

## CONCRETE GRAVITY DAMS STABILITY: A COMPARATIVE STUDY BETWEEN UNKEYED AND KEYED FOUNDATIONS



**Renato Pereira<sup>1</sup>**  
Laboratório Nacional de  
Engenharia Civil (LNEC)



**António Lopes  
Batista**  
Laboratório Nacional de  
Engenharia Civil (LNEC)



**Luís Canhoto Neves**  
University of Nottingham

### ABSTRACT

Concrete gravity dams are mass concrete structures which resist to external loads mainly by their dead weight. The geometrical solution currently considered, which evolved from the reasoned application of mathematical theory to structural engineering, are characterized by right-angled triangular profiles with downstream face slopes of 0.7 to 0.8.

In medium to high seismic intensity zones, such as Portugal, other constructive dispositions may be necessary. Under seismic loadings, it is generally accepted that the gravity profile keyed into the foundation at a depth corresponding to 10% of the dam height is a crucial contribution to ensure structural stability conditions. However, this detail is often not considered in stability analyses, which is generally understood as a conservative strategy.

In this work, the benefits from considering the keyed depth in stability analyses are evaluated. For that, 100-meter-high hypothetical gravity profiles, keyed at a depth of 10 meters, are considered. To allow the development of rigid-body failure mechanisms, a downstream rock wedge, inclined at a critical angle, is assumed. At first, the analytical expressions that describe the failure mechanisms identified, considering the dam-foundation interface as a dominant failure surface, are deduced and validated through numerical modelling. Afterwards, the frictional properties of the interface, for several loading conditions which result in different total net forces and the corresponding application points, are computed. Lastly, the stability benefit is evaluated by comparing the safety factor obtained with the correspondent of an unkeyed profile.

When explicitly considering the keyed depth in stability analyses, higher safety levels are obtained which can be crucial to ensure stability conditions. It was proved that, under the same load conditions, the consideration of unkeyed profiles would demand higher values of the friction angle, up to 12° more than considering the correspondent keyed profile. Moreover, this also ensures stability conditions for more inclined resultant net forces.

**Keywords:** Concrete gravity dams, keyed profiles, stability analysis, numerical modelling, safety factor.

## 1. INTRODUCTION

The structural solution adopted nowadays for the design of concrete gravity dams evolved from rudimentary practices, in times immemorial, to modern conceptions based not only on engineering but also on economic concerns. The rudimentary embankment structures, with trapezoidal sections, had not changed considerably until the Roman era when the construction of masonry dams became widespread. From that time on, based upon accumulated experience, the quantities of masonry gradually decreased leading to more triangular sections than trapezoidal.

At the 19<sup>th</sup> century, rationality was introduced into the dam design, after the linkage between mathematical theory and structural engineering have been established [1, 2]. Dam design consisted in searching for the most economical profile that ensures certain performance requirements, specifically that applied stresses are kept below a practical limit, which ultimately led to curved-face triangular profiles with laborious execution. This was performed considering both trial-and-error techniques and classical methods of structural analysis under limiting assumptions. The dam was considered homogeneous and continuous, the method of sections was used to obtain the maximum stresses and the structural properties were taken from the masonry material itself.

At the end of the 19<sup>th</sup> century, given the widespread use of Portland cement [3] and the development of auxiliary means of transport [4], masonry was being replaced by concrete. Given the development of concrete manufacturing technology, the mechanized concrete placement and formwork techniques, as well as the higher material capacity, there were no practical or theoretical objections to the right-angled triangular profile [5] which lasted to current days. The downstream face slope would depend on the coefficient of friction necessary to prevent sliding as a rigid body.

Dams were designed considering first only the hydrostatic pressure from the reservoir and, later, also the uplift pressures due to the water seepage through higher permeability zones of the dam-foundation system. Since the 1950s, given the increasing height of built dams, the effects of earthquakes in the structural safety were also taken into account. In that case, the typical gravity profiles, characterized by vertical or near-vertical upstream faces and downstream faces with slopes of 0.7 to 0.8 [6], may not be sufficient to ensure stability conditions in medium to higher intensity seismic zones [7]. While, in some locations, the trend to increase the inclination of the upstream face was followed [7], in other locations it is generally accepted that the gravity profile keyed into the foundation is a crucial contribution to ensure structural stability conditions. In fact, this practice had been already suggested by Wegmann [6], who stated that the excavation of the foundation shall continue to a certain depth after bed

rock is reached, even if the rock is perfectly solid, in order to lock the foundation into the rock and prevent dam sliding. A compilatory study concluded that, in Portugal, the keyed depth is, in average, 10% of the dam height [8]. However, this detail is often not explicitly considered in stability analyses, which is generally understood as a conservative strategy. In this work, the benefits from considering the keyed depth in stability calculations are evaluated. Firstly, the principles for the stability analysis of concrete gravity dams are revised and the analytical expressions that describe the failure mechanisms are deduced and numerically validated. Lastly, the stability benefits are evaluated by comparing the safety factor obtained with the correspondent of an unkeyed profile, considering several loading conditions which result in different total net forces and application points.

