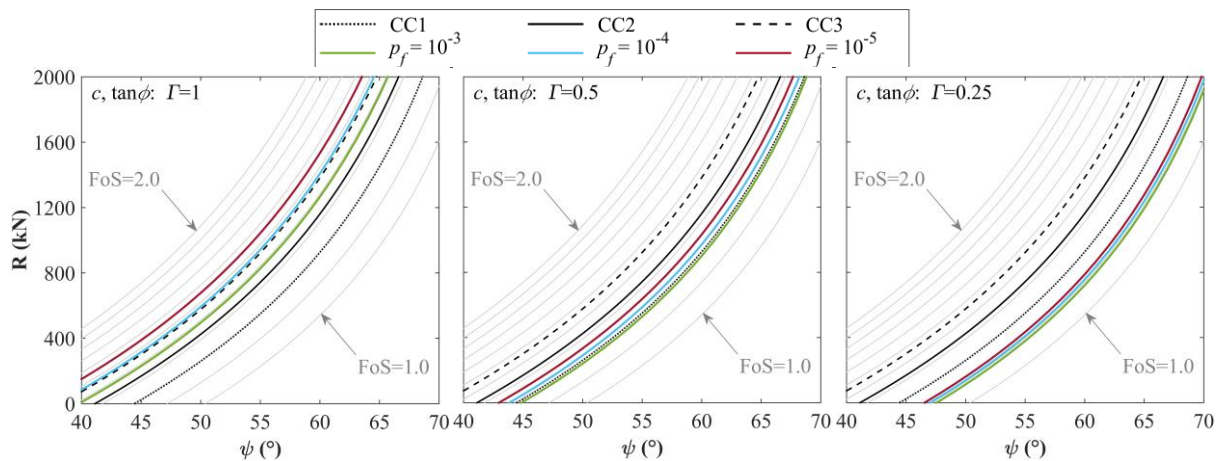
Figure 2. Anchor force obtained in the deterministic, semi-probabilistic and probabilistic approaches, considering the shear strength parameters as random variables.

As expected, the variability of the strength parameters plays a major role on the results obtained with the probabilistic analysis. The smaller the total coefficient of variation considered, the closer are the results for the three values of p_f and the closer they are to lower values of the FoS. For $\Gamma = 0.25$ the results for the three values of p_f (with FoS around 1.1) are lower than the semi-probabilistic results for CC1, while for $\Gamma = 0.5$ they are between those for CC1 and CC2 (with FoS of 1.2-1.25), and for $\Gamma = 1.0$ they are close to those for CC3 (with FoS of 1.4-1.55).

4.3. Sensitivity analysis of the variability of the rock weight density

According to EN 1990, the representative value of the weight of a structure or structural member may be calculated as the product of its nominal dimensions and the characteristic value of the weight density. The rock mass weight density γ presents an inherent variability, but due to the large dimensions of the rock block, in deterministic analyses it is reasonable to use its mean value as the characteristic value. In probabilistic calculations, a normal distribution is usually assumed, such as for most permanent actions.

The graphs in figure 3 present the results of the sensitivity tests for the three values of the coefficient of variation V_γ (10, 7.5 and 5%). For comparison purposes, the same curves as in figure 2 are presented for the factors of safety from 1.0 to 2.0 used in the deterministic calculations, and for the three consequence classes considered in the Eurocode and for the corresponding probabilities of failure.
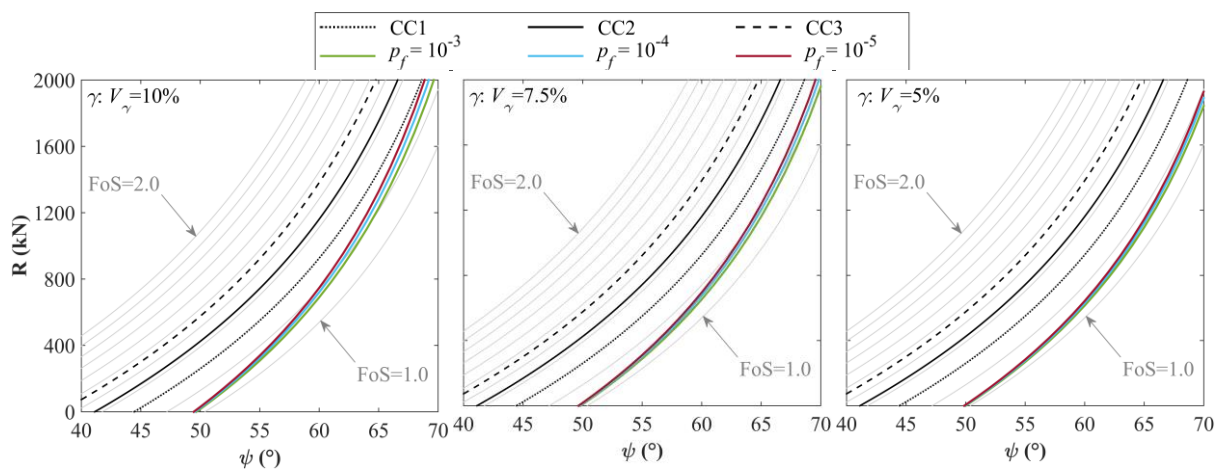

