SHEET-FLOW SEDIMENT TRANSPORT FORMULAE VERIFICATION UNDER A NEW SET OF EXPERIMENTS

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Abstract

This paper intends to increase the understanding of cross-shore sediment transport processes under combined nonlinear waves plus currents. The recent whole set of Dong et al. (2013) is used for this purpose. The experiments were performed in the Oscillating Flow Tunnel at the University of Tokyo, under sheet flow conditions. The flow conditions contemplate different wave periods, different degrees of velocity- and acceleration-skewness and different median grain sizes. The measured net transport rates are predicted with four sediment transport models, assessing their range and limits of applicability. The results evidence two important mechanisms which play an important role and affect the correct prediction of sediment transport modeling, namely, skewed bed shear stresses and unsteady phase-lag effects between the velocity and sediment concentration.

1. Introduction

The seashore and nearshore morphological changes, resulting from erosion and accretion processes, either in the long-term and the short-term manifestations, have been a constant societal concern. Particularly, the technical and scientific community has addressed the analysis of the near-bed hydrodynamic and sediment transport processes, which are fundamental to understand and predict the bed-level changes. The actions of coastal waves and currents, and their interaction, are the main responsible for coastal sediment transport. Various researchers have performed several laboratory experiments aiming at analyzing and quantifying the role of wave-form asymmetries (around a vertical and a horizontal axis) and wave-current interaction, contributing to a better description of the near-bed sediment dynamics (e.g., Watanabe and Sato, 2004; Silva *et al*., 2011; Dong *et al*., 2013).

The recognition of the importance of those asymmetries has motivated the development of empirical and semi-empirical sediment transport models that include the non-linear effects of the oscillatory flow. For example, some authors developed sediment transport models from the knowledge (or prediction) of the bed shear stresses, estimated from flow velocity and acceleration time series or from conspicuous points of these properties in the corresponding time series (e.g., Drake and Calantoni, 2001; Hoefel and Elgar, 2003; Nielsen, 2006; Silva *et al*., 2006; Abreu *et al*., 2013).

In this paper, the whole set of Dong *et al*. (2013) experiments is analyzed, in order to deduce on the role of the underlying sediment transport mechanisms. Moreover, the measured net sediment transport rates are predicted using different sediment transport models, assessing their range and limits of applicability. The application of different sediment transport models to this recent experimental data strongly contributes to the validation and further development of these models.

2. Dong *et al*. (2013) experiments

Dong *et al*. (2013) experiments were performed in the Oscillating Flow Tunnel at the University of Tokyo (TOFT), Japan. The experimental facility consists of a loop-shaped closed conduit controlled by a piston capable of simulating velocity- and acceleration-skewed oscillatory flows (Figure 1). The TOFT is fitted with a recirculation system that enables the generation of a mean

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current U_0 that is superimposed on the oscillatory flow. The horizontal test section is rectangular (5.7 m long, 0.076 m wide and 0.233 m height) and the central part of the test section can be filled with a 0.04 m thick sand bed.

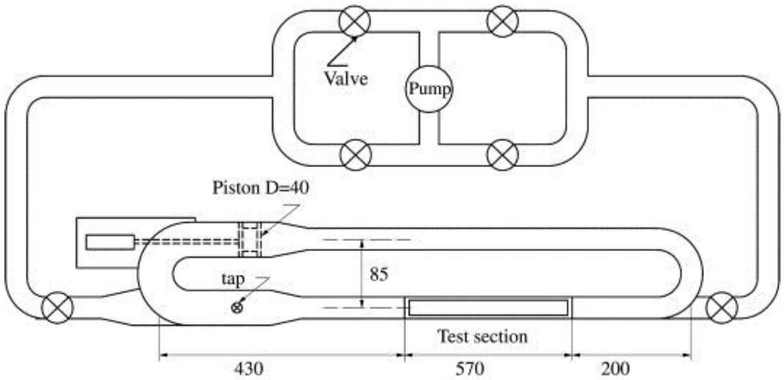

Figure 1. Schematic diagram of the Oscillatory Flow Tunnel at the University of Tokyo (adapted from Dong. *et al*., 2013) (dimensions in cm).

