Output-Only Modal Identification on Multi-Span Continuous Viaducts

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ABSTRACT

This paper calls the attention to a type of civil engineering structures – multi-span continuous viaducts – that can have natural vibration modes with very close frequencies, which are as close as greater is the number of spans. This, off course, creates difficulties in what concerns the experimental evaluation of the dynamic properties of that type of structures, especially when using natural input testing and output-only modal identification methods. In the paper, a brief review is made of a few aspects concerning the dynamic properties of multi-span continuous beams with constant cross section. The tests and modal properties extraction analysis performed in two highway viaducts with continuous decks (with 4 and 16 spans) are also presented. Considering the data from those tests, the performance of an output-only modal identification method is analyzed and the identified dynamic characteristics are compared with the ones computed with finite element models of the viaducts.

1. Introduction

One of the difficulties involved in natural input testing and output-only modal identification is the capacity to extract the modal parameters of structures with very close modes, since there is no control on the excitation forces and only structural responses are analyzed. Some works [1, 2] on output-only modal identification of civil engineering structures, exemplify with tower like structures a practical situation where close modes can appear. Another situation, that eventually can become even more difficult in terms of modal identification, is the case of multi-span continuous decks, a structural solution that is common to find in highway viaducts.

Theoretically a multi-span continuous beam with constant cross section has its modes of vibration in concentrated zones and within each zone there are as many modes as the spans of the beam, with frequencies that are very close to each other [3]. If this happens with both the flexural and torsional modes, the experimental evaluation of the modal characteristics of a multi-span viaduct deck can become in fact a difficult task.

In this paper, after addressing a few aspects about the dynamic properties of multi-span continuous beams with constant cross section, a presentation is made of the dynamic tests and modal properties extraction analysis performed in two highway viaducts with continuous decks (with 4 and 16 spans). The tests that were performed in those structures consisted in the measurement of accelerations induced by the traffic of loaded trucks. The measured structural responses were analyzed using an output-only modal analysis method. The performance of the method is analyzed and the identified dynamic characteristics are also compared with the ones computed with finite element models of the viaducts.

2. Dynamic characteristics of multi-span continuous beams with constant cross section

The natural frequencies and mode shapes of the vertical modes of multi-span continuous beams with constant cross section can be determined analytically from the solution of the homogeneous differential equation of motion:

$$EI\,\frac{\partial^4 v(x,t)}{\partial x^4} + \mu\,\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \qquad (1)$$

where: EI is the flexural stiffness of the beam; μ is the mass per unit length of the beam; $v(x,t)$ is the vertical deflection of the beam at the point x and at the time instant t.

Considering harmonic vibrations with frequencies ω_j, the solution of equation (1) can be assumed to be:

$$v(x,t) = \sum_{j=1}^{\infty} v_j(x)\,\sin(\omega_j\,t) \tag{2}$$

where $v_j(x)$ are the modes of natural vibration, which have the following general form:

$$v_j(x) = A_1 \sin\left(\frac{\lambda_j\,x}{l}\right) + A_2 \cos\left(\frac{\lambda_j\,x}{l}\right) + A_3 \sinh\left(\frac{\lambda_j\,x}{l}\right) + A_4 \cosh\left(\frac{\lambda_j\,x}{l}\right) \tag{3}$$

In equation (3) the coefficients A_i are integration constants, which are determined from the boundary conditions of the beam. In the case of multi-span continuous beams the integration constants A_i are different from span to span. Thus in an n span continuous beam there are $4n$ constants to be determined. These integration constants are found from the solution of a homogeneous system of linear algebraic equations, whose non-trivial solution requires that the corresponding determinant should be equal to zero. This enables to formulate a frequency equation from which the parameters λ_j are found. The frequencies f_j of the natural vibration modes of a beam are then determined from the parameters λ_j considering the relation:

$$f_j = \frac{\lambda_j^2}{2\pi\,l^2}\sqrt{\frac{EI}{\mu}} \tag{4}$$

In the case of multi-span continuous beams with constant cross section and equal spans, the solution of the equations presented above, shows that the natural vibration modes appear in groups or concentrated zones of frequencies. Each group contains as many modes as the spans of the continuous beam and in each dense zone the natural frequencies are very close to each other. This fact is illustrated in figure 1 where for beams with 1, 5, 10 and 15 spans, the order n of their natural vibration modes is plotted as a function of the corresponding normalized frequency, which is given by the relation of the frequency f_n of each mode with the frequency f_1 of the first mode of the beam. Notice that the purpose of figure 1 is just to show the closeness of the frequencies and the fact that the modes appear in groups, and not the actual values of the frequencies, which off course depend on the span l, the flexural stiffness EI and the mass per unit length μ.

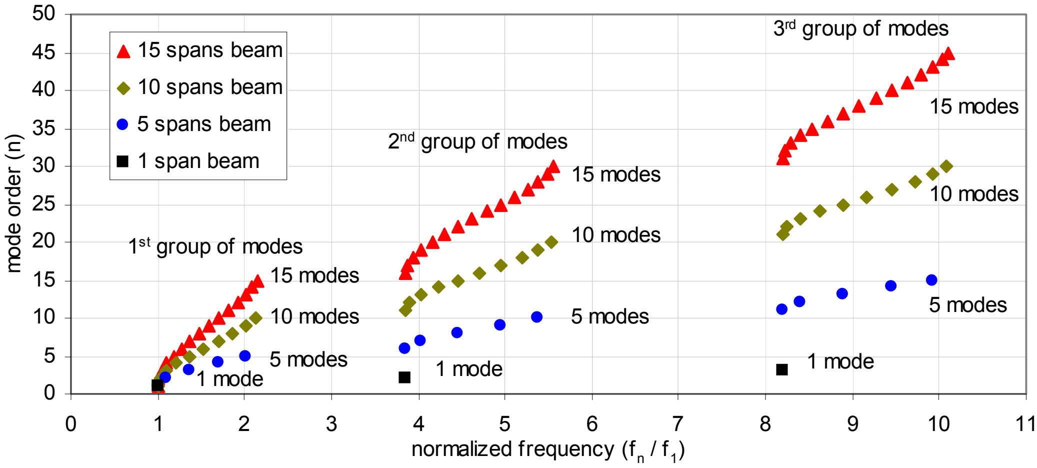


Fig. 1 – Natural frequencies of continuous beams with constant cross section.

The plot in figure 1 was made only up to the third group of modes of a beam, but it could have been continued indefinitely. Figure 1 shows quite clearly that in each group of modes the frequencies are very close to each other, especially when the number of spans is high.

