UNIVERSITY OF LIVERPOOL

Department of Civil Engineering

PROBABILISTIC ASSESSMENT OF THE SAFETY OF COASTAL STRUCTURES

Thesis submitted in accordance with the requirements of the University of Liverpool for the Degree of Doctor in Philosophy

by

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TO TERRY

For all those times you stood by me
For all the truth you made me see...
Seawall overtopping: Cascais, Portugal.

Seawall overtopping: north Wirral coast, UK.
Dune erosion: Formby Point, UK.

Dune erosion: east coast, USA.
This thesis is about the safety of coastal structures. To date, applications of probabilistic methods in the design of coastal engineering works have been very limited. The present study concentrates on the probabilistic assessment of single failure modes using First Order Reliability Methods (FORM). FORM have been implemented in a computer program called PARASODE which has been developed as part of this research. The program has been used to study the failure mechanisms of seawall overtopping by waves and of dune erosion during a storm surge.

A brief review of available equations for predicting wave overtopping is followed by the development of a new model and re-analysis of a large set of existing overtopping data for simple seawalls having uniform seaward slopes of 1:1, 1:2 and 1:4. Both the new model and an earlier formulation are used as input to PARASODE. It is suggested that, in both cases, regression coefficients contained within the overtopping equations should be established using a robust regression technique such as Least Absolute Deviations (LAD). It is shown that the two overtopping models are little different in their ability to represent the data, but the new model is inherently better suited to describing low overtopping rates.

An example is given of the application of the new and existing overtopping models in predicting the freeboards necessary to limit discharges to specified values. This example shows that, for the small allowable rates associated with normal design conditions, the new model predicts seawall crest elevations which may be several metres lower than the values from the earlier model. Such differences may have significant financial and environmental consequences and are worthy of further investigation. Calculations using PARASODE show that the choice of overtopping model is also very important in the probability assessment of the safety of seawalls. The FORM sensitivity parameters demonstrate that the main influence on the variability of the probability of failure is generally provided by the uncertainty in the sea state. The accuracy of the FORM reliability algorithms used in PARASODE is confirmed by comparing with results provided by the simulation method of Latin Hypercube Sampling (LHS).

Dutch experience with regard to the probabilistic design of dunes is also examined. The computational procedures currently used in The Netherlands are based on an equilibrium profile model. They are not directly applicable to conditions along coasts such as that in Sefton, UK, where there is a much
weaker correlation that in The Netherlands between wave heights and water levels. Consequently, only some features of the Dutch methods are incorporated in PARASODE. Examples illustrate how nourishment can be used to decrease a dune’s failure probability caused by erosion during a storm surge. PARASODE is run in two modes. In mode 1, a nourishment width is chosen and the corresponding probability of failure is calculated. In mode 2, a probability of failure is input and a corresponding nourishment width is computed. These tests demonstrate the converse nature of modes 1 and 2 and show consistency between the results. The FORM sensitivity parameters show that the most important contributions to the resulting variance in the probability of failure are provided by uncertainties in the maximum water level during surge and the sea state.
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Wherever possible, the notation used in the text of this thesis follows the recommendations of the International Association of Hydraulic Research (IAHR, 1989). The notation used in PARASODE is defined within the program.

