



Second International  
Dam World Conference  
PORTUGAL • LISBON • LNEC • April 21-24, 2015

## 3D ANALYSIS OF MONTESINHO CFRD USING CODE-ASTER FEM PROGRAM

João Marcelino<sup>\*</sup>, André Serrano<sup>2</sup>, João Manso<sup>3</sup>, Laura Caldeira<sup>3</sup>, José Paixão<sup>4</sup>

<sup>\*</sup> Laboratório Nacional de Engenharia Civil (LNEC)  
Av. do Brasil, 101, 1700-066 Lisboa, Portugal  
e-mail: marcelino@lnec.pt, webpage: <http://dw2015.lnec.pt>

**Keywords:** Dams, CFRD, Rockfill, FEM, Open Source

**Abstract.** *This paper describes the setup of a finite element method (FEM) model for the three dimensional simulation of the construction of a concrete face rockfill dam and filling of the its reservoir using the Code-Aster code. The prototype of the study is the 36.5 m high Montesinho dam, located in the north of Portugal near the Spain border, witch is finishing its construction, at the time of the paper writing. The dam is located in the Sabor river in the Montesinho Natural Reserve and it is a concrete face rockfill dam. Its main purpose is to provide water supply to Bragança town reinforcing the current reserve of the Serra Serrada dam located 3 km west of Montesinho, which is not sufficient to supply Bragança during a normal summer. Due to the fact that the dam is located in Montesinho Natural Reserve, which is a very important and sensitive ecological reserve, very special measures has been taken by the dam owner – ATMAD (Águas de Trás-os-Montes e Alto Douro) to minimize the ecological impacts on the environment.*

*Code-Aster is a general purpose FEM software package, developed by Eléctricité de France (EDF) during many years, for the expertise and the maintenance of EDF's power plants and electrical networks. In 2001 it was decided by EDF to release Code-Aster to the public under the GNU General Public License. Since that it has been under constant development making it more capable do address a wide range of phenomena. Code-Aster is written in Fortran and Python and has more than 1.5 million lines of code. In this study, a finite element procedure was developed to simulate the construction process of the dam and its first filling. To model the behaviour of the rockfill material a elastic model with a simple Drucker-Prager model was used. This is a first approach to further developments. The model parameters were calibrated by large-scale triaxial tests performed on materials used in the dam. The step-by-step construction followed by subsequent impounding of the reservoir was simulated in the numerical procedure. The numerical results agree well with in situ monitoring records of dam settlements, indicating that the three-dimensional finite element procedure applied here can be used to evaluate the deformation of CFRDs.*

---

<sup>2</sup> IST

<sup>3</sup> LNEC

<sup>4</sup> Águas de Trás-os-Montes e Alto Douro

## 1 INTRODUCTION

The modeling of the construction and filling of the reservoir of embankment dams is usually made by using either commercial *FEM* packages or by custom in-house code. Each approach has advantages and shortcomings. Commercial *FEM* packages are normally built for simplicity, easy to use and fast development of models permitting to obtain some results using a modest amount of both geotechnical expertise and computer code knowledge. Another important characteristic of commercial *FEM* packages is that they are very expensive (sometimes in the range of 10 K to 30 K euros) and its license model is in closed source, not allowing any or with very limited customization. Nevertheless, if used wisely, those programs can be very useful tools specially for design companies or contractors where time for studies and model development is usually limited. On the other hand, as those programs are simple and focused on general purpose models usually they don't include some special needs of some special structures. This is the case of the study of embankment dams where the variety of phenomena and the complexity of the models can be overwhelming. For example, to simulate the construction of a embankment dam some special care is needed to correctly account for the construction deformation. Sometimes the use of custom-made programs can provide a good solution to those situations. For example, Marcelino<sup>1</sup> has developed a efficient code to model both collapse and creep phenomena on rockfill using a viscoelastoplastic approach and an in-house computer program. The code was capable of accounting for the construction and first filling of the reservoir, but it was only able to address plane strain equilibrium.

*EDF*, an important French company, which business is related with the production of electricity, as developed trough the years very powerful in-house *FEM* code to suit its needs. Its development began in 1988 in order to be applied to all of the *EDF* special problems<sup>2</sup>. In 1994, the first PhD using Code-Aster calculations was delivered<sup>3</sup>. In 2001, it was decided by *EDF* to deliver the software and the code to the general public under the Richard Stallman *GNU*<sup>4</sup> license model. This allowed the code to be tested by users all over the world, while being actively developed to incorporate some of the feed-back received. In 2007, a special software bundle – Salome Meca – was delivered, making it more accessible and increasing even more is ability to solve a wide range of phenomena.

By the same time Code-Aster was ready for parallel processing, either for multi-processor computers and for computer clusters, taking advantage of computing capabilities some orders of magnitude higher than what is currently available in desktop computers.

As it is available in the *GNU* software license mode, Code-Aster<sup>3</sup> and Salome-Meca are free to use, free to study and free to modify.