## 2. STABILITY OF CONCRETE GRAVITY DAMS

Gravity dams are mass concrete structures which resist to external loads mainly by their dead weight. They are divided into independent monoliths, intersected by vertical transverse contraction joints. Each monolith corresponds to a concrete block, placed in batches, originating construction or lift joints which, if not carefully treated, may compromise the stability of the blocks above and be a preferential path for water transfers with serious consequences regarding structural integrity. Given the existence of transverse contraction joints, the safety analysis of concrete gravity dams is advantageously conducted based on two-dimensional simplifications, neglecting conservatively any three-dimensional effect, by considering the most conditioning monolith typically located on the riverbed.

Due to their dimensions, internal stresses in gravity dams are, in general, much smaller than the concrete strength. The safety of concrete gravity dams is therefore usually considered practically independent on the mechanical strength of the concrete, since sliding or overturning failures generally occur before a conditioning stress field is achieved [9]. Thus, the design of concrete gravity dams is based on the stability analysis of the cross-section profile, considering rigid body mechanisms. For that, any potential failure surface shall be tested, either in the dam body, the dam-foundation interface or within the rock mass foundation. Stability analysis is primarily performed on the dam-foundation interface. Although they could attain some relevancy for large concrete dams, lift joints are not usually conditioning on the stability evaluation, since good construction strategy shall ensure adequate resistance properties. Rock joints may compromise the overall stability mostly in cases when unsafe geometry and/or mechanical properties are identified.

The stability conditions are evaluated worldwide mainly through the concept of safety factor [10] that relates the stabilizing and destabilizing actions, i.e.,

$$FS = R/S \quad (1)$$

Depending on whether evaluating safety against sliding or overturning,  $R$  and  $S$  are the driving force and bearing capacity or the stabilizing and destabilizing moments around the rotation point, respectively. Stability conditions are ensured if the safety factor is greater than one.

For unkeyed gravity profiles (Figure 1a), considering the sliding along the dam-foundation interface, the shear strength is evaluated, in its simplest form, by the Mohr-Coulomb failure criterion which takes the contribution of both cohesive and frictional components into account,

$$R = c \cdot A + N \cdot \tan \varphi \quad (2)$$

where  $c$  is the cohesion,  $A$  is the contact area,  $N$  is the normal force and  $\varphi$  is the friction angle.

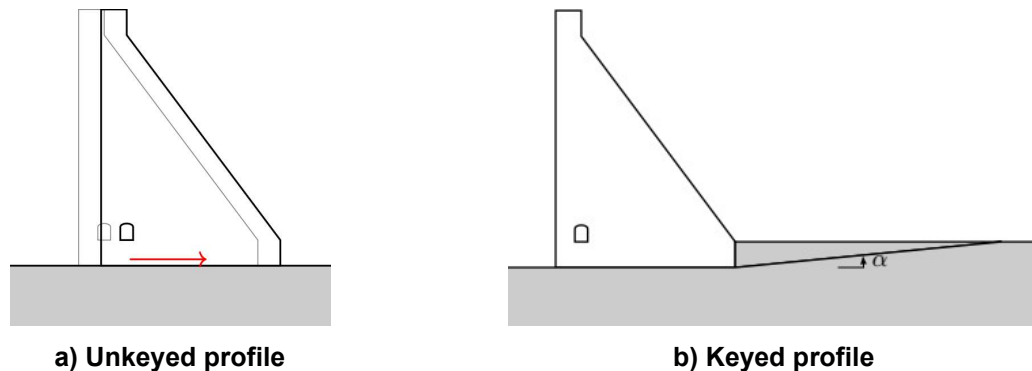


Fig. 1 – Unkeyed vs keyed gravity profile

Expression (3) models the shear behaviour of intact sliding surfaces that, if considered in stability analysis, represents a perfect-plastic behaviour with brittle failure. However, in most cases, failure occurs after a gradual degradation involving a progressive and more complex mechanism often exhibiting a near-elastic pre-peak behaviour and a softened post-peak behaviour towards a residual strength. Therefore, the Portuguese dam safety regulation [11], considers, by conservatism, that, for ultimate equilibrium conditions, only the residual capacity of the frictional component shall be considered.

