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Non-Linear Behaviour of Geomaterials in Railway Tracks under Different Loading Conditions

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Abstract

The resilient behaviour of the geomaterials used in railway tracks, particularly the ballast layer, is mostly non-linear and depends mainly on the loading stress path. However geomaterials are frequently considered as linear elastic in structural analyses, assuming that it somewhat reproduces the results of non-linear models for a given load amplitude. This study focuses on whether this consideration is adequate to simulate not only the overall track behaviour, but also the response of the ballast layer, considering different loading conditions. The authors used three-dimensional train-track-soil system models, validated with experimental data, and the results of linear-elastic models are compared against non-linear models. Although the linear elastic models required significantly lower computational effort and can provide accurate estimates of the overall track response, they strongly underestimate the stress levels inside the ballast layer. This aspect can be an important hindrance to studies using linear-elastic models to analyse resilient and plastic deformations of the ballast layer in railway tracks.

Keywords: railway tracks, resilient modulus, FEM modelling, non-linear constitutive laws, $K-\theta$ model

1 Introduction

The structural behaviour of railways tracks is rather complex, both in terms of its transient response, under the dynamic loads of the trains, as in terms of its long-term behaviour, with the accumulation of loading cycles due to traffic (Varandas, 2013; Paixão, 2014). Numerical models have long been valuable tools to provide insight into the railway track structural behaviour and to optimize its design. Such models often require adequate calibration and validation with experimental data in order to provide valuable and reliable information. The FEM modelling technique has been widely used to simulate various aspects of the structural behaviour of railway tracks, including the behaviour of the geomaterials that comprise the supporting layers of the track, such as the ballast, sub-ballast,

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capping and other layers of the track substructure. It is well known that the resilient behaviour of these geomaterials is mostly non-linear and depends mainly on the loading stress path, degree of compaction, water content, among other aspects (Lekarp *et al.*, 2000). Despite this fact, these geomaterials are frequently modelled considering a linear elastic behaviour. This assumption may be acceptable for the soils that comprise the deeper layers of the track, because those materials undergo very low strain increments under the successive load cycles of the trains. However, such consideration for the upper layers, particularly the ballast, is far from consensual, mainly due to the higher stress amplitude acting on these materials. While linear elastic models may be calibrated to approximately reproduce the results of non-linear models for a given load amplitude acting on the track, it is not clear if they are adequate to reproduce the structural behaviour of the track for a wider range of load amplitudes, resulting from different train axle loads or different dynamic load amplitudes.

To address this issue, the authors used a three-dimensional non-linear numerical program to analyse the dynamic response of the track, either using linear and non-linear constitutive laws to simulate the behaviour of the ballast layer and track measurements to validate the models.

2 The Numerical Modelling Approach

The three-dimensional numerical program that was used - *Pegasus* - was developed and fully coded in MATLAB® environment by Varandas (2013). Details and further developments of the program can be found in (Varandas *et al.*, 2014; Varandas *et al.*, 2016). The vehicle, the track, and the ballast/soil layers form three distinct structural systems, as represented in Figure 1. These three systems interact by means of interaction forces, considered only in the vertical direction.

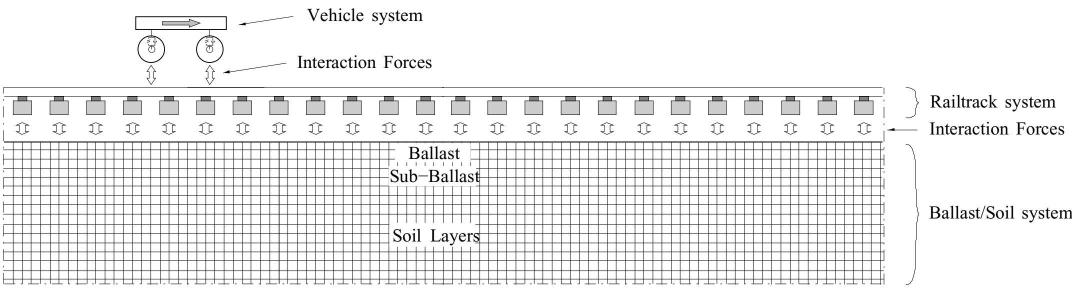

Figure 1: Vehicle system, rail track system, and ballast/soil system, shown in the direction of the track (after Varandas *et al.*, 2016)**.**

