Bayesian probabilistic assessment of in-situ concrete strength

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Summary

Core testing is considered the most accurate technique for the assessment of in-situ concrete strength. EN 13791:2007 gives guidance for estimating in-situ compressive strength in existing structures and states that core testing is the reference method. However, the number of cores that can be taken from a structure is usually limited, so it may be advantageous to supplement the core tests with some type of indirect test. The standard mentioned above establishes two alternatives for the calibration of indirect tests, both based on core tests results taken from the structure being assessed. One of them requires at least 18 core tests. But if it is available 18 core tests, it is only natural to ask if it is really necessary to supplement those core tests with an indirect test. This question motivates the study here presented. Specifically, this study deals with the determination of the number of cores above which the use of an indirect test, as a supplement to core tests, is no longer attractive.

Keywords: core tests; indirect tests; concrete strength; statistical uncertainty; Bayesian approach; calibration.

1. Introduction

Currently the most accurate method to assess the concrete strength of an existing structure is the assessment directly from core tests. However, the number of cores that can be taken from a structure is in general limited, not only because it introduces damage into the structure, but also because it is a time consuming and expensive technique. Thus, if it is required to estimate, for example, the characteristic value of the concrete strength from that small sample of cores, the statistical uncertainty will be large and reduce such an estimate.

This drawback can be overcome by supplementing the core tests with indirect tests, such as rebound hammer tests, ultrasonic pulse velocity tests and pull-out tests. These tests are much more economic than core tests and furthermore, with exception of pull-out tests, do not introduce any damage into the structure. With these indirect tests it is possible to obtain a large number of results, virtually eliminating the statistical uncertainty.

Nevertheless, these tests need a previous calibration, which, according to EN 13791:2007 [1] must be carried out specifically for the structure being analyzed. In fact such tests depends not only on the equipment itself, but also on the properties of the concrete being analysed, such as the concrete age, the type of aggregates, the condition in terms of durability, among others [2]. If indirect tests are used without a previous calibration specifically for the structure under study, there is a risk of introducing systematic errors.

According to EN 13791:2007 [1] the calibration must be carried out using cores taken from the structure. But, again, since the number of cores is limited, there will be (statistical) uncertainty in calibration. On the other hand, it is necessary to take into account the lack of precision of the indirect test, because they measure a property not fully correlated with the concrete strength. Thus, by using indirect tests as a supplement to the direct test (core tests), even though the statistical uncertainty is eliminated because of the high number of results that can be obtained, they introduce two new sources of uncertainty: one due to the fact that the calibration is carried out from a limited number of cores and other due to the lack of precision of the indirect test.

In this paper the statistical uncertainty associated to core tests will be called *direct test uncertainty* and the two sources of uncertainty associated to indirect tests mentioned above will be called together *indirect test uncertainty*. By balancing the direct test uncertainty and the indirect test uncertainty it can be decided if it is beneficial to use indirect tests as a supplement to core tests. From a probabilistic point of view, it can be expected that there is a number of cores above which it is not worth using indirect tests. This happens when the *direct test uncertainty* becomes smaller than the *indirect test uncertainty*.

As it will be seen, the number of cores above which it is not worth using indirect tests depends basically on two parameters: the precision of the indirect test and the variability (coefficient of variation) of the concrete.

2. Assessment of direct test uncertainty

Suppose that the compressive strength f_c of an existing structure follows a normal distribution. The population f_c is identified here as the values that f_c takes from point to point in that structure. The objective is to estimate the characteristic value of f_c, corresponding to the quantile 0.05 of his probability distribution. Let the characteristic value be denoted by f_{ck} and an estimate of f_{ck} by $\hat{f}_{ck}$. With the objective of determining an estimate $\hat{f}_{ck}$, suppose that a sample of n cores was taken, which, after being tested in laboratory, resulted in a sample $\{f_{c1},..., f_{cn}\}$ of n values of the compressive strength f_c. Let $\bar{f}_c$ and s be, respectively, the mean and standard deviation of that sample. Let $V = s / \bar{f}_c$ be the coefficient of variation. Thus, since it was assumed that f_c is normally distributed, an estimate of f_{ck} would be given simply by:

