Long-term geodetic displacements. Evaluating the quality of measured displacements

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SUMMARY

The displacements measured by geodetic methods have long been used by civil engineers to the structural safety control. For a normal structural behaviour it is possible to establish a simple model for describing the relations between the loads (gravity, temperature, hydrostatic pressure, etc.) and the structural response (e.g. displacements). Using data from many geodetic campaigns – displacements, campaigns dates, age of the structure, water level (in the case of dams), etc. – it is possible to create simple statistical/empirical models (linear regression models for effects separation) for supporting the analysis of the observed behaviour. Once the model is established it can be used to predict the displacements.

The quality control of the displacements measured by geodetic methods should be based on the analysis of the compatibility of observations. Under some conditions the quality control cannot be performed due to the lack of redundancy of observations. In this case information about the expected displacements can help us to decide about the quality of the measurements. This paper presents the regression models usually considered in common structures (dams, buildings) to analyze the observed behaviour. A simple method using Excel functions to perform the regression and to build the functions for displacements prediction is presented. Some results from the application of this method on some large structures, two dams and a pavilion with a glulam structure, will be also presented.

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1. INTRODUCTION

Generally the displacements of a structure are mainly due to load variations. However, for long monitoring periods the time effects like creep and material deterioration should also be took into account. A structure has a normal behaviour when its response to the same loads remains unchanged. This fact enables the development of simple statistical/empirical models that can be applied, for instance, to analyse the displacements of a structure.

For the development of reliable models is useful to have data from a large number of campaigns, along with information of the loads at the date of the campaigns. For instance, in case of dams, reservoir water level and temperature can be related with displacements as well as ageing effects. Once establish, the model can be applied to predict values of displacements. Generally, the model displacements can be used to evaluate the quality of the results obtained in a campaign. However this method should only be applied when, for lack of observations, there is not enough redundancy to apply the usual quality control based on residuals analysis.

In this paper is presented a method to calculate simple statistical/empirical models based on regression methods. As examples were chosen three structures that are subject to regular monitoring: a concrete dam, an embankment dam and a roof structure of a pavilion. To become easy to anyone to use the methodology presented, the examples were build using Excel.

2. LONG-TERM GEODETIC DISPLACEMENTS

Quite often, monitoring follows large periods of the life of a structure. In Portugal, large dams are regularly monitored, being that, usually, there is a geodetic surveying campaign every year. This means that there is a large amount of data on the behaviour of these structures, data that can contribute for a better analysis of the latest campaigns.

In Figures 1 and 2 are presented vertical displacements of some structures, chosen because it was easy to one notice a regular behaviour. These structures are an embankment dam and a glulam (glued laminated timber) roof structure of a pavilion. The displacements show that there is a factor that is becoming less important with time since its effect is decreasing. In the case of the pavilion there is also a factor that has an annual effect, clearly evident because there are two campaigns a year, one during winter other during the summer.

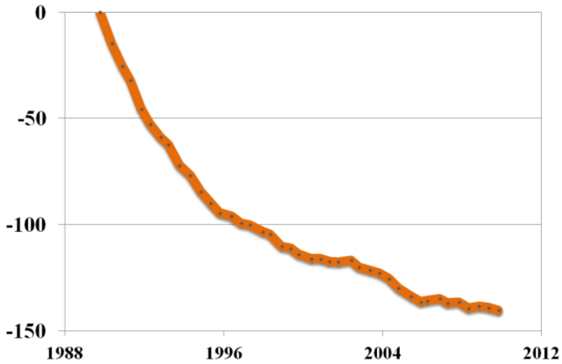


Figure 1 – Vertical displacements (in mm) of an embankment dam.

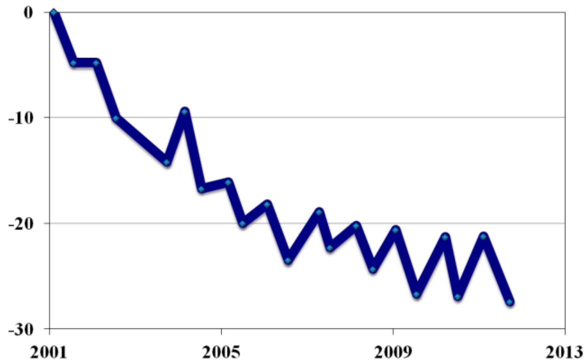


Figure 2 – Vertical displacements (in mm) of a glulam roof structure of a pavilion.

Looking to the behaviour of these structures is expectable that, taking into account these historic displacements, it will be possible to predict the displacements of these structures during the following years, and compare these with the displacements measured.

In the next chapters is presented a method to build simple statistical/empirical models. This method uses data like the age of the structure, the date of the observation campaign, the reservoir water level (in the case of dams), and the values of the displacements measured.

3. REGRESSION MODEL

A regression model may be used to establish the relation between the variables measured and loads on the structure:

$$y(x_1, x_2,.., x_k) = a_0 + a_1\, f(x_1) + b_2\, f(x_2) + \ldots + a_k\, f(x_k) \qquad (1)$$

In this equation y, often called response variable, stands for the variable measured (displacement component, inclination, etc.), x, the predictor variable, stands for the loads on the structure (temperature, reservoir water level, age, etc). The coefficients a_0, …, a_k are the unknowns, and are estimated by solving a system of equations (2). To solve this system is necessary a minimum number of equations, which is determined by the number of coefficients of (1): it is necessary a minimum of k+1 equations.

$$\begin{aligned} y_1 &= a_0 + a_1\, f_1(x_{11}) + a_2\, f_2(x_{21}) + \ldots + a_k\, f_k(x_{k1}) \\ y_2 &= a_0 + a_1\, f_1(x_{12}) + a_2\, f_2(x_{22}) + \ldots + a_k\, f_k(x_{k2}) \\ &\ldots \\ y_n &= a_0 + a_1\, f_1(x_{1n}) + a_2\, f_2(x_{2n}) + \ldots + a_k\, f_k(x_{kn}) \end{aligned} \qquad (2)$$

Each equation establishes a relation between the variable measured at t, the date of a campaign, for instance, and values related with the loads (at t).