The collected data concerns different velocity- and acceleration-skewed oscillatory flows with and without the presence of collinear currents. This experimental data complements previous experiments (e.g., Watanabe and Sato, 2004; Silva *et al*., 2011), introducing 53 new hydrodynamic conditions obtained in the sheet flow regime. Therefore, the collected data is attractive for the validation of sediment transport formulae since it uses three different median grain sizes (d_{50}=0.16, 0.2 e 0.3 mm), with oscillating periods ranging from 3 to 7 s, and because part of the velocity- and acceleration-skewed oscillatory flows also includes the presence of strong currents, opposing the (implied) direction of the wave propagation, similar to undertow currents.

Part of the experiments consider, for the description of the free-stream horizontal velocity, $u(t)$, the oscillatory motion described by Abreu *et al*. (2010). The formula contains four free parameters: two related to the orbital velocity amplitude U_w (=(u_{max}-u_{min})/2) and wave period T and two related to the velocity and acceleration skewnesses, namely, an index of skewness, r, and a waveform parameter, ϕ:

$$u(t) = U_w f \frac{\sin(\omega t) + \frac{r \sin\phi}{1+f}}{1 - r\cos(\omega t + \phi)} \quad . \tag{1}$$

Here, ω is the angular frequency (ω=2π/T) and f is a dimensionless factor that allows the velocity amplitude to be equal to U_w ($f = \sqrt{1-r^2}$).

An example of the application of this expression can be seen in Figure 1 for the test case W11. This test corresponds to a velocity- and acceleration-skewed oscillatory flow (r= 0.587 and ϕ=-0.684 rad). It should be mentioned that, in order to characterize the effects of the velocity and acceleration skewness of the experiments, Dong *et al*. (2013) set the flow conditions in terms of the velocity skewness coefficient (R=u_{max}/(u_{max}-u_{min})) and the "forward leaning" index (β_c=1-T_{pc}/T_c), where T_{pc} is the time interval measured from the zero up-cross point to u_{max} (see Figure 1). Since they use Eq. (1) to describe the velocity time series, it is important to convert both parameters into r and ϕ. Following Abreu *et al*. (2010), each combination of (R, α) can be transformed in pairs of (r, ϕ), where α represents the "acceleration skewness parameter" (α=2T_{pc}/T). The parameter α can be directly related to β_c and R:

$$\alpha = 2\left(1-\beta_c\right)\frac{T_c}{T} \ , \tag{2}$$

and, according to Malarkey and Davies (2012), T_c can be defined as

$$T_c = \left(\pi - 2\arcsin\left(2R-1\right)\right)\frac{T}{2\pi} \ . \tag{3}$$

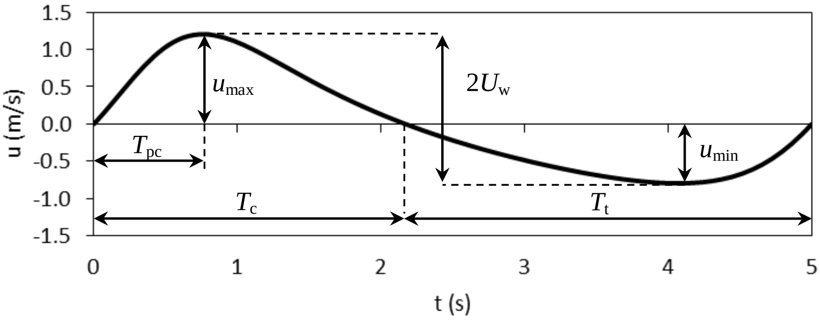

Figure 1. Free-stream velocity for test case W11 (U_w=1 m/s, U_0=0, T=5 s, R=0.60 and β_c=0.65).

Figure 2 shows the measured net transport rates for the three different sand sizes as a function of the root mean square velocity, U_{rms}. Only the tests with d_{50}=0.2 mm (Figure 2c) contemplate opposing currents (U_0≈-0.3 and 0.5 m/s), leading to negative values (offshore transport) of the net sediment transport rates q_s.