### Table 1: Notation.

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| a                | • Constant (Chapter 2)  
|                  | • Coefficient (Chapter 4) |
| A                | • Regression coefficient (Chapters 3, 5 & 6; Appendix D)  
|                  | • Area between the surge level and the parabolic part of Vellinga's profile (Appendix C) |
| Ac               | Accuracy of Vellinga's computational model |
| AN                | Anderson-Darling statistic |
| b                | • Coefficient (Chapter 4)  
|                  | • Regression parameter considered, b₀ or b₁ (Appendix A) |
| b₀               | Estimate of parameter β₀ |
| b₁               | Estimate of parameter β₁ |
| B                | • Regression coefficient (Chapters 3, 5 & 6; Appendix D)  
|                  | • Area between the surge level and the gradient 1:mt of Vellinga's profile (Appendix C) |
| BD               | Area of a depression |
| BetaAcc          | Relative accuracy of the reliability index |
| BH               | Area of a hump |
| C                | • Coefficient (Chapter 3)  
|                  | • Ratio of the maximum run-up to the significant height of the incident waves (=Rₘₐₓ/Hₛ) (Chapters 3 & 5)  
<p>|                  | • Erosion quantity (m³/m) above storm surge level (Chapter 4 &amp; Appendix C5) |
| CL               | Crest level above datum |
| Cd               | Discharge coefficient |
| Cov[X]           | Covariance matrix of X=(X₁,...,Xₙ) |
| Cov[X,Y]         | Covariance of X and Y |
| d                | Depth |
| dX               | Infinitely small integration step on variable X |
| dS               | Still-water-depth at the toe of the seawall |</p>
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| D               | • Design point (Chapter 2)  
|                 | • Matrix of eigenvalues of $C_{\text{ov}}[X]$ (Chapter 2)  
|                 | • Area between the two 1:md gradients, the surge level and the nourished profile (Appendix C) |
| Depth           | Depth of the most seaward point of the parabolic part of Vellinga's post-storm profile |
| DF              | Degrees of freedom |
| DP              | • Change in the initial profile (Chapter 4; Appendices B, D3 & D4)  
|                 | • Design Point (Appendix D2) |
| $D_N$           | Kolmogorov-Smirnov statistic |
| $D_{50}$        | Median grain size diameter (50% of the weight being finer) |
| e               | Estimate of the error term $\varepsilon$; also called residual from the regression line |
| $e_A$           | Parameter representing the degree of variability in regression coefficient $A$ |
| $e_B$           | Parameter representing the degree of variability in regression coefficient $B$ |
| E               | Area which lies between points (S9,T9), (S2,T2), the surge level and the nourished profile |
| $E[\ldots]$    | Expected value operator; it represents the expected value of its argument |
| Err             | Error in the balance between erosion and accretion |
| $\text{Err}_1$ | Error in the balance between $T_{\text{Surch}}$ and area D |
| $E_{\text{nc}}$| Encounter probability (probability that the $T_r$-year return load will be exceeded at least once during the design life or reference period) |
| $f(X_1,\ldots,X_N)$ | A function of the N variables, $X_i$, $i=1,\ldots,N$ |
| $f_X$           | Probability density function of variable $X$ |
| $f_X(x)$        | Probability density function of variable $X$ evaluated at point $X=x$ |
| $f_{X_1,\ldots,X_n}$ | Joint probability density function of variables $X_1,\ldots,X_N$ |
| $F$             | F statistic |
| $F(t)$          | A function of time (in the description of water surface elevation) |
| $F_{\text{Crit}}$ | Value of the F statistic corresponding to the one-tailed F distribution with $p$ and $N-p-1$ degrees of freedom, for a specific level of significance $\alpha$ |
| $F_X$           | • Cumulative distribution function of variable $X$  
|                 | • Input data cumulative distribution function of variable $X$ (Appendix A) |
| $F_X(x)$        | Cumulative distribution function of variable $X$ evaluated at point $X=x$ |
| $F_{X_1,\ldots,X_n}$ | The cumulative distribution function of the maximum intensity of action $X_i$ within the reference period, $T_{\text{ref}}$, which is subdivided into a number, $r_i$, of elementary time intervals for action $X_i$ |

**Table 1:** Notation (continued).

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<td>Fitted cumulative distribution function of variable $X$</td>
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<td>$F_X^F(x)$</td>
<td>Fitted cumulative distribution function of variable $X$ evaluated at point $X=x$</td>
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<tr>
<td>$F_{X_i}^r$</td>
<td>Cumulative distribution function of the intensity of the action $X_i$ raised to the power $r_i$</td>
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<td>$F_X^{-1}$</td>
<td>Inverse of the cumulative distribution function of variable $X$</td>
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<td>$F_X^{-1}(x)$</td>
<td>Inverse of the cumulative distribution function of variable $X$ evaluated at point $X=x$</td>
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</tr>
<tr>
<td>$GB$</td>
<td>Gust bumps</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Reference value for $G$</td>
</tr>
<tr>
<td>$h$</td>
<td>Maximum water level during storm surge</td>
</tr>
<tr>
<td>$H$</td>
<td>Wave height</td>
</tr>
<tr>
<td>$H_0$</td>
<td>Null hypothesis</td>
</tr>
<tr>
<td>$H_{rms}$</td>
<td>Root mean square wave height</td>
</tr>
<tr>
<td>$H_S$</td>
<td>Significant wave height</td>
</tr>
<tr>
<td>$H_S</td>
<td>H$</td>
</tr>
<tr>
<td>$H_{max}$</td>
<td>Upper limit on $H_R, H$ to account for depth limitations</td>
</tr>
<tr>
<td>$i, j$</td>
<td>Integers</td>
</tr>
<tr>
<td>$k$</td>
<td>An integer</td>
</tr>
<tr>
<td></td>
<td>Number of time-varying actions (Chapter 2 &amp; Appendix C6)</td>
</tr>
<tr>
<td></td>
<td>Coefficient (in the description of water surface elevation) (Chapter 3)</td>
</tr>
<tr>
<td></td>
<td>Number of classes into which the data is grouped for use of the Chi-Square test (Appendix A)</td>
</tr>
<tr>
<td>$K$</td>
<td>Armour stability coefficient (in Iribarren's equation)</td>
</tr>
<tr>
<td>$K_D$</td>
<td>Armour stability coefficient (in Hudson's equation)</td>
</tr>
<tr>
<td>$K_{X_R}$</td>
<td>Coefficient defining the fractile which corresponds to the characteristic value of the resistance variable, $X_R$</td>
</tr>
<tr>
<td>$K_{X_S}$</td>
<td>Coefficient defining the fractile which corresponds to the characteristic value of the load variable, $X_S$</td>
</tr>
<tr>
<td>$\ell$</td>
<td>An integer</td>
</tr>
<tr>
<td>$Le$</td>
<td>Length of the parabolic part of Vellinga's post-storm profile</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Airy wavelength at the toe of the seawall calculated using the mean zero-crossing wave period</td>
</tr>
<tr>
<td>$L_{op}$</td>
<td>Airy wavelength in deep water calculated using the period of peak spectral density ($= gT_p^2 / 2\pi$)</td>
</tr>
</tbody>
</table>