Using vector notation, the system (2) is expressed by

$$Y = A\ B \qquad (3)$$

being Y, A and B arrays: Y an (1,n) array, A an (1, k+1) array, and B an (n,k+1) array.

$$Y = [y_1 \quad y_2 \quad \cdots \quad y_n]; \qquad A = [a_0 \;\; a_1 \quad a_2 \quad \cdots \quad a_k\,];$$

$$B = \begin{bmatrix} 1 & f_1(x_{11}) & \cdots & f_k(x_{k1}) \\ 1 & f_1(x_{12}) & \cdots & f_k(x_{k2}) \\ 1 & \vdots & \ddots & \ddots \\ 1 & f_1(x_{1n}) & \cdots & f_k(x_{kn}) \end{bmatrix}. \qquad (4)$$

To solve the system of equations (3), called regression model, is usually applied the least squares method:

$$A = Y\,B\,\left(B^T B\right)^{-1} = Y\,B\,C \qquad (5)$$

where B^T represents the transpose of the array B and $\left(B\,B^T\right)^{-1}$ is the inverse of the product of B by its transpose. The system must be overdetermined (the number of equations must be larger than the number of unknowns) to give a reliable solution. Once the coefficients are estimated it is possible to predict the displacements,$\hat{y}$, given values to the loads (x1, x2,…):

The difference between the measured value (y) and the predicted value ($\hat{y}$) is an error, called residue:

$$v_i = y_i - \hat{y}_i \,. \qquad (6) \qquad\qquad \text{Vector notation:} \qquad V = Y - \hat{Y} \qquad (7)$$

This variable is helps to evaluate the goodness of fit, or helps to find the answers to questions like "are all the terms of (1), used in the model, needed?". The following procedures can be applied whenever random errors are normally distributed.

Let

$SSE = V^T V = \sum_{i=1}^{m} v_i^2$	sum of squared errors	(8)
$s_e = \sqrt{V^T V / m\text{-}n}$		(9)
$SSY = Y^T Y$		(10)
$\bar{y} = \dfrac{\sum y}{m}$	average of the displacements	(11)
$SS0 = n\,\bar{y}$		(12)
$SST = SSY - SS0$	total sum of the squares	(13)
$SSR = SST - SSE$		(14)
$s_{ai} = s_e\sqrt{c_{ii}}$, ($c_{ii:}$ element i,i of the array C)	estimated standard deviation of the regression parameters.	(15)

To evaluate the goodness of fit, one can use the coefficient of determination (R^2) that varies between 0 and 1: when $R^2 = 1$ the fit is perfect; when R^2=0 the model doesn't fit to the data

$$R^2 = \frac{SSR}{SST}\,. \qquad (16)$$

To evaluate when a term of the model is necessary (i.e., contributes to the model), one can use the statistic T_i, which has a t-student distribution with m-n degrees of freedom and a significance level of α/2:

$$T_i = \frac{\hat{a}_i}{s_{ai}} \in t\left(1 - {}^{\alpha}\!/_{2}\,;\; m - n\right) \qquad (17)$$

being $\hat{a}_i$ the value of the estimated coefficient and s_{ai} the estimated standard deviation of this coefficient. When $T_i > t$ the term i doesn't contribute to the model.

It is important to notice that, although tests are important to evaluate the quality of the model, sometimes is no less important to plot charts, like the ones that will be presented in the examples of this paper.

4. EVALUATING DISPLACEMENTS

Assigning values to the predictor variables and knowing the values of the coefficients, makes it possible to predict values of y. Sometimes it might be important to evaluate if the result of an observation campaign (for instance, a displacement measured) is "equal" to the one predicted.

Let y_{obs} be the displacement observed and $\hat{y}$ the displacement predicted. After the evaluation of the coefficients (to create the array A) and building array B based on values related with the loads is easy to determine the value for the predicted variable $\hat{y}$. To accept the equality of the observed and predicted displacements, the difference between y_{obs} and $\hat{y}$ should lie within an interval: the predicting interval. To calculate the upper and lower limits of this interval, the following expression is used:

$$y_{obs} \epsilon \left[\hat{y} - s_e\, t_{1-\alpha/2,m-n}\ \sqrt{X\,(B^T B)^{-1}\,X};\ \hat{y} + s_e\, t_{1-\alpha/2,m-n}\ \sqrt{X\,(B^T B)^{-1}\,X}\right] \qquad (18)$$

where $t_{1-\alpha/2,m-n}$ is the 1-α/2 quantile of the t-student distribution with (m–n) degrees of freedom.

One can make another kind of study which is to evaluate the value of α that turns

$$|y_{obs} - \hat{y}| = s_e\, t_{1-\alpha/2,m-n}\ \sqrt{X\,(B^T B)^{-1}\,X} \qquad (19)$$

Low values of α (large confidence intervals $1\text{-}\alpha/2$, near 0.99 for instance) mean that y_{obs} is far from the predicted value.