For the analysis of the fine sand cases (d_{50}=0.16 mm), Figure 2 is divided in two separate panels. Panel a) contains the pure acceleration-skewed flows (R=0.5 and β_c≠0.5) and panel b) contains velocity-skewed oscillatory flows (R≠0.5). Figure 2a evidences that acceleration skewness leads to positive values of q_s and its magnitudes increase with U_{rms}. This corroborates van der A *et al.* (2010) findings using an almost similar fine sand (d_{50}=0.15 mm). However, velocity-skewed flows (panel b), particularly for decreasing flow periods (T=3 and T=5 s), present a reverse trend where the values of q_s become negative. In such cases, significant suspension of fine sediments occur and unsteady phase-lag effects between the velocity and sediment concentration play an important role (e.g., Dohmen-Janssen *et al.*, 2002). Indeed, for small periods, Dong *et al.* (2013) observed important fractions of sand remaining in suspension after flow reversal, promoting offshore sand transport.

Focusing on d_{50}=0.2 mm (Figure 2c), it is mentioned that all the tests consider T=3 s. The increase in U_0 from -0.3 m/s to -0.5 m/s, accentuates the negative values of the transport q_s. However, the increase of the acceleration skewness from β_c=0.55 to 0.68 results in the decrease of the offshore transport. Moreover, the increase of velocity skewness from R=0.6 to 0.7 also leads to a reduction in magnitude of q_s. Therefore, the introduction of nonlinearities balances the negative values of q_s associated to the opposing currents. As suggested by Dong *et al.* (2013) the mechanism leading to these reductions can be attributed to the skewed bed shear stresses.

The coarser sand d_{50}=0.3 mm (Figure 2d) presents positive values of q_s, i.e., onshore transport. Here, the phase-lag effects also play their role. For instance, for the pure acceleration-skewed flow (R=0.5 and β_c =0.65), the positive transport is enhanced with the decrease of the flow period from T=5 to 3 s. One observes the same trend for the other velocity- and acceleration-skewed flows if the same hydrodynamic conditions are compared, i.e., the same R and β_c for T=3 and 5 s. Concerning the test cases with T=5 s, it is perceivable that the values of q_s increase with the growth of R and β_c. For the same U_{rms} (≈0.8 m/s), it is seen that the pure acceleration-skewed flow (R=0.5 and β_c =0.65) presents lower values of q_s compared to the pure velocity-skewed flow (R=0.6 and β_c =0.5), which in turn also exhibits lower values than the velocity- and acceleration-skewed flow (R=0.6 and β_c =0.65). The reasoning behind these observations is attributed to the skewed bed shear stresses that become an important mechanism for coarser sand grains (Dong *et al.*, 2013).

