Assessment of characteristic failure envelopes for intact rock using results from triaxial tests

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ABSTRACT: The paper presents contributions to the statistical study of the parameters of the Mohr-Coulomb and Hoek-Brown strength criteria, in order to assess the characteristic failure envelopes for intact rock, based on the results of several sets of triaxial tests performed by LNEC.

1 INTRODUCTION

Introduction of Eurocode 7 (EN1997-1), bringing structural safety concepts in geotechnical design, was an important step forward in many European countries. Eurocode 7 (EC7) deals with constructions in or on the ground, which is defined as "soil, rock and fill in place prior to the execution of the construction works". Rock engineering design is, therefore, included in the scope of EC7, but this is often overlooked.

At present, application of EC7, though with a number of difficulties, can be considered at a cruise speed for many types of geotechnical engineering problems dealing with soils. Unfortunately, the same cannot be said regarding rock engineering problems, where major difficulties still have to be overcome.

One of the main issues regarding applicability of the semi-probabilistic approach of EC7 to rock engineering has to do with an essential feature of rock masses, which is their discontinuous nature. This has a number of major implications regarding, for instance, the definition of failure modes or the validity of the assumption of the aleatory nature of rock mass parameters.

The basis of the limit state design philosophy adopted in EC7 is that, for each particular design situation, all the possible limit states for a structure, or part of it, shall be considered and that it shall be demonstrated that the likelihood of any limit state being exceeded is sufficiently small.

A distinctive feature of EC7, when compared with the Eurocodes for other types of structures, is that the limit states shall be verified by one or a combination of the following methods: use of calculations, prescriptive measures, experimental models and load tests, and observational method.

The first method of safety verification, using calculations, is by far the most used one, and it is often confused with the EC7 itself. It involves using characteristic values of actions, ground properties and geometrical data, as well as obtaining their design values by the partial factor method.

EC7 gives rules for obtaining characteristic values of geotechnical parameters in general, which take into consideration that geotechnical design does not deal with manufactured materials, with relatively well known parameter values, but with natural materials, of a great diversity as regards their origin and the condition in which they are found in nature.

Also given in EC7 are recommended values of the partial factors to use for some specific ground parameters: angle of shear resistance, effective cohesion, undrained shear strength, unconfined strength and weight density. These values indicate the minimum level of safety for conventional design.

It is easy to recognise that these parameters were chosen having in mind soil properties. It is doubtful that the partial factors to use with Mohr-Coulomb strength parameters or with unconfined strength for rock masses should have the same value as for soils.

2 CHARACTERISTIC VALUES OF ROCK MASS PARAMETERS

It is useful to revise the exact meaning given in the Eurocode (EN 1990) to characteristic value and to design value of material properties.

The Eurocode defines characteristic value as: "*value of a material or product property having a prescribed probability of not being attained in a hypothetical unlimited test series. This value generally corresponds to a specified fractile of the assumed statistical distribution of the particular property of the material or product*."

As regards the design value of a material or product property, its definition according to the

Eurocode is: "*value obtained by dividing the characteristic value by a partial factor*".

EC7 defines how characteristic values of geotechnical parameters are obtained. Characteristic values "*shall be selected as a cautious estimate of the value affecting the occurrence of the limit state*". This value depends on the zone of ground governing the behaviour of the geotechnical structure. Usually it is much larger than the volume affected in an *in situ* or laboratory test, and the characteristic value should be "*a cautious estimate of the mean value*" or of the range of values covering that whole zone of ground. However, if the behaviour of the geotechnical structure is governed by the lowest or highest value of the ground property, the characteristic value should be "*a cautious estimate of the lowest or highest value*".

If statistical methods are used, the characteristic value is "*a selection of the mean value of the limited set of geotechnical parameter values, with a confidence interval of 95%*", in the first case, or "*a 5% fractile*" in the second case.

The following sections of this paper present contributions to the statistical study of the parameters of the Mohr-Coulomb and Hoek-Brown strength criteria, in order to assess the characteristic failure envelopes for intact rock, based on the results of several sets of triaxial tests performed by LNEC.

