On pressure disturbance waves

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ABSTRACT: Pressure disturbance waves are computed via a fully nonlinear, unsteady, boundary integral formulation for various Froude numbers. Three moving pressure distributions are introduced in the numerical model to evaluate the produced wave patterns in a channel. For Froude numbers equal to one, classical runaway solitons are obtained on the fore of the moving pressure patch whereas "stern" waves are radiated away. "Step-like" pressure distributions give different responses to the free-surface flow, with upward breaker jets and steeper "stern" waves. For supercritical and subcritical flows, steady solitons and stationary trenches moving at the same speed of the pressure distribution are obtained, respectively. Nonlinear results show that the wave field is significantly affected by the chosen moving pressure distribution, with breaking "bow" waves and steeper "stern" waves when "step-like" pressure functions are used.

KEY-WORDS: Free-surface flows; Ship waves; Boundary integral method.

1 INTRODUCTION

The representation of a ship's motion by a moving pressure distribution was first proposed by Havelock in 1909 [1], followed by a series of papers published in the early 20th century [2, 3, 4]. Since then, researchers have used the pressure disturbance approach to model ship waves in shallow, intermediate and deep water, with open and bounded domains. Several one-dimensional (1D) pressure functions have been proposed aiming to model the far-field wave pattern. A sinusoidal pressure field distribution was introduced by Ertekin, Webster and Wehausen [5] and later followed by several authors [6, 7, 8, 9, 10, 11, 12],

$$p_0(x,t)=\begin{cases}\frac{1}{2}p_m\left\{1+\cos\left[\frac{2\pi}{l}(x+Ut)\right]\right\}, & \text{if } |x+Ut|<\frac{l}{2},\\ 0, \quad \text{if } |x+Ut|\geq\frac{l}{2}. & \end{cases} \tag{1}$$

The x axis is in the direction of the motion i.e. the centerline of the ship; l is the length of the pressure distribution in the x direction; p_m is the pressure amplitude; U is the velocity of the left-going (positive) or right-going (negative) surface pressure. Grimshaw, Maleewong and Asavanant [13] used a hyperbolic function to study the stability of gravity-capillary waves,

$$p_0(x,t)=p_m\,\text{sech}^2(x+Ut), \quad \text{for } |x+Ut|<\infty. \tag{2}$$

Some authors suggested a two-dimensional (2D) moving pressure distribution to represent the effect of the ship's hull at the free surface. The product of harmonic functions for the pressure distribution was proposed by Ertekin, Webster and Wehausen [14] and followed by Liu and Wu [10],

$$p_0(x,y,t)=p_m f(x,t)\,g(y), \tag{3}$$

where

$$f(x,t)=\begin{cases}1, & \text{if } 0\le |x+Ut|\le \dfrac{\alpha l}{2},\\ \cos^2\left[\dfrac{\pi}{l}\left(\dfrac{|x+Ut|-\alpha l/2}{1-\alpha}\right)\right], & \text{if } \dfrac{\alpha l}{2}<|x+Ut|<\dfrac{l}{2},\\ 0, & \text{if } |x+Ut|\ge \dfrac{l}{2},\end{cases}$$

$$g(y)=\begin{cases}1, & \text{if } 0\le |y|\le \dfrac{\beta b}{2},\\ \cos^2\left[\dfrac{\pi}{b}\left(\dfrac{|y|-\beta b/2}{1-\beta}\right)\right], & \text{if } \dfrac{\beta b}{2}<|y|<\dfrac{b}{2},\\ 0, & \text{if } |y|\ge \dfrac{b}{2}.\end{cases}$$

Here l and b are the length and the width of the pressure distribution in the x (centerline) and y (transverse) directions; α and β are dimensionless parameters, which are respectively equal to 0.7 and 0.4 according to Ertekin, Webster and Wehausen [14]. The pressure acts inside a rectangle of $l\times b$ dimensions, vanishing outside it. Li and Sclavounos [15] followed by other researchers [12, 16, 17, 18] also used equation (3) to model ship waves though with $\alpha=\beta=0$. For a 1D case, expression (3) is equivalent to equation (1).