In summary, Code-Aster is a general purpose *FEM* program ready to solve 1D/2D/3D models of mechanical, thermal and associated phenomena. The program includes several types of loading models, nonlinear material models (including Cam-Clay, *CJS* and Barcelona for soils), nonlinearities in the geometry, interactions for fluid-structure or soil-fluid-structure, about 400 different finite element types including special absorbing boundaries, etc.

All these characteristics of Code-Aster allowed the authors to develop the tools needed to make the 3D model and study of a rockfill dam currently under construction.

Concrete-faced rockfill dams (*CFRD*) are a major type of rockfill dams, whose structures consist of cushion, transition, main rockfill, and secondary rockfill zones. Due to their good adaptability to topography, geology and climate, use of locally available materials, cost-effectiveness, simple construction and short construction period, they have been repeatedly constructed in recent decades, in some cases higher than 200 m (Table 1).

Name	Height	Opening date	Country
Shuibuya Dam	233 m	2008	China
Bakun Dam	205 m	2011	Malaysia
Aguamilpa Dam	187 m	1993	Mexico
Pubugou Dam	186 m	2010	China
Sanbanxi Dam	185.6 m	2006	China
Barra Grande Dam	185 m	2005	Brazil
Hongjiadu Dam	179.5 m	2005	China
Tankeng Dam	162 m	2008	China
Foz do Areia	160 m	1980	Brazil

Table 1: Tallest CFRD in the world

The major concerns regarding the design and operation of these structures are related to the deformations of rockfill zones and to the stresses in concrete slabs and slab joints. Numerical methods, such as finite element method, can be used to predict dam deformation during construction and operation. However, the reliability of the results for the adopted model depends significantly on its suitability to model rockfill materials.

Several methods have been adopted for the modeling of rockfill. The nonlinear elastic Duncan–Chang E–B model<sup>5</sup> has been widely used to model *CFRD* construction, due to the simplicity and clear physical meaning of its parameters. Xing et al.<sup>6</sup> studied the mechanical and hydraulic properties of weak rockfill during placement and compaction in three different dam projects. The stresses and the strains in the dams were evaluated using a two-dimensional finite element software, where they implemented the nonlinear hyperbolic model Duncan–Chang E–B, and compared to the field measurements. Zhou et al.<sup>7</sup> also applied this model to analyse the measured deformations resulting from continuous monitoring of the Shuibuya *CFRD*. They performed a displacement back-analysis for parameters using a hybrid generic algorithms (*HGAs*), allowing the prediction of the long-term deformation of the dam. Their simulations were performed in two-dimensional plain-strain conditions, considering the time-dependent deformation, the construction process and water storage. They showed that the settlement rate decreased with time and tended to stabilize, and also that the material's deformation modulus was smaller than those obtained from the corresponding laboratory tests. Once again they showed that the parameters in Duncan and Chang E–B model could be evaluated using a group of conventional triaxial tests.

Li and Desai<sup>8</sup> developed a finite element procedure for stress-deformation analysis of dams, including sequential embankment construction, seepage analysis (including transient and steady free surface), and a combination of both. They modeled the mechanical behavior using linear elastic, nonlinear or piecewise linear elastic (hyperbolic) and plasticity (Drucker-Prager) models. They adopted the same mesh for both seepage and for the stress analysis and conveniently superimposed the two effects. Providing the proper conditions, it was possible to incorporate effects of partial saturation during construction and to include changes in the dam geometry during the deformation process. This procedure provided satisfactory correlation with analytical solutions and field observations, which could be useful for nonlinear stress, seepage and stability analysis of dams. Xu et al.<sup>9</sup> modified the generalized plasticity model for sand, which was based on the work of Pastor et al.<sup>10</sup> and Ling and Liu<sup>11</sup>, in order to describe the behavior of rockfill materials, particularly their unique pressure dependency due to particle crushing. Then they incorporated the model into a three-dimensional *FEM* program and simulated the construction process and reservoir impounding of the Zipingpu *CFRD*, comparing the numerical results to field measurements. This proposed numerical procedure does not consider the creep deformation of rockfill, nor the influence of very large particles on

rockfill behaviour, and thus may underestimate dam deformation and slab stresses, particularly during the period of reservoir operation. However, these underestimations were somehow compensated by the use of a lower-density rockfill material to calibrate the model parameters, due to the laboratory apparatus dimensions.

In the following paragraphs it is shown that a successful modeling of Montesinho dam was achieved, and that the results for the construction phase are in good agreement with the observed behavior. This allows prediction of the expected response for the first filling.

## 2 DESCRIPTION OF MONTESINHO DAM

The Montesinho dam is located in the Sabor river, which is in the Montesinho Natural Reserve. It is a *CFRD* built to provide water supply to Bragança reinforcing the current reserve of the Serra Serrada dam located approximately 3 km west of Montesinho.

Montesinho dam has 36.5 m of height and a crest with a length of about 310 m with 7 m of width. The total volume of the embankment is of about 174 000 m<sup>3</sup> and consists of granitic rockfill obtained from the quarries located upstream of the dam in the reservoir area.