3 TRIAXIAL TESTS OF ROCK SAMPLES

To describe the strength of intact rock under triaxial conditions, the most frequently used strength criteria are the well-known Mohr-Coulomb and Hoek-Brown criteria. The Mohr-Coulomb criterion can be appropriately used to model the relation between the principal stresses at failure using a linear relation, with the parameters c and ϕ (cohesion and internal friction angle), as long as small ranges of the confining stresses are involved (Labuz & Zang 2012). Hoek & Brown (1980) developed a nonlinear relationship between the principal stresses at failure. For a given rock type, this relationship is characterized by the parameters m_i and σ_{ci} (uniaxial compressive strength), where the index i stands for intact rock (Eberhart 2012).

Values for m_i and σ_{ci}, which can be used as initial estimates, are available in the literature for a variety of rock types. However, important projects require specific triaxial tests to be performed to determine the actual values. For this purpose, a statistically significant set of triaxial tests should be performed, under confining stresses that cover the expected range of stresses. In order to assess parameter variability, the rock specimens to be tested under all the confining stresses should be prepared from a homogeneous sample of rock cores. In the tests used for the analysis presented here, a minimum of 20 specimens was collected from each rock sample.

Test results used in this paper correspond to granite samples from dam sites in the north of Portugal, tested between 2008 and 2012. The specimens have all of the same dimensions: 51 mm diameter and 125 mm height. They were tested in a stiff testing machine, with controlled deformation, using a procedure adapted from the standard ASTM D7012-07, which corresponds to "type I" test of the ISRM Suggested Method (ISRM, 1978). The ranges of confining stresses (σ_3) varied between sets of tests, and the values of the axial stress at failure (σ_1) were peak values.

4 STATISTICAL ANALYSIS OF THE RESULTS

4.1 *Detailed analysis of one set of triaxial tests*

In this section, the statistical analysis of the results of a chosen set of triaxial tests will be presented as an example. This particular set comprises 21 results of medium-grained granite that were tested under the following confining stresses σ_3: 0, 2, 5, 10 and 15 MPa.

To estimate the parameters of the Mohr-Coulomb criterion (c and ϕ), firstly, it is necessary to perform a linear regression of the maximum principal stress σ_1 versus the confining stress σ_3 (and also minimum principal stress). Figure 1 shows, in the $\sigma_2 = \sigma_3$ plane of the principal stress space, the results of the triaxial tests (dots) and their best fit straight line.

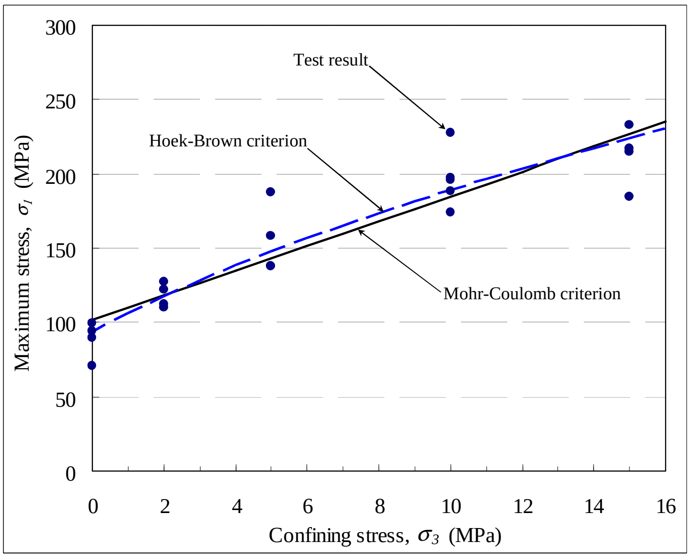

Figure 1. Mohr-Coulomb and Hoek-Brown failure criteria in the $\sigma_2 = \sigma_3$ plane of the principal stress space.

Then, from the slope of the straight line (tan β) and the y-axis intercept, that in this case is the uniaxial compressive strength of the intact rock σ_{ci}, it is possible to calculate the internal friction angle and the cohesion for the mean Mohr-Coulomb failure envelope using the following equations:

$$\phi = \sin^{-1} \frac{\tan \beta - 1}{\tan \beta + 1} \tag{1}$$

$$c = \sigma_{ci} \frac{1 - \sin \phi}{2 \cos \phi} \tag{2}$$

In the Mohr diagram (σ_n–τ space), the Coulomb failure criterion is also a linear relation:

$$|\tau| = c + \sigma_n \tan\phi \tag{3}$$

Any kind of regression in the Mohr diagram is very difficult to perform since it means evaluating the “best” tangent straight line to a set of Mohr circles each representing a triaxial test result. As a consequence, all variability analyses have to be performed in terms of σ_1 versus σ_3, which are the direct results of the triaxial tests.

