Non-linear infra-gravity and sea-swell wave-wave interactions at S. Jacinto beach

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Abstract: Non-linear wave-wave interactions between sea-swell waves ($0.5 < f < 0.04$ Hz) and infra-gravity waves ($0.04 < f < 0.004$ Hz) are studied in detail from field data using bispectral analysis. This methodology allowed to understand the direction and magnitude of the non-linear energy transfers within the spectrum. Field measurements of pressure data were collected at S. Jacinto beach during a full tidal cycle on a cross-shore transect at the subtidal and intertidal zone. Bispectral estimates showed triad interactions between two sea-swell waves and one infra-gravity wave at the surf zone. The non-linear transfers were out of phase which indicates the presence of bound waves. These results evidenced the presence of energy transfers from infra-gravity to sea-swell band where the sea-swell waves are more energetic, during shoaling and breaking.

Key words: Infra-gravity waves, non-linear wave-wave interactions, bispectral analysis, wave observations.

1. INTRODUCTION

Low-frequency motions with periods between 25 and 250 s are known as infra-gravity waves (IG). They were first observed in the field by Munk (1949) and Tucker (1950). Since then, many field campaigns, laboratory experiments and numerical modelling were performed to study these small-amplitude waves and to assess their impact on coastal circulation (e.g., Herbers *et al*., 1995; Van Dongeren *et al*., 2007; Ruju *et al*., 2012).

Currently, it is widely accepted that IG waves become dominant in the inner surf zone (De Bakker *et al*., 2014). In that zone, the energy of sea-swell waves has already been dissipated and IG waves start to shoal towards the shore. At the coastline, it is expected that such small amplitude waves are capable to reflect seawards without losing energy. However, IG dissipation was observed and several mechanisms were proposed, such as bottom friction (Henderson and Bowen, 2002) and depth-induced breaking (Van Dongeren *et al*., 2007).

Ruju *et al*. (2012) also proposed that non-linear energy transfers between IG waves and sea-swell waves are important where the sea-swell waves are dominant. This study was performed based on numerical modelling using the IH-2VOF model (Lara *et al*., 2011). Such an accurate model can provide a good approximation but this conclusion should be verified with field measurements.

Therefore, the study presented herein aims to verify the non-linear wave-wave interactions at the shoaling zone, thereby extending the study of Ruju *et al*. (2012) with field measurements.

2. FIELD CAMPAIGN

The field experiment was performed from 17 to 19 of June 2015 at S. Jacinto beach, NW Portugal. This beach is located northwards of the Aveiro inlet jetties. The sediments are characterized by fine sand with a mean sediment diameter of 0.2 mm.

Several instruments were deployed during this field campaign (Mendes *et al*., 2015). For the purpose of this study, we are going to focus only on two of the eight pressure sensors moored from 8h00 of 18 June to 2h00 of 19 June, 2015 (Figure 1).

These sensors were moored in a cross-shore transect from 6 m to 2.5 m depth chart datum (CD). They had a sampling frequency of 2 Hz and they were attached to a steel tube fixed to a concrete base. Bathymetry was collected by boat and topography with a real time kinematic GPS mounted on a quad-bike.

3. METHODS

Non-linear interactions between waves with different frequencies cannot be detected by using the traditional Fast Fourier Transform (FFT). However, they can be detected by using bispectral analysis which is a spectral analysis similar to the FFT but using second-order theory (Hasselman *et al*., 1963; Collis *et al*., 1998). The discrete bispectrum is defined as:

$$B_{f1,f2} = E[A_{f1}A_{f2}A_{f1+f2}]$$

where $E[]$ is the ensemble average of the triple product of complex Fourier coefficients A at the frequencies $f1$, $f2$ and their sum.

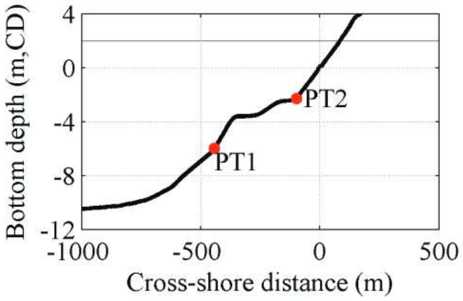

Fig. 1. Instruments position, cross-shore profile and mean sea level (horizontal grey line) at S. Jacinto beach in meters and referenced to chart datum.