5. LOADS ON THE STRUCTURE

To build the array B or, by other words, to choose the functions $f_1,\ldots,f_k$ one need to know the characteristics of the system, namely the loads that most influence the structural behaviour like reservoir water level and temperature, or material creep parameters.

The relations between the variable measured and the load values are expressed by empirical expressions, some of which are presented in the next paragraphs (Oliveira, 2006). Many of these are time dependent. Usually the time is measured using as reference: the date when the main building work has finished, the date of the middle of the construction period, the date of the first observation campaign, the date the reservoir started to fill (dams).

5.1 Annual Periodic Changes

Most of the engineering structures show a behaviour that follows a regular periodic pattern along the year. The vertical displacements present in Figure 2 show this kind of behaviour: upwards during winter, downwards during summer. This pattern is usually well represented by a sinusoidal wave, with period 365.25 days (each year is, in average, 365.25 days long) represented by the sum of two terms:

$$a_k \sin\frac{2\pi\, t'}{365.25} + a_j \cos\frac{2\pi\, t'}{365.25} \qquad (20)$$

where t' stands for the number of days since the beginning of the year. In this expression t' is expressed in days. This function is often used to represent the effect of the annual variation of the temperature or humidity of the structure.

5.2 Daily Periodic Changes

Some structures might present periodic variations in a daily basis. The equation that represents this parcel of the behaviour is very similar to the last equation:

$$a_k \sin\frac{2\pi\, t'}{24} + a_j \cos\frac{2\pi\, t'}{24} \qquad (21)$$

where t' stands for the time of the day (in hours) since the beginning of the day. Other periods can be considered (see an example in Henriques et al., 2012), being that is easy to change the equation accordingly. In Figure 3 is presented a graphical representation of the values of equation (21), considering a 24h period (green line), a 12h period (black line), and the superposition (sum) of these two waves (24h+12h; orange line). It was considered $a_k=a_j=1$.

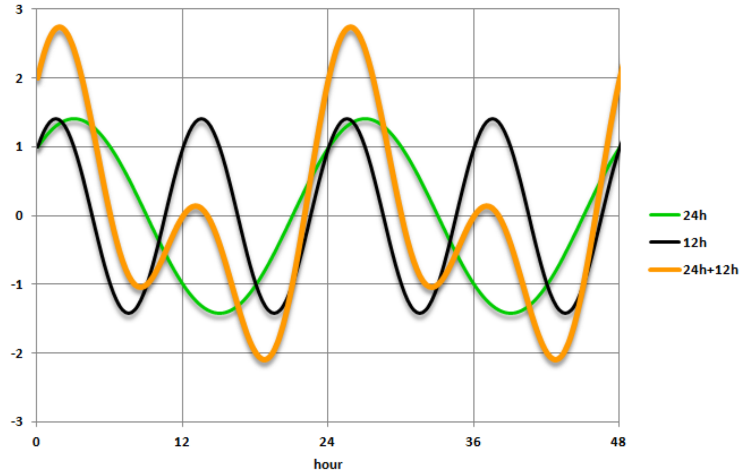


Figure 3 – Periodic effect with a periods of 24 and 12 hours and the superposition of both

5.3 Water Level in Reservoirs

In the case of dams, water of reservoirs exerts hydrostatic pressures not only on the floor and walls of the reservoir but also on the upstream face of a dam. The rise of the level of the water is the origin of: i) horizontal displacements in the downstream direction; ii) vertical displacements in the upward direction in some dams with double-curvature in both horizontal and vertical planes (dome dams); iii) horizontal displacements in the upstream direction can be noticed in some arch dams. Both horizontal and vertical displacements are modelled by the same function

$$a_k h^n \qquad (22)$$

where h represents the reservoir level, with n usually equal to 3 or to 4. In Figure 5 are represented the displacements of points distributed along a vertical plane in the central cantilever of an arch dam (Figure 4), displacements in the radial and vertical directions, due to a raise of the water level. All points face downstream displacements: the majority of the points also face upwards displacements while the points at lower levels have downwards displacements.

In concrete dams is usual to include in the equation two terms related with the effect of the water level, one with n=1 and another with n=3 or n=4.

5.4 Temperature

When the temperature of the structure is measured on some representative points, it can be included in the regression model. The temperature effects are usually described by a linear term

$$a_k T \qquad (23)$$

where T stands for the temperature measured. Large structures as concrete dams react to annual temperature waves, always with a delay that can be of several months. For instance, the largest vertical upwards displacement at the dam crest, due to seasonal heating, is not in July/August (the hottest months in the northern hemisphere) but a couple months later (September/October).

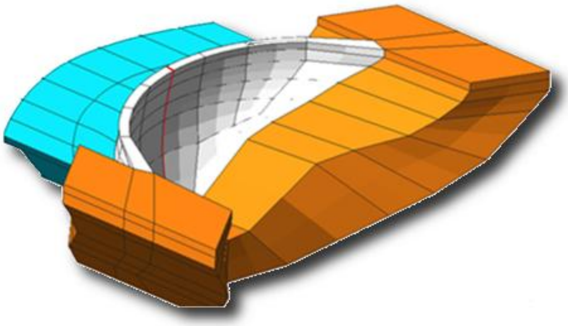


Figure 4 – Numerical model of an arch dam.