The vehicle system is an assemblage of rigid bodies, springs and dampers. The track system and the ballast/soil system are spatially discretized using the Finite Element Method (FEM). The track is built with Euler-Bernoulli beam elements representing the rails and the sleepers. The rails are connected to the sleepers with 3-D spring-damper elements, representing the rail pads. The ballast-soil system is discretized with low-order eight-node solid hexahedral elements. The wheel-rail interaction forces are determined using the Hertzian contact theory and the interaction forces between the sleepers and the underlying ballast are due to vertical contact between the sleeper's base and the ballast, and friction between the sleeper's lateral faces and the confining ballast. The definition of the interaction forces is non-linear due to the on/off contact distinction.

At the lateral boundaries of the model local transmitting boundaries, consisting of visco-elastic dampers (dashpots), are placed to absorb impinging waves generated during the dynamic simulations.

The method used to integrate the spatially discretized equations with respect to time (time integration) is the explicit integration scheme described in (Zhai, 1996). This method is conditionally stable, and therefore the integration time step must be less than a critical value for convergence of the solution ($\Delta t \leq \Delta t_{crit}$). In present and previous analyses made with this program, typical values for Δt vary between 1.25×10^{-5} s and 2.50×10^{-5} s, depending on the geometrical and material properties of the finite elements composing the model.

3 Model Validation with Experimental Track Measurements

3.1 The Case Study

The case study adopted in this study is a track section on a 4.5 m high embankment. This section is located in Portugal, in the recent line of the Alcácer bypass described in (Paixão, 2014). The track is a single ballasted railway line (Figure 2), with Iberian gauge (1.668 m), comprising UIC60 rails, resting on concrete monoblock sleepers, spaced 0.6m, and with fastening system Vossloh W14 with elastomer railpads Zw700/148/165. This line allows mixed traffic, with maximum axle loads of 25 t, maximum speeds of 220 km/h for tilting passenger trains, and was opened to traffic by the end of 2010.

The ballast and the sub-ballast layers were made of crushed granite aggregate. The capping layer was made with well-graded crushed limestone aggregate (Fortunato *et al.*, 2012). The natural soil, classified as QS2 according to UIC719R, corresponds to a gypsum-clay geologic formation from the Mio-Pliocene, predominantly comprising sands, and also silts and clays.

3.2 Description of the Numerical Model

The three-dimensional model consists of 1074 frame elements and 1031 nodes to model the track frame and 32 032 solid elements and 39 486 nodes to represent the ballast-soil system using the mesh depicted in Figure 3. The rails were modelled with vertical bending stiffness of 6380 kNm^2, transverse bending stiffness of 1076 kNm^2 and mass of 60.3 kg/m. The sleepers, with Young's modulus of 30 GPa and mass density of 1.95 t/m^3 (total mass equals 322 kg), were assumed to be rectangular prisms of 2.6 m by 0.30 m, with equivalent height of 0.212 m. The stiffness and damping of the rail pads were estimated from receptance tests in the track, yielding a stiffness value of 160 kN/mm and a damping constant of 17 kNs/m (Paixão *et al.*, 2014). The properties of the geomaterials from the ballast/soil system are presented in Table 1. Initially, all materials were considered with linear elastic behaviour, but the non-linear resilient behaviour of the ballast layer will be introduced in the following section. It is noted that, in earlier studies (Varandas, 2013), it was found that the value of 130 MPa for the ballast modulus yields very good approximations for the overall track behaviour.

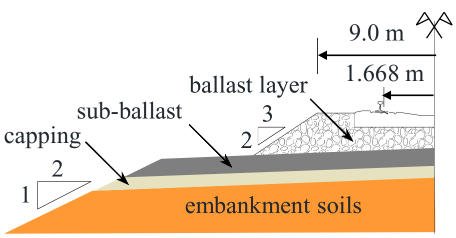

Figure 2: Schematic track cross-section (after Paixão, 2014).

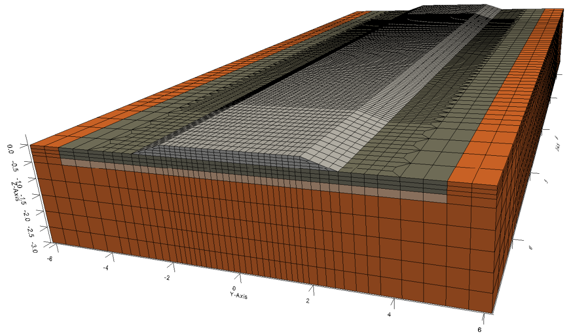

Figure 3: Representation of the numerical model and respective FEM mesh.