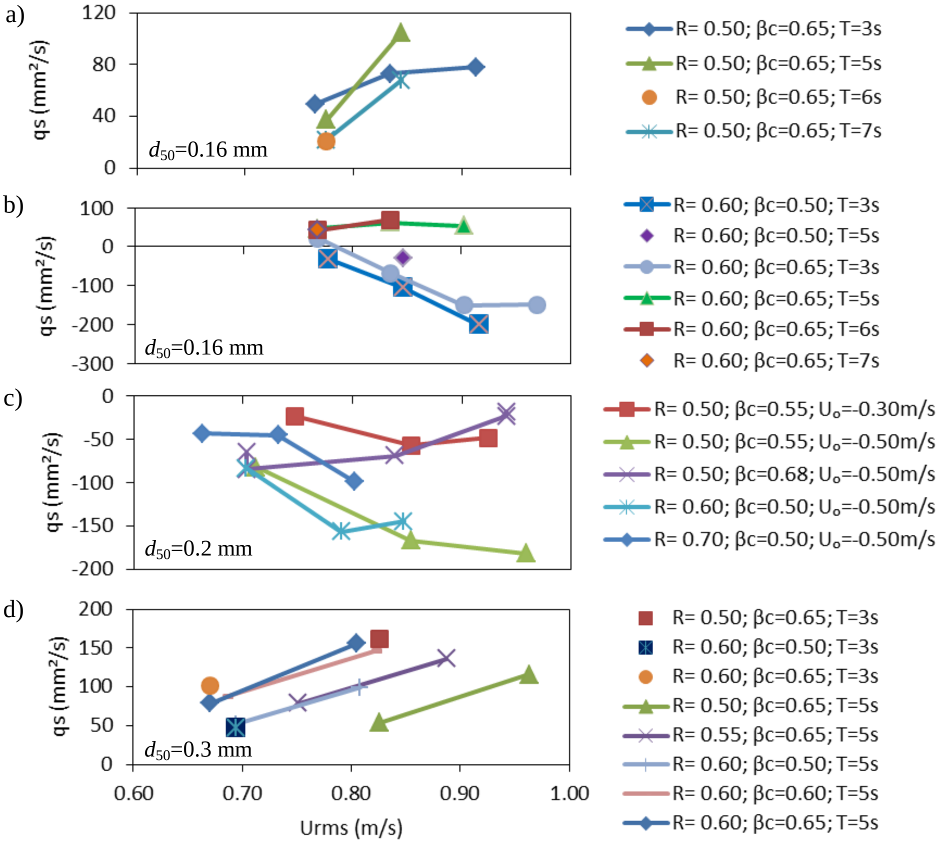

Figure 2. Dong *et al*. (2013) measured transport rates for: (a) d_{50}=0.16 mm with R=0.50, (b) d_{50}=0.16 mm with R=0.60, (c) d_{50}=0.2 mm and (c) d_{50}=0.3 mm.

3. Sediment Transport Models

The existing literature presents different formulations that claim to be able to estimate sediment transport rates under velocity- and acceleration-skewed oscillatory flows in the presence or absence of collinear currents. Here, some empirical sediment transport formulae were selected and its ability to predict Dong *et al.*'s net sediment transport rates is assessed. These formulations use different model concepts and are briefly described in the following. Further details can be found in the original papers.

Hoefel and Elgar (2003) – HE03

Following Drake and Calantoni (2001), Hoefel and Elgar (2003) extended the classical energetics-type sediment transport model of Bailard (1981), accounting for acceleration effects, random waves and mean currents. The modification to the classical formulation is achieved through the inclusion of an additional term, q_{ba}, in the bedload q_b component of the sediment transport, representing the effects of the fluid flow acceleration, a:

$$q_{ba} = \begin{cases} K_a \left(a_{spike} - a_{cr} \times \left(a_{spike} / \left| a_{spike} \right| \right) \right) & , \left| a_{spike} \right| > a_{cr} \\ 0 & , \left| a_{spike} \right| < a_{cr} \end{cases} . \tag{4}$$

The dimensional descriptor of the acceleration skewness, a_{skipe} (= <a^3>/<a^2>), and its critical threshold, a_{cr}, enhancing acceleration effects, were introduced by Drake and Calantoni (2001). Through the comparison of model results with field observations of a sandbar migration (Duck94 field data), Hoefel and Elgar (2003) proposed the optimal values $K_a = 1.4\times10^{-4}$ ms and $a_{cr} = 0.2$ m/s^2. In this work, the suspended load, q_{ss}, was also computed according to Bailard (1981) in order to give the total transport q_s (= $q_b + q_{ss}$).

Silva et al. (2006) – S06

Following the work of Dibajnia and Watanabe (1992), Silva *et al*. (2006) developed a semi-unsteady, practical model, to predict the total sediment transport rates in wave or combined wave-current flows. The predicted sediment transport rates, q_s, are computed from:

$$q_s = \upsilon \, |\Gamma|^{\gamma} \frac{\Gamma}{|\Gamma|} \sqrt{(s-1) g d_{50}^{\,3}} \;, \tag{5}$$

with

$$\Gamma = \frac{u_c T_c (\Omega_c^3 + \Omega_t'^3) - u_t T_t (\Omega_t^3 + \Omega_c'^3)}{2(u_c T_c + u_t T_t)} \;. \tag{6}$$

In these equations $s = \rho_s / \rho$, where ρ and ρ_s are the water and sediment density, respectively, g is the gravitational acceleration, T_c and T_t are the time duration of the positive and negative half cycle of the near bed velocity, respectively, with equivalent velocities u_c and u_t (the subscript c stands for crest and t for trough). The quantities Ω_i and Ω_i' ($i = c, t$) represent the amount of sediment that is entrained, transported and settled in the i half cycle, and the amount of sediment still in suspension from the i half cycle that will be transported in the next half cycle, respectively. The values of Ω_i are computed from the bed shear stress. The non-steady processes are taken into account through the exchange of sediment fluxes between the two half cycles (Ω_i' quantities). The parameters υ and γ are two empirical constants. Their values were determined by fitting the numerical solutions to a large data set, $\upsilon = 3.2$ and $\gamma = 0.55$.