The reservoir has a capacity of 3.69 hm<sup>3</sup> (net volume of 3.53 hm<sup>3</sup>) with a flooded area of 35.8 ha and a catchment area of 10.1 km<sup>2</sup>. The normal water level is at 1217.50 m and the maximum water level is at 1219.73 m. The freeboard has 1.37 m, therefore the crest is located at level 1221.10 m. The upstream and downstream embankments originally had a slope of 1:1.5 (v:h), however, during the construction, it was decided to include a berm in the downstream shell.

In the area of the dam and its reservoir, the outcropping blocks and top layers of the bedrock consists essentially of a two-mica granite with coarse grain. The rock presents a generalized and mild to medium kaolinization of the feldspars, therefore sometimes its mechanical characteristics correspond to a weathered granite (W3 and W4), with low mechanical resistance and a low deformability modulus. At greater depths its quality increases (W2 to W3).

All the foundation has a thin coating of top soil or organic soil which was removed prior to the construction. Figures 1 and 2 present the plan and the cross section of the dam. As it can be seen from the figures, the valley is asymmetrical with a average slope of 1:6,5 (v:h) above level 1200 on the right bank and 1:2,6 (v:h) on both left and right banks below that level.

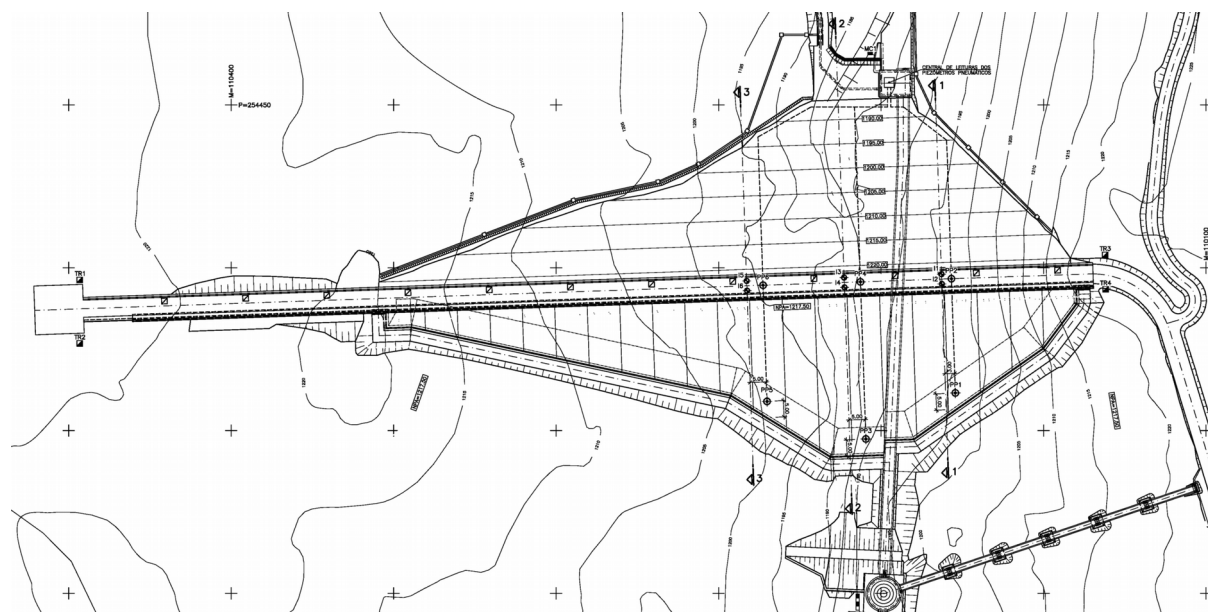


Figure 1: Plan of Montesinho dam



allows for the creation of groups of elements, faces or nodes, to specify different materials and boundary conditions. Figure 3 shows a Salome-Meca session in the mesh module. The mesh refers to Montesinho dam.