Another interesting way of showing the closeness of the frequencies of the vertical vibration modes of continuous beams is through the analytically evaluated spectra of their response to white noise input loads. This was done for beams with 1, 5, 10 and 15 spans assuming damping coefficients of 1% for all the modes and considering the vertical responses at the mid-span and quarter-span points of all the spans; the corresponding averaged spectra that were obtained are represented in figure 2. In this figure the above mentioned features of the vibration modes of continuous beams are quite evident: the modes appear in groups; in each group there are as many modes as the number of spans; and in each group the frequencies of the vibration modes are very close to each other.

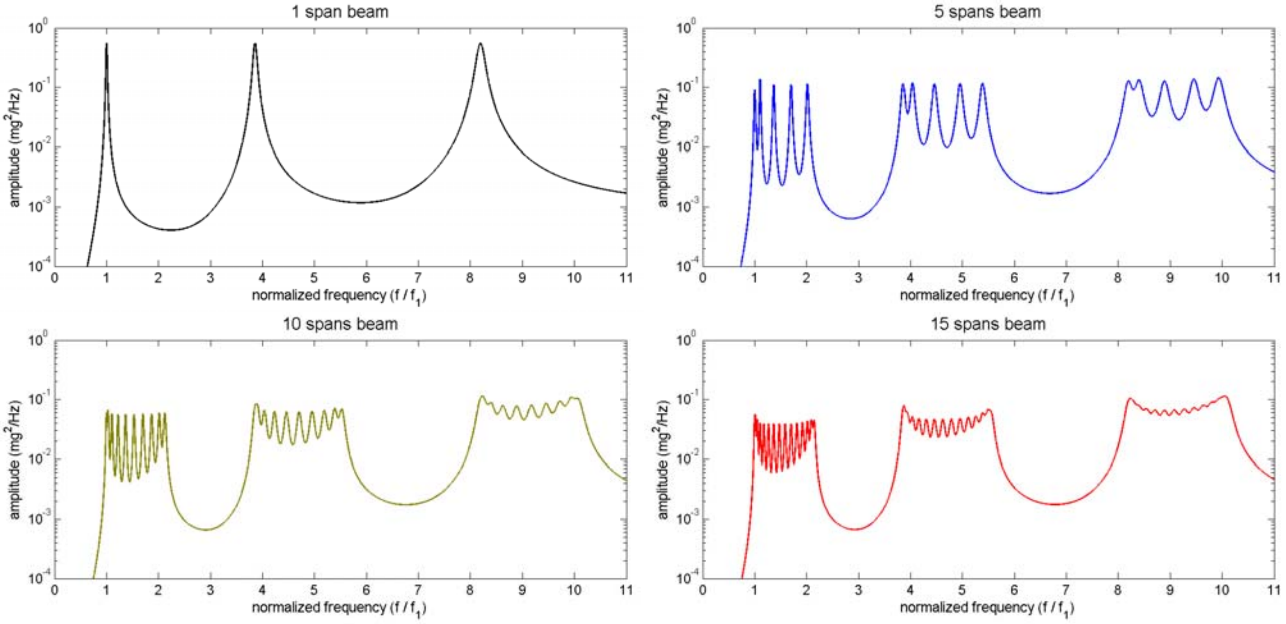


Fig. 2 – Analytical spectra of the vertical response of continuous beams with constant cross section.

The mode shapes for a few cases of continuous beams with constant cross section are illustrated in figures 3 to 5.

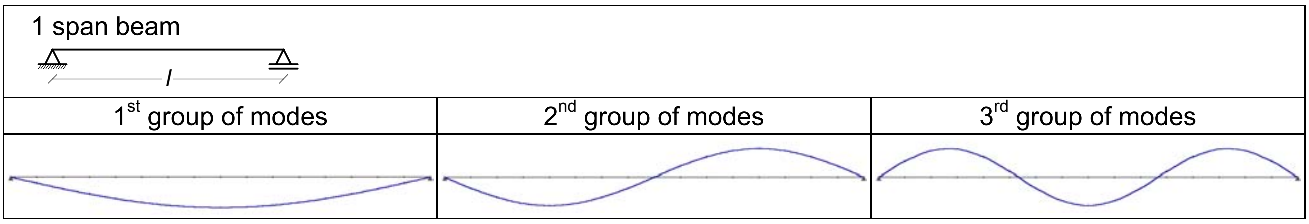


Fig. 3 – First three groups of modes for a 1 span beam with constant cross section.

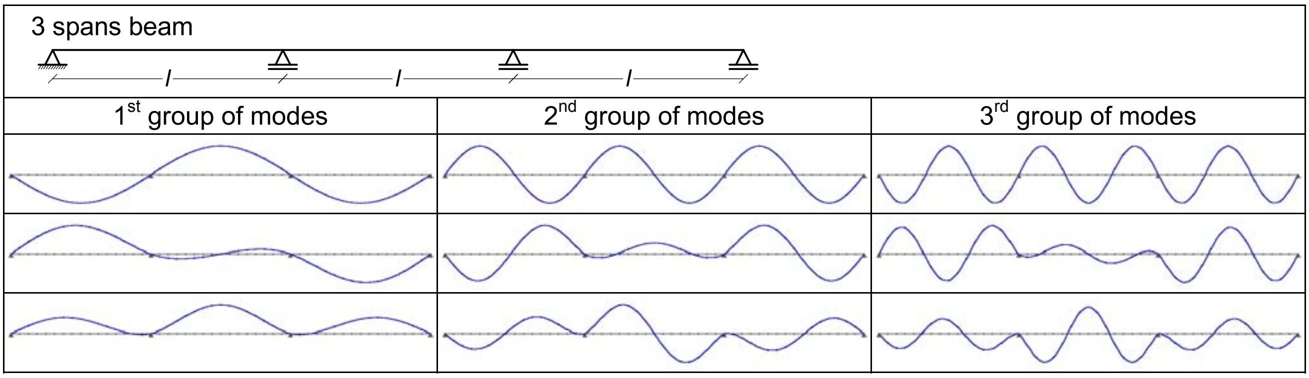


Fig. 4 – First three groups of modes for a 3 spans beam with constant cross section.