For keyed gravity profiles (Figure 1b), an extra downstream passive resistance is mobilized. This contribution can also be expressed analytically considering that the development of failure mechanisms implies the mobilization of a downstream rock wedge. In that case, the shear strength is given by [12],

$$R = \overbrace{c_1 \cdot A_1 + N \cdot \tan \varphi_1}^{\text{Interface}} + \overbrace{\left( \frac{c_2 \cdot A_2}{\cos \alpha \cdot [1 - \tan \alpha \cdot \tan \alpha]} \right) + W_w \cdot \tan(\alpha + \varphi_2)}^{\text{Passive wedge resistance}} \quad (3)$$

where  $c_i$ ,  $A_i$ ,  $\varphi_i$  are the cohesion, contact area and friction angle, respectively, of the dam-foundation interface (index 1) and the passive wedge slope (index 2),  $N$  is the normal force to the dam-foundation interface and  $W_w$  is the downstream wedge dead weight.  $\alpha$  is the downstream face slope which is usually taken as  $45^\circ - \varphi/2$  [12, 13]. Again,  $c$  may be absent and  $\varphi$  may be residual when past movement has occurred.

Expressions (3) and (4) represent the limit conditions before destabilization corresponding to a small displacement analysis. However, for keyed profiles, after the movement has been initiated, new equilibrium configurations are immediately achieved such as reported in experimental studies of numerical and physical models under static [14, 15] and dynamic loadings [16]. The type of mechanism following the plastification of the sliding surfaces (Figure 2), corresponding to a large displacement analysis, depends on the direction of the total net force [16]:

- In failure mechanism 1, which shall occur when the direction of the total net force intersects the dam-foundation interface (Plane A-B), the gravity profile slides along the dam-foundation interface, pushing the downstream rock wedge;
- In failure mechanism 2, which shall occur when the direction of the total net force intersects the Plane B-C originating only compressions stresses, the gravity profile and the downstream rock wedge slide together along the downstream rock slope;
- In failure mechanism 3, which shall occur also when the direction of the total net force also intersects the Plane B-C but originating tensile stresses near the dam toe (point B), the gravity profile rotates pushing the downstream rock wedge; and
- In failure mechanism 4, which shall occur when the total net force passes above the point C, the profile rotates over the downstream rock wedge.