Geomaterials	Young's modulus E_i (MPa)	Poisson's ratio ν_i (-)	Damping* ξ_i (%)	Density ρ_i (kg/m^3)	Thickness h (m)
Ballast	130	0.20	3	1530	0.30
Sub-ballast	200	0.30	3	1935	0.30
Capping layer	1000	0.30	3	1935	0.20
Embankment soils	100	0.30	3	2040	4.5$^+$

Notes: *Damping coefficients for frequencies 2 Hz and 100 Hz, according to the Rayleigh damping concept; $^+$The bottom 2.0 m of the model were replaced by vertical springs following the procedure described in (Varandas, 2013).

Table 1: Properties of the materials of the geomaterials (Paixão *et al.*, 2014; Varandas *et al.*, 2016).

3.3 Different Loading Conditions due to Different Passenger Trains

To assess the influence of different loading conditions, two types of passenger trains were considered in the analysis (Figure 4): a) the Alfa Pendular, running at 220 km/h with average axle loads of 133 kN; b) the Intercity locomotive, at 200 km/h with 213.4 kN/axle. Only the leading bogies were modelled, using the parameters presented in (Calçada, 1995; Ribeiro *et al.*, 2013).

Figure 5 compares the experimental results against the numerical results of the vertical rail displacements (a and b) and the vertical sleeper accelerations (c and d) when the Alfa Pendular (AP) and the Intercity (IC) passed by the track section under study. Rail displacements and sleeper accelerometer were respectively measured with a laser diode-PSD transducer and a piezoelectric accelerometer described in (Paixão *et al.*, 2014). Both the numerical and experimental accelerations were filtered by a low-pass filter with cut frequency of 80 Hz. The comparison of the results suggests that the model is somewhat adequate to simulate the track behaviour, especially the accelerations measured on the sleepers. The higher differences on the displacements are probably due to the greater variability normally obtained with these measurements, even for the same loading conditions, as evidenced in earlier studies (Paixão *et al.*, 2014).

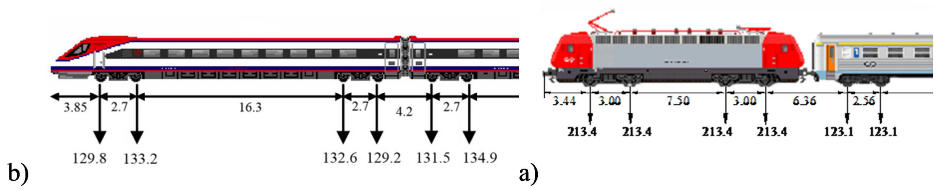

Figure 4: Schematic representation of the axle arrangement (in m) and respective load estimates (in kN) of the passenger trains considered in the study: a) Alfa Pendular train: b) Intercity passenger train.

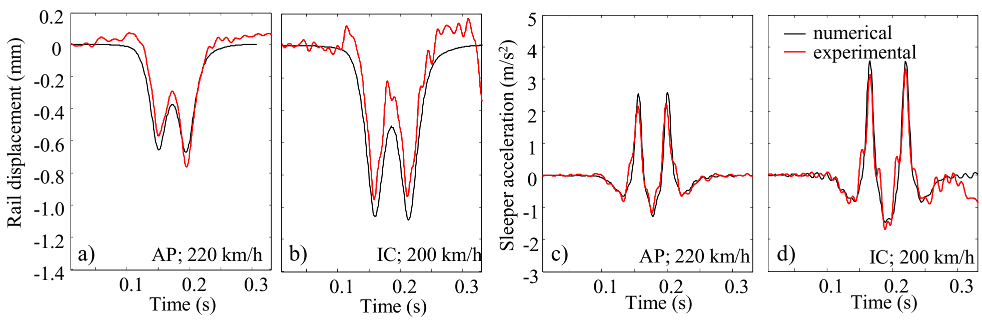

Figure 5: Vertical rail displacements (a and b) and vertical sleeper accelerations (c and d) due to the leading bogies of the Alfa Pendular (AP) and Intercity trains (IC).