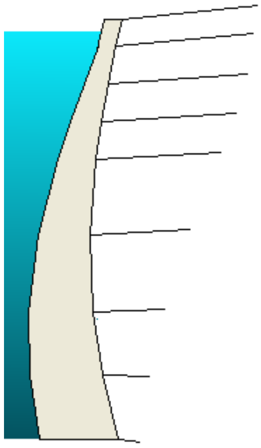


Figure 5 – Displacements (radial and vertical) along a vertical plan, normal to the surface of the dam.

5.5 Time effects

Creep, swelling of the material (e.g. concrete), foundation movements, compactation (e.g. embankment dams), etc., causes the structure to move slowly or deform permanently. In wood and embankment structures is usual to have large displacements during the first years after the construction, effect that slows down and, quite often became null, after some years (see Figures 1 and 2). In the case of earth fill dams this is caused by the compaction of the earth due to its wetting during the first filling. The time effects are usually described or by an exponential term or a logarithmic term like

$$a_k \frac{1}{(t-t_0)^n} \qquad (24) \qquad\qquad a_k \ln\left(1+\frac{t-t_0}{\alpha}\right) \qquad (25)$$

being t the date of the observation and t_0 the date of the beginning of the phenomena ($t-t_0$ is the age of the phenomena). The parameters n and α are chosen by the user.

5.5 Concrete Swelling

The water in contact with concrete surface, concrete that are a mixture of reactive aggregates and alkali-rich cements, is responsible by the formation of expansive hydration products (hydrophilic gel) that causes the expansion of the concrete, a process that is irreversible. The overall development of the expansion has, usually, two phases, as seen in Figure 6. During the first stage the effect increases exponentially with time; during the second it decreases, following a symmetric rule. After some time the effect becomes null.

This effect is described by

$$a_k\, e^{\frac{(t-t_0)^n}{\beta}} \qquad (26)$$

where t is the date of the observation, t_0 is the date of the beginning of the expansion of the concrete, n=3.258, being β calculated by the expression

$$\beta = (t_{hs} - t_0)^n \frac{n}{n-1} \qquad (27)$$

where t_{hs} is the date of the ”half swelling” (the date of half of the full expansive effect, red dot in Figure 6).

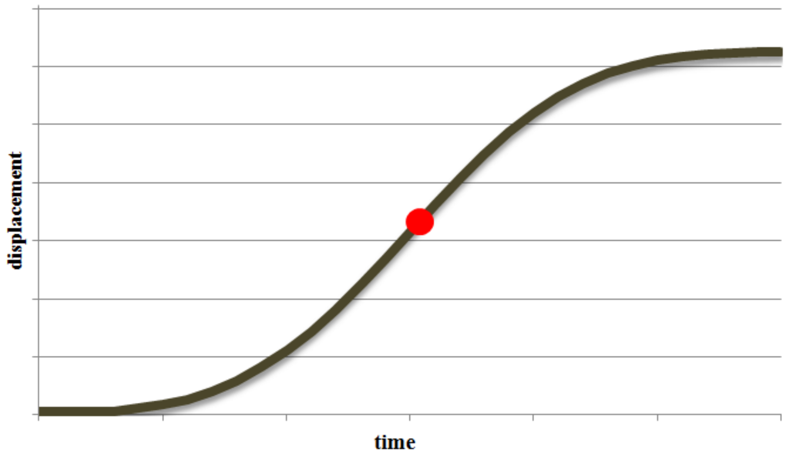


Figure 6 – Displacements variation due to concrete swelling

6. REGRESSION MODELS APPLIED TO CIVIL ENGINEERING STRUCTURES

The regression model that is applied to describe the displacements of civil engineering structures depends on the structure. For instance, displacements of dams usually reflect the level of the water in the reservoir, especially when the dam is a thin arch. Concrete structures respond to annual variations of temperatures; wooden structures to humidity, although the variation along the year is difficult to model due to the fact that humidity has large variations and it's very temperature dependent. This paragraph presents the model used in a concrete arch dam.

To calculate the coefficients a_0, a_1, …, a_k is necessary to solve the system of equations (3). Many computer softwares can be used to solve such a system. Spreadsheets like Excel, LibreOffice Calc or Google Sheets, are accessible applications, which can be used to save the data, to solve the system of equations and to draw charts. In this chapter will be presented the expressions applied to determine the coefficients of equation (1), using Excel notation. The structure chosen to present the functions applied in a Excel spreadsheets is *Vilarinho das Furnas* dam.

Figure 7 – *Vilarinho das Furnas* dam

Vilarinho das Furnas is located in the north of Portugal. The construction finished in 1971; it is 94 m hight and 385 m long (crest). The crest is at an altitude of 570 m. The displacements of this dam are measured by geodetic methods since 1971 (beginning of the first filling). The monitoring system consists of: i) a network used to determine horizontal displacements (radial, d_rad, and tangential, d_tan, components) with 19 targets (object point), on the downstream face of the dam, and four pillars; ii) two levelling lines, used to determine vertical (d_z) displacements, one on the crest, another in a gallery. The data here analysed corresponds to 35 observation campaigns.

It is necessary to compute the coefficients by component (d_rad, d_tan, and d_z) and by point, so each component of the displacement was recorded in a different worksheet of the Excel file. In this paper it is presented, in Figure 8, the first cells of the worksheet "d_rad", a sheet were the variations of the displacements in the radial direction were recorded. The major loads on the structure are: temperature (an annual periodic load), hydrostatic pressures (related with the water level) and time effects (age). For this reason it was also recorded the date of the campaign and the altitude of the surface of the reservoir in that date. In this sheet, each of the 19 columns (columns C to U) is an array Y.