This methodology can detect the phase coupling between the three frequency components in a triad (*f1*, *f2* and *f1*+ *f2*). Since the coefficients A are complex, $B_{f1,f2}$ has a real and an imaginary part. According to Doering and Bowen (1986), the imaginary part provides a measure of the non-linear energy transfers and this part will be used hereafter. A positive value indicates energy transfer from *f1* and *f2* to *f3* and a negative value, from *f3* to both *f1* and *f2* (De Bakker *et al*., 2015).

Here, we analyzed 2 h of data centred on the high tide as in De Bakker *et al*. (2014). These data were divided into blocks of 20 min and the bispectral estimates were averaged over 7 frequencies. This resulted in a frequency resolution of 0.0058 Hz, 154 degrees of freedom and 95% confidence interval for non-bicoherence of 0.197 (Haubrich, 1965).

4. RESULTS

Bispectral analysis was applied to both PTs during high-tide (Figures 2 and 3). Non-linear energy transfers were stronger for PT2 (top panels of Figures 2 and 3). These interactions in PT2 between peak frequencies (*fp*) at B(0.09, 0.09) induced a positive transfer to higher harmonics at *f* = 0.18 Hz. Contrastingly, at PT1 the negative value at B(0.09,0.09) corresponds to transfer energy back from the higher harmonic (*f* = 0.18 Hz) to *fp*.

There were also interactions between *fp* and IG frequencies (*f* < 0.04 Hz). While it seems that for PT1, the positive are similar to negative transfers, for PT2, they were mainly positive with a value close to the transfer around *fp*. In other words, at B(0.09, 0.005-0.008) energy is transfer to *f* = 0.095-0.098 Hz.

Bicoherence gives a normalized measure of the triad interactions (middle panels of Figures 2 and 3). These interactions were stronger for *fp* and super-harmonics of *fp*. Triad interactions between *fp* and IG frequencies were null.

Finally, biphase provides a normalized measure of the phase relation between the several frequencies (bottom panels of Figures 2 and 3) but their value is only significant if bicoherence is greater than zero (Elgar and Guza, 1985). Frequencies of *fp* and their super-harmonics are interacting with zero phase lag.

There were some interactions between *f* = 0.05 Hz and several IG frequencies in PT1 that occurred in phase.