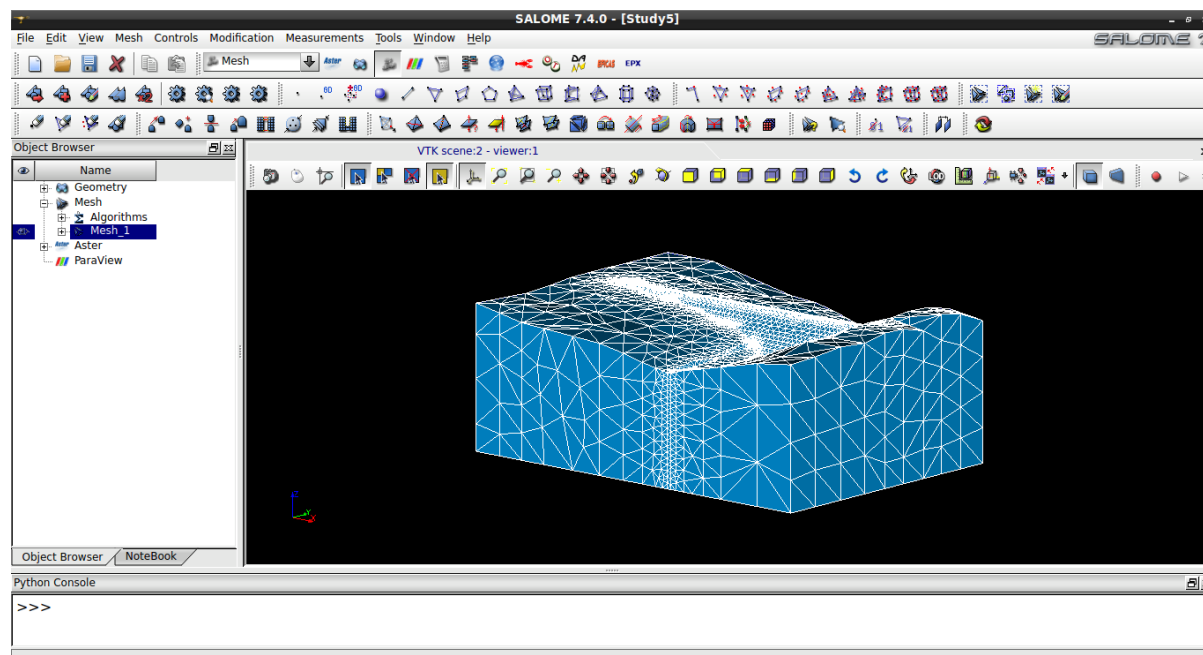


Figure 3: Salome-Meca including python console

The Efficas module is dedicated to the creation of the script containing all the commands to the Code-aster modelation. This module is fairly complex and requires a good knowledge of the finite element method because it permits to specify very complex analysis and operations to the model.

The Aster module is just a graphical user interface to launch the *FEM* calculations. It allows for the specification of the computing machine (local, remote or remote cluster) the specification of the number of cpus, memory allocation and total cpu time.

Finally, to view the results Salome-Meca uses the Paravis module. Paravis derives from ParaView that is is an extensible and very configurable framework used to view data in many forms.

## 5 MODEL SETUP

### 5.1 Geometry, scripts and automation

The geometry of the model consists of 2 parts. The foundation is generated from a set of blocks with a triangular base and top. The foundation surface is obtained from a simple point sampling of the dam foundation surface. Those points are submitted to a Delaunay triangulation creating the block set. All the blocks are finally fused together giving place to the geometry of the foundation. All the process is easily obtained by means of a python script written by the authors. If required, another tool permits the generation of the bottom outlet pipe. The dam geometry is highly parametrized in the script. The user has to specify 2 points from the crest (the upstream edge) and all the relevant features of the dam geometry, crest and bank width, slopes, etc. The dam is then generated as a regular solid independent from the foundation. Using a boolean cut geometric operation this block is then cutted by the foundation, adjusting its geometry to the real surface of the foundation. Because the stress and strain paths are relevant to the soils and rockfill behavior the

construction of the dam has to be made in layers<sup>13</sup>. On the other hand, to correctly account for the deformation of a fill being built, the simulation of the construction is supposed to account for the displacements of the existing layers, not the one being placed. Thus the dam model is also split in layers by means of several cutting planes, allowing for an arbitrary user-defined number of layers.

## 5.2 Finite element mesh

The mesh was generated using the simplest algorithm available in the program – Netgen<sup>14</sup> and all the default hypothesis in the Mesh module. In the case of Montesinho dam analysis the generated mesh has about 17k nodes and 83k tetrahedral elements. From those, about 30k elements are used to build the 15 layers of the dam itself. In this module it is also advisable to define groups of nodes, faces and elements, to permit the imposition of boundary conditions and the construction sequence.

## 5.3 Modeling of the construction and the filling

The simulation of the construction of the dam is made in layers because it is necessary to account for the construction deformations and also because soil and rockfill materials behavior is dependent on the stress (and strain) paths. To account correctly to construction deformations one follows a Naylor proposal, where the displacements of a newly constructed layer are dismissed. Using this method, when the dam is finally completed the displacements in the crest are null as in the reality, because the dam is always built up to the designed level. Figure 4 shows the actual construction history of the dam. Because the model used does not take in account the time in the material behavior these values were not taken in to consideration. Instead, a constant construction rate was considered. To model the filling of the reservoir two alternatives were considered. The first one consists of modeling the water as special incompressible finite elements having and the second one consists on applying the water load as surface loads. Considering all the implications of the former hypothesis namely a larger mesh, it was decided to consider the filling of the reservoir as loads applied in accordance with the reservoir level.

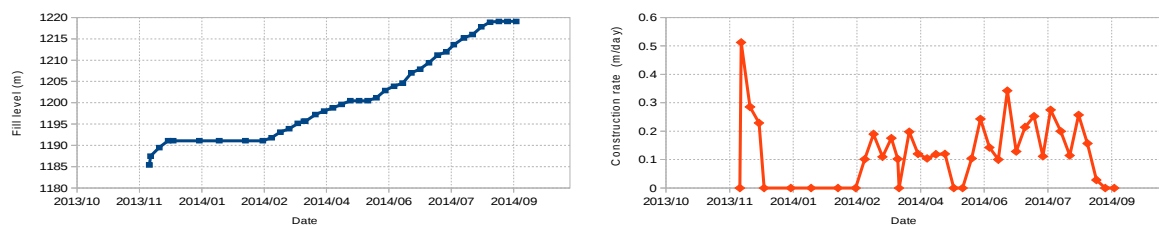


Figure 4: Construction rate (actual values)

## 6 ROCKFILL PARAMETERS IDENTIFICATION

The maximum particle size of the rockfill materials applied in the construction of the dam was 800 mm, making it impossible to calibrate the model parameters based on the actual materials. In order to test the rockfill using laboratory tests with normal dimensions, such as oedometric or triaxial tests, the parallel gradation technique was applied. This technique uses the same rockfill, but considers only smaller particles.