Test results displayed in Figure 1 show that a curved function may yield a better relation between σ_1 and σ_3. To consider this type of negative curvature between the principal stresses at failure that several types of rocks often display, the Hoek-Brown a criterion uses the following equation:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_i \frac{\sigma_3}{\sigma_{ci}} + 1} \tag{4}$$

It also requires only two parameters, the uniaxial compressive strength σ_{ci} and m_i. These parameters have to be determined from triaxial tests results by non-linear regression. Figure 1 also includes the best Hoek-Brown equation (dashed line).

In both cases, regression was used to calculate the criterion parameters. It is a statistical procedure for fitting data to any selected criterion equation by minimizing the residual sum of squares RSS

$$RSS = \sum_i r_i^2 = \sum_i (\sigma_{1i} - \hat{\sigma}_{1i})^2 \tag{5}$$

where r_i are the residuals, σ_{1i} are the test results and $\hat{\sigma}_{1i}$ are the corresponding model predicted values.

In assessing the goodness of fit of the criteria, it is essential to examine several graphs: the criteria curves superimposed on the data points, the residuals versus the independent variable σ_3 and the predicted values $\hat{\sigma}_1$ (Draper & Smith 1998). These graphs allow to easily recognize if a model is inappropriate, or if the variance of the residuals is constant across observations (homoscedasticity). In addition, several statistical methods can be used to quantify goodness of fit. Generally, all take into account s^2, an unbiased estimator of the variance of the residuals, also known as the residual mean square, given by

$$s^2 = \frac{RSS}{n-p} \tag{6}$$

where n is the number of experimental values and p is the number of parameters in the model (two in both cases). In sequence, standard errors of the regressions s can be easily calculated. In linear regressions (Mohr-Coulomb criterion), this estimator and appropriate values of the Sudent’s t distribution, allow predicting, for a given confining stress σ_3, 95% confidence limits for the true mean value of the maximum principal stress $\hat{\sigma}_1$, and 5% fractiles:

$$\sigma_1^{95\%} = \hat{\sigma}_1 \pm t_{(n-p,0.95)}\, s \left(\frac{1}{n} + \frac{(\sigma_3 - \bar{\sigma}_3)^2}{\sum_i (\sigma_{3i} - \bar{\sigma}_3)^2} \right) \tag{7}$$

$$\sigma_1^{>5\%} = \hat{\sigma}_1 - t_{(n-p,0.90)}\, s \left(1 + \frac{1}{n} + \frac{(\sigma_3 - \bar{\sigma}_3)^2}{\sum_i (\sigma_{3i} - \bar{\sigma}_3)^2} \right) \tag{8}$$

In non-linear regression, exact definition of confidence intervals is seldom possible. So, to determine the 95% confidence limits and the 5% fractiles for the Hoek-Brown criterion the bootstrap method (Efron 1979) was used. This procedure considers the sample as the population and draws with replacement samples with the same size n. With these samples, almost all statistical inference calculations can be carried out. Performing a sufficiently large number of draws will allow the bootstrap estimates to asymptotically tend to the correct values.

Evaluating equations (7) and (8) and using the bootstrap method for any required σ_3 value within the range of applied confining stresses, 95% confidence intervals for the fitted values and 5% failure envelopes were determined. They are displayed in Figure 2 for the Mohr-Coulomb and Hoek-Brown criteria, respectively.