$$\hat{f}_{ck0} = (1 - 1.645V)\,\bar{f}_c \,. \tag{1}$$

This estimate, however, does not include the statistical uncertainty, i.e., the uncertainty arising by the fact that the parameters $\bar{f}_c$ and V was estimated from a sample with finite size. It has been widely accepted that a Bayesian approach is the appropriate way to deal with statistical uncertainty [3]. According to the predictive model for a normal distributed variable corresponding to an non-informative prior distribution on the parameters, the Bayesian estimate for f_{ck}, here denoted by $\hat{f}_{ck1}$, is given by:

$$\hat{f}_{ck1} = \left(1 + t_{0.05,n-1}\sqrt{1+\frac{1}{n}}V\right)\bar{f}_c\,, \tag{2}$$

where $t_{0.05,n-1}$ denotes the inverse of the t distribution with $\nu = n-1$ degrees of freedom, computed at $p = 0.05$.

Dividing (1) by (2) it is obtained a factor which reflects the extent to which the estimate $\hat{f}_{ck0}$ must be reduced in order to take into account the statistical uncertainty. This factor, here denoted by α_1 and called *direct test uncertainty factor*, is always greater than one and tends to one as the sample size of cores grows. The *direct test uncertainty factor* is then given by:

$$\alpha_1 = \frac{\hat{f}_{ck0}}{\hat{f}_{ck1}} = \frac{1-1.645V}{1+t_{0.05,n-1}\sqrt{1+1/n}\,V}\,, \tag{3}$$

Note that the *direct test uncertainty factor* depends only on the number of cores and the estimated coefficient of variation of f_c. The factor α_1 is the factor by means of which $\hat{f}_{ck0}$ must be divided in order to obtain the estimate $\hat{f}_{ck1}$ which, as mentioned above, properly include the statistical uncertainty. Note that when $n \to \infty$, $\alpha_1 \to 1$. So the case with $\alpha_1 = 1$ corresponds to the case where there is no statistical uncertainty.

As an example, consider a structure from which 5 cores was taken, later tested in laboratory. Suppose that those tests yielded $\bar{f}_c = 40$ MPa and $V = 0.12$. The estimate of f_{ck} without take into account the statistical uncertainty is then $\hat{f}_{ck0} = (1-1,645\times 0,12)40 = 32,1$ MPa . The direct test uncertainty factor, (Eq. (3), is equal to 1,115, which leads to $\hat{f}_{ck1} = 32/1,115 = 29,7$ MPa .

In Figure 1 α_1 is plotted as a function of the number of cores n for a concrete with $V = 0.12$. According to that graphic (concrete with $V = 0.12$), for $n > 20$ the factor α_1 is less than 1.02. So, for this concrete the statistical uncertainty associated to a sample with size of about 20 is almost zero (for the purpose of estimating f_{ck}). Therefore, from a probabilistic point of view, there would be no need to supplement the core tests with an indirect test.

But, suppose that it would be feasible to take from the structure only 5 cores, which, as seen above, leads to $\alpha_1 = 1,115$. In this case, probably it would be worth supplementing the direct tests with indirect test in order to reduce the statistical uncertainty. As it will be seen in next section, whether it would be worth or not depends on the precision of indirect test (and also on the coefficient of variation of f_c) .