There are some factors that affect rockfill behavior, such as strength and stiffness. They are the nature of particle minerals, grain shape and size, uniformity coefficient ( $C_u$ ), relative density, the stress trajectories, the presence of water, the gradation and the effect of time<sup>15,16,17,18,19,20</sup>. Some of these factors can be taken into account by employing the

parallel gradation technique and by maintaining similar grain angularity. Although it is known that the smaller particle size influence the material mechanical and hydraulic<sup>21</sup> responses, there are some difficulties to quantify the relation between particle size and the strength and stiffness of rockfill. Some authors<sup>18,19</sup> showed that the empirical stiffness of rockfill may decrease up to 50 % if the maximum particle size increases by a factor of ten<sup>22</sup>. On the contrary, some studies have shown that coarse grains lead to larger dilatancy<sup>23</sup>, which may slightly increase the overall stiffness of a rockfill dam because of the restriction effect.

Considering this aspects, the tested materials chosen to calibrate the model parameters had a smaller density. The void ratio of the *in situ* rockfill tested was 0.14, with a density of 2.30 g/cm<sup>3</sup> after compaction.

On the basis of these considerations, a smaller density of tested materials was chosen to calibrate the model parameters. The actual main rockfill had a void ratio of 0.259 and a density of 2.16 g/cm<sup>3</sup> after compaction. In this study, the void ratio of the main rockfill tested was 0.319, with a density of 2.06 g/cm<sup>3</sup>. The lower void ratio results in a lower modulus and shear strength of the rockfill<sup>24,25</sup>. It also reduces the dilatancy of the rockfill and further contributes to a decreased overall stiffness of the rockfill dam because of smaller restriction effects.

## 7 RESULTS ANALYSIS AND DISCUSSION

According to the model and in close agreement to other cases, the dam exhibits a low level of deformation during the construction and also during the filling of the reservoir. Concerning the construction phase this obtained results are in close agreement with the actual values.

Figure 5 presents the displacement records in the vertical internal settlement gauges. The diagrams show actual values recorded during the construction and the results from FEM calculations. In the case of gauge I3 (near the higher cross-section) two different calculations are available. One is from the 3D model, described in this article and the other is from a 2D model using the same set of mechanical parameters. The actual values were obtained in August 2014, before the final 2 meters of rockfill. Presently the upstream face is being constructed and only after that the final campaign of settlements will be available. The maximum settlement is expected to be in the range of 25 +/- 5 mm

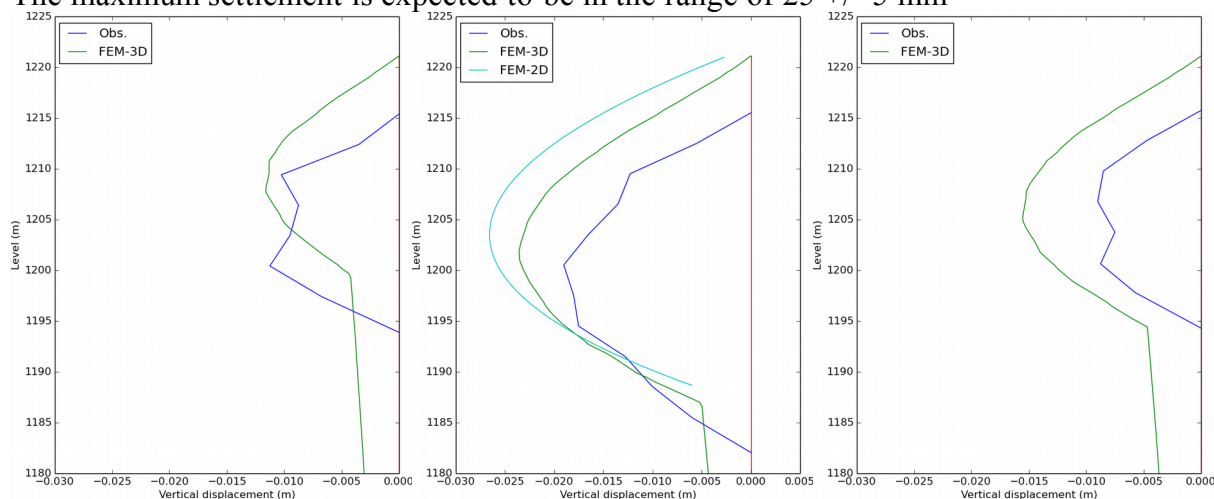


Figure 5: Internal settlements I1, I3 and I5 (actual and calculated)

Using the 3D model it is possible to predict the behavior during the first filling. This phase of the loading was performed in 5 steps. Figure 6 Shows the displacements in the



dam only due to its filling. The following conclusions can be derived: a) the maximum displacement is less than 1 cm, and is expected to occur in the lower third of the dam and near the highest cross-section. This displacement will be recorded in inclinometer I4. Near the abutments (where inclinometers I2 and I6 are placed) a lower level of deformation in the range on 2-3 mm is expected. The dam may exhibit a overall downstream movement of about 2 mm near the highest (central) zone. The estimative of the horizontal displacements in the longitudinal section is presented in Figure 7.