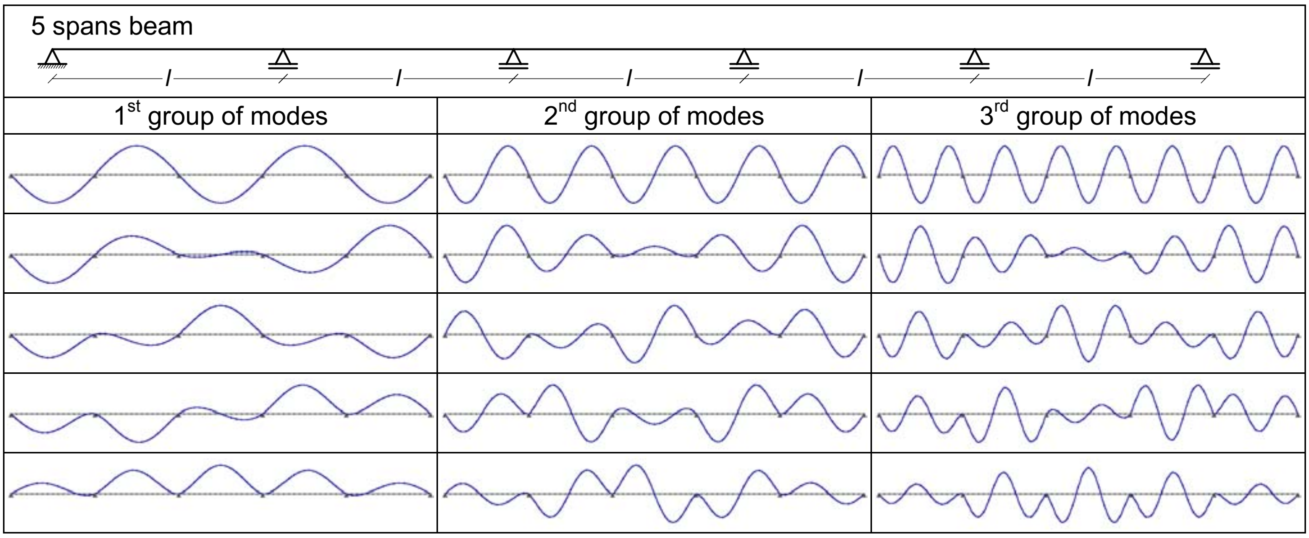


Fig. 5 – First three groups of modes for a 5 spans beam with constant cross section.

The above results show that in fact multi-span continuous beams with equal spans and constant cross section have vibration modes that appear in groups or dense regions of modes, with very close frequencies in each group. Those results correspond of course to a theoretical model that considers a beam as a structural element with small transverse dimensions when compared to its length, assumes the theory of small deformations, the validity of Hooke's law, the Navier hypothesis and the Saint-Venant principle [3]. Some cases of civil engineering structures, like the deck of multi-span continuous viaducts, are close to that theoretical model, although all the referred assumptions may not be completely fulfilled.

One should notice also that in the above results, only the vertical modes were considered, in practice the deck of a viaduct will have also torsional modes exhibiting as well the features of grouping of the modes and closeness of the frequencies. All this considered leads us to the conclusion that multi-span continuous viaducts may be a really difficult situation in terms of experimental modal analysis, especially when they have a large number of spans.

The examples that will be presented bellow illustrate some results that were obtained and the difficulties that were encountered with natural input dynamic tests and output-only modal analysis performed on multi-span continuous viaducts. Two viaducts will be considered, respectively with 4 and 16 spans. Only the results in terms of vertical and torsional modes will be presented and discussed, although the tests also involved the measurement and analysis of transverse and longitudinal accelerations.

3. Dynamic tests and output-only modal identification on a 4 spans continuous viaduct

The first example that is presented in this paper is the V2 viaduct of the A14 highway that connects the cities of Coimbra and Figueira da Foz in Portugal. It is a 4 spans continuous viaduct with equal spans of 34 m each one. The general characteristics of the V2 viaduct are illustrated in figures 6 and 7.

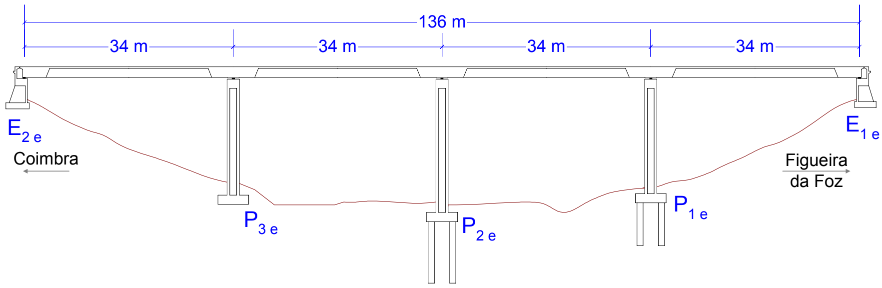


Fig. 6 – Longitudinal section of the V2 viaduct.

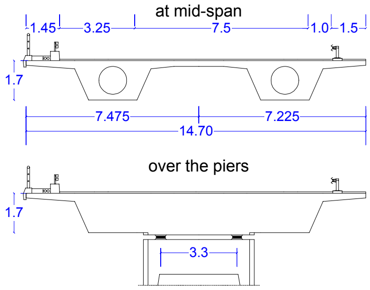


Fig. 7 – Cross sections of the V2 viaduct.

The dynamic tests [4] of the V2 viaduct were conducted as part of its reception load tests that also included static tests [5] and were performed at the end of the construction of the viaduct, before it was opened to the traffic. The dynamic tests consisted in the measurement of accelerations in the structure, induced by the traffic of loaded trucks (the same that were used in the static load tests). The measurements were taken in all the spans at the mid-span and quarter-span sections and also at the sections over the piers. A total of three set-ups were performed, as it is represented in figure 8. In each set-up, the accelerations were measured with 15 Kinemetrics ES-U accelerometers, during a total time of about 14 minutes and using a sampling frequency of 200 Hz. During the tests, the ES-U's and the signal conditioning equipment were configured for a sensitivity of 0.125 Volt/mg.