Nielsen (2006) – N06; Abreu et al. – A13

Nielsen (2006) proposed a quasi-steady bedload formula to estimate sediment transport rates, which is a modified version of the Meyer-Peter Müller (1948) bedload-type formula:

$$q_s = 12\sqrt{(s-1) g d^3}\,\left(\theta(t) - \theta_{cr}\right)\sqrt{\theta(t)}\, u_* / |u_*| \;,\; \theta > \theta_{cr} \,. \tag{7}$$

The Shields parameter θ is defined by $\theta(t) = \tau(t) / \left(\rho(\rho_s/\rho - 1) g d_{50}\right)$ and τ is the instantaneous bottom shear stress ($\tau(t) = \rho u_*(t)|u_*(t)|$) computed as a function of the shear velocity, u_* (Nielsen, 1992, 2002):

$$u_*(t) = \sqrt{\frac{f_w}{2}} \left(\cos(\varphi)\; u(t) + \frac{\sin(\varphi)}{\omega} \frac{du(t)}{dt} \right) . \tag{8}$$

The angle φ represents a calibrating parameter that, in the case of a single harmonic, roughly corresponds to the phase-lead of the bed shear stress over the free-stream velocity. The parameter establishes the balance between drag forces and pressure gradients associated with the cosine and sine of $\varphi \in [0^o, 90^o]$, respectively. An optimal value of $\varphi = 51^o$ was proposed, which optimizes Nielsen's net transport predictions for the data of Watanabe and Sato (2004). To compute the wave friction factor, f_w, Nielsen's (1992) formulation is applied.

Recently, Abreu *et al*. (2013) extended the work of Nielsen (1992, 2002), proposing a new formulation to predict the bed shear stress under skewed/asymmetric oscillatory flows. The shear velocity, u_*, incorporates the nonlinearity of the oscillatory flow through the inclusion of r and ϕ:

$$u_*(t) = \sqrt{\frac{f_w}{2}}\left(\cos(\varphi)\ u(t) + \frac{\sin(\varphi)}{\omega}\left[\frac{du(t)}{dt} - S(t,\phi,r)\right]\right), \tag{9}$$

with

$$S(t,\phi,r) = \omega \cdot f \cdot U_w \frac{r\left[-(-1+f)\cos\phi - 2r\cos(\omega t) + (1+f)\cos(2\omega t+\phi)\right]}{2(1+f)\left[-1+r\cos(\omega t+\phi)\right]^2}. \tag{10}$$

Eq. (9) can be expressed as

$$u_*(t) = \sqrt{\frac{f_w}{2}}\left(\cos(\varphi)\ u(t) - \sin(\varphi)\mathscr{H}(u(t))\right), \tag{11}$$

where $\mathscr{H}(u(t))$ is the Hilbert transform of $u(t)$.

The advantage to rewrite Eq. (9) into Eq. (11) allows the method to be applied to any $u(t)$ for which a Hilbert transform can be defined.

Whereas Nielsen (2006) recommends the use of a phase φ = 51° with a constant bed roughness of $k_s = 2.5d_{50}$ for sediment transport rate estimates, Abreu *et al*. (2013) show that the new bed shear stress predictor improve the results using φ = 51° and $k_s = 15d_{50}$.