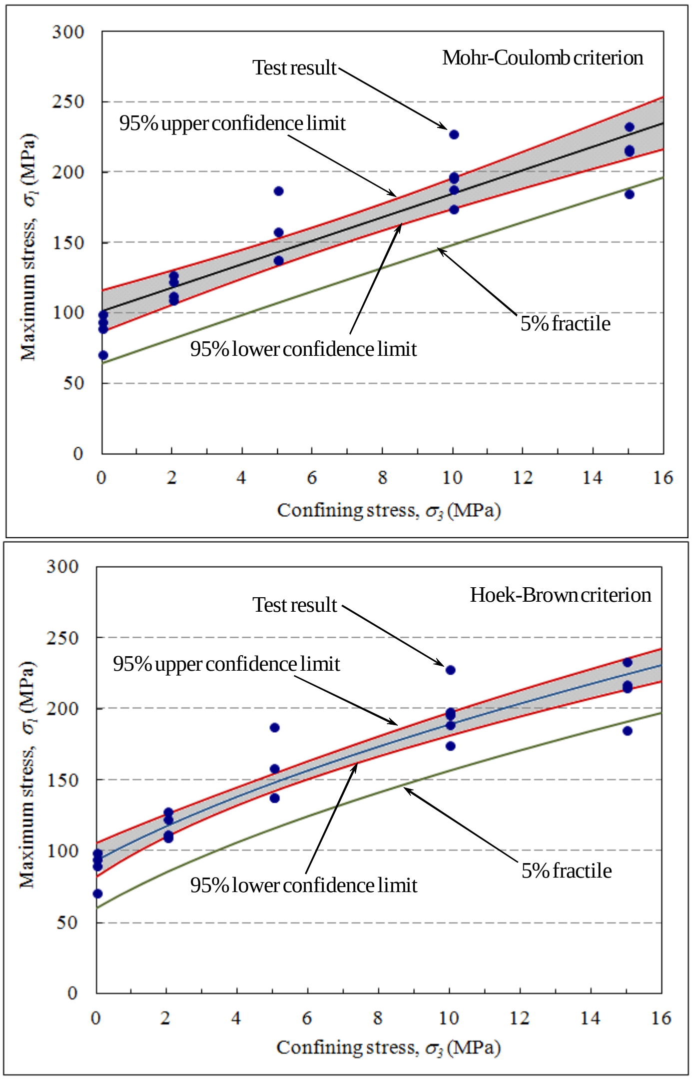

Figure 2. 95% confidence intervals and 5% failure envelopes for the Mohr-Coulomb and Hoek-Brown models.

These plots show that the Hoek-Brown criterion provides a better fit to the test results, since its 95% confidence interval is clearly narrower. This conclusion can also be recognized by the regression standard error values *s*: 20.4 MPa and 18.2 MPa, for the Mohr-Coulomb and Hoek-Brown models, respectively. The plots also reveal that the 95% confidence limits and the 5% envelope are not straight lines in the case of the Mohr-Coulomb criterion, nor parabolas in the case of the Hoek-Brown criterion. However, both criteria could still provide good approximations to these curves. As could be expected from the analysis of equations (7) and (8), the curves are closer to the models when the confining stress is equal to its average value $\overline{\sigma}_3$ (6.57 MPa, in this example). All these curves can be transferred to the $\sigma_n - \tau$ space evaluating numerically at each point their tangents and then using equations (1) and (2) to plot the respective values in the Mohr diagram (Figure 3). The normal stress range tries to cover approximately the principal stresses of the tests.

For the Mohr-Coulomb (linear) criterion, results displayed in Figure 2 can also be easily transferred to the space of the estimated parameters (tan β–σ_{ci}) by evaluating the tangents to each curve for any given σ_3 value (Figure 4). The red ellipse corresponds to all tangents to the 95% confidence limits. The black dot represents the mean Mohr-Coulomb envelope determined by the linear regression (tan β = 8.34 and σ_{ci} = 101.5 MPa), and therefore defines the centre of the ellipse. The green ellipse segment represents the 5% fractiles for positive σ_3 values. The remaining part of this ellipse corresponds to the 95% fractile, and therefore it was not plotted.

As already mentioned, though the model is linear, the 95% confidence limits and the 5% fractiles are not. So, in order to assess the reduction from the mean value they lead to, linear approximations were calculated within the range of the triaxial tests (0-15 MPa, thicker parts of the ellipses). These averaged values are (tan β =8.14; σ_{ci} = 90.9 MPa) and (tan β =8.29; σ_{ci} = 65.2 MPa), respectively, and are indicated in Figure 4.