Tests performed by the authors with other ballast moduli, namely 100 MPa and 160 MPa, yielded very similar displacement and acceleration results. In particular, the reduction to 100 MPa resulted in an increase in the rail displacement by about 4 %, while the value of 160 MPa resulted in 2% and 3% smaller displacements for the case of the Alfa Pendular and Intercity, respectively.

4 Consideration of the Non-Linear Resilient Behaviour of the Ballast Layer

The ballast layer experiences considerable stress changes during loading from passing trains, resulting in significant stiffness variations in correspondence with the applied stress level. It is therefore important to consider the non-linear response when studying stress paths in this material.

In this section, the resilient behaviour of the ballast layer, namely its resilient modulus E_r, is determined by the numerical program using the nonlinear-elastic $K-\theta$ model (Brown & Pell, 1967), generally expressed by the well-known formulation $E_r = K_1\theta^{k_2}$, where θ is the sum of the principal stresses, and K_1 and K_2 are model parameters. The $K-\theta$ model was implemented in the numerical code *Pegasus* using an adapted formulation described in detail by Varandas (2013), following the parameter calibration performed by Aursudkij et al (2009), with K_1 = 110 MPa and K_2 = 0.6, assuming minimum value of E_r = 16 MPa and a constant Poisson's ratio of 0.20.

Figure 6 compares the vertical displacements obtained with the linear-elastic approach (presented in Figure 5a) with those obtained with the considered $K-\theta$ model in the ballast layer. The results show that the rail displacements are practically coincident for the two train types. Although not presented here, the same applies to sleeper accelerations. As highlighted in other studies (Varandas *et al.*, 2014), the consideration of the linear-elastic behaviour of the ballast layer can yield very good estimations of the overall behaviour of the track, assessed in terms of vertical rail displacements. In addition, the authors showed here that the same may also apply for different loading conditions, namely for the Alfa Pendular and Intercity trains with significantly different axle loads (Figure 4).

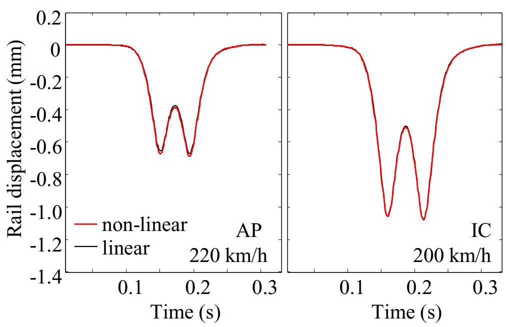

Figure 6: Vertical rail displacements obtained with the linear and non-linear models for the two trains: a) Alfa Pendular (AP); b) Intercity (IC).

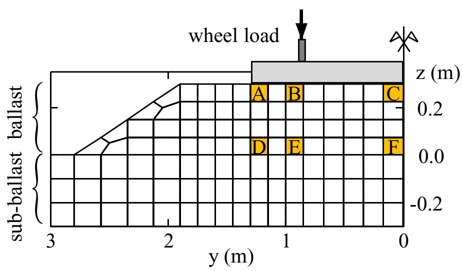

Figure 7: Location of the selected elements under analysis.

As regards stresses inside the ballast layer, for example at the finite elements identified in Figure 7, time history variations of the mean stress, p, and deviatoric stress, q, are presented in Figure 8. Figure 9a presents colour maps of the peak vertical stresses, σ_z, obtained with the linear and non-linear models when the trains crossed the central section of the model. The figure presents, for each case and train type, sectional views at x = 0.0 m (aligned with the central sleeper), z = 0.0 m and z = 0.3 m.

It is visible that the linear elastic models not only fail in estimating the peak stresses (p, q and σ_z) at various positions, but also that the stress evolution is somewhat different, particularly before and after the axles cross the section under analysis (more pronounced in terms of q at positions A, B and C in Figure 7). This is the consequence of the different load transfer from the sleepers' base to the ballast layer that is achieved with the non-linear model because higher stresses are concentrated in the finite elements undergoing higher modulus variations, resulting from the higher loading conditions experienced in these elements. This is quite noticeable at the elements under the sleepers (Figure 9a).