Figure 3. Anchor force obtained in the deterministic, semi-probabilistic and probabilistic approaches, considering the weight density as a random variable.

As a single source action, the weight of the wedge has correlated favourable and unfavourable effects. This causes the solution of the probabilistic approach to move away from the deterministic solution for FoS = 1 as the dip of the rock discontinuity increases. However, the coefficient of variation is low, making the influence of the rock weight density variability on the probability of failure small.

4.4. Sensitivity analysis of the variability of the dip of the rock discontinuity

Although recognizing the variability of the geometric properties, the use of nominal values is recommended in the Eurocodes. Their inherent variability is supposedly absorbed by partial factors used to calculate the design values of the effects of actions and of the resistances. However, the geometric properties play an instrumental role in rock masses where, on the contrary of soils, discontinuities often govern the occurrence of limit states. A conservative approach, selecting the worst foreseeable case regarding the geometry, is usually tentatively made. In practice, geotechnical surveys often allow to estimate the mean value of the dip of the rock discontinuity (ψ), but some dispersion (Δ) is allowed. Based on that, in stability analysis, the worst case, ψ - Δ or $\psi + \Delta$, is conservatively assumed.

For the deterministic and semi-probabilistic approaches of this case study, the nominal value of the dip of the rock discontinuity is assumed as $\psi + \Delta$. In the probabilistic calculations, the dip of the rock discontinuity is considered as a random variable following the Fisher distribution centred at ψ. The value of the Fisher constant (k) is such that the Fisher distribution encompasses a probability of occurrence of 95% in the interval [ψ-Δ, ψ+Δ]. The outcome of the sensitivity tests on the value of Δ is shown in figure 4. Note that the deterministic and semi-probabilistic curves are shifted Δ to the left, according to the above mentioned method.
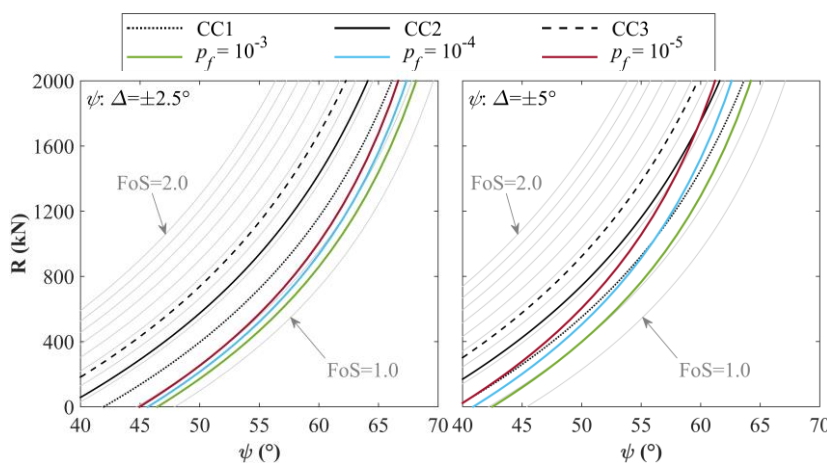

Figure 4. Anchor force obtained in the deterministic, semi-probabilistic and probabilistic approaches, considering the dip of the rock discontinuity as a random variable.