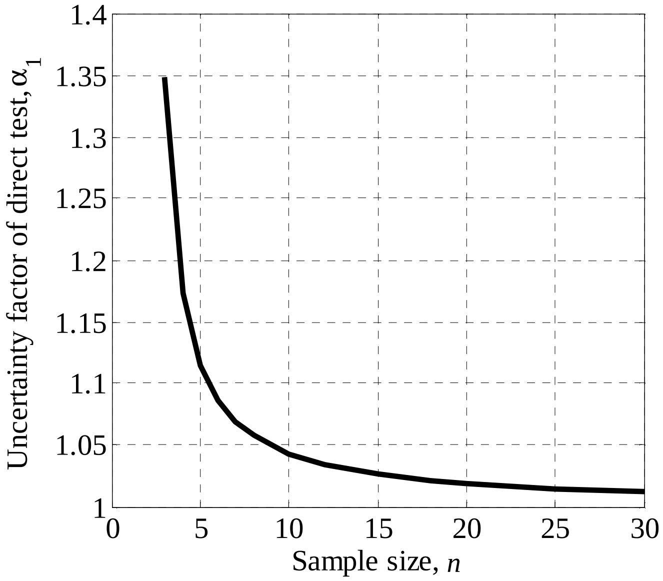

Figure 1: Direct test uncertainty factor for a concrete with $V = 0.12$

3. Assessment of indirect test uncertainty

Using indirect tests, the compressive strength of the concrete is evaluated from a property correlated with the strength, such as the superficial hardness in the case of the rebound hammer. As mentioned before, indirect tests need a previous calibration performed from a number of cores taken from the structure under evaluation.

Suppose that the correlation between the property X measured by the indirect test and the compressive strength f_c of concrete meets the requirements of the linear regression model, that is, the compressive strength can be predicted by a model of the form:

$$f_c = \beta_0 + \beta_1 X + \sigma Z , \tag{4}$$

where Z is a standardized normal variable, i.e., $Z \sim N(0,1)$.

The parameters of such a model (β_0 , β_1 and σ) are estimated from a sample of n pairs $\{(x_1, f_{c1}),...,(x_n, f_{cn})\}$, where x_i represents the value measured by the indirect test at location i and f_{ci} the strength of the core taken from that location. The calibration of the indirect test consists of estimating the parameters β_0 , β_1 and σ , whose estimates here denoted by $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$, are in general performed by the least squares method [4]. The parameter σ measures the mean deviation of the points (x_i, f_{ci}) from the regression line, thereby constituting a measure of the precision of the indirect test, or still, a measure of the ability of the indirect test in predicting the compressive strength f_c .

But, since the parameters β_0, β_1 and σ are estimated from a sample $\{(x_1, f_{c1}),...,(x_n, f_{cn})\}$ of finite size (n, in this case) statistical uncertainty will exist, which is properly taken into account by the following predictive model [5]:

$$f_c = \hat{\beta}_0 + \hat{\beta}_1 X + T_{n-2}\sqrt{1+\frac{1}{n}+\frac{(X-\overline{x})^2}{S_{xx}}}\,\hat{\sigma} \tag{5}$$

where T_{n-2} denotes a t distributed variable with $\nu = n-2$ degrees of freedom; $x = \Sigma_{i=1}^{n} x_i$; $S_{xx} = \Sigma_{i=1}^{n}(x_i - \overline{x})^2$.

The predictive model expressed by (5) is the Bayesian model obtained considering the Jeffreys' prior distribution for the parameters β_0, β_1 and σ, which is a (improper) distribution of the non-informative type. It must be said that the model above coincides with the classic one. The advantage of the Bayesian approach, however, is that it would be able to include prior knowledge about the parameters β_0, β_1 and σ, which could reduce the statistical uncertainty. Nevertheless in the present study it will be considered the situation where there is no previous knowledge.

Based on the predictive model above, the objective is now to determine an estimate of f_{ck}, which will be denoted by $\hat{f}_{ck2}$. This will be then the estimate given by the indirect test. Unfortunately, it is not possible to derive an expression for $\hat{f}_{ck2}$, as was done for $\hat{f}_{ck1}$ (see Eq. (2)), since it is not possible to obtain a closed form for the predictive distribution associated to the Eq. (5). However, $\hat{f}_{ck2}$ can be obtained trough the Monte Carlo Method (MCM), as explained in the following.

Suppose that an indirect test properly calibrated for a given structure was performed m times on that structure, leading to a sample $\{x_1,...,x_m\}$. Thus, generating trough MCM a sample $\{t_1,...,t_m\}$ of the variable T_{n-2}, where n is the number of cores used in calibration, Eq. (5) can be used to generate a sample $\{f_{c1},..., f_{cm}\}$, from which $\hat{f}_{ck2}$ can be computed.

Note that in general m is high, of the order of several dozens or even hundreds, because of the ease use of the indirect test. So, for m large there is no statistical uncertainty in the sample $\{f_{c1},..., f_{cm}\}$ itself, except of course the uncertainty in calibration and the uncertainty due to the lack of precision of the indirect test.