Student's *t* probability distributions of the mean values are also plotted for both parameters showing, that the horizontal and vertical tangents to the 95% confidence limits ellipse define 2.5% tail areas of the independent probability distributions of each parameter.

Considering separately the parameters tan β and σ_{ci}, 95% lower confidence limits for the means would lead to use the low-left corner of the rectangle tangent to the red ellipse, instead of the red dot. The figure allows to recognize that considering separately the variability of the parameters imposes a strong reduction for the characteristic values.

Though tan β and σ_{ci} are the intrinsic regression parameters, they are not commonly used. So, it is essential to define the same results in terms of internal friction angle (tan ϕ) and cohesion *c*.

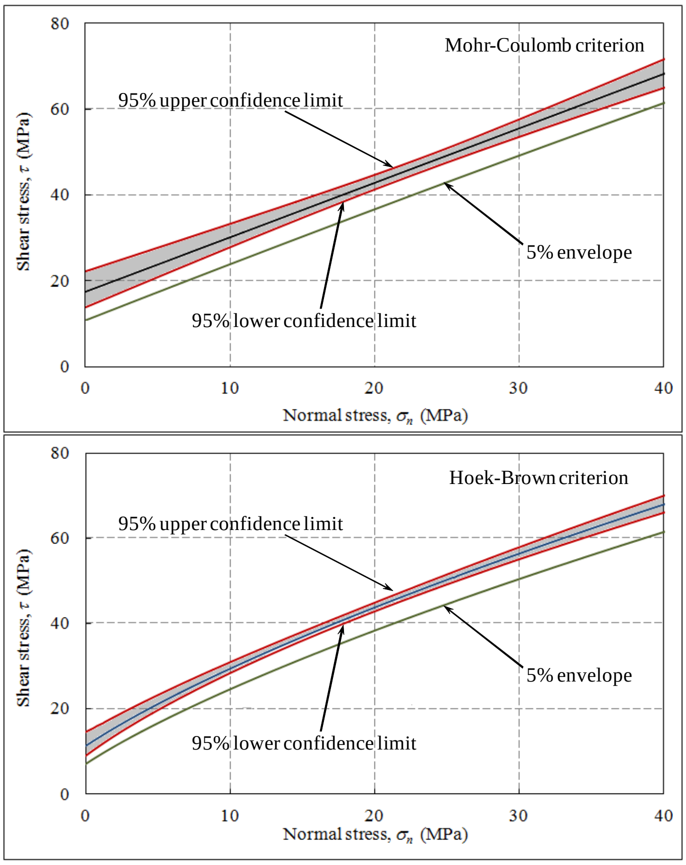

Figure 3. 95% confidence intervals and 5% envelopes for the Mohr-Coulomb and Hoek-Brown models in Mohr diagrams.

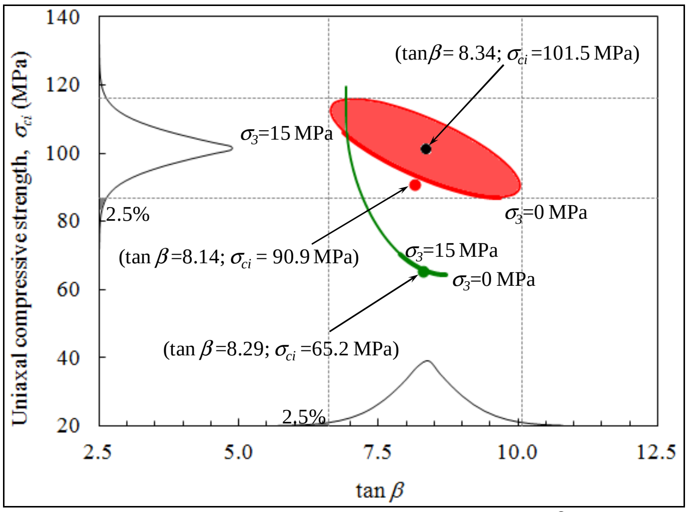

Figure 4. Confidence limits and 5% fractile for tan β and σ_{ci}.