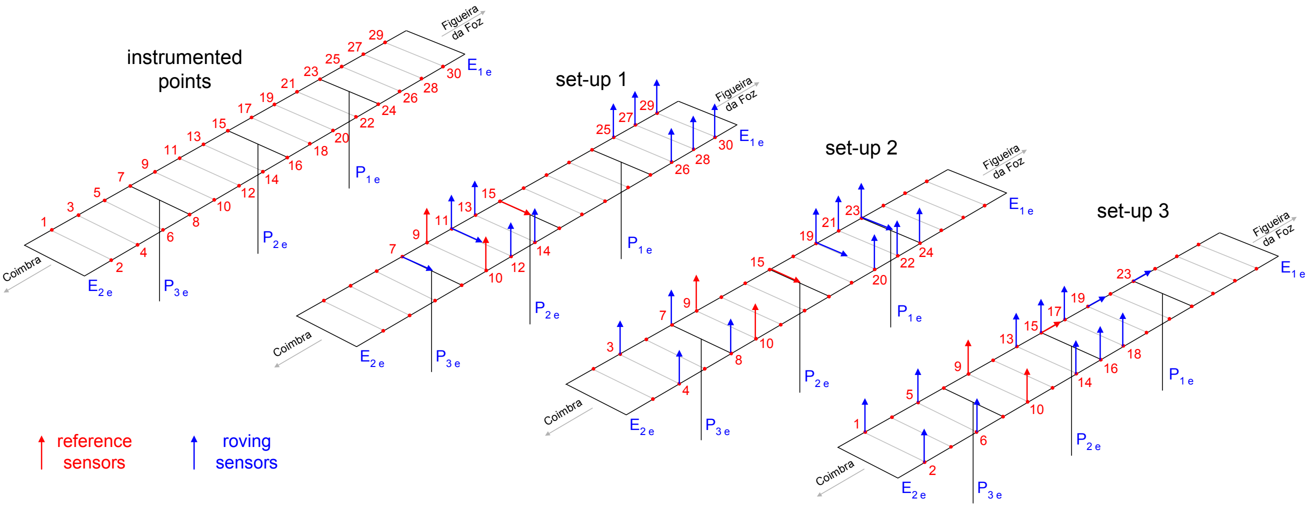


Fig. 8 – Instrumented points and set-ups in the dynamic tests of the V2 viaduct.

The acceleration data collected in the dynamic tests was first pre-processed, with low-pass digital filtering at 20 Hz with a 4 poles Butterworth filter and decimation to a sampling frequency of 50 Hz.

The modal identification analysis was performed using the basic frequency domain method (BFD) [6] or peak picking method (PP). For that purpose the spectral density functions of the response of the structure, were estimated using the technique [7] that involves the application of the FFT algorithm. Data segments with 2048 values at 50 Hz were considered for the application of the FFT, thus the frequency resolution of the estimated spectral density functions is $\Delta f = 0.024$ Hz. Figures 9 and 10 show the averaged normalized power spectral density functions (ANPSD) of, respectively, the semi-sum and the semi-difference, of the vertical accelerations measured in the deck of the viaduct (as referred above, the results that are presented in this paper will be limited to the vertical and torsional modes).

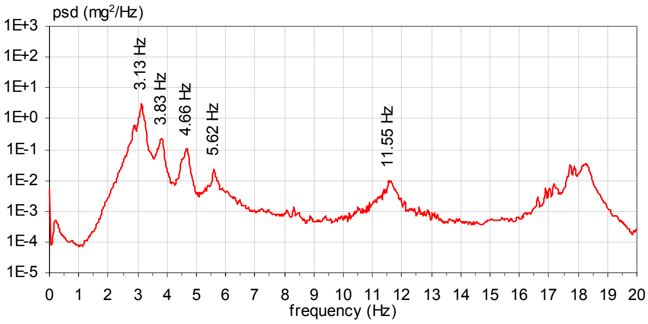


Fig. 9 – ANPSD of the semi-sum of the vertical accelerations.

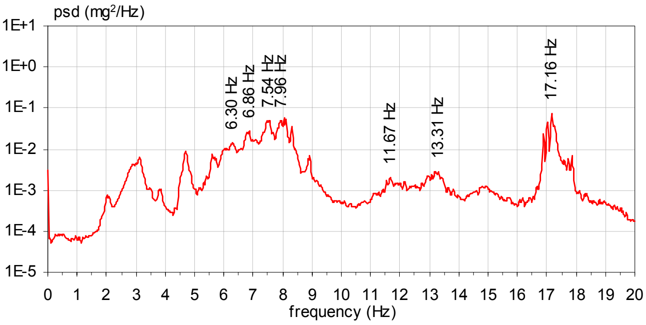


Fig. 10 – ANPSD of the semi-difference of the vertical accelerations.

The values of the frequencies of the vibration modes that were identified for the V2 viaduct are also included in the ANPSD's of figures 9 and 10. With the BFD method, the identification of those modes involved not only the selection of the peaks in the spectra but also the analysis of the coherence functions between the different measurement points and of the corresponding mode shapes extracted from the H_1 estimate of the transmissibility frequency response function also between the different measurement points.

In terms of the vertical modes of vibration, it was possible to identify all the 4 modes of the 1st group of modes (the V2 viaduct has 4 spans so, in principle, each group of vertical modes has 4 modes) since the corresponding peaks are quite evident in the ANPSD of the semi-sum of the vertical accelerations (figure 9). It was also possible to identify the 1st mode of the 2nd group of modes. In the ANPSD of figure 9, there is also a peak at a frequency of about 18.2 Hz which eventually corresponds to the 1st mode of the 3rd group of modes, however the quality of the results at that frequency didn't allowed to take that conclusion completely.

In what concerns the torsional modes (see the ANPSD of figure 10) it was harder to identify them, since the corresponding peaks aren't as evident as the peaks of the vertical modes in the ANPSD of figure 9. However, analysing also the coherence function and the mode shapes, it was possible to identify 7 torsional modes.

Besides the experimental modal identification through the application of the BFD method, a finite element (FE) model of the viaduct was also developed in order to compare the experimentally identified frequencies and mode shapes with the ones computed with the model. That comparison can be seen in figures 11 and 12.