As before, let us define the factor:

$$\alpha_2 = \frac{\hat{f}_{ck0}}{\hat{f}_{ck2}}, \tag{6}$$

where, remember, $\hat{f}_{ck0} = (1-1.645V)\overline{f_c}$ which is the estimate of f_{ck} from core tests without taking into account the statistical uncertainty. The factor α_2 reflects the extent to which the estimate $\hat{f}_{ck0}$ must be reduced in order to account the uncertainty associated to the indirect test. Evidently, from a probabilistic point of view, it only makes sense to use the indirect test as a supplement to the direct test if $\alpha_2 < \alpha_1$. Similar to the factor α_1, the factor α_2 will be called *indirect test uncertainty factor*.

In order to determine α_2 using the MCM, the following algorithm was developed. Suppose that it was taken from a given structure n cores, which, once tested in laboratory, gave a mean strength $\overline{f_c}$ and a coefficient of variation of V. Based on those tests, the chosen indirect test was calibrated, giving the estimates $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$. The factor α_2 can be computed as follows:

1) draw a sample $\{f_{c1},..., f_{cm}\} \sim N(\overline{f_c}, V \cdot \overline{f_c})$;

2) draw a sample $\{z_1,..., z_n\} \sim N(0,1)$;

3) simulate the measurements with the indirect test, computing $x_i = (\overline{f_c} - \hat{\beta}_0 - \hat{\sigma} z_i)/\hat{\beta}_1, i = 1,..., m$;

4) evaluate $\overline{x} = \sum_{i=1}^{m} x_i$ and $s_X^2 = [1/(m-1)]\sum_{i=1}^{m}(x_i - \overline{x})^2$;

5) compute $S_{xx} = (n-1)s_X^2$;

6) draw a sample $\{t_1,...,t_m\} \sim \text{student}(n-2)$;

7) generate $f_{ci} = \hat{\beta}_0 + \hat{\beta}_1 x_i + t_i \sqrt{1 + \frac{1}{n} + \frac{(x_i - \overline{x})^2}{Sxx}}\, \hat{\sigma}$, $i = 1,\ldots,m$;

8) from the sample $\{f_{c1},\ldots, f_{cm}\}$ generated in the previous step, compute the quantile 5%, $\hat{f}_{ck2}$;

9) compute $\alpha_2 = \frac{\hat{f}_{ck0}}{\hat{f}_{ck2}} = \frac{(1-1.645V)\,\overline{f_c}}{\hat{f}_{ck2}}$.

It should be assigned to m a value sufficiently high so that α_2 stabilizes in successive runs of the routine. The systematic use of this routine showed that α_2 does not depend on the parameters $\hat{\beta}_0$ and $\hat{\beta}_1$, nor the mean $\overline{f_c}$. So, α_2 depends only on the sample size n, on the indirect test precision $\hat{\sigma}$, and on the estimated coefficient of variation V of the concrete strength.

Consider again the example of the previous section, where, remember, the concrete strength of a existing structure is being assessed. From that structure it was taken $n = 5$ cores, which, after tested in laboratory, gave $\overline{f_c} = 40$ MPa and $V = 0.12$. Suppose that, based on those core tests, the indirect test chosen was calibrated, giving the parameters $\hat{\beta}_0 = -24$ MPa, $\hat{\beta}_1 = 1.2$ MPa and $\hat{\sigma} = 2.0$ MPa. These are reasonable values for rebound hammer tests [2]. The routine described above gave $\alpha_2 = 1.08$, that is lower than α_1. Remember that in this example $\alpha_1 = 1{,}115$. So, in this case it is worth supplementing the core tests with the indirect test being considered, since $\alpha_2 < \alpha_1$. Obviously it is being assumed that the cores available are representative for the structure or structural element under study.

In Figure α_2 is plotted as a function of the number of cores n used to calibrate the indirect test. Figure 2 also shows α_1 in order to compare it with α_1. It can be observed that, for an indirect test with $\hat{\sigma} = 2.0$ MPa and for a concrete with $V = 0.12$, such an indirect test would only be useful if it was feasible to take from that structure up to about 8 cores. Above 8 cores the uncertainty associated to the core tests becomes smaller than the uncertainty of the indirect test, even for a large number of measurements with the indirect test.