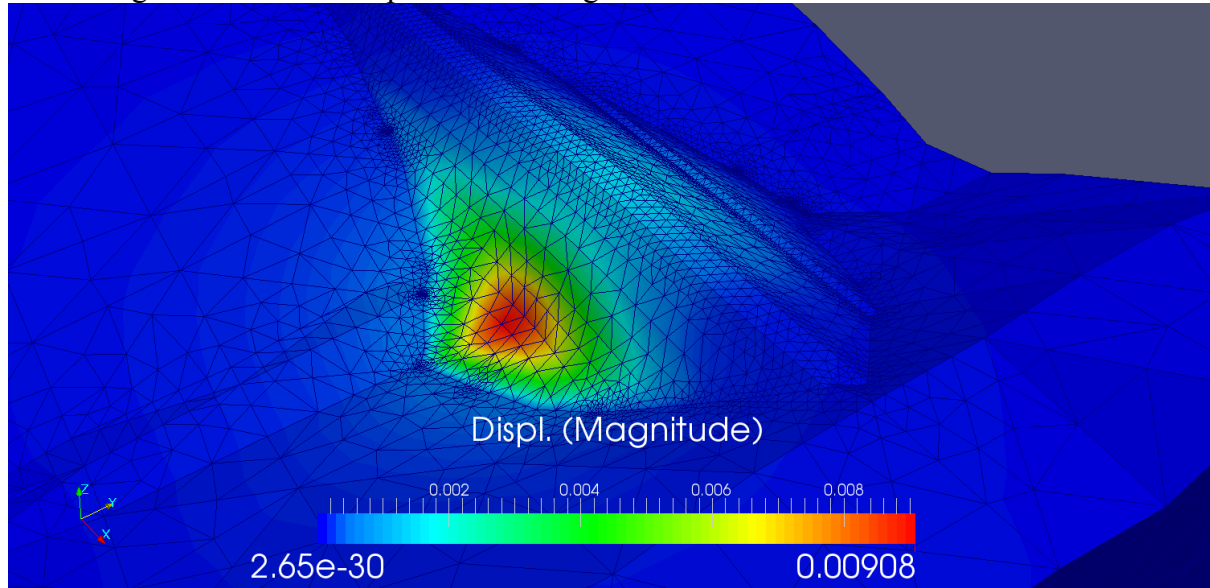


Figure 6: Displacements due to the first filling

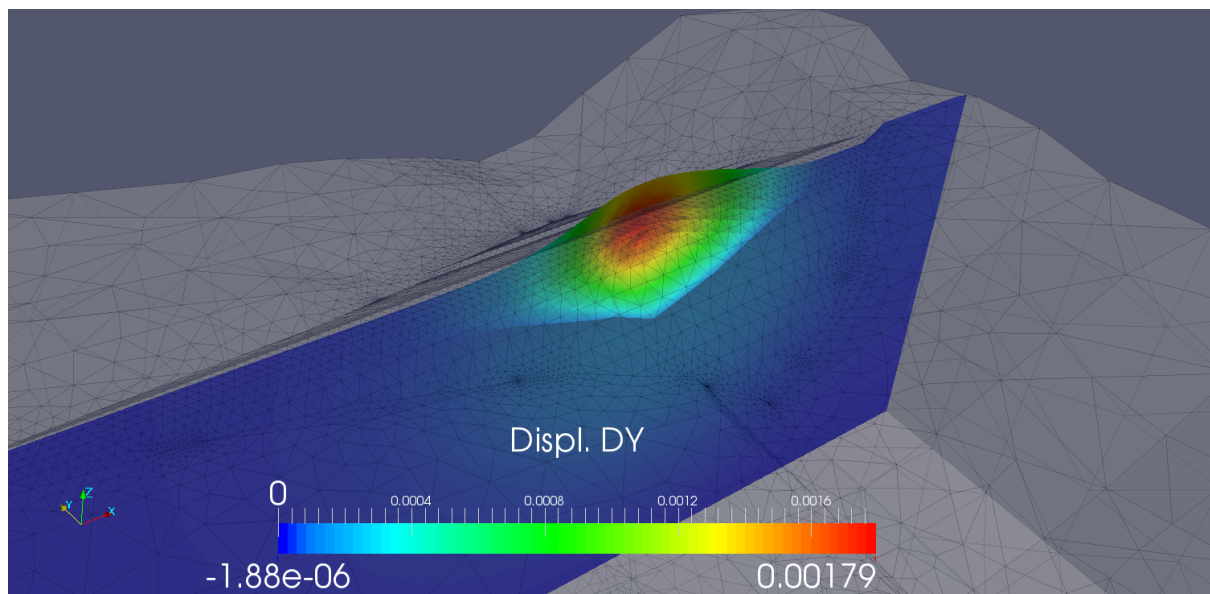


Figure 7: Longitudinal section with upstream-downstream displacements

## 8 CONCLUSIONS AND FURTHER DEVELOPMENTS

This paper presents the development of a 3D finite element model to study the construction of a concrete face rockfill dam. All the resources and tools used are available under the *GNU* licensing statement and therefore are freely available. The model and general procedure presented in this paper is seems to be in good agreement with the actual behavior of the dam in this particular case. Because the dam is still under construction,