4. Results

Figure 3 presents the net transport rate predictions obtained using the models described above. In order to complement the analysis, Table 1 lists the performance of the models in terms of the root mean square error (RMSE) and the determination coefficient (cor^2). In general, a good result is perceivable for the two coarser grains (d_{50}=0.2 and 0.3 mm). However, for the finest sediment (d_{50}=0.16 mm), some experimental conditions indicate predicted transport rate directions opposite to those measured. That happens in the cases where the oscillating period is small (T=3 s) and the unsteady phase-lag effects between the velocity and sediment concentration are significant. In such cases, the semi-unsteady model S06 produces better results since the model involves parameters that quantify the amount of sediment mobilized from the bed during each half-wave cycle. The other formulations do not contemplate such mechanism and are not able to predict the correct direction of the sediment transport. For example, HE03 formulation does not take into account the influence of the flow period because this conceptual model only accounts for the near-bed statistical moments of the velocity and acceleration. A13 and N06 formulations accounts for the flow period in the computation of the wave friction factor. However, one notes that A13 and N06 are bedload formulas and if suspended load becomes significant and, in addition, unsteady phase-lag effects are important, as it is expected for fine sands, it is anticipated that both models fail the predictions. This can be confirmed in Table 1 since, for d_{50}=0.16 mm, S06 provides the lowest RMSE and the highest determination coefficient, whereas HE03 presents the worst results. For the two coarser grains (d_{50}=0.2 and 0.3 mm) A13 give the best overall results, suggesting that the skewed bed shear stresses are reasonably captured by the new parameterization.

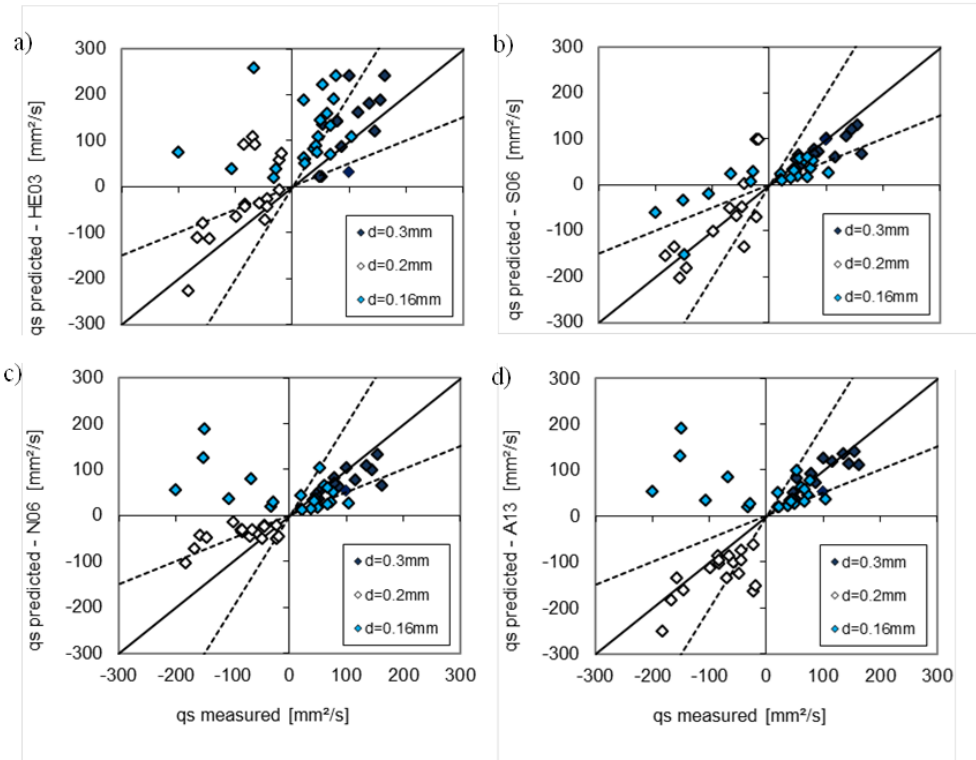

Figure 3. Comparison between predicted and measured net transport rates: (a) HE03; (b) S06; (c) N06 and (d) A13. Envelope lines represent transports within a factor of ½ and 2.

Table 1. Performance of the models in terms of the root mean square error (RMSE) and the determination coefficient (r^2).