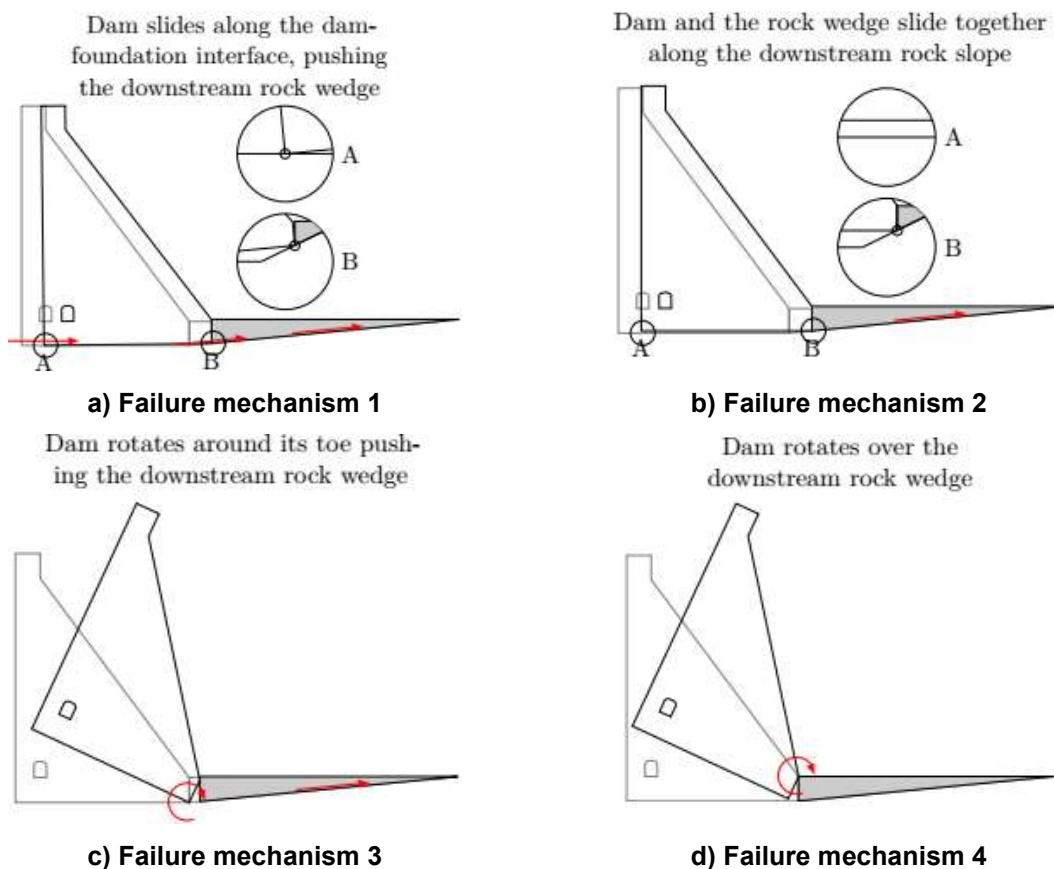


Fig. 2 – Ultimate failure mechanism for keyed profiles

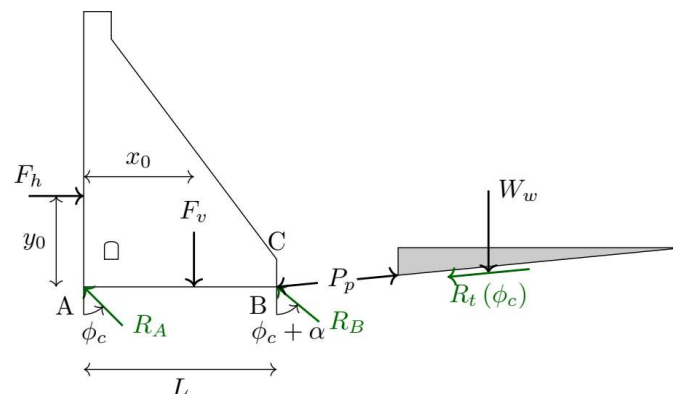
### 3. FAILURE MODE MODELING

The failure mechanisms identified can be analytically expressed considering the reactions produced on contact points. For that, limit equilibrium is then characterized by the critical shear strength defined as the minimum shear strength below which stability is not verified, considering the same failure criterion for all surfaces. By simplicity, since limit analysis is being performed, only the residual frictional component of the critical shear strength, given in terms of the critical friction angle ( $\varphi_c$ ), is considered such as recommended in the Portuguese dam safety regulation [10]. Thus, the safety factor, in accordance with the former principles, is taken as,

$$FS = \varphi / \varphi_c \quad (4)$$

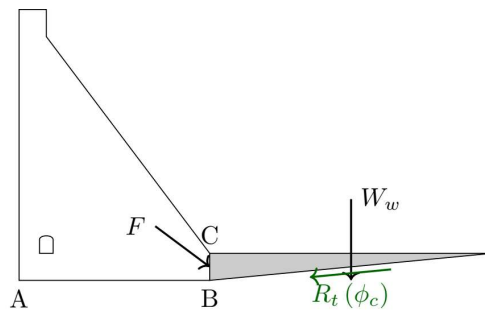
where  $\varphi$  is the existing friction angle.

Failure mechanism 1 illustrates the limit equilibrium conditions characterized by the sliding of the gravity profile which climbs the downstream wedge slope while pushing the rock wedge. The gravity profile rests on the foundation in two contact points (A and B) producing punctual reactions ( $R_A$  and  $R_B$ ). Given the expected movement, an unbalanced force ( $P_p$ ) is transmitted to the rock wedge which ultimately slides along the downstream slope. Accordingly, to reproduce the movement, the ultimate shear strength must be achieved in both surfaces. This mechanism would occur if the total net force intersects the dam-foundation interface (plane A-B). Figure 3 shows the corresponding free-body diagram. The critical friction ( $\varphi_c$ ) angle can be deduced from a system of equilibrium equations.



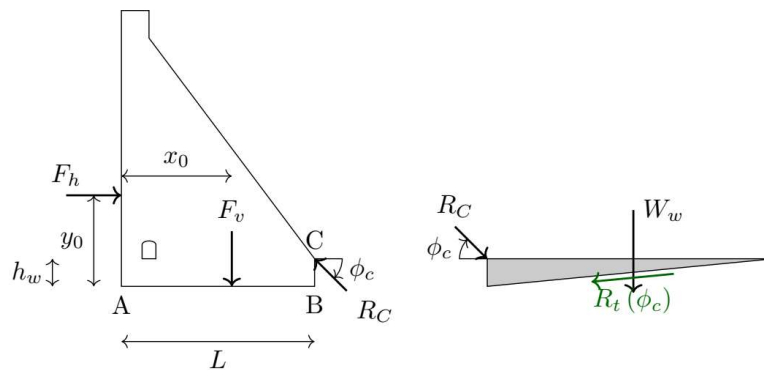
**Fig. 3 – Free-body diagram of the failure mechanism 1**

Failure mechanism 2 illustrates the limit equilibrium conditions characterized by the sliding of both the gravity profile together and the rock wedge along the downstream wedge slope. To reproduce the movement, the ultimate shear strength must be achieved in this surface. This mechanism would occur if the total net force intersects the plane B-C. Figure 4 shows the corresponding free-body diagram. The critical friction angle ( $\varphi_c$ ) can be deduced considering the equilibrium of the dam-rock wedge set.