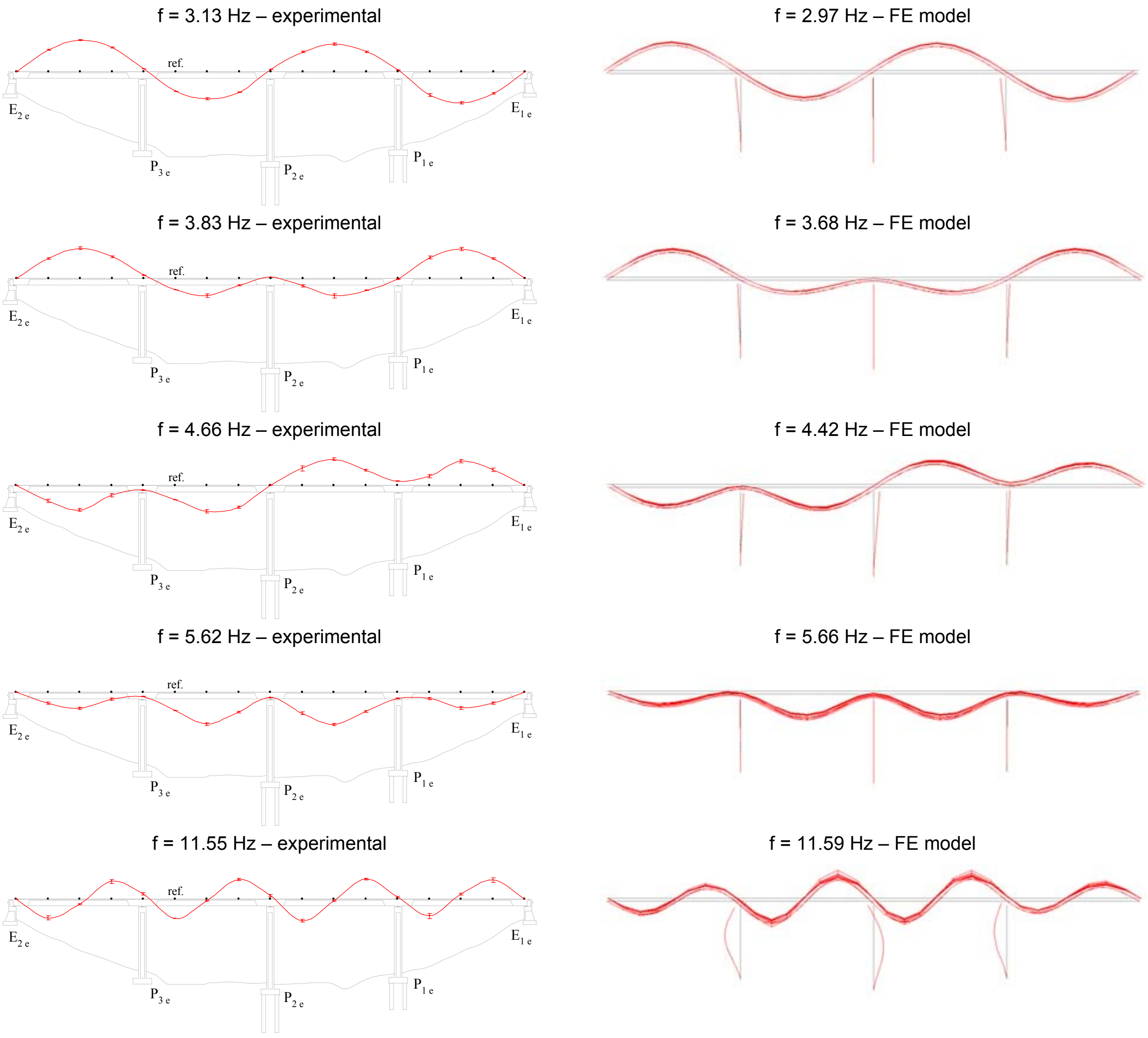
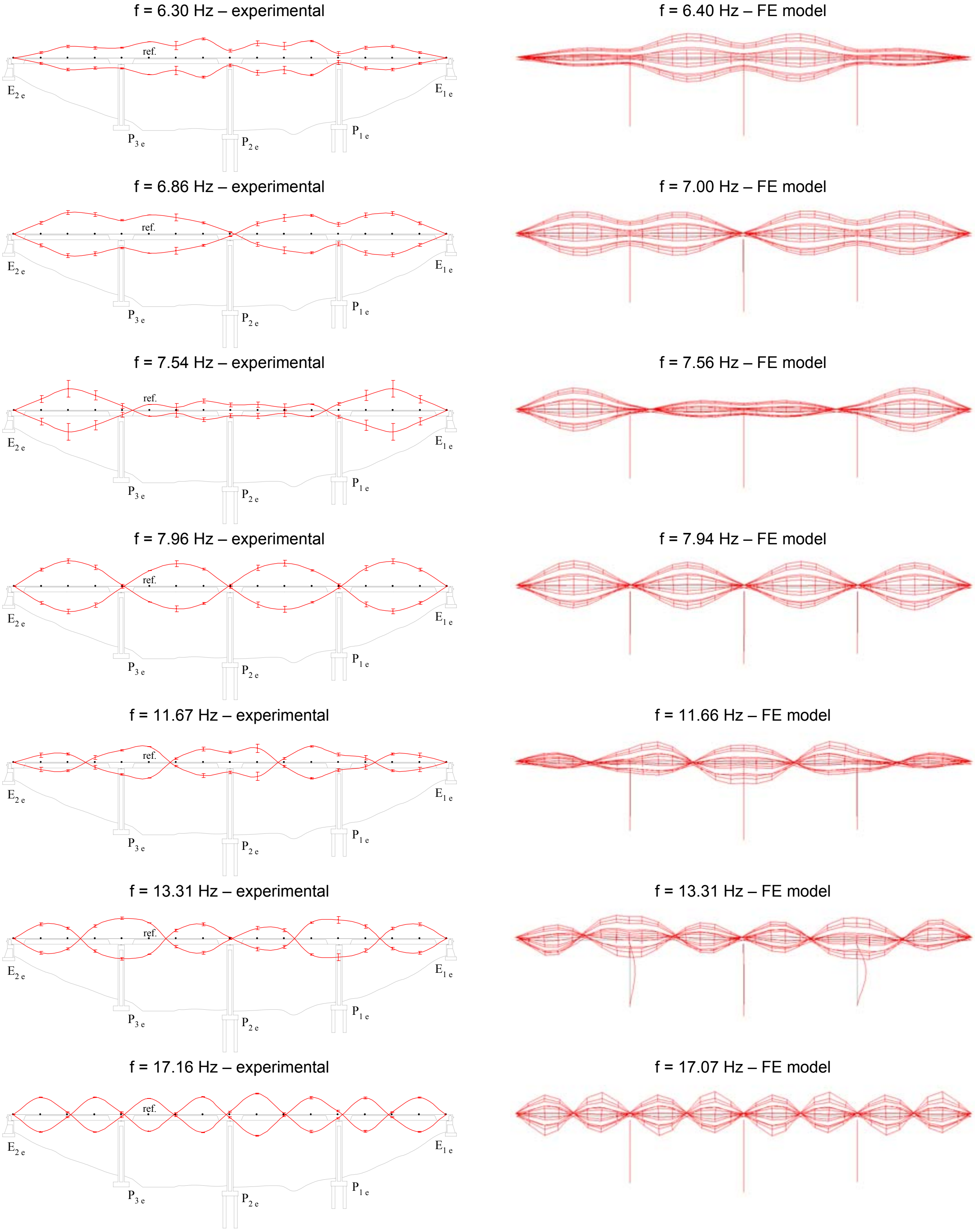


Fig. 11 – Experimental and FE model frequencies and mode shapes for the vertical modes of the V2 viaduct.



f = 6.30 Hz – experimental

f = 6.40 Hz – FE model

f = 6.86 Hz – experimental

f = 7.00 Hz – FE model

f = 7.54 Hz – experimental

f = 7.56 Hz – FE model

f = 7.96 Hz – experimental

f = 7.94 Hz – FE model

f = 11.67 Hz – experimental

f = 11.66 Hz – FE model

f = 13.31 Hz – experimental

f = 13.31 Hz – FE model

f = 17.16 Hz – experimental

f = 17.07 Hz – FE model

Fig. 12 – Experimental and FE model frequencies and mode shapes for the torsional modes of the V2 viaduct.