	HE03		S06		N06		A13	
	RMSE	*cor*²	RMSE	*cor*²	RMSE	*cor*²	RMSE	*cor*²
d_{50}=0.16 mm	2,46	0.09	0,65	0.64	1,45	0.28	1,45	0.23
d_{50}=0.2 mm	0,86	0.48	2,34	0.06	0,60	0.35	0,64	0.39
d_{50}=0.3 mm	0,57	0.42	0,33	0.46	0,34	0.51	0,23	0.61
All sediments	1,54	0.16	1,43	0.36	0,92	0.24	0,92	0.37

In order to assess the influence of some parameters like d_{50}, R, β_c and T using A13, Figure 4 presents a more detailed comparison between predicted and measured net transport rates in terms of these parameters. The results concern only A13 since this model provided the best overall results. The first row corresponds to the finest sediment (d_{50}=0.16 mm) and evidences that the pure acceleration-skewed flows (i.e., R=0.5 and $\beta_c \neq 0.5$) lead to positive sediment transports. The predictions present a linear trend but they are under-estimated by a factor of 2. The same tendency was observed by Abreu *et al*. (2011) for van der A *et al*. (2010) data with d_{50}=0.15 mm. The results could be improved if a higher roughness was applied. As mentioned before, it is not surprising that the q_s values of the finer sand are under-estimated. The estimations do not account for suspended transport loads and the measured transport rates are subject to significant phase lag effects, which are not being captured in the present quasi-steady approach. It is also perceivable that the phase lag effects are clearly reduced for the larger periods where a clear linear trend is observed.

The second row of Figure 4 corresponds to d_{50}=0.2 mm (T=3 s). Though most of the points lie within a factor of 2, one observes two clear outliers where the predictions are severely overestimated. These predictions correspond to pure acceleration-skewed flow cases with β_c=0.68. One notes that, in total, the data contains 5 points under such conditions (R=0.5 and β_c=0.68), but these two outliers are the ones with larger velocity amplitude, U_w (=1.3 m/s). Again, the differences might be explained by a significant suspension of sediments combined with unsteady phase-lag effects due to the small oscillating period (T=3 s).

This agrees with Ruessink *et al*. (2009) outcomes about phase-lag effects being particularly important in fine-medium sands (d_{50}<≈ 0.25 mm). According to the authors, phase-lag effects become more pronounced with an increase in wave nonlinearity and in velocity amplitude, and with a decrease in wave period. Therefore, for the coarser grain (d_{50}=0.3 mm), phase-lag effects are expected to be weakened. Indeed, the third row of the figure shows that all the points lie within a factor of 2, confirming that nearshore sediment transport equations based on the instantaneous bed shear stress may produce good results. This is hereby corroborated by Table 1 that lists a low RMSE value and a good determination coefficient for A13. Again the skewed bed shear stresses seem to be reasonably captured by the new parameterization.