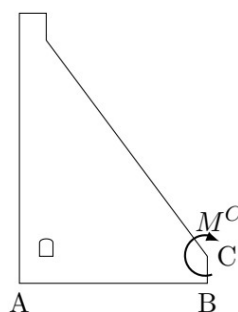
**Fig. 4 – Free-body diagram of the failure mechanism 2**

Failure mechanism 3 illustrates the limit equilibrium conditions characterized by the rotation of the gravity profile around the dam toe pushing the rock wedge which slides along the downstream wedge slope. The rotation of the gravity profile implies that the contact between it and the rock wedge is made on point C producing a punctual reaction ( $R_c$ ). The rock wedge would then slide along the downstream wedge slope in which the ultimate strength is achieved. This mechanism would occur if the total net force intersects the plane B-C originating tensile stresses near the dam toe. Figure 5 shows the corresponding free-body diagram. The critical friction ( $\phi_c$ ) angle can be simply deduced considering the equilibrium of the rock wedge.



**Fig. 5 – Free-body diagram of the failure mechanism 3**

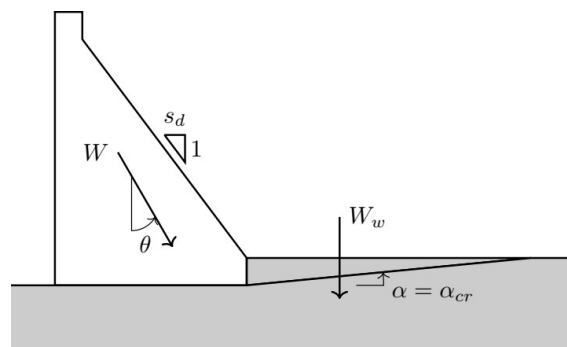
Failure mechanism 4 illustrates the limit equilibrium conditions characterized by the rotation of the gravity profile over the rock wedge (around point C). This mechanism would occur if the total net force passes above point C. Figure 6 shows the corresponding free-body diagram. In this case, the safety factor is given by the relation between the stabilizing and destabilizing moments.



**Fig. 6 – Free-body diagram of the failure mechanism 4**

#### 4. NUMERICAL VALIDATION

The analytical solution of the stability analysis of keyed gravity profiles, deduced from the equilibrium equations which reproduce the failure mechanisms presented, was here validated. For that, the critical friction angle obtained analytically was compared to the critical friction angle obtained in numerical discrete-element models [17], since these are particularly appropriate to perform large displacement analysis. To manipulate the direction of the total net force, the dead weight of the gravity profile ( $W$ ) was artificially applied at different inclinations according to angle  $\theta$ . Four hypothetical 100-meter-high gravity dam monoliths (Figure 7) keyed into the foundation at a depth of 10 meters, with downstream face slopes ( $s_d$ ) of 0.25, 0.50, 0.75 and 1.00 were considered. The inclination of the downstream rock wedge is assumed as the value ( $\alpha_{cr}$ ) that results in higher critical friction angles. The critical friction angle of the numerical models is obtained using the strength reduction method, considering the Mohr-Coulomb failure criterion, with only the frictional component, for all surfaces.



**Fig. 7 – Representation of the quantities involved in the numerical model**

Figure 8 compares the critical friction angle obtained analytically (different colours for different failure mechanism) and numerically (black circles) for the cases analysed. The critical friction angle obtained for unkeyed gravity profiles are also shown.

In all cases, the critical friction angle and the failure mechanism match the results obtained in numerical models, proving the correctness of the corresponding analytical description. When the resultant net force intersects the dam-foundation interface (plane A-B), i.e., on the left of the first vertical dotted line, failure mechanism 1 is expectedly the most conditioning. When the resultant net force intersects the plane B-C, i.e., between the two vertical dotted lines, failure mechanism 2 or 3 are confirmed to be the most conditioning. Failure mechanism 3 would only be the most conditioning in some cases with more inclined downstream face slopes which may produce tensile stresses near the dam toe. In fact, the profiles with downstream face slopes of 0.25 and 0.50, clearly outside the range of practical values, were tested to force failure mechanism 3 to be the most conditioning. When the resultant net force passes above the point C, i.e., on the right of the second vertical dotted



line, failure mechanism 4 will occur, independently on the friction angle. These observations confirm what was expected and validate the analytical description of the failure mechanisms identified which are then representative of the ultimate limit states referring to loss of static equilibrium.