Another 2D moving pressure distribution was suggested by Pedersen [19],

$$p_0(x,y,t)=\begin{cases}\dfrac{1}{2}p_m\left\{1+\cos\left[\pi\sqrt{\left(\dfrac{x+Ut}{l}\right)^2+\left(\dfrac{y}{b}\right)^2}\right]\right\}, & \text{for } \left(\dfrac{x+Ut}{l}\right)^2+\left(\dfrac{y}{b}\right)^2<1,\\ 0, & \text{for } \left(\dfrac{x+Ut}{l}\right)^2+\left(\dfrac{y}{b}\right)^2\ge 1.\end{cases} \tag{4}$$

In this case the pressure acts inside an ellipse with major and minor radii l and b. For a 1D case, expression (4) resembles equation (1) with the pressure distributed along a length $2l$ in the x direction. More recently 2D constant-pressure rectangular patches and non-uniform pressure distributions have been proposed to represent the effects of different hulls at a free surface. For instance, the following constant-pressure rectangular patch was proposed by Scullen and Tuck [20],

$$p_0(x,y,t)=\begin{cases}p_m, & \text{if } |x+Ut|<\dfrac{l}{2} \text{ and } |y|<\dfrac{b}{2},\\ 0, & \text{elsewhere.}\end{cases} \tag{5}$$

Figure 1 compares the pressure distribution functions proposed by [5, 14, 20] in 1D (see equations (1), (3) and (5)). Despite the step-function character along the sides of the patch, Scullen and Tuck [20] surprisingly found that no discontinuity was observed at the free surface, at least for subcritical flows. Far-field and local components at the leading and trailing edges cancel each other, such that the total free surface elevation is continuous everywhere. Accordingly Trinh, Chapman and Vanden-Broeck [21] show that the formation of waves near a ship is a necessary consequence of singularities in the ship's geometry. The use of "step-like" functions in the pressure distribution then seems to be a more realistic way to represent the resultant free-surface flow produced by certain hulls. According to Yih and Zhu [22], the geometric details of the hull modelled by a pressure disturbance are not relevant if the region under consideration is sufficiently far from the track of the ship; then the far-field wave pattern will be the same despite the pressure distribution used. However it seems quite reasonable to expect that at least the properties of the near-field waves would be directly influenced by the pressure distribution function used.

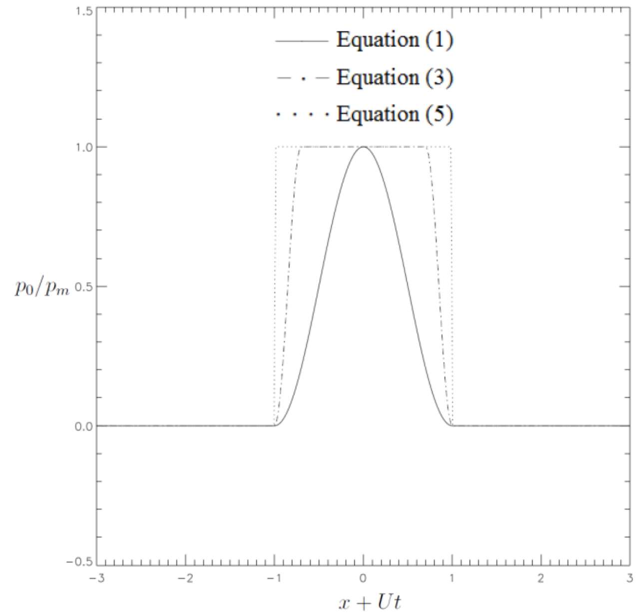


Figure 1: 1D pressure distribution functions (l = 2m).

Despite all the theoretical and experimental works on this field, none of them have studied in what extent the pressure disturbance model can be used to represent the geometric details of certain hulls and whether the ship waves would be equivalent. This paper aims to shed further light on the generation and propagation of free-surface waves due to pressure disturbance models in a channel. The fully nonlinear boundary-integral potential flow solver is adapted in order to include the 1D pressure distributions shown in Figure 1. "Bow" and "stern" waves are obtained and discussed. Our purpose is to determine numerically the wave field induced by pressure disturbance models and to present possible differences between their wave patterns.