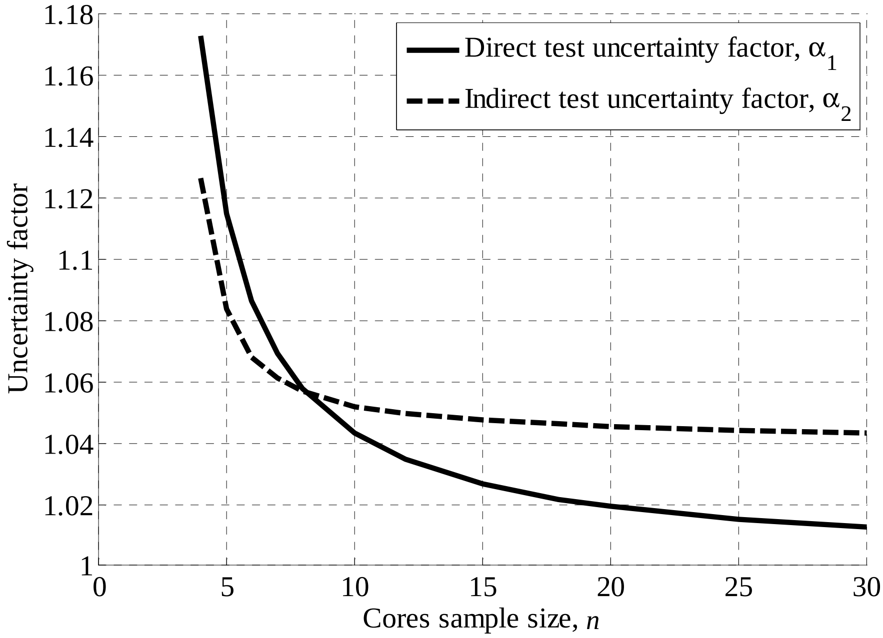

Figure 2: Direct and indirect test uncertainty factors for a concrete with $V = 0.12$ *and a indirect test with* $\hat{\sigma} = 2.0$ MPa.

In the following, similar curves will be drawn, but considering that the concrete is of inferior quality, with say $V = 0.18$. Figure 3 shows the results for an indirect test with $\hat{\sigma} = 2.0$ MPa. As it can be seen, the number of cores above which the indirect test is no longer beneficial rose to about 20. So, concrete with poor quality favours the use of indirect tests.

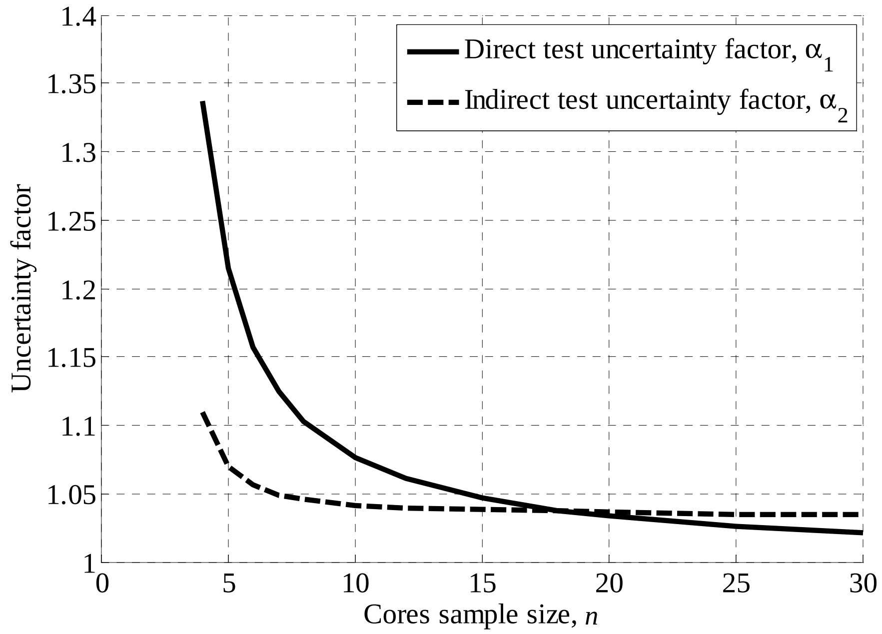

Figure 3: Direct and indirect test uncertainty factors for a concrete with $V = 0.18$ *and a indirect test with* $\hat{\sigma} = 2.0$ MPa.