This variation in the ballast modulus, resulting from the consideration of the $K-\theta$ model, is clear in Figure 9b, showing the maximum resilient modulus at the same sectional views. For example, in the case of the Intercity, it is visible that under the sleepers the modulus may reach up to 220 MPa, but between sleepers significantly lower values can be expected, mostly at the centre of the track.

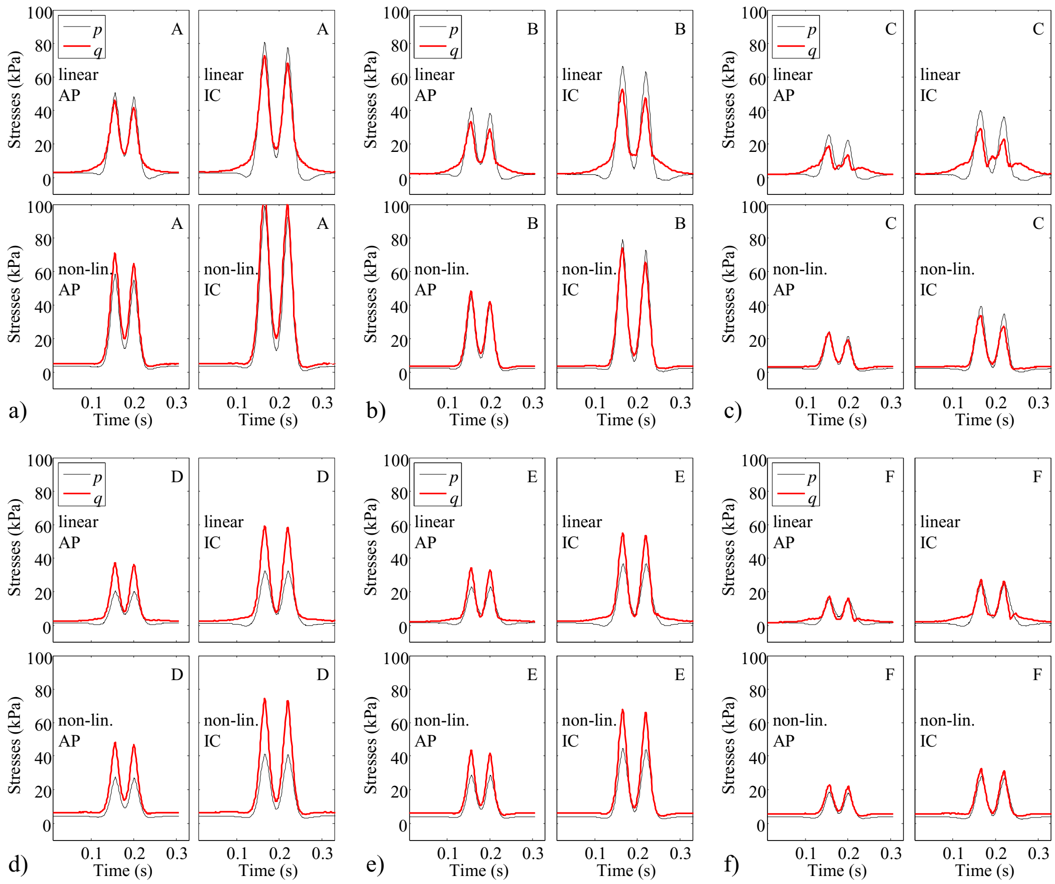

Figure 8: Variation of p and q stresses, considering the linear and non-linear (non-lin.) resilient behaviour of the ballast layer under the Alfa Pendular (AP) and Intercity (IC), at the locations A to F identified in Figure 7.

These results show that the linear-elastic models underestimate the stress levels inside the ballast layer. This may undermine simulation studies that depend on the accurate assessment of stresses or deformations of geomaterials. For instance, they should be used with caution when studying the long-term degradation behaviour of railway tracks that involve empirical formulations relating plastic strains with stress or strain levels. Nevertheless, the elastic models still yield a very good estimate of the overall track behaviour and require significantly lower computational effort. For example, in the studied cases the authors used a 2.66 GHz Intel® Core™ i7 Processor and the ratio between the simulation duration and the necessary calculation time was about 1:3600 for the linear elastic models, but 1:10800 in the non-linear approach (though these ratios depend on numerous factors).