2 UNSTEADY NONLINEAR MODEL

We consider a 2D incompressible, inviscid, fluid flow in the (x,z) plane. The *x* axis is located at the still-water surface, parallel to the ship's centerline, and the *z* axis points vertically upwards, such that the fluid occupies $-h<z<0$ when at rest, with a flat impermeable bottom at $z=-h$. Following Havelock's method, the ship's motion is represented by a 1D pressure disturbance function, which moves at a constant speed U in the *x* direction. The velocity field $\vec{u}(x,z,t)$ is given by the gradient of a velocity potential $\varphi(x,z,t)$, which satisfies Laplace's equation.

At the free surface, the kinematic and dynamic boundary conditions are applied

$$\frac{D\vec{r}}{Dt}=\nabla\varphi, \qquad \frac{D\varphi}{Dt}=\frac{1}{2}\left|\nabla\varphi\right|^2-gz-\frac{p}{\rho}, \tag{6}$$

where $\vec{r}=(x,z,t)$; $z=\eta(x,t)$ is the free-surface elevation; g and ρ are the gravitational acceleration and the fluid's density. The pressure exerted at the free surface is given by

$$p=p_0(x,t), \tag{7}$$

where $p_0(x,t)$ is the moving pressure distribution given by expressions (1), (3) or (5) (see Figure 1); surface tension is neglected.

As time evolves, pressure disturbance waves are formed and radiate away from the source. The desired "infinite" domain is then approximated by a finite extension in *x*, satisfying the condition $\nabla\varphi(\pm x_\infty,z,t)<\varepsilon$, valid for $-h\le z\le\eta(x,t)$ and $t>0$; ε is a specified small "precision" parameter; x_∞ is a finite distance, which is determined iteratively by the numerical scheme. To complete the model, a flat free surface is used as the initial condition of the fully nonlinear unsteady problem i.e. $\eta(x,0)=0$ and $\varphi(x,\eta,0)=0$.

The solution method is based on solving the principal value of an integral equation that arises from Cauchy's integral theorem of a complex variable function. The boundary-integral scheme described by Moreira and Peregrine [23] is modified in order to include the pressure disturbance models. For each time, φ is used to calculate the velocity $\nabla\varphi$ on the free surface. Then the surface profile and the velocity potential are stepped in time using a truncated Taylor series. Note that *p* and its corresponding Lagrangian time derivatives affect directly the calculation of the higher order time derivatives of φ, which are used in the time stepping process. Such stages are repeated until either the final time is reached, or the algorithm breaks down. This means that an insufficient number of points could be found if, for example, the wave approaches Stokes' limiting shape (120º at the crest of the calculated wave) or the crest overturns.

With the introduction of *p* and its Lagrangian derivatives, sawtooth numerical instabilities appear more frequently in the surface variables, which eventually cause the breakdown of computations. A low-order smoothing formula is employed. This proves to be efficient in the removal of unstable modes and also gives a slightly larger maximum steepness of surface waves rather than when employing a higher-order formula. A sufficient number of surface calculation points is then necessary in order to have any wave well resolved and thus not being smoothed away by the numerical scheme. All the computations presented in this paper have the same initial discretization and smoothing parameter. For more details of the numerical method see e.g. [24].

All variables are non-dimensionalized in such a way that $X = x/h$, $T = t\sqrt{g/h}$, $P = p/(\rho g h)$, and $Fr = U/\sqrt{gh}$; *X*, *T* and *P* are the dimensionless space, time and pressure; *Fr* is the Froude number, which is used to characterize the free-surface flow. Following the works [7,9,10], 2D unsteady nonlinear free-surface profiles are computed with the 1D pressure distributions illustrated in Figure 1 for various Froude numbers.