Consider once again a concrete with $V = 0.12$, but suppose now that the precision of the indirect test is estimated by $\hat{\sigma} = 3.0$ MPa. As it can be seen from Figure 4, the uncertainty associated to the core tests solely is lower than the indirect test uncertainty, even for few cores. So, for the purpose of assessment the concrete strength, such an indirect test should not be used, at least as a supplement of core tests.

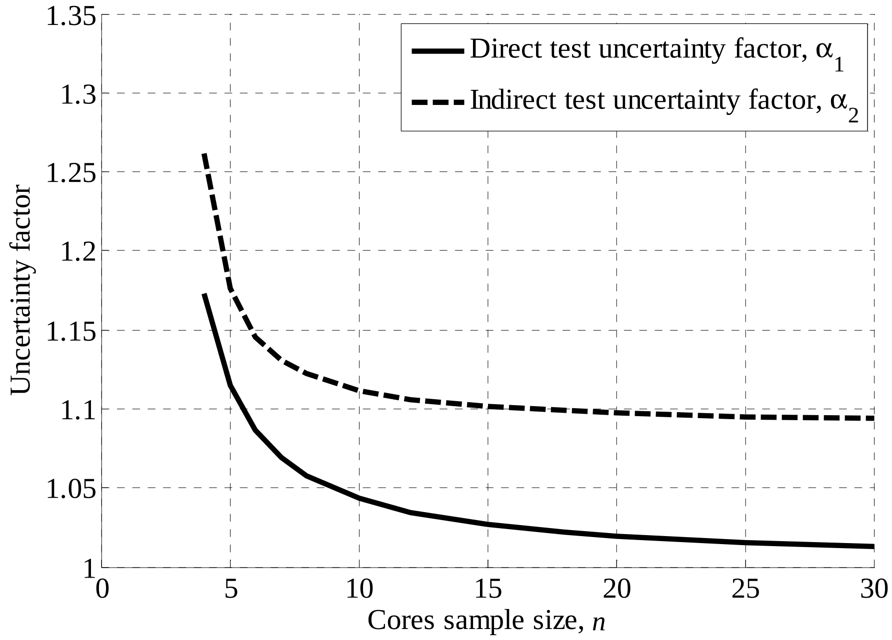

Figure 4: Direct and indirect test uncertainty factors for a concrete with $V = 0.12$ *and a indirect test with* $\hat{\sigma} = 3.0$ MPa.

4. Conclusions

Whenever the concrete strength of an existing structure is to be assessed, core testing constitutes the reference method, not only because it is the most accurate method, but also because it constitutes the base for calibration of indirect tests. In this paper it was shown how characteristic value of the concrete strength can be estimated taking into account the statistical uncertainty associated to a limited number of cores.

When the number of cores is small and the coefficient of variation of the concrete strength is high, the statistical uncertainty can be significant, reducing the estimate of the characteristic concrete strength. In this case it can be advantageous to supplement the core tests with an indirect test, properly calibrated using the core tests results. As shown in this paper, for a given concrete (characterized by a certain coefficient of variation) and for a given indirect test (characterized by a certain precision) there is a number of cores above which the statistical uncertainty associated to core tests solely is lower than the uncertainty introduced by the indirect test. So the use of that indirect test is only attractive if the number of cores that is practicable to take from the structure is smaller than that number.

It should be emphasized, however, that the conclusions drawn in this study are valid under two basic assumptions: first, the indirect test satisfies the requirements of the linear regression model and, second, the calibration is carried out specifically for the concrete to be assessed, i.e., any previous knowledge about the calibration parameters gained from others calibration operations is not used.

It is evident that if the indirect test being used had been calibrated for a similar structure, it would be reasonable to use the knowledge gained. This study can be easily extended to cover that situation. For example, suppose that a previous calibration was performed on a similar structure, having been used m cores. Thus the uncertainty factor associated to the indirect test could be re-evaluated using $n+m$ instead n in the predictive model.

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