	A	B	C	D	E	F	G
1	date	water level	AB568	DE568	IJ568	MN568	PQ568
2	1971-01-27	488.13	0.0	0.0	0.0	0.0	0.0
3	1972-03-08	569.49	1.4	3.5	19.4	36.7	38.5
4	1972-10-24	486.70	2.1	0.7	-2.4	-4.6	-9.0
5	1973-05-31	567.53	1.2	1.8	11.2	23.6	24.9
6	1974-02-20	568.67	3.8	7.0	24.2	42.3	44.7
7	1975-05-16	552.02	0.7	0.9	6.8	16.6	19.1

Figure 8 – Radial displacements of *Vilarinho das Furnas* dam

The equation chosen is:

$$y= a_0 + a_1 \cos\left(2\pi \frac{t'}{365.25}\right) + a_2 \sin\left(2\pi \frac{t'}{365.25}\right) + a_3 h + a_4 h^n + a_5 \ln\left(1+\frac{t-t_0}{\alpha}\right) \qquad (28)$$

where t represents the date of the observation, h the water level of the reservoir, t' the number of days since the beginning of the year, t_0 the date that the dam was finished; n and α were chosen: n=3 and α=300.

The next step is to build array B using the values of the functions. In the worksheet presented, array B (an array 35×6) starts in column Y and ends in column AD. According to equation (28) it is necessary some auxiliary data (constant values): i) t_0, that was recorded in cell w2; ii) altitude of the surface of the reservoir at the lowest level, recorded in cell W4, necessary to calculate the water height (h); iii) n, recorded in cell W6; iv) α, recorded in cell W8. Columns V, X and AE were intentionally left blank. To populate columns Y to AD (array B) is necessary data from the columns A, B and W. The expressions used in range Y2:AD2 are:

cell	Excel's formula	cell	Excel's formula
Y2	=1	AB2	=A2-W$2
Z2	=COS(2*PI()*(A2-DATE(YEAR(A2);1;1)+1)/365.25)	AC2	=(A2-W$2)^W$6
AA2	=SIN(2*PI()*(A2-DATE(YEAR(A2);1;1)+1)/365.25)	AD2	=LN(1+(A2-W$2)/W$8)

	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK
1	construction finishing		const	f1	f2	f3	f4	f5			AB568	DE568	IJ568	MN568	PQ568
2	1970-06-15		1	0.89	0.45	9	642.7	0.56		a0	1.93	1.59	-0.38	-5.79	-11.32
3	minimum water level		1	0.39	0.92	90	728757.0	1.13		a1	0.94	1.61	4.07	7.07	6.90
4	479.5		1	0.40	-0.92	7	373.2	1.35		a2	0.09	1.16	4.30	6.38	7.39
5	n		1	-0.86	0.52	88	682169.2	1.53		a3	7.81E-02	3.03E-02	-1.22E-01	-9.15E-02	-2.74E-(
6	3		1	0.64	0.77	89	709016.4	1.70		a4	-7.20E-06	6.77E-07	3.54E-05	5.58E-05	5.34E-0
7	α		1	-0.70	0.72	73	381393.6	1.94		a5	-1.59	-1.66	0.24	2.98	4.93
8	300		1	-0.73	0.68	68	311665.8	2.11							
9			1	-0.14	-0.99	56	179501.6	2.16		se	1.9	1.7	2.4	3.3	3.3
10	m		1	0.84	0.54	87	660093.8	2.21		R2	0.33	0.66	0.92	0.95	0.96
11	36		1	-0.96	-0.28	89	707109.8	2.38							
12	n		1	-0.10	1.00	89	701884.3	2.46		a0/sa0	1.4	1.3	-0.2	-2.4	-4.7
13	6		1	-0.98	0.20	78	476379.5	2.48		a1/sa1	2.0	3.9	6.9	8.7	8.5
14			1	-0.62	-0.78	15	3281.4	2.50		a2/sa2	0.2	2.2	5.6	6.0	7.0
15	t (student)		1	0.14	-0.99	16	4251.5	2.51		a3/sa3	1.6	0.7	-2.0	-1.1	-0.3
16	2.4		1	0.06	1.00	84	588269.8	2.56		a4/sa4	-1.4	0.1	5.4	6.2	5.9
17			1	-0.99	-0.13	78	482628.3	2.67		a5/sa5	-2.9	-3.4	0.4	3.2	5.2
18			1	0.12	-0.99	61	231023.4	2.77							
19			1	0.98	-0.18	87	658503.0	2.93		sa0	1.4	1.2	1.8	2.4	2.4
20			1	0.97	-0.23	74	402764.8	2.99		sa1	0.5	0.4	0.6	0.8	0.8

Figure 9 – Auxiliary data and values of the functions and of the coefficients.

To calculate the coefficients (elements of array A presented in (4)), is necessary to apply the array operations “multiplication” and “inverse” to the arrays Y and B, as seen in (5). The Excel array formulas used are: MMULT (multiplication), TRANSPOSE (transposition) and MINVERSE (inversion). In the example, the elements of array Y of the first object point (point AB568) are written in the range C2:C36 (an array 35×1), the array B is written in the range Y2:AD36 (an array 35×6). The elements of array A (an array 6×1), i.e., the array of the coefficients a_0 to a_5, will be saved in column AF, lines 2 to 7 (see Figure 9).

cells	Excel’s formula
AF2:AF7	=TRANSPOSE(MMULT(MMULT(TRANSPOSE(C2:C36);($Y2:$AD36)); MINVERSE(MMULT(TRANSPOSE($Y2:$AD36);($Y2:$AD36)))))

It will be important to remember that AF2:AF7 is an array. So, after selecting the range AF2:AF7 and written the mathematical expression just presented in cell AF2 is mandatory to hit simultaneously the keys CTRL+SHIFT+ENTER to enter the array formula.