3 FULLY NONLINEAR RESULTS

Figures 2 and 3 show the pressure disturbance waves (with different vertical exaggerations) obtained with the pressure distributions illustrated in Figure 1 for $Fr = 0.1, 1.0, 1.5$ and $P_m = 0.10, 0.15$. Results are plotted in a reference frame moving with the left-going pressure distribution. The shaded area represents the region where the pressure models are applied. A good agreement is found between the present fully nonlinear unsteady computations and those obtained by [7]: runaway solitons are formed on the fore of the pressure distribution whereas "stern" waves are radiated away (see solid lines in Figure 2a); as the

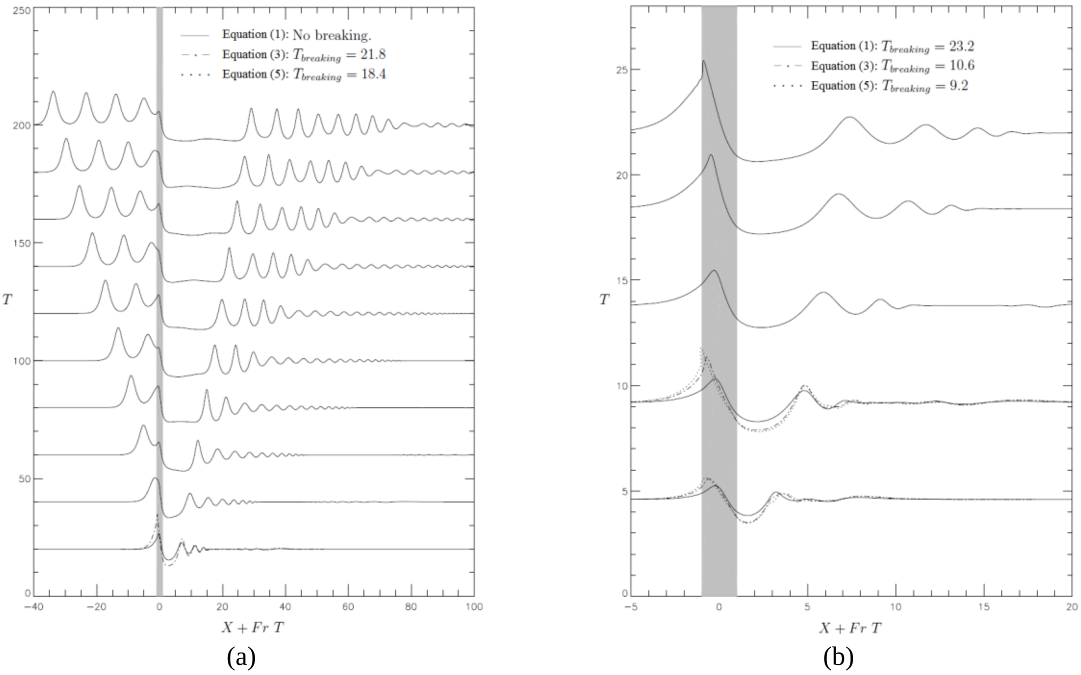
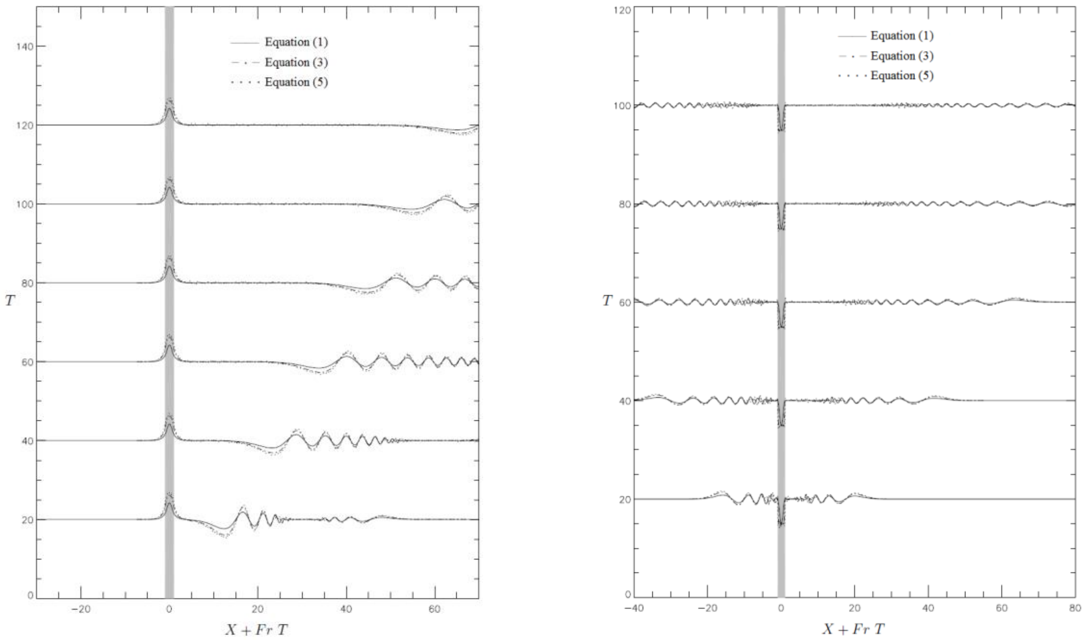