As expected, the consideration of the dip of the rock discontinuity as a random variable has a relevant impact on the results and causes stronger non-linearity on the outcome of the probabilistic analyses. Such non-linearity becomes more prominent for larger values of the dip dispersion. For $\Delta = 2.5°$ the results of the probabilistic analyses for the three consequence classes are close to the semi-probabilistic results for CC1 (with FoS of 1.05-1.15), while for higher dispersion, with $\Delta = 5°$ they are in the range from CC1 to CC3 (with FoS of 1.1-1.4). This demonstrates the importance of the effect of the uncertainties associated with the estimation of the orientation of the rock discontinuities.

4.5. Fully probabilistic analysis

The sensitivity analyses presented above provided information about the effect of the variation of each source of uncertainty on the probability of failure. Considering all sources of uncertainty simultaneously in the analysis allows comparing the solution obtained with the probabilistic, semi-probabilistic and deterministic approaches, and making some remarks on the adequacy of the application of the semi-probabilistic approach of the Eurocodes in rock engineering.

Since the role played by the uncertainty of the rock weight density is low, only the case with $V_\gamma = 5\%$ was considered. For the shear strength the same three cases were considered, with three values of Γ. For

the dip of the rock discontinuity, the same two cases were considered, with different values of Δ. The results obtained for the resulting six combinations are presented in figure 5. Note that the deterministic and semi-probabilistic solutions are shifted Δ to the left, as explained above.
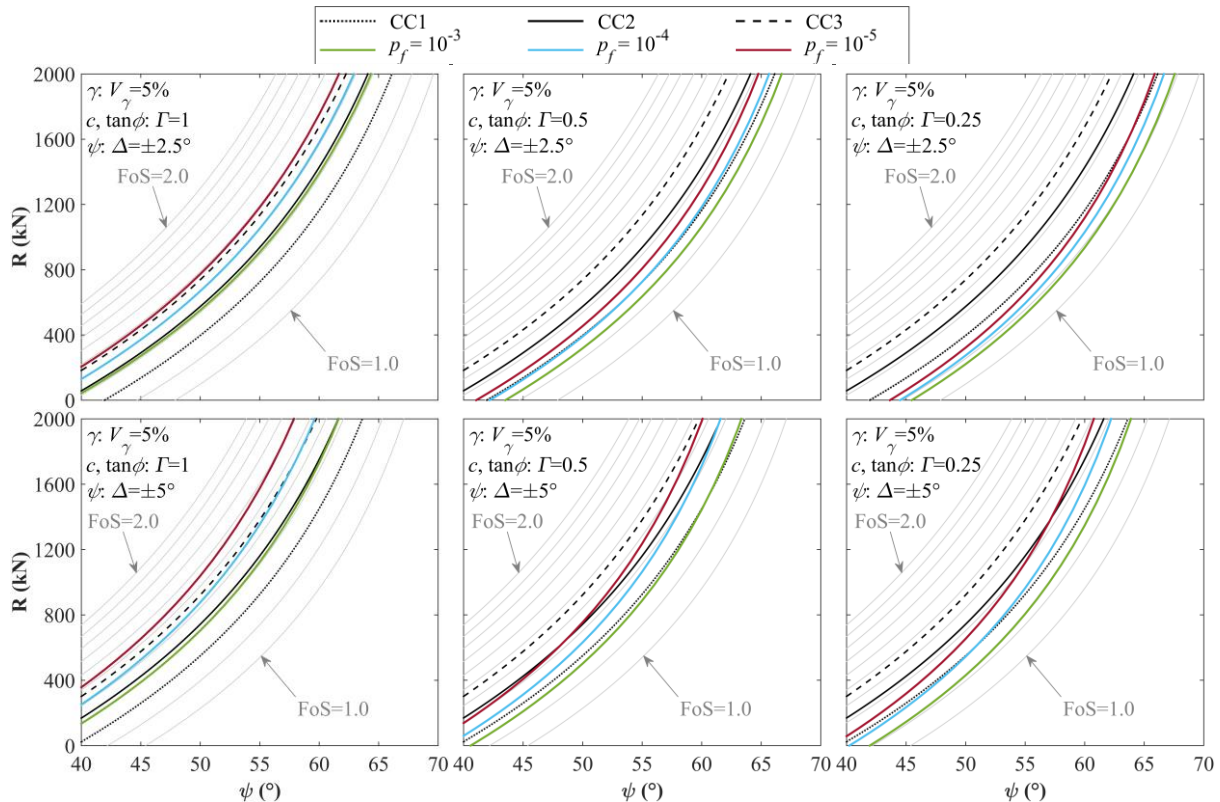

Figure 5. Anchor force obtained in the deterministic, semi-probabilistic and probabilistic approaches, considering the shear strength parameters, the dip of the discontinuity and the weight density as random variables.