**Table 1:** Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{os}$</td>
<td>Airy wavelength in deep water calculated using the significant wave period ($= gT_s^2 / 2\pi$)</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Airy wavelength at the toe of the seawall calculated using the period of peak spectral density</td>
</tr>
<tr>
<td>$L_S$</td>
<td>Airy wavelength at the toe of the seawall calculated using the significant wave period</td>
</tr>
</tbody>
</table>
| $m$             | - Total number of time intervals per year (Chapter 2)
|                 | - Number of parameters of a probability distribution estimated from a data set (Appendix A) |
| $1:md$          | Gradient of the eroded dune face |
| $1:mnour$       | Gradient of the nourished face |
| $1:mt$          | Gradient of the toe of the post-storm profile |
| MaxIter         | Maximum number of iterations in a FORM calculation |
| MAD$_{Reg}$     | Mean absolute deviations regression |
| MAD$_{Res}$     | Mean absolute deviations residual |
| MS$_{Reg}$      | Mean square regression |
| MS$_{Res}$      | Mean square residual |
| $n$             | Ratio of the prototype value to the model value |
| nourtlev        | Nourishment top level |
| nourwidt        | Nourishment width at top level |
| $n_d$           | Depth scale (for beach profile and hydraulic conditions) |
| $n_l$           | Length scale for beach profile |
| $n_t$           | Time scale |
| $n_w$           | Scale for the fall velocity of the sand |
| $n_H$           | Wave height scale |
| $n_L$           | Wavelength scale |
| $n_T$           | Wave period scale |
| $N$             | - Number of variables (Chapter 2)
|                 | - Number of run-up values (Chapter 3)
|                 | - Number of observed data points in a sample (Appendix A) |
| NPch            | Number of points to be changed in the initial profile |
| NPD             | Initially, the number of points defining the initial profile; then, the number of points defining the nourished profile |
| NPV             | Total number of points defining Vellinga's profile |
| NumDep          | Number of depressions |
| NumHump         | Number of humps |
| $N_j$           | Number of data points in the $j$th class (in the Chi-Square test) |
| $N_S$           | Stability number of armour stones |
| $N_{WO\%}$      | Percentage of waves passing over the crest of a structure |

Table 1: Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_X$</td>
<td>Number of data points, $X_i$'s, less than or equal to $x$, $i=1,...,N$</td>
</tr>
<tr>
<td>$NR_{ji}$</td>
<td>Power to which each basic distribution, $F_{X_i}$, should be raised, for each time-varying action $X_i$, $i=1,...,k$, and for each combination $j$, $j=1,...,k$</td>
</tr>
</tbody>
</table>
| $p$              | • Probability of occurrence of a time-varying action in each elementary time interval, $\tau$ (Chapter 2)  
                              • Number of independent variables in the model (Appendix A) |
| $P_i$            | Expected proportion of the data points that would fall in the $j$th class if sampling was done from the fitted distribution (in the Chi-Square test) |
| $P_f$            | Probability of failure |
| $q$              | Instantaneous discharge of water over unit length of seawall |
| $Q$              | • Mean overtopping discharge over unit length of seawall (Chapter 3)  
                              • Area between the surge level and the nourished profile below surge (Appendix C) |
| $Q_p$            | Peak overtopping discharge |
| $Q_{PRED}$       | Predicted mean overtopping discharge over unit length of seawall |
| $Q_*$            | Dimensionless overtopping discharge |
| $r$              | Effective roughness of the seawall's front slope |
| $r_i$            | Number of elementary time intervals during the design life or reference period of a structure for the time-varying action $X_i$ |
| $R$              | • Reliability (Chapter 2)  
                              • Most landward position to which a dune profile has been eroded during a storm surge (Chapter 4) |
| RD               | Retreat distance |
| ReqBetaAcc       | Required relative accuracy of the reliability index |
| ReqZAcc          | Required accuracy of the failure function |
| $R_c$            | Seawall's freeboard (the height of the crest of the structure above the still-water-level) |
| $R_{max}$        | Maximum run-up (=CH$_s$) |
| $(R_{max})_{p\%}$| $p\%$ confidence value of the estimated maximum run-up |
| $R_S$            | Significant wave run-up |
| $R_2$            | Run-up exceeded by only 2% of the incident waves |
| $R^2$            | Coefficient of determination |
| $R_a^2$          | Adjusted statistic which attempts to correct $R^2$ to more closely reflect the goodness of fit of the model in the population |
| $R_*$            | Dimensionless freeboard |