part of the results obtained serve also as calibration for the model. In particular, the settlement obtained by the model is in good agreement with the actual values. Maximum settlement recorded so far is about 2 cm, while for the end of construction, the model forecasts 2,5 cm.

For the first filling the forecast indicates that the the displacements due to the rise of the reservoir will be small and fully compatible with a normal performance of the upstream concrete slab. Maximum displacement of the concrete face for the full water level will be close to 1 cm.

Finally, although a good agreement was obtained, the model and the general procedure for analysis are still under development and some improvements as follows are needed:

- explicit inclusion of the upstream face using shell elements;
- inclusion of appurtenant works in the model;
- implementation and testing of more complete rheological models (such as Cam-clay and Barcelona models);
- coupling of termo-hydronechanical behavior.

## REFERENCES

- [1] Marcelino, J. (1996) – “Modelação do colapso e da fluência em aterros”. Dissertação de doutoramento, FEUP Porto
- [2] Code\_Aster Open Source - general FEA software - [www.code-aster.org](http://www.code-aster.org) (accessed in 2014-10-15)
- [3] <http://www.code-aster.org/V2/spip.php?article806> (accessed in 2014-10-15)
- [4] [www.gnu.org](http://www.gnu.org) (accessed in 2014-10-15)
- [5] Duncan JM, Chang CY. Nonlinear analysis of stress and strain in soils. *J Soil Mech Found Div, ASCE* 1970;96(SM5):1629–53.
- [6] Xing Hao-Feng, Gong Xiao-Nan, Zhou Xiao-Guang, Fu Hai-Feng. Construction of concrete-faced rockfill dams with weak rocks. *J Geotech Geoenviron Eng, ASCE* 2006;132(6):778–85.
- [7] Zhou Wei, Hua Jun-Jie, Chang Xiao-Lin, Zhou Chuangbing. Settlement analysis of the Shuibuya concrete-face rockfill dam. *Comput Geotech* 2011;38(2):269–80.
- [8] G.C. Li, C.S. Desai. Stress and seepage analysis of earth dams. *J GE, Proc ASCE*, 103 (GT7) (1983), pp. 667–676.
- [9] Xu, B., Zou, D., Liu, H. Three-dimensional simulation of the construction process of the Zipingpu concrete face rockfill dam based on a generalized plasticity model. *Computers and Geotechnics* 43 (2012) 143–154.
- [10] Pastor M, Zienkiewicz OC, Chan AHC. Generalized plasticity and the modeling of soil behavior. *Int J Numer Anal Methods Geomech* 1990;14(3):151–90.
- [11] Ling HI, Liu H. Pressure dependency and densification behavior of sand through a generalized plasticity model. *J Eng Mech, ASCE* 2003;129(8):851–60.
- [12] <https://www.python.org/> (accessed in 2014-10-15)
- [13] Naylor, D. J., Pande, G. N.; Simpson B. & Tabb, R. (1981)- "Finite elements in geotechnical engineering". Pineridge Press, Swansea, U.K.
- [14] Schoberl, Joachim (1997) . NETGEN. An advancing front 2D/3D-mesh generator based on abstract rules. *Computing and Visualization in Science*. Springer-Verlag. pp 41-52
- [15] Gamboa, C. (2011). Mechanical behaviour of rockfill materials application to concrete face rockfill dams. PhD thesis.
- [16] Hardin, B. O. (1985). Crushing of soil particles. *Journal of Geotechnical and Engineering, ASCE*, 111(10):1177–1192.
- [17] Fumagalli E. Tests on cohesionless materials for rockfill dams. *J Soil Mech Found Div* 1969;95(1):313–32.