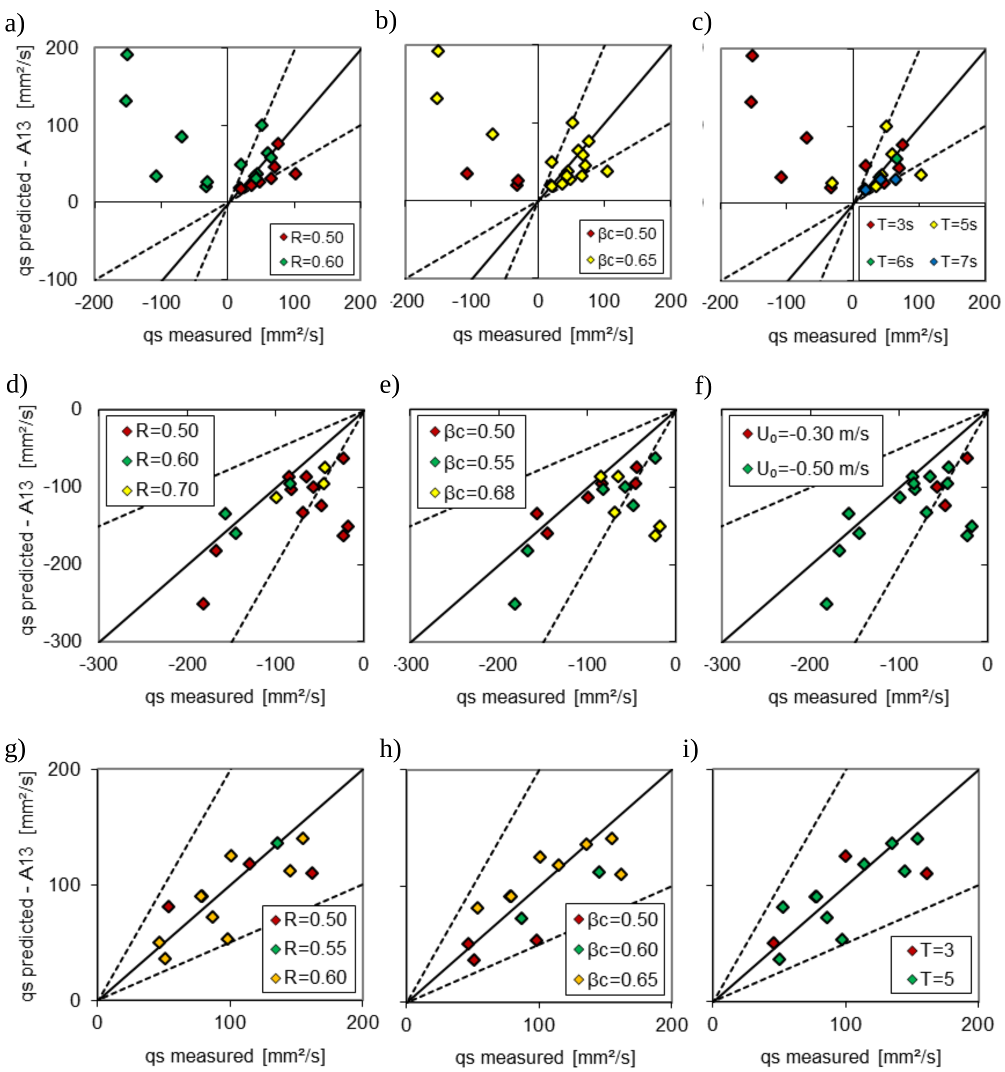

Figure 4. Comparison between predicted and measured net transport rates using A13 for:d_{50}=0.16 mm (first row); d_{50}=0.2 mm (second row) and d_{50}=0.30 mm (third row).

5. Conclusion

In this paper the new dataset of net sediment transport rates obtained in the Oscillating Flow Tunnel at the University of Tokyo (TOFT) under sheet flow conditions is analyzed. The data is compared with the predictions from four simple sediment transport models. The experiments performed in the tunnel contemplate different wave periods and different conditions of velocity- and acceleration-skewness using three different median grain sizes. Also, superimposed net currents opposing the wave direction were tested.

The new dataset evidences two mechanisms that play a key role in the sediment transport, namely, skewed bed shear stresses and unsteady phase-lag effects between the velocity and sediment concentration. It is seen that phase lag effects are particularly important in fine-medium sands and its effects become more pronounced with an increase in wave nonlinearity and in velocity amplitude, and with a decrease in wave period.

The accuracy of four practical sediment transport models is tested against the new available data. The comparison shows that the best overall results are obtained using Abreu *et al*.'s (2013) new bed shear stress parameterization in a modified version of the Meyer-Peter Müller (1948) bedload-type formula (Nielsen, 2006). However, the predictions fail when unsteady phase-lag effects become dominant, i.e., for fine sands and small oscillating periods. In such cases, the semi-unsteady model of Silva *et al*. (2006) produces better results since the model involves parameters that quantify the amount of sediment mobilized from the bed during each half-wave cycle.

Finally, it should be noted that all the knowledge acquired by the new data set and from the ability of the practical transport models to predict sediment transport rates, accounting for the effect of nonlinear waves, provides further insights into morphological modelling in engineering applications.

Acknowledgements

Part of this work is funded by the Research Projects PTDC/CTE-GIX/111230/2009 (EROS) and PTDC/ECM/103801/2008 (3D-MOWADI), supported by the Portuguese Foundation for Science and Technology (FCT).

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