Figure 2: Pressure disturbance waves for $Fr = 1.0$: (a) $P_m = 0.1$, vertical exaggeration 30:1; (b) $P_m = 0.15$, vertical exaggeration 5:1. Pressure distributions are applied in the shaded region.



(a) (b)

Figure 3: Pressure disturbance waves for $P_m = 0.10$: (a) $Fr = 1.5$, vertical exaggeration 100:1; (b) $Fr = 0.1$, vertical exaggeration 50:1.

pressure amplitude P_m increases, nonlinear effects take over causing wave breaking (see solid lines in Figure 2b). For weaker pressure disturbances e.g. $P_m = 0.02$ (not shown), runaway solitons were also observed after long computational run-times. Regardless the magnitude of P_m, the "step-like" pressure functions proposed by [14] and [20] (expressions 3 and 5) cause an earlier breaking of the "bow" wave in the form of an upward breaker jet (see the dashed-dotted and dotted lines in Figure 2).

For supercritical flows (see Figure 3a), a single steady solitary wave is formed, travelling at the same speed of the pressure distribution, with no breaking being observed at least for $P_m = 0.10$. Again a good agreement is found with [7]. "Step-like" pressure distributions give steeper solitons and "stern" waves when compared to the free-surface profiles obtained with expression (1) (compare the solid, dashed-dotted and dotted lines in Figure 3a). The appearance of wave instabilities when using the pressure distribution (5) was also observed, which prohibited longer computational runs. For subcritical flows (see Figure 3b), a single steady trench is formed in the region where the pressure distributions are applied, with "bow" and "stern" waves being radiated away. Accordingly pressure patches given by expressions (3) and (5) produce wider depressions when compared to expression (1).

4 CONCLUSIONS

Pressure disturbance waves are computed via a fully nonlinear, unsteady, boundary integral formulation for various Froude numbers. Classical runaway and single steady solitons are formed for critical and supercritical flows when applying the moving pressure distribution (1), while "stern" waves propagate away, agreeing with [7]. The introduction of "step-like" pressure functions, as suggested [14] and [20] (expressions 3 and 5), gives different responses to the free-surface flows, with upward breaker jets on the fore and steeper "stern" waves on the rear of the pressure patches. The use of "step-like" functions to supercritical flows proves to be more numerically unstable for the same discretization and smoothing parameter. For subcritical flows, "bow" and "stern" waves are radiated away while different trenches are formed in the region where the pressure distributions are applied. Nonlinear results show that wave patterns are significantly affected by moving pressure distributions. Further investigations are necessary to conclude whether the near and/or far-field ship waves would be equivalent to the pressure disturbance waves here presented. In this sense the introduction in our model of an impermeable 2D surface to represent the ship's hull is under development.

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