To calculate the coefficients of the other points one uses the fill handle to copy the formulas to adjacent columns (until AY: 18 columns=18 points). As the array B is constant (it will change only when a new line, i.e. a new epoch, is added), one must fixed the range Y2:AD36 when the expressions are copied. For this reason it was used absolute cell references ($Y2:$AD36) turning this range constant when the fill handle moves to the next columns. In the lines that follow this area it was written some statistics that help to evaluate the quality of the model: the goodness of fit (coefficient of determination, R^2) the contribution of the different terms to the model (a_0/s_{a0}, …).

Suppose that one would like to calculate de radial displacement of a point for a given date. Let’s suppose that that date, and the water level measured (or expected), were recorded in cells BA2 and BB2 (see Figure 10). Adapting the expressions previously used in columns Y-AD to the columns BC-BH, is no difficult to calculate the displacements of the points in that date (columns BI and following). It was also used absolute cell references.

cell	Excel's formula
BI2	=$BC2*AG$2+$BD2*AG$3+$BE2*AG$4+$BF2*AG$5+$BG2*AG$6+$BH2*AG$7

	AZ	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ
1		date	water level	const	f1	f2	f3	f4	f5	AB568	DE56
2		1981-06-13	564.97	1	-0.95	0.31	85	624368.0	2.67	-1.0	-1.0
3		1989-12-30	567.40	1	1.00	-0.02	88	679151.4	3.21	-0.3	1.0

Figure 10 – Calculating displacements using the model.

These expressions are also useful to analyse the quality of the model chosen, by calculating the displacements and comparing them with the measured ones. In Figure 11 are presented the radial displacements, calculated by regression, and the ones measured. As, since 1 January 1971, there is daily information of the water level it was drawn a continuous line of the expected displacements. In the same graph is also included the displacement measured (data from geodetic campaigns; red diamond). In the days of the geodetic campaigns the displacement calculated by regression were highlighted (a black diamond was used).

The lines that follow the area with the coefficients (see Figure 10) have some statistics that help to evaluate the quality of the model (see Table 1): the goodness of fit (coefficient of determination, R^2) and the contribution of the different terms to the model (a_0/s_{a0}, …).

Of the six object points presented in Table 1, one can see that the functions chosen allow a goodness of fit of the equation (1) to the data (radial displacements) of the points IJ568, MN568, PQ568 and ST568. The cells highlighted in the table alert to coefficients that don't contribute to the model. The bad fit related with the point AB568 is usual: the behaviour of points near the abutment is not modelled by the same kind equations that the points in the middle of dam.

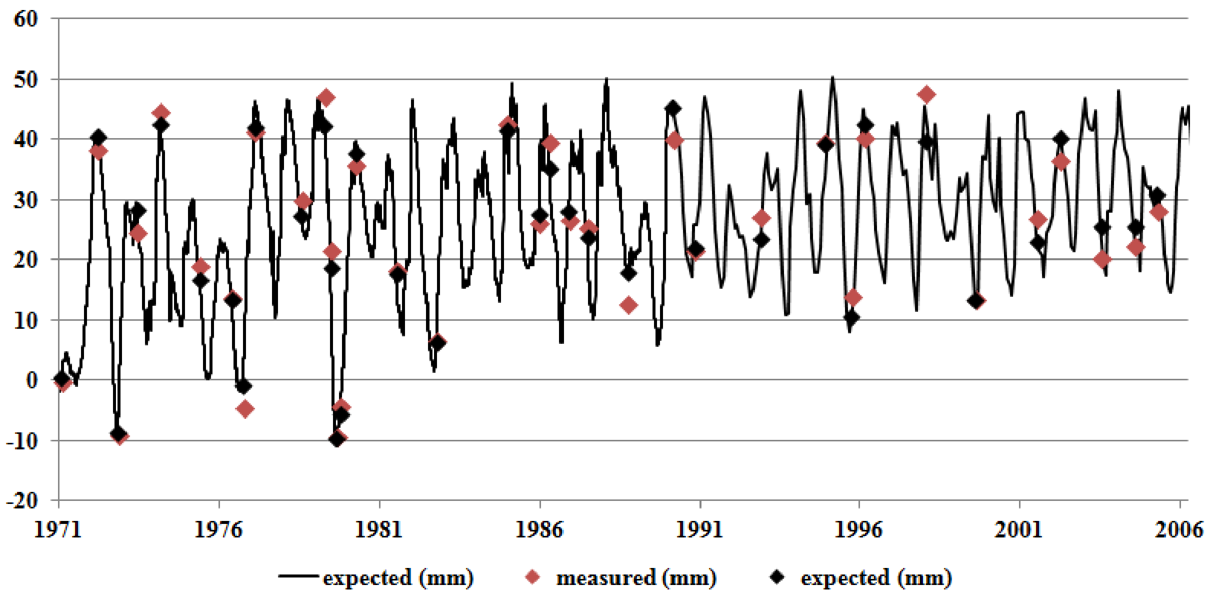


Figure 11 – Radial displacements (in mm) of the highest point in the central cantilever.

Table 1 – Evaluating the quality of the regression model.