For the analysis of the results, it should be stressed that the only realistic cases are those with $\Gamma = 0.25$, because they consider, in a more adequate way, the effect of spatial variability on the reduction of the total coefficient of variation of the strength parameters. The other cases are only presented to illustrate the influence of the dispersion of the strength parameters on the results of the probabilistic analyses.

As expected, figure 5 shows that, as the total coefficient of variation of the ground strength properties decreases, the lower are the values obtained for the force R in the probabilistic analyses, and they correspond to lower values of the equivalent FoS. The same occurs when the dip angle range decreases. It can be noticed that for higher mean values of ψ the results of the probabilistic analyses increase over-proportionally when compared with the deterministic and semi-probabilistic results. However, for more realistic cases with $\Gamma = 0.25$, the probabilistic results are lower than the semi-probabilistic results, except for higher values of ψ and Δ.

As an aid for the analysis of the results, sensitivity factors (α) were calculated for these six cases. They are the direction cosines of a unit-length vector pointing to the design or most probable point obtained in the inverse FORM algorithm used in the probabilistic calculations. They represent the relative influence of each random variable on the solution of the problem, conditioned by both the limit state and the target reliability index or probability of failure. Figure 6 shows the sensitivity factors obtained, for the six cases, considering $p_f = 10^{-4}$. Note that random variables with negative (destabilizing) effect have positive values (γ and ψ), and are read in the left y-axis, whereas random variables with positive (stabilizing) effect have negative values (c and ϕ), and are read in the right y-axis.
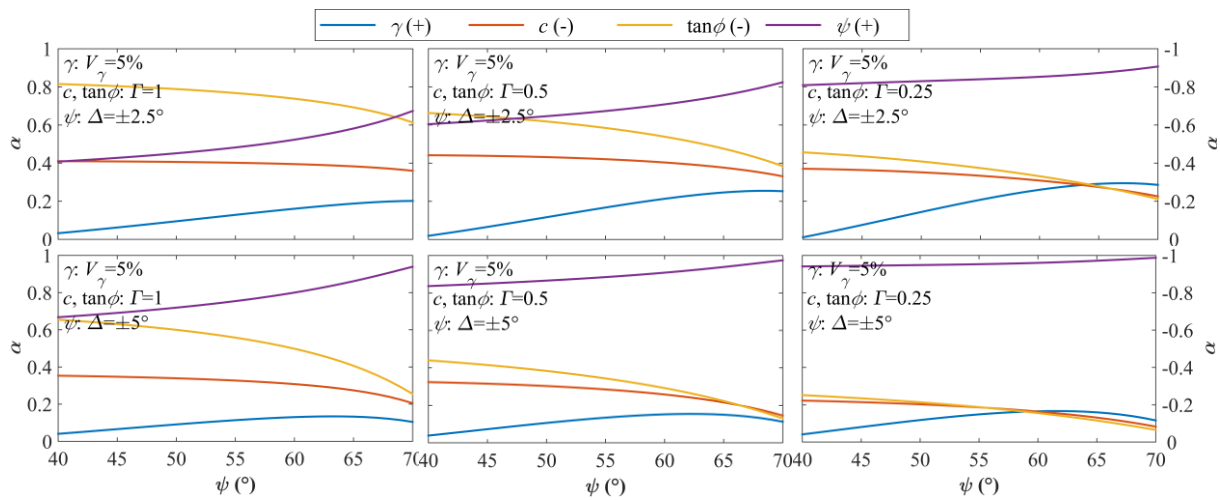
Figure 6. Sensitivity factors considering $p_f = 10^{-4}$.

Figure 6 shows how the sensitivity of the results to the strength parameters decreases when their dispersion decreases (Γ decreases), and simultaneously the sensitivity to the angle ψ increases. For the more realistic cases with $\Gamma = 0.25$, the sensitivity of the results to the angle ψ is clearly dominant, while the relative influences of the strength parameters and of the weight density are lower and reach similar values for mean dip angles ψ higher than 55º.