It should be noted that 95% confidence limits no longer yield an ellipse and the mean values are not in the centre, meaning that the joint distribution of tan ϕ and *c* is skewed. Considering normalized values for these parameters (ratios with the respective average values), Figure 5 shows clearly that the reductions from the mean values almost only affect the cohesion. This conclusion could also be drawn from Figure 2, as the 95% confidence lower limit and the 5% fractile curves are almost parallel to the mean envelope. This approach to calculate characteristic values is a consequence of all hypotheses regarding linear regression, namely homoscedasticity, and so it is much related with the confining stress ranges that are considered.

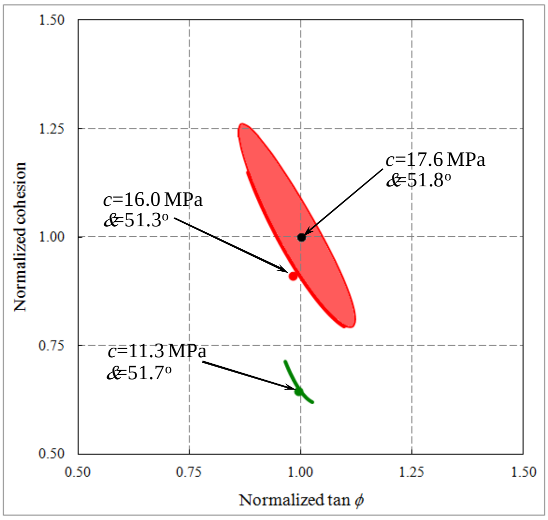

Figure 5. 95% confidence region and 5% fractile for normalized Mohr-Coulomb parameters.

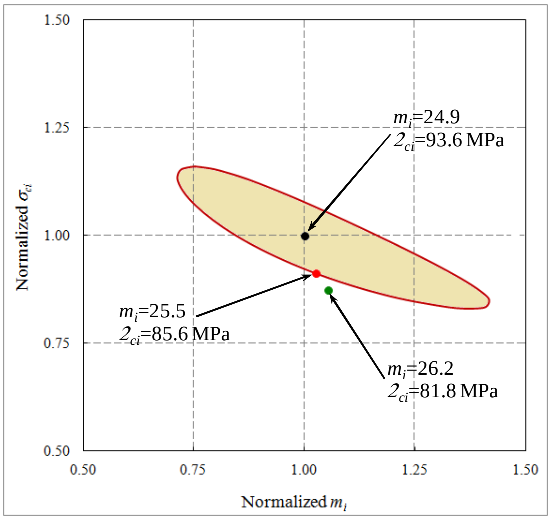

Figure 6. 95% confidence regions and 5% fractile for normalized Hoek-Brown parameters.

Considering a range of confining stresses centred around the mean $\bar{\sigma}_3$, the values of σ_{ci} and c would be reduced but the values of tan β or ϕ would remain constant. In the case of a particular rock engineering project, if foreseen stress values are higher than the average confining stress, this procedure would render characteristic values for the cohesion higher than the mean value, and lower than the mean for the friction angle. In the opposite case, if stresses were lower than those used in the tests, reverse results should be calculated (lower cohesion and higher friction angle).

For the Hoek-Brown criterion a similar approach can be used. In this case, since it is a non-linear criterion, the 95% confidence limits for the mean values of the parameters, σ_{ci} and m_i, do not produce an ellipse but an elongated closed curve (Figure 6). As for the linear model, the Hoek-Brown equation (4) was also used to model the 95% lower confidence limit and the 5% fractile within the range of applied confining stresses. Figure 6 displays the normalized values of the parameters of the Hoek-Brown criterion. Since the strength reductions are almost independent of the confining stress (in Figure 2 the curves are almost parallel), the characteristic value for the uniaxial compressive strength reflects a similar behaviour as in the linear model. However, due to particular features of the non-linear equation, the Hoek-Brown constant m_i shows an increase.

Finally, as already mentioned, all these statistical analyses are strongly influenced by the assumptions underlying regressions, namely the independence of residuals and homoscedasticity.

Simple procedures to verify these hypotheses rely on the inspection of plots of the residuals *vs* independent variable σ_3 and *vs* predicted values $\hat{\sigma}_1$ (Draper & Smith 1998). This first plot is presented in Figure 7, showing that dispersion of the residuals is not much influenced by σ_3, and that the Hoek-Brown criterion fits better the experimental results, as the regression standard error values s already indicated.