Figure 13 shows a comparison of the experimental and FE model frequencies using 45°-plots. The MAC coefficients between the experimental and the FE model mode shapes are presented in figure 14.

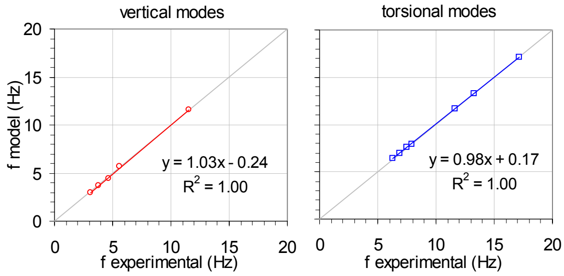


Fig. 13 – Experimental and FE model frequencies.

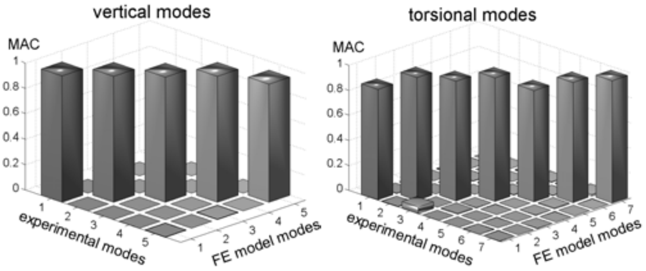


Fig. 14 – MAC coefficients.

4. Dynamic tests and output-only modal identification on a 16 spans continuous viaduct

The second example that is presented in this paper is the V1 viaduct also of the A14 highway, the same highway of the V2 viaduct considered in the previous example. It is a 16 spans continuous viaduct with 2 extreme spans with 20 m and 14 intermediate spans with 30.5 m. The general characteristics of the V1 viaduct are illustrated in figures 15 and 16.

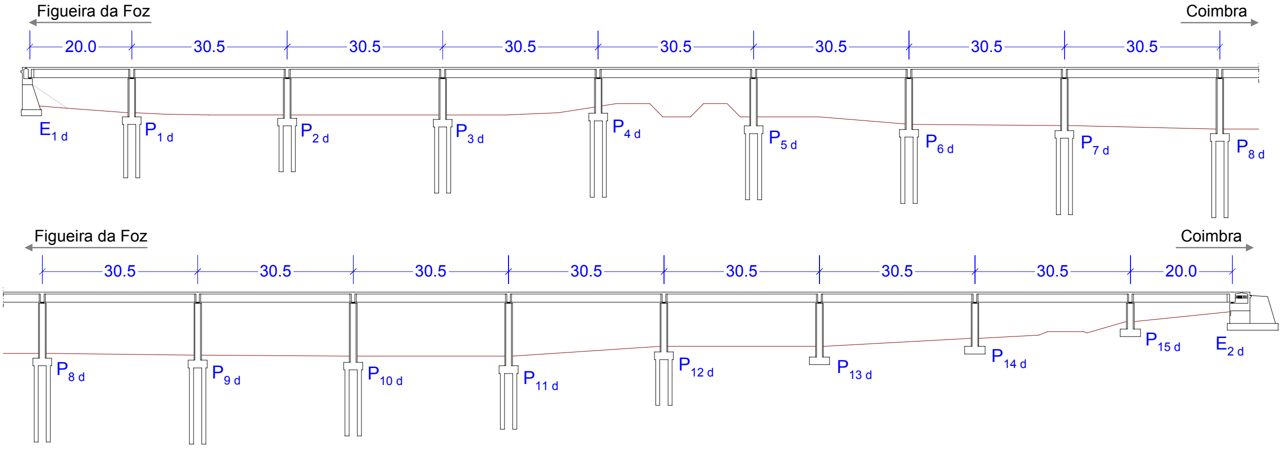


Fig. 15 – Longitudinal section of the V1 viaduct.

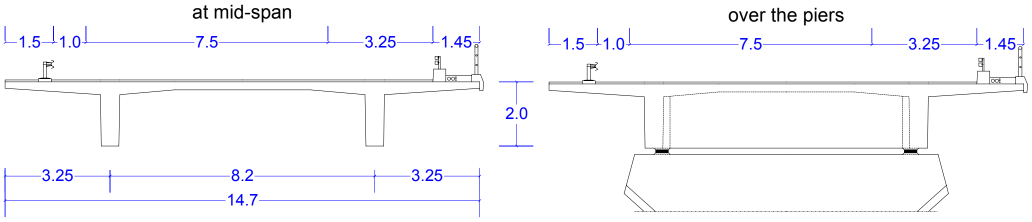


Fig. 16 – Cross sections of the V1 viaduct.

The dynamic tests [8] of the V1 viaduct were performed at the end of its construction, before it was opened to the traffic, as part of its reception load tests that also included static tests [5]. As for the previous example, the dynamic tests of the V1 viaduct consisted in the measurement of accelerations in the structure, induced by the traffic of loaded trucks.

Due to the large number of spans of the V1 viaduct and to the limited time to do the tests, only four spans were instrumented (the spans between piers P_{10d} and P_{14d}). On those spans the measurements were taken at the mid-span sections, close to the quarter-span sections and also at the sections over the piers (see figure 17). The testing procedure was quite similar to the one adopted for the V2 viaduct, with a total of three set-ups and a total recording time in each set-up of about 14 minutes. The same equipment was also used, configured in a similar manner.

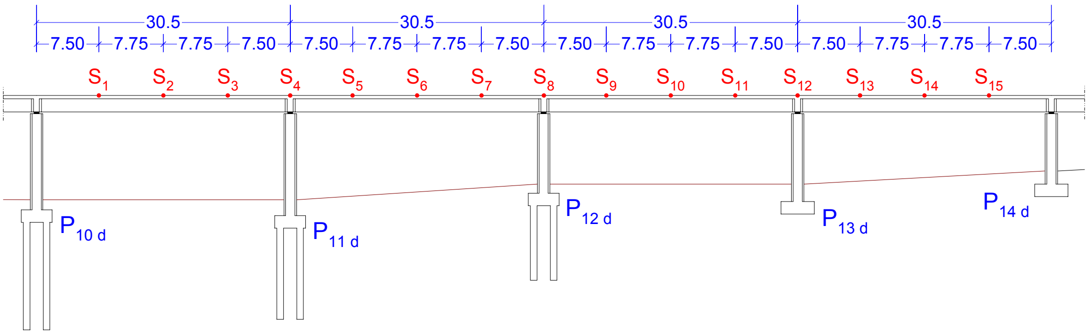


Fig. 17 – Sections that were instrumented in the dynamic tests of the V1 viaduct.