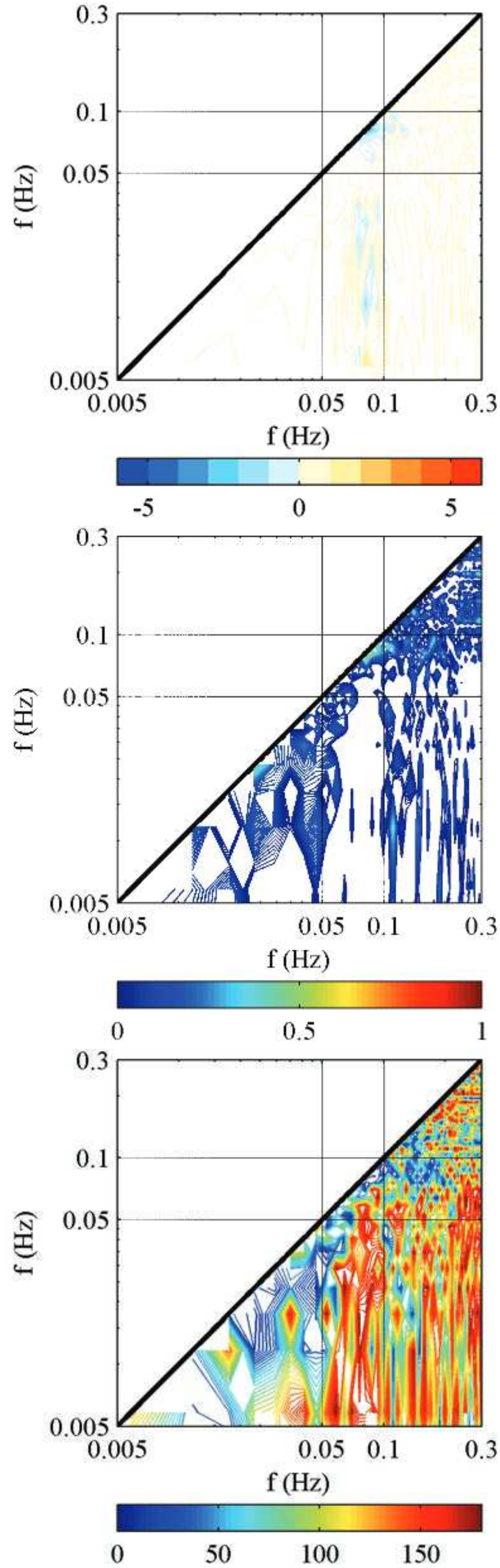

Fig. 2. Imaginary part (x10^{-5}) of the bispectrum (top), bicoherence values within 95% confidence interval (middle) and absolute value of the biphase (bottom) for PT1 around high-tide.

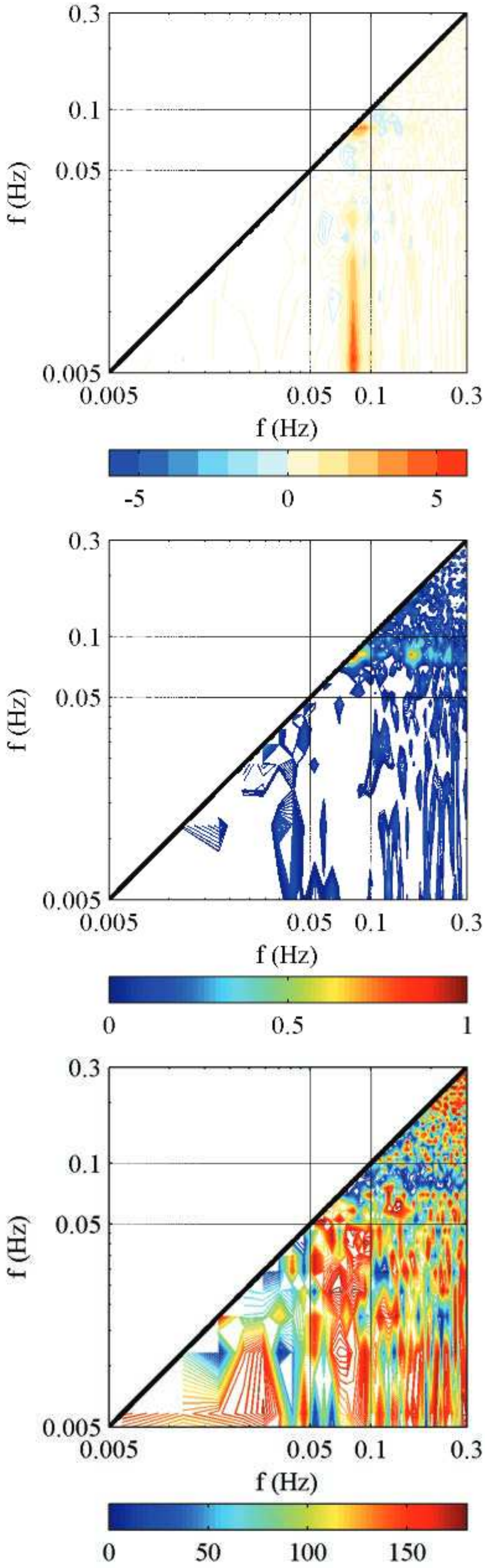

Fig. 3. Imaginary part (x10^{-5}) of the bispectrum (top), bicoherence values within 95% confidence interval (middle) and absolute value of the biphase (bottom) for PT2 around high-tide.

5. DISCUSSION

Non-linear interactions around *fp* and their super-harmonics were as expected in such depths. These interactions are characterized by triad interactions in phase which indicates that super-harmonics are generated from the *fp* interactions.

Non-linear interactions between IG and *fp* frequencies were stronger and positive for PT2. These interactions are a clear evidence of non-linear interactions between *fp* and IG waves that are transferring energy around *fp*.

Biphase estimates for *fp* were in phase and it seems that some IG frequencies occurred also in phase. Last evidence may correspond to interactions between two IG waves.

6. CONCLUSIONS

Non-linear wave-wave interactions between SS and IG waves at the surf zone were verified using bispectral analysis. Non-linear interactions between SS waves and IG waves occurred indeed where SS waves still dominate. Further integration over each frequency should be calculated in order to quantify the non-linear energy source term input for wave spectral numerical models.

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