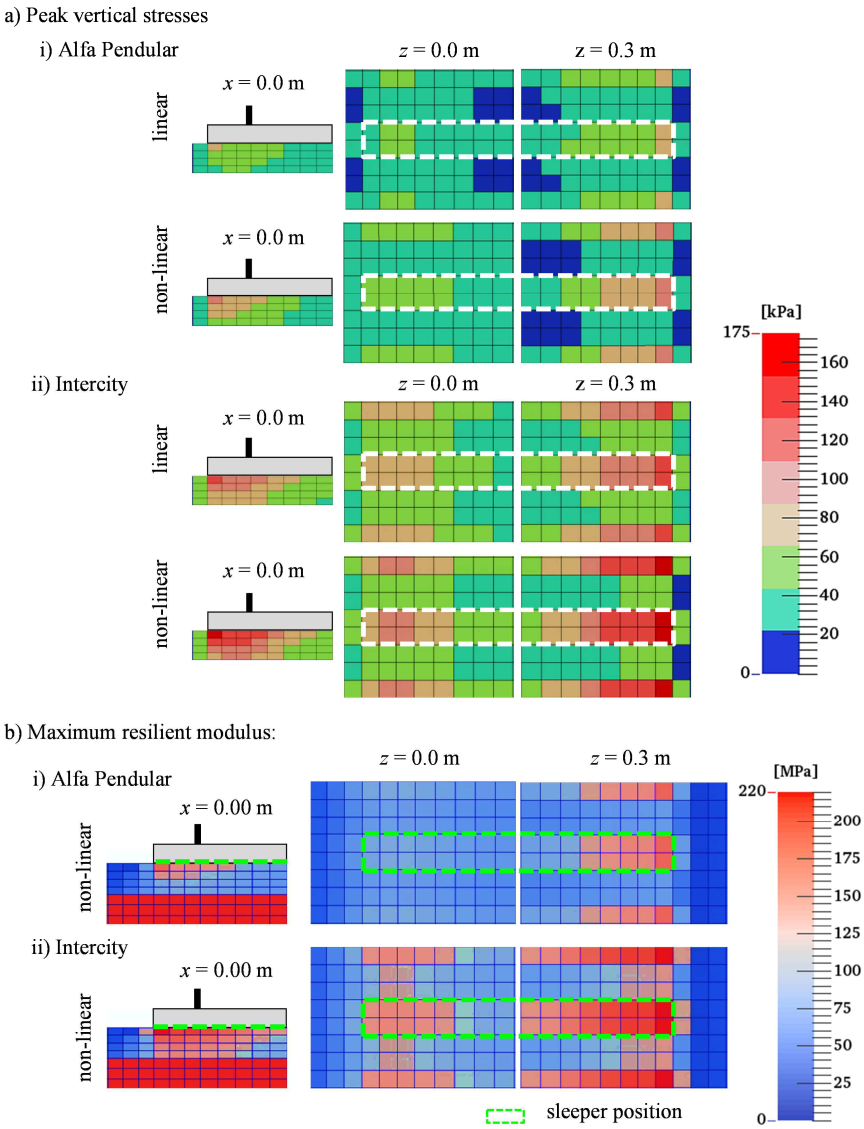

Figure 9: Peak vertical stresses (a) and maximum resilient modulus (b) obtained in the ballast layer.

5 Final Remarks

In railway track structural analysis (static or dynamic) the resilient behaviour of the ballast layer is normally considered to be linear elastic, although it is well known that it strongly depends on the stress level, among other factors. Using a three-dimensional numerical model of the train-track-soil system validated with field measurements, the authors focused on evaluating the adequacy of this consideration to simulate the track behaviour considering trains at different speeds and with different axle loads. Both linear and non-linear constitutive laws of the ballast material were considered in the analysis. The authors concluded that the linear elastic modelling approach, when compared with the non-linear approach, can provide very good overall track behaviour results, particularly in terms of rail displacements and sleeper accelerations, even under different loading conditions. However, it strongly underestimates stress levels inside the ballast layer, because the non-linear approach yields a different three-dimensional loading transfer between the sleepers and the ballast that is caused by the increase of the resilient modulus on the ballast material, mostly developing under the sleepers under each load cycle. On the other hand, the linear elastic models required significantly lower computational effort, as was expected. Based on the results, the authors suggest caution when considering the linear-elastic behaviour of geomaterials to study aspects of the railway track that require an accurate assessment of stresses and strains.

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