	AB568	DE568	IJ568	MN568	PQ568	ST568
R2	0.33	0.66	0.92	0.95	0.96	0.93
a0/sa0	1.38	1.28	-0.21	-2.38	-4.66	-2.23
a1/sa1	2.02	3.89	6.92	8.71	8.50	6.59
a2/sa2	0.15	2.16	5.61	6.04	6.99	5.36
a3/sa3	1.59	0.69	-1.97	-1.07	-0.32	-0.89
a4/sa4	-1.39	0.15	5.44	6.21	5.94	4.81
a5/sa5	-2.95	-3.44	0.36	3.16	5.25	2.29

The same method of calculation was applied to displacements measured in other structures: a embankment dam (Figure 12) and the glulam roof structure of a pavilion (Figure 13). In Table 2 are presented the correlation coefficient and the coefficient of determination of one point of each structure – concrete arch dam (point on the central cantilever, highest altitude); embankment dam (point on the crest, highest altitude); glulam structure (point in the middle of an arch) – points that, due to their positions on the structure, face major displacements.

Correlation coefficients were calculated using Excel's CORREL function. This function is used to determine the relationship between two properties, quantified in two arrays. In the case presented before (concrete dam), the two arrays are: i) displacements; ii) values of the functions. For instance, to calculate the correlation coefficient between displacements and age function (f5), the Excel expression would be, to point AB568: =CORREL(AC2:AC36; C2:C36).

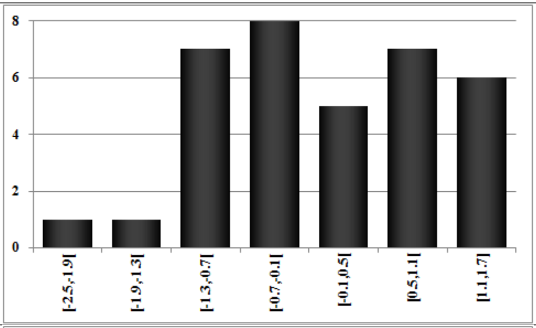
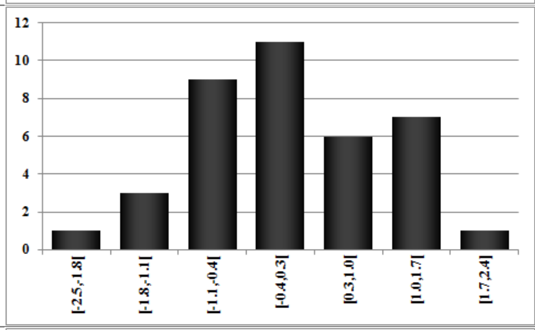
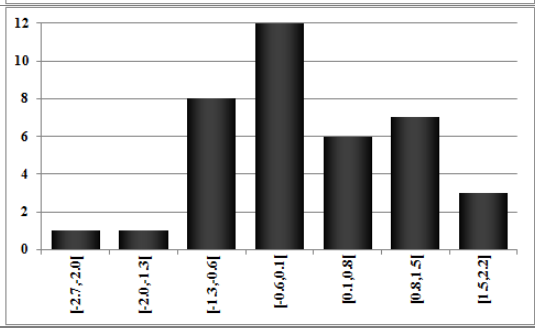
Figure 12 – Embankment dam.

Figure 13 – Glulam roof structure of a pavilion.

Table 2 – Correlation coefficient and coefficient of determination

		a1	a2	a3	a4	a5	R^2	f_5
concrete dam	radial disp.	0.30	0.63	0.83	0.90	0.28	0.96	$\ln\left(1+\frac{t-t_0}{300}\right)$
concrete dam	vertical disp.	0.68	0.42	-0.19	-0.18	-0.24	0.66	
embankment dam	vertical disp.	0.01	-0.04	0.16	0.15	1.00	1.00	$\frac{1}{t-t_0}$
glulam structure	vertical disp.	0.34	0.16	–	–	0.94	0.97	

Table 3 – Frequency chart of the residuals.

<table>
<tr><th>Equation</th><th>Frequency chart of the residuals</th></tr>
<tr><td>$$y=a_1 \cos\left(2\pi \frac{t'}{365.25}\right)+a_2 \sin\left(2\pi \frac{t'}{365.25}\right)+$$ $$+a_3 h+a_4 h^3+a_5 \ln\left(1+\frac{t-t_0}{300}\right)+c$$</td><td>
</td></tr>
<tr><td>$$y=a_1 \cos\left(2\pi \frac{t'}{365.25}\right)+a_2 \sin\left(2\pi \frac{t'}{365.25}\right)+$$ $$+a_4 h^3+a_5 \ln\left(1+\frac{t-t_0}{300}\right)+c$$</td><td>
</td></tr>
<tr><td>$$y=a_1 \cos\left(2\pi \frac{t'}{365.25}\right)+a_2 \sin\left(2\pi \frac{t'}{365.25}\right)+$$ $$+a_3 h+a_4 h^3+a_5 \frac{1}{t-t_0}+c$$</td><td>
</td></tr>
</table>

One can also ask if the model chosen (functions selected) was good. One empirical method is to draw a chart with residuals. Other kind of chart that can give good information is the frequency chart of the residuals. In the Table 3 are presented three frequency charts of residuals of vertical displacements of the concrete dam and also the equation used. The number of classes (seven in the charts presented) were chosen by using the Sturge's rule. To analyse the charts one must takes into account the chart of the normal probability plot (a bell-shaped curve). The analysis of the three charts suggest that the second type of equation fits better the data measured due to the fact that the second chart is the one that has a shape nearer a bell.