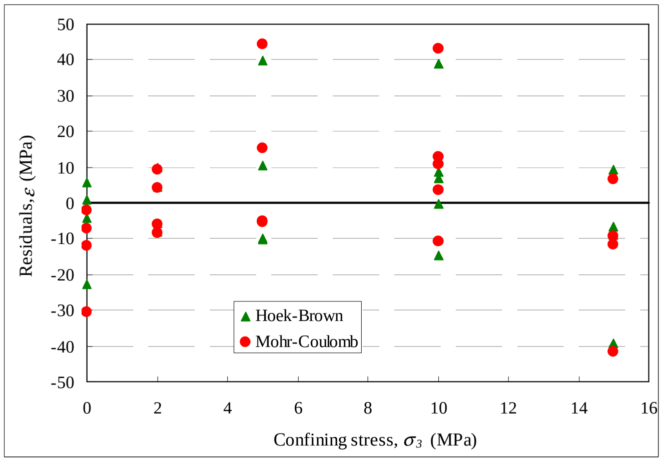

Figure 7. Relation between residuals and confining stress.

4.2 *Results of several sets of triaxial tests*

In this section, the results of the statistical analyses of seven sets of triaxial tests, referred to as A to G, are presented. These tests were performed to support the design of several projects (dam foundations and underground caverns) all of them in granitic rock masses. Confining stress ranges were 0.5 to 5 MPa (set A), 0 to 10 MPa (sets B, C and D) and 0 to 15 MPa (sets E, F and G). Sets C and D were from the same rock mass but with different weathering degrees: W1 for set C and W2 to W3 for set D. Sets E and F were from the same project; set E consisted of fine-grained granite samples, and set F of medium-grained granite. In Table 1, the mean regression values of the Mohr-Coulomb and Hoek-Brown criteria are presented.

The standard errors s allow comparing the fit of the criteria. It can be concluded that both models reach similar approximations to the test results. This can be attributed to the limited range of confining stresses that were used in the tests. It is likely that applying higher σ_3 values would lead to a non-linear increase of the maximum principal stress. However, it should be stressed that the confining stress ranges that were used cover the design situations that can be found in most rock engineering projects.

Table 1. Mean regression parameters

Set	Mohr-Coulomb					Hoek-Brown		
	$\tan\beta$	σ_{ci} (MPa)	s (MPa)	ϕ (°)	c (MPa)	m_i	σ_{ci} (MPa)	s (MPa)
A	10.80	63.8	15.3	56.2	9.7	29.5	60.6	15.2
B	12.30	110.4	22.1	58.2	15.7	34.2	106.7	22.7
C	9.44	119.0	16.0	53.9	19.4	23.7	115.6	15.4
D	4.78	81.1	12.4	40.8	18.5	9.4	80.2	12.4
E	6.69	139.9	27.9	47.7	27.0	15.1	137.3	27.8
F	8.34	101.5	20.4	51.8	17.6	24.9	93.6	18.2
G	12.98	129.2	19.1	59.0	17.9	41.6	120.6	19.9

Figure 8 shows the 95% joint confidence regions for the internal friction coefficient and the cohesion. In the previous section, it was shown that selection of *cautious estimates* within the confining stress ranges of the tests corresponded to reduce just σ_{ci} or the cohesion. In this figure the red dots refer to selecting the characteristic values as a 95% confidence limit for the mean $c^{95\%}$, and the green dots as a 5% fractile $c^{>5\%}$. These values are included in Table 2, along with the respective percentage reductions, which are closely related to the regression dispersion.

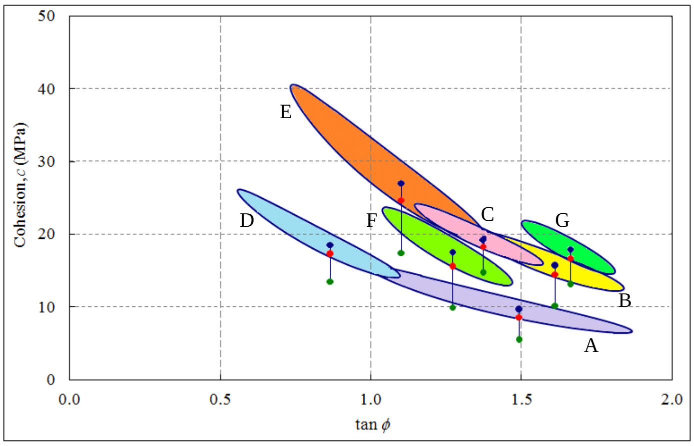

Figure 8. 95% joint confidence regions for the Mohr-Coulomb parameters.