Finally, figure 7 presents the equivalent global FoS values obtained in this case study, following the semi-probabilistic and probabilistic approaches, as a function of the mean dip angle ψ of the discontinuity. The values for $\psi = 45º$, $\psi = 55º$ and $\psi = 65º$ are presented in the tables of the figure. For the lower dispersion of the dip of the discontinuity, with $\Delta = 2.5º$, the equivalent global FoS values for the three values of p_f are always lower than those obtained with the semi-probabilistic method. For $\Delta = 5.0º$ the equivalent global FoS values for the three values of p_f increase rapidly and for mean dip angles ψ above 65º become higher than those obtained with the semi-probabilistic method. This means that the mean value and the dispersion of the dip of the discontinuity influence significantly the equivalent global FoS values and that it is not possible to directly compare the results of the three approaches independently of these geometric properties.
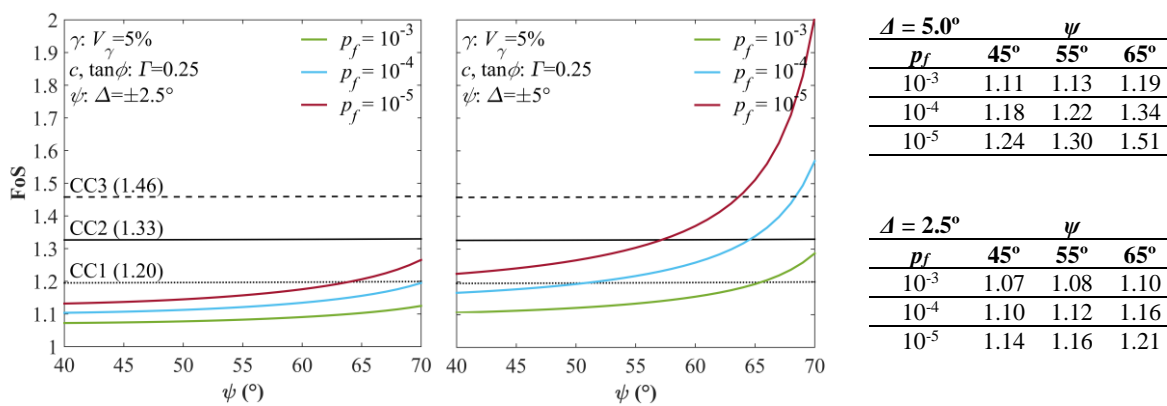

$\Delta = 5.0º$	ψ		
p_f	45º	55º	65º
10^{-3}	1.11	1.13	1.19
10^{-4}	1.18	1.22	1.34
10^{-5}	1.24	1.30	1.51

$\Delta = 2.5º$	ψ		
p_f	45º	55º	65º
10^{-3}	1.07	1.08	1.10
10^{-4}	1.10	1.12	1.16
10^{-5}	1.14	1.16	1.21

Figure 7. Equivalent factors of safety obtained with the semi- and probabilistic approaches.

5. Conclusions

Though the case study is particularly simple, it allows drawing interesting conclusions. Firstly, the reliability levels obtained with the semi-probabilistic approach, with partial factors provided by Eurocode 7, and the deterministic approach, with the traditional FoS values, do not differ substantially. In both approaches the most unfavourable dip angle of the discontinuity was used in the calculations. It

should be noted that the characteristic values of the strength parameters are estimates of the mean values corresponding to a 95% confidence level and are lower than the actual mean values obtained from the test results. By performing more tests and improving the knowledge about the strength parameters, reduction of the anchor forces could be achieved.

The fully probabilistic analyses consider the variability of the strength parameters, the dip angle of the discontinuity and the rock weight density, and lead to a significant reduction of the anchor forces. The results are much more sensitive to the variability of the dip angle of the discontinuity than to that of the strength parameters, while the sensitivity to the rock weight density is almost negligible. This means that carrying out adequate surveys of the geometrical properties of rock discontinuities, to reduce their associated uncertainty, can have a relevant impact on the results obtained. Considering the coefficient of variation of the strength parameters by combining the inherent variability with the effect of the spatial variability, as a function of the relation between their fluctuation scale and the extent of the failure surface, has a great influence on the results of the probabilistic calculations.

These conclusions give important indications concerning the results of the three approaches, but their validity cannot be simply extrapolated. Many other similar exercises should be done, and particular attention should be paid to the calibration of partial factors values used in the Eurocode for safety verification of limit states in rock mass structures.

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