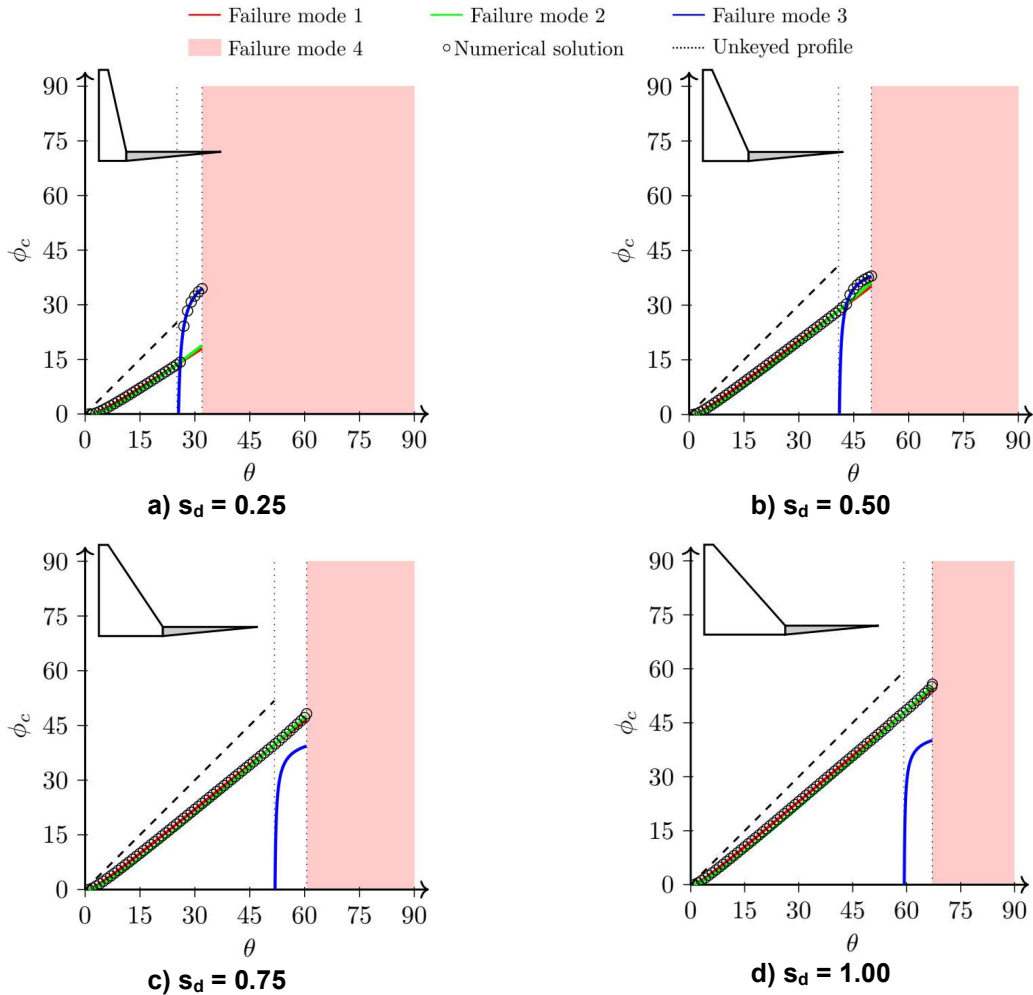


Fig. 8 – Analytical and numerical solutions of the cases analysed

### 5. STABILITY CONDITIONS OF KEYED AND UNKEYED FOUNDATIONS

The stability conditions of gravity profiles keyed into the foundation are here compared to the correspondent of unkeyed profiles. For that, assuming safety factors given by expression (4), the benefits from considering explicitly the keyed depth into the calculations are measured by an increment of the critical friction angle achieved ( $\delta_1$ ), under the same loading conditions, given by,

$$\delta_1 = \varphi_c^{Keyed} - \varphi_c^{Unkeyed} \tag{5}$$

where  $\varphi_c^{Keyed}$  and  $\varphi_c^{Unkeyed}$  are the critical shear strength, defined as the minimum shear strength below which stability is not verified, computed for the keyed and unkeyed profile stability conditions, respectively.

Moreover, there is an extra benefit due to the ability of the keyed profile to sustain positive moments around B, as opposite as unkeyed profiles which would destabilize by overturning in that case. This benefit is quantified through an increment of the critical stability conditions ( $\delta_2$ ) given by,

$$\delta_2 = \theta(M_C = 0) - \theta(M_B = 0) \tag{6}$$

where  $\theta(M_i=0)$  is the angle of the resultant net force to the vertical that produces null moments around point  $i$ , i.e., when the resultant net force intersects point  $i$ .

Figure 9 illustrates the benefits achieved when considering explicitly the keyed depth into stability analyses.