The acceleration records collected in the dynamic tests of the V1 viaduct were first pre-processed, with low-pass digital filtering at 20 Hz with a 4 poles Butterworth filter and decimation to a sampling frequency of 50 Hz.

The modal identification analysis was also performed using the BFD method. For that purpose the spectral density functions of the response of the structure, were estimated using the technique based on the application of the FFT algorithm. Data segments with 4096 values at 50 Hz were considered for the application of the FFT, thus the frequency resolution of the estimated spectral density functions is $\Delta f = 0.012$ Hz. Figures 18 and 19 show the ANPSD's of, respectively, the semi-sum and the semi-difference, of the vertical accelerations measured in the deck of the viaduct.

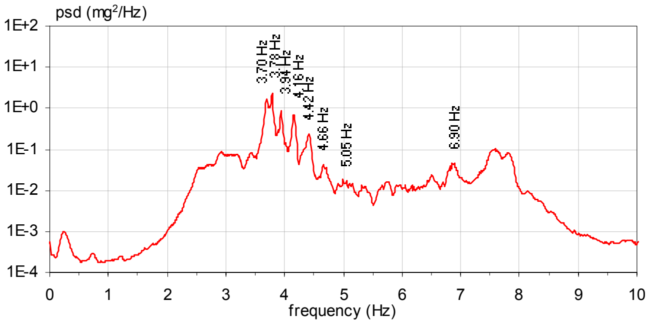


Fig. 18 – ANPSD of the semi-sum of the vertical accelerations.

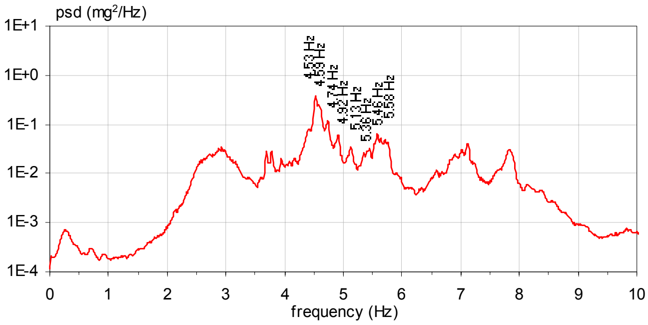


Fig. 19 – ANPSD of the semi-difference of the vertical accelerations.

The ANPSD's presented in figures 18 and 19 were analysed in order to select the peaks corresponding to the natural vibration modes of the viaduct. In order to do that, especially in this case were several modes have very close frequencies, it was important to analyse the coherence functions between the different measurement points and also the corresponding mode shapes extracted from the H_1 estimate of the transmissibility frequency response function also between the different measurement points. Due to the closeness of the frequencies it was also useful to compare the experimental frequencies and mode shapes with the ones computed with a FE model of the viaduct.

From the analysis that was performed using the BFD method, it was possible to identify 8 vertical modes of vibration and 8 torsional modes of vibration. The values of the frequencies of those modes are also included in the ANPSD's of figures 18 and 19.

One should notice that the output-only modal identification method that was adopted for the analysis of the V1 viaduct data – the BFD method – is not the best one for situations of close modes, other methods exist that, in principle, would provide better results, like for instance the stochastic subspace identification method (SSI) [2]. However, the results that were obtained with the BFD method can be considered as good ones, as it will be more evident in the mode shape plots that will be presented bellow.

A comparison of the experimentally identified frequencies and mode shapes with the ones computed with a FE model is presented in figures 20 and 21.