- [18]Frost RJ. Some testing experiences and characteristics of boulder-gravel fills in earth dams. STP 523:207. Philadelphia: ASTM; 1973.
- [19]Saboya F, Byrne PM. Parameters for stress and deformation analysis of rockfill dams. *Can Geotech J* 1993;30(4):690–701.
- [20]Hunter G, Fell R. Rockfill modulus and settlement of concrete face rockfill dams. *J Geotech Geoenviron Eng, ASCE* 2003;129(10):909–17.
- [21]Einav, I. (2007). Breakage mechanics - Part I: Theory. *Journal of the Mechanics and Physics of Solids*, 55(6):1274–1297.
- [22]Alonso EE, Oldecop L, Pinyol NM. Long term behavior and size effects of coarse granular media. *Mech Nat Solids* 2009:255–81.
- [23]Evans TM, Mojarrad H, Cunningham C, Tayebali AA. Grain size distribution effects in 2D discrete numerical experiments. In: *Geotechnical Special Publication No. 186: contemporary topics in situ testing, analysis, and reliability of foundation*, Reston, VA; 2009. p. 58–65.
- [24]Hardin BO, Drnevich V. Shear modulus and damping in soils. *J Soil Mech Found Div* 1972;98(7):667–92.
- [25]Kokusho T, Esashi Y. Cyclic triaxial test on sands and coarse materials. In: *Proceedings of the 10th international conference on soil mechanics and foundation engineering*, Stockholm, vol. 1; 1981. p. 673–6.

- [1] Marcelino, J. (1996) – “Modelação do colapso e da fluência em aterros”. Dissertação de doutoramento, FEUP Porto
- [2] Code\_Aster Open Source - general FEA software - [www.code-aster.org](http://www.code-aster.org) (accessed in 2014-10-15)
- [3] <http://www.code-aster.org/V2/spip.php?article806> (accessed in 2014-10-15)
- [4] [www.gnu.org](http://www.gnu.org) (accessed in 2014-10-15)
- [5] Duncan JM, Chang CY. Nonlinear analysis of stress and strain in soils. *J Soil Mech Found Div, ASCE* 1970;96(SM5):1629–53.
- [6] Xing Hao-Feng, Gong Xiao-Nan, Zhou Xiao-Guang, Fu Hai-Feng. Construction of concrete-faced rockfill dams with weak rocks. *J Geotech Geoenviron Eng, ASCE* 2006;132(6):778–85.
- [7] Zhou Wei, Hua Jun-Jie, Chang Xiao-Lin, Zhou Chuangbing. Settlement analysis of the Shuibuya concrete-face rockfill dam. *Comput Geotech* 2011;38(2):269–80.
- [8] G.C. Li, C.S. Desai. Stress and seepage analysis of earth dams. *J GE, Proc ASCE*, 103 (GT7) (1983), pp. 667–676.
- [9] Xu, B., Zou, D., Liu, H. Three-dimensional simulation of the construction process of the Zipingpu concrete face rockfill dam based on a generalized plasticity model. *Computers and Geotechnics* 43 (2012) 143–154.
- [10] Pastor M, Zienkiewicz OC, Chan AHC. Generalized plasticity and the modeling of soil behavior. *Int J Numer Anal Methods Geomech* 1990;14(3):151–90.
- [11] Ling HI, Liu H. Pressure dependency and densification behavior of sand through a generalized plasticity model. *J Eng Mech, ASCE* 2003;129(8):851–60.
- [12] <https://www.python.org/> (accessed in 2014-10-15)
- [13] Naylor, D. J., Pande, G. N.; Simpson B. & Tabb, R. (1981)- "Finite elements in geotechnical engineering". Pineridge Press, Swansea, U.K.
- [14] Schoberl, Joachim (1997) . NETGEN. An advancing front 2D/3D-mesh generator based on abstract rules. *Computing and Visualization in Science*. Springer-Verlag. pp 41-52
- [15] Gamboa, C. (2011). Mechanical behaviour of rockfill materials application to concrete face rockfill dams. PhD thesis.
- [16] Hardin, B. O. (1985). Crushing of soil particles. *Journal of Geotechnical and Engineering, ASCE*, 111(10):1177–1192.
- [17] Fumagalli E. Tests on cohesionless materials for rockfill dams. *J Soil Mech Found Div* 1969;95(1):313–32.
- [18] Frost RJ. Some testing experiences and characteristics of boulder-gravel fills in earth dams. STP 523:207. Philadelphia: ASTM; 1973.
- [19] Saboya F, Byrne PM. Parameters for stress and deformation analysis of rockfill dams. *Can Geotech J* 1993;30(4):690–701.
- [20] Hunter G, Fell R. Rockfill modulus and settlement of concrete face rockfill dams. *J Geotech Geoenviron Eng, ASCE* 2003;129(10):909–17.
- [21] Einav, I. (2007). Breakage mechanics - Part I: Theory. *Journal of the Mechanics and Physics of Solids*, 55(6):1274–1297.
- [22] Alonso EE, Oldecop L, Pinyol NM. Long term behavior and size effects of coarse granular media. *Mech Nat Solids* 2009:255–81.
- [23] Evans TM, Mojarrad H, Cunningham C, Tayebali AA. Grain size distribution effects in 2D discrete numerical experiments. In: *Geotechnical Special Publication No. 186: contemporary topics in situ testing, analysis, and reliability of foundation*, Reston, VA; 2009. p. 58–65.
- [24] Hardin BO, Drnevich V. Shear modulus and damping in soils. *J Soil Mech Found Div* 1972;98(7):667–92.
- [25] Kokusho T, Esashi Y. Cyclic triaxial test on sands and coarse materials. In: *Proceedings of the 10th international conference on soil mechanics and foundation*

engineering, Stockholm, vol. 1; 1981. p. 673–6.