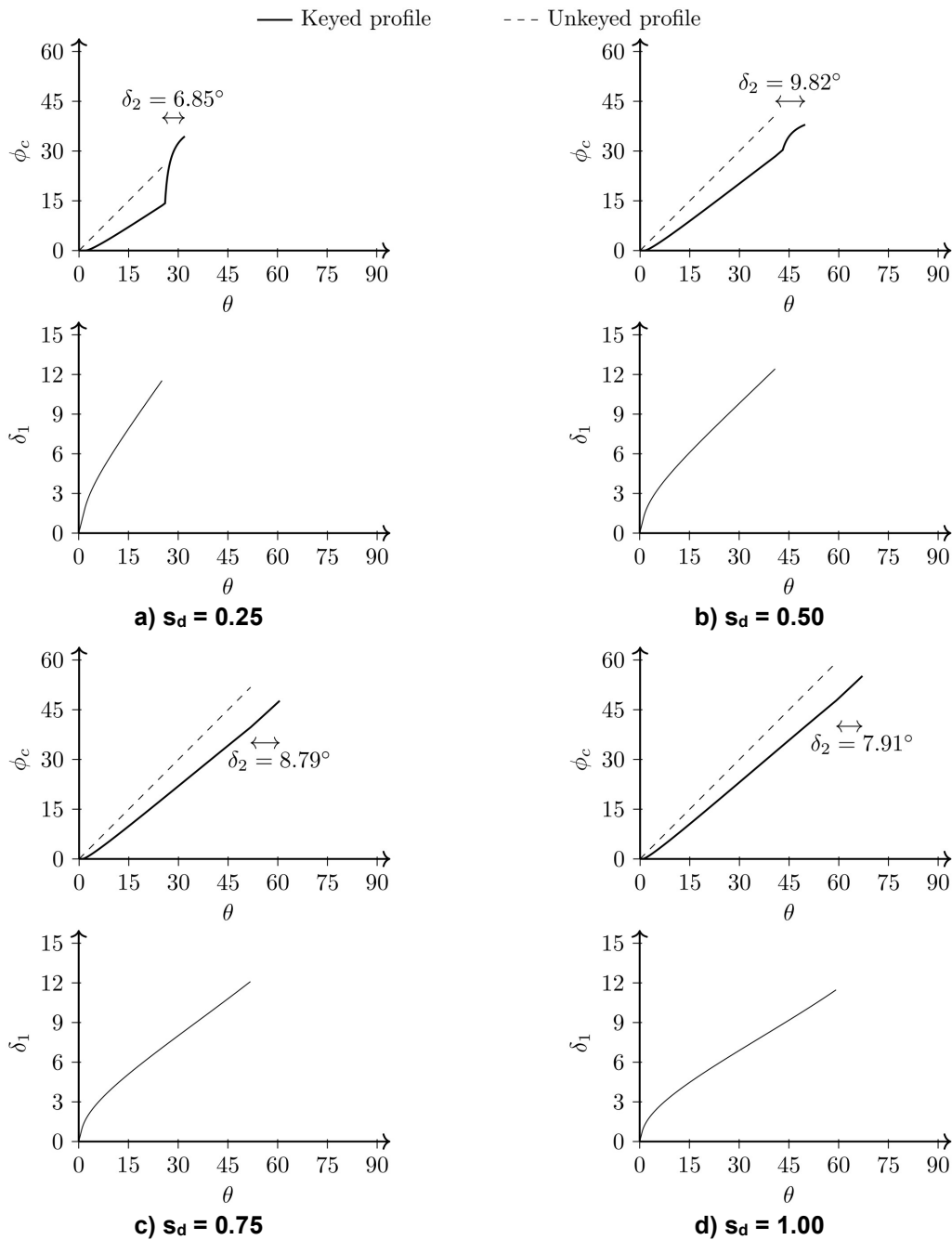


Fig. 9 – Benefits from considering the keyed depth into the stability analyses

The figure above clearly shows that with the consideration of the keyed depth in stability analyses, higher safety levels are obtained. Although varying with the downstream face slope and the angle of the resultant net force to the vertical, the benefits achieved, characterized by the increments  $\delta_1$  and  $\delta_2$ , can be crucial to ensure stability conditions. In fact, under the same load conditions, the consideration of unkeyed profiles would demand higher values of the friction angle, up to  $\delta_1=12^\circ$  more than considering the correspondent keyed profile. Moreover, this also ensures stability conditions for more inclined resultant net forces whose benefit would depend on the downstream face slope. For the slopes analysed, values of  $\delta_2$  between  $6.85^\circ$  and  $9.82^\circ$  were obtained.

## 6. CONCLUSION

The structural solution adopted nowadays for the design of concrete gravity dams is characterized by right-angled triangular profiles with vertical or near-vertical upstream faces and downstream face slopes of 0.7 to 0.8. In medium to higher seismic zones, large concrete gravity dams would also be keyed into the foundation at a depth corresponding to 10% of the dam height, to prevent loss of equilibrium. However, this aspect is frequently not considered in stability analyses, which is generally understood as a conservative strategy.

In this work, the benefits from considering the keyed depth in stability analyses were evaluated. For that, 100-meter-high gravity profiles keyed into the foundation at a depth of 10 meters were considered. A downstream rock wedge, inclined at a critical angle, was considered to allow the development of rigid-body failure mechanisms. Firstly, the failure mechanisms cinematically compatible with these geometric features were identified, and their analytical descriptions were deduced and validated through comparison with discrete-element models. Lastly, the stability benefits were evaluated by comparing the safety factor obtained with the correspondent of an unkeyed profile, considering several loading conditions which result in different total net forces and application points.