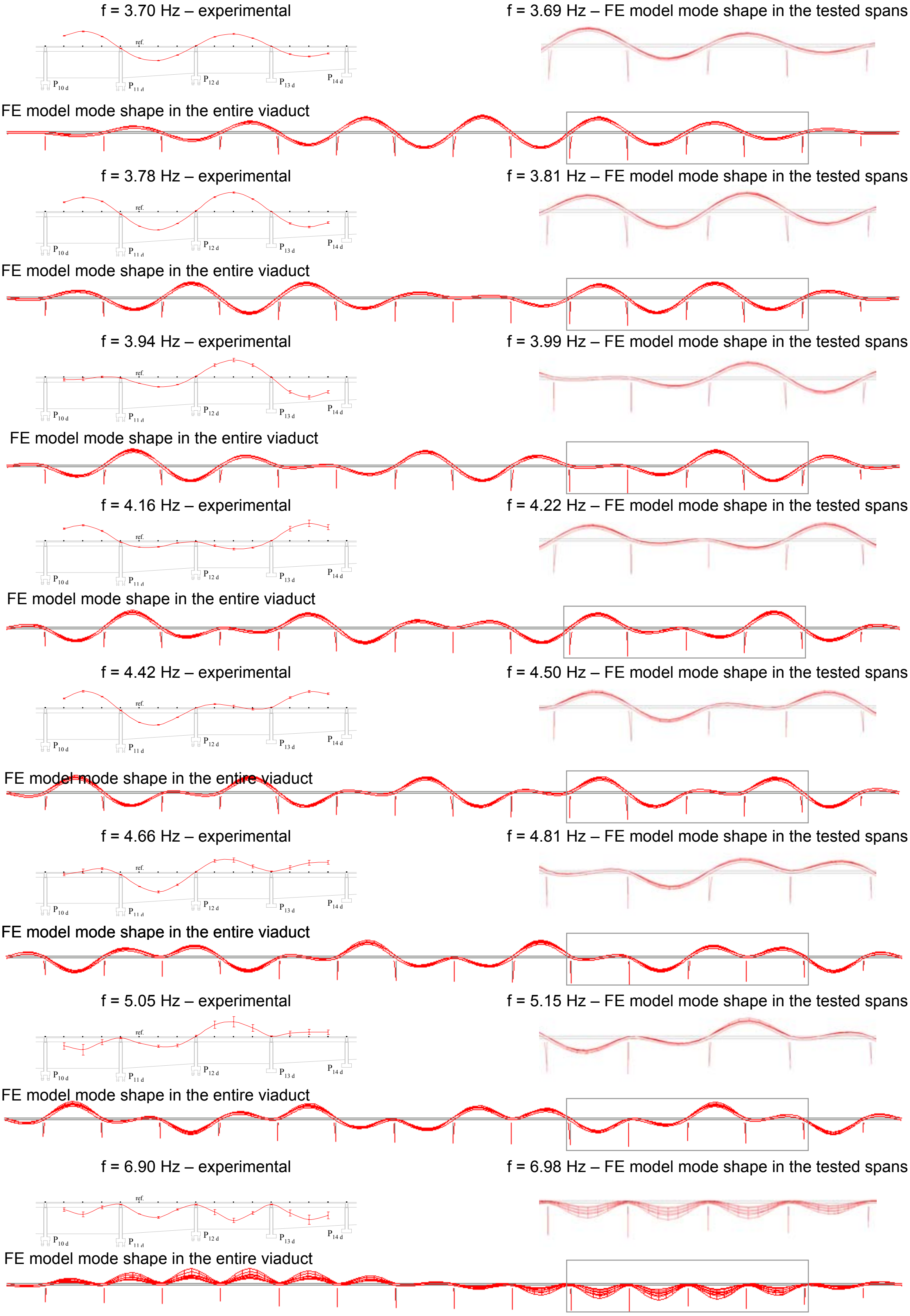


Fig. 20 – Experimental and FE model frequencies and mode shapes for the vertical modes of the V1 viaduct.

f = 4.53 Hz – configuração experimental

f = 4.41 Hz – FE model mode shape in the tested spans

FE model mode shape in the entire viaduct

f = 4.59 Hz – experimental

f = 4.49 Hz – FE model mode shape in the tested spans

FE model mode shape in the entire viaduct

f = 4.74 Hz – experimental

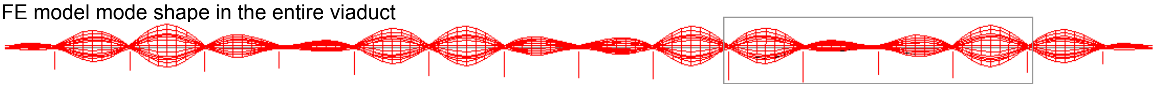
f = 4.62 Hz – FE model mode shape in the tested spans

FE model mode shape in the entire viaduct

f = 4.92 Hz – experimental

f = 4.78 Hz – FE model mode shape in the tested spans

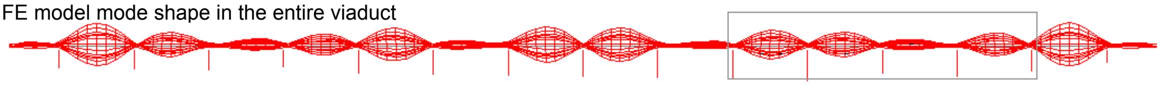
FE model mode shape in the entire viaduct



f = 5.13 Hz – experimental

f = 4.97 Hz – FE model mode shape in the tested spans

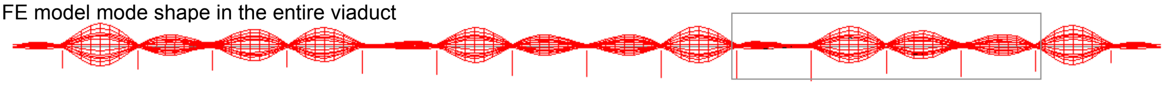
FE model mode shape in the entire viaduct



f = 5.36 Hz – experimental

f = 5.18 Hz – FE model mode shape in the tested spans

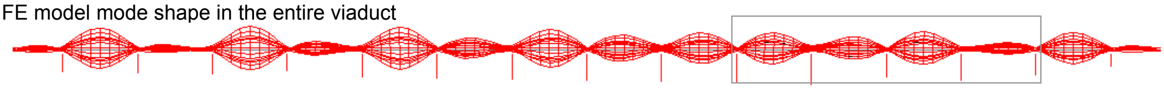
FE model mode shape in the entire viaduct



f = 5.46 Hz – experimental

f = 5.37 Hz – FE model mode shape in the tested spans

FE model mode shape in the entire viaduct



f = 5.58 Hz – experimental

f = 5.40 Hz – FE model mode shape in the tested spans

FE model mode shape in the entire viaduct

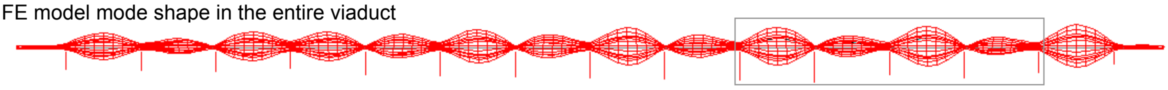


Fig. 21 – Experimental and FE model frequencies and mode shapes for the torsional modes of the V1 viaduct.

Figure 22 shows a comparison of the experimental and FE model frequencies using 45°-plots.

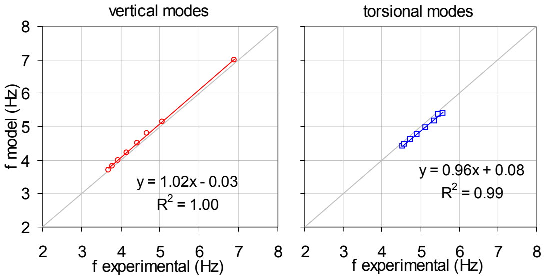


Fig. 22 – Experimental and FE model frequencies.

5. Conclusions

The present paper called the attention to a type of civil engineering structures – multi-span continuous viaducts – that can have vibration modes with very close frequencies, these being as close as greater is the number of spans of the viaduct.

A brief review was made on the theoretical modal characteristics of multi-span continuous beams with constant cross section, showing that in those structures the vertical vibration modes appear in groups or dense regions of modes, each region having as many modes as the spans of the beam.

The results obtained from natural input tests, performed on two highway viaducts, and output-only modal identification carried out with the data collected in those tests, were also presented. One of the viaducts had 4 spans while the other had 16 spans. The closeness of the frequencies of the vertical and torsional modes of vibration of the viaducts could be observed in the experimental results, especially in the 16 spans one. The output-only modal identification method that was used – the basic frequency domain method – is not the most adequate one for systems with close modes, however the results that were obtained can be considered as good ones. A good agreement was also achieved between the experimental frequencies and mode shapes with the modal properties computed with finite element models that were developed for the two viaducts.

Acknowledgements

The work presented in this paper has been developed within the LNEC research project 305/11/14745 on *Dynamic Identification of Civil Engineering Structures*.

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