Table 2. Characteristic values for the Mohr-Coulomb criterion

Set	$\tan\phi$	ϕ (°)	c (MPa)	$c^{95\%}$ (MPa)	$c^{>5\%}$ (MPa)
A	1.49	56.2	9.7	8.6 (11%)	5.5 (43%)
B	1.61	58.2	15.7	14.5 (8%)	10.2 (35%)
C	1.37	53.9	19.4	18.3 (5%)	14.8 (24%)
D	0.86	40.8	18.5	17.3 (6%)	13.5 (27%)
E	1.10	47.7	27.0	24.7 (9%)	17.4 (35%)
F	1.27	51.8	17.6	15.6 (11%)	9.9 (43%)
G	1.66	59.0	17.9	16.7 (7%)	13.2 (26%)

5 DISCUSSION AND CONCLUSIONS

This paper presents a statistical approach for the determination of characteristic values as cautious estimates of the mean or lowest values. Though it is well-known that more competent rocks exhibit higher values of both cohesion and friction angle, the impact these parameters jointly have in rock strength is seldom assessed. The importance of the relation between this joint influence and the stress ranges was also recognized, and new discussion topics are put forward. Since it is a linear criterion, the Mohr-Coulomb parameters are easily transferred between the principal stresses space and the Mohr diagrams, and each Coulomb envelope corresponds to a given joint confidence probability of the parameters. On the other hand, the Hoek-Brown criterion does not allow similar simplifications. In the example, cautious estimates led to a reduction of σ_{ci}, but an increase of m_i. So, this parameter is not suited for this type of joint statistical analyses. It is possible that the parameter $m_i\sigma_{ci}$ may yield better results.

The paths for the reduction from mean to characteristic values are a field of discussion. Though it seems obvious that the joint probability of the parameters have to be taken into account, the precise way is not evident, as the stress range that is considered affects the outcome. Considering that the range of confining stresses is centred around $\overline{\sigma}_3$, the reduction only affects the cohesion; however, for confining stresses lower than average, cohesion decreases and friction angle increases, and, for higher than average confining stresses, cohesion increases and the friction angle decreases. This shows the importance of the choice of the confining stresses for the tests.

In the future, as the variance of the results is fully characterized, it will be possible to discuss other possibilities to optimize the testing procedures (spread of the confining stresses along the foreseen stress range, define a minimum number of tests, optimize the number of tests), and to address issues referring to rock mass strength and to discuss how subjective parameters, such as GSI, with very difficult to evaluate effects in the end result, have to be considered in the frame of EC7.

REFERENCES

ASTM D7012-07. *Standard Test Method for Compressive Strength and Elastic Moduli of Intact Rock Core Specimens under Varying States of Stress and Temperatures*. ASTM Intl.

Draper, N.R. & Smith, H. 1988. *Applied regression analysis*. New York: John Wiley & Sons.

Eberhardt, E. 2012. ISRM Suggested Method. The Hoek-Brown failure criterion. *Rock Mech Rock Eng*: 45: 981-988.

Efron, B. 1976. Bootstrap methods: another look at the jackknife. *The Annals of Statistics*: 7: 1-26.

EN1990: 2002. *Eurocode: Basis of structural design*. CEN

EN1997-1: 2004. *Eurocode 7: Geotechnical design. Part 1: General rules*. CEN.

Hoek, E. & Brown, E.T. 1980. Empirical strength criterion for rock masses. *J. Geot. Eng. Div. ASCE*: 106(GT9): 1013–1035.

ISRM, 1978. Suggested methods for determining the strength of rock materials in triaxial compression. In *The Complete ISRM Suggested Methods for Rock Characterization, Testing and Monitoring: 1974:2006*. ISRM, Ankara.

Labuz, J.F. & Zang, A. 2012. ISRM Suggested Method. Mohr-Coulomb failure criterion. *Rock Mech Rock Eng*: 45: 975-979.