This section will end with an evaluation of the displacements of a point, by comparing them with the predicted ones. It was chosen the vertical displacements of one point of the glulam structure monitoring system, measured in 2011 and 2012 (four campaigns). The value of coefficient of determination, R^2, is 0.97 (the regression model fits well). In Figure 14 is presented a graph with the differences between the observed and predicted displacements (red diamonds). The area in blue represents the prediction interval. The amplitude of this interval varies along the year, due to the terms of the annual periodic changes, and it increases when the prediction is performed for a distant date (the green line on the graph helps one to realize the increasing of the amplitude of the prediction interval). Concerning the displacements differences, the result of the first campaign presents, clearly, an anomaly. As the three other differences are in the prediction interval, there is a high probability that it's due to errors in the campaign and not an error in the estimation of the regression model or a structural problem.

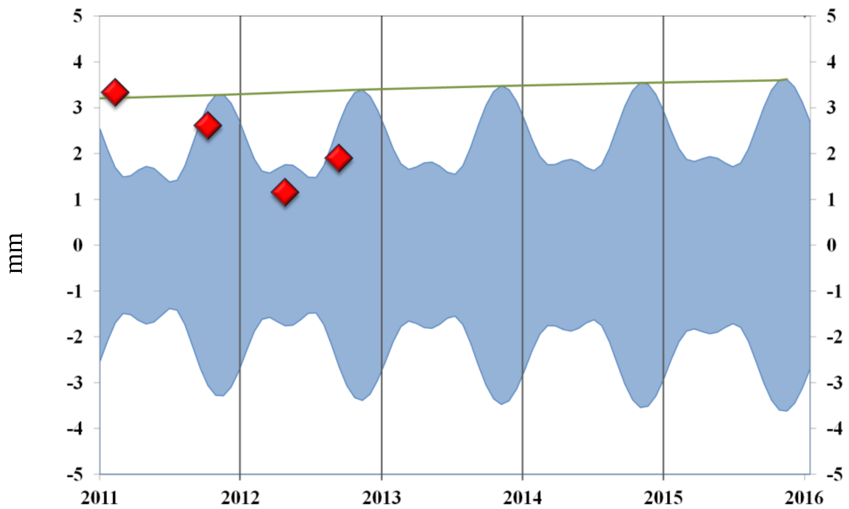


Figure 14 – Analysis of vertical displacements on a glulam structure. Differences between observed and predicted values in mm (red) and prediction band (blue).

7. CONCLUSIONS

It was presented a simple method to build statistical/empirical models using functions of a well known spreadsheet application (Excel). The regression model used, that can be applied to analyse displacements, establishes linear relations between loads, time effects and structural response.

FINAL REMARK

Excel macros (Figure 14) developed to compute the statistical model coefficients are available, email: mjoao@lnec.pt.

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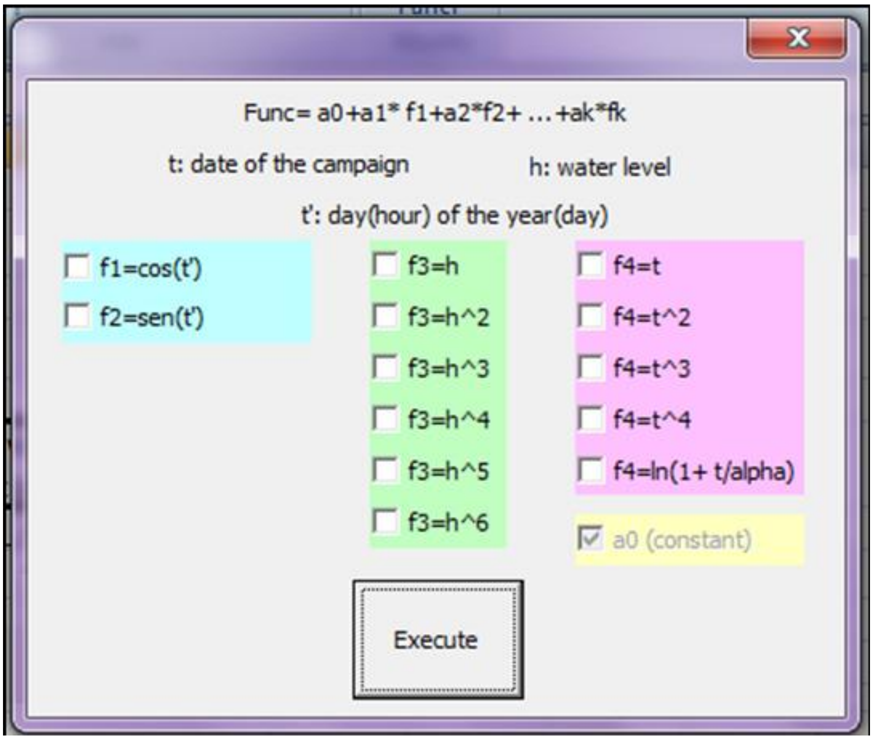

Figure 14 – Screenshot of the first form of the macro developed

BIOGRAPHICAL NOTES

Maria João Henriques is a Senior Research Officer at the Applied Geodesy Division of LNEC. Her research activities include geodetic surveying systems design and quality control, atmospheric effects on the measurements, calibration of equipment and automation of field observations and office work.

Sérgio Oliveira is a Research Officer at the Division of Physical and Mathematical Modelling of LNEC Concrete Dams Department. His areas of research include, Finite Element Models for static and dynamic analysis of dam-reservoir-foundation systems, use of constitutive damage models, and development of systems for continuous dynamic monitoring of large dams, including the development of computational applications for automatic modal identification.

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