When the resultant net force intersects the dam-foundation interface, failure mechanism 1, describing the sliding of the gravity profile which climbs the downstream wedge slope while pushing the rock wedge, is the most conditioning. When the moment around the dam toe is positive, failure mechanism 2, describing the sliding of both the gravity profile together and the rock wedge along the downstream wedge slope, is frequently the most conditioning. However, if the resultant net force is such that produces tensile stresses near the dam toe, failure mechanism 3, describing the rotation of the gravity profile around the dam toe pushing the rock, may be the most conditioning, especially for more incline downstream face slopes. When the result net force passes above the downstream face slope, the gravity profile would rotate over the rock wedge (failure mechanism 4). These observations confirmed what was

expected and validated the analytical description of the failure mechanisms identified which are then representative of the ultimate limit states referring to loss of static equilibrium.

When explicitly considering the keyed depth in stability analyses, higher safety levels were obtained which can be crucial to ensure stability conditions. It was proved that, under the same load conditions, the consideration of unkeyed profiles would demand higher values of the friction angle, up to 12° more than considering the correspondent keyed profile. Moreover, this also ensures stability conditions for more inclined resultant net forces.

## REFERENCES

- [1] Sazilly, J. A. T. (1853). Sur un type de profil d'égal résistance proposé pour les murs des réservoirs d'eau. *Annales de Ponts et Chaussées*: 191-222.
- [2] Rankine, W. J. M. (1871). Report on the design and construction of masonry dams. *The Engineer* (33): 1-2.
- [3] Bruggemann, D. A., K. J. Hollock, and G. P. Sims (2001). Historic concrete in dams. In: *Historic Concrete: Background to Appraisal* (Eds. J. Sutherland, D. Humm and M. Chrimes), Thomas Telford: 343-357.
- [4] Arce, D. S., and A. B. Herrera (2003). Construction techniques and auxiliary facilities used in the construction of masonry and early concrete dams. *Proceedings of the First International Congress on Construction History*, January, Madrid, Spain: 1783-1794.
- [5] Wegmann, E. (1888). *The design and construction of dams*. John Wiley & Sons.
- [6] Corns, C. F., G. S. Tarbox, and E. K. Schrader (1988). Gravity dam design and analysis. In: *Gravity dam design and analysis* (Ed. R. B. Jansen), Van Nostrand Reinhold: 466-492.
- [7] ICOLD (2000). *The gravity dam: a dam for the future. Review and recommendations. Bulletin 117*. International Committee on Large Dams.
- [8] Pedro, J. O. (2018). *As barragens portuguesas enquadradas pelos regulamentos de segurança*. Laboratório Nacional de Engenharia Civil, Lisboa, Portugal.
- [9] Chopra, A. K., and L. Zhang (1991). Earthquake-induced base sliding of concrete gravity dams. *Journal of the Structural Division* (117), 12: 3698–3719.
- [10] Ruggeri, G. (2004). *Sliding safety of existing gravity dams. Final Report*. International Commission on Large Dams (ICOLD) - European Club.
- [11] RSB (2018). *Regulamento de segurança de barragens. Decreto-Lei 21/2018*. Diário da República, Lisboa, Portugal.
- [12] Underwood, L. B., N. A. Dixon (1976). Dams on rock foundations. In: *Rock Engineering for Foundation and Slopes Vol. 2*: 125-146.

- [13] Leclerc, M., P. Léger, and R. Tinawi (2001). CADAM User's manual. Version 1.4.3. École Polytechnique de Montréal, Canada.
- [14] Pina, C. A. B, C. P. Costa, J. V. Lemos, and J. P. Gomes (1994). An experimental study of failure of a gravity dam on a jointed rock foundation. Memória 798. Laboratório Nacional de Engenharia Civil, Lisboa, Portugal.
- [15] Gomes, J. P., C. P. Costa, J. V. Lemos, and C. A. B. Pina (1997). Study on geomechanical model of the failure of a gravity dam due to sliding along the foundation. Memória 810. Laboratório Nacional de Engenharia Civil, Lisboa, Portugal.
- [16] Gomes, J. P. (2005). Análise experimental de cenários de rotura em fundações de barragens de betão. Ensaios estáticos e dinâmicos, PhD Thesis, Universidade Federal do Rio de Janeiro, Brazil.
- [17] Pereira, R. (2019). Probabilistic-based structural safety analysis of concrete gravity dams. PhD Thesis, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Portugal.
- [18] Itasca (2011). UDEC - Universal Distinct Element Code, Version 5.0, User's manual. Itasca Consulting Group, Minneapolis, USA.