**Table 1:** Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>sup[...]</td>
<td>Smallest upper bound on members of the set [...]</td>
</tr>
<tr>
<td>si</td>
<td>Number of elementary time intervals during a period of time shorter than the reference period of a structure for the time-varying action X_i</td>
</tr>
<tr>
<td>sm</td>
<td>Deep water wave steepness calculated using T_m</td>
</tr>
<tr>
<td>sp</td>
<td>Deep water wave steepness calculated using T_p</td>
</tr>
<tr>
<td>S</td>
<td>Sample standard error of the estimate</td>
</tr>
<tr>
<td>SD</td>
<td>Surge duration</td>
</tr>
<tr>
<td>SDep</td>
<td>Cumulative area of the depressions starting from the seaward end of the profiles</td>
</tr>
<tr>
<td>SEb</td>
<td>Standard error of the parameter b considered, b_0 or b_1, (or estimate of the standard error)</td>
</tr>
<tr>
<td>SEb_0</td>
<td>Standard error of b_0 (or estimate of the standard error of b_0)</td>
</tr>
<tr>
<td>SEb_1</td>
<td>Standard error of b_1 (or estimate of the standard error of b_1)</td>
</tr>
<tr>
<td>SEInd^Y</td>
<td>Standard error of the individual prediction at a specific value X_o of X (or estimated standard error of the individual prediction at a specific value X_o of X)</td>
</tr>
<tr>
<td>SE^Y</td>
<td>Standard error of the predicted mean value of Y at a specific value X_o of X (or estimated standard error of the predicted mean value of Y at a specific value X_o of X)</td>
</tr>
<tr>
<td>SHump</td>
<td>Cumulative area of the humps starting from the seaward end of the profiles</td>
</tr>
<tr>
<td>Smooth</td>
<td>Smoothing coefficient for the iteration process</td>
</tr>
<tr>
<td>Surcharge</td>
<td>A coefficient for the surcharge on the erosion area above surge level</td>
</tr>
<tr>
<td>SurchEros</td>
<td>Surcharge on erosion area C above surge level to take into account the effects of the storm surge duration, of the gust bumps and of the accuracy of the computation</td>
</tr>
<tr>
<td>SurchLongT</td>
<td>Surcharge on erosion area C to take into account the effect of a gradient in the longshore transport rate</td>
</tr>
<tr>
<td>SurD</td>
<td>Total surcharge distance</td>
</tr>
<tr>
<td>S1</td>
<td>X-coordinate of the intersection point between the nourished profile and the surge level</td>
</tr>
<tr>
<td>S2</td>
<td>X-coordinate of the intersection point between the nourished profile and the gradient 1:mt of Vellinga's profile</td>
</tr>
<tr>
<td>S3</td>
<td>X-coordinate of the intersection point between the nourished profile and the gradient 1:md of Vellinga's profile</td>
</tr>
<tr>
<td>S4</td>
<td>X-coordinate of the intersection point between the nourished profile and the surcharge gradient, 1:md</td>
</tr>
<tr>
<td>S8</td>
<td>X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile</td>
</tr>
<tr>
<td>S9</td>
<td>X-coordinate of the point where the parabolic part of Vellinga's profile finishes</td>
</tr>
</tbody>
</table>

Table 1: Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10 X</td>
<td>X-coordinate of the point of intersection between the surge level and the gradient 1:md of the surcharge</td>
</tr>
<tr>
<td>SADReg</td>
<td>Regression sum of absolute deviations</td>
</tr>
<tr>
<td>SADRes</td>
<td>Residual sum of absolute deviations</td>
</tr>
<tr>
<td>SSReg</td>
<td>Regression sum of squares</td>
</tr>
<tr>
<td>SSRes</td>
<td>Residual sum of squares</td>
</tr>
<tr>
<td>SX</td>
<td>Standard deviation of the X values</td>
</tr>
<tr>
<td>S1</td>
<td>Parallel-series system</td>
</tr>
<tr>
<td>S11, S12, S13</td>
<td>Sub-systems of system S1</td>
</tr>
<tr>
<td></td>
<td>t Time (Chapters 2, 3 &amp; 4)</td>
</tr>
<tr>
<td></td>
<td>t statistic (Appendix A)</td>
</tr>
<tr>
<td>tanα</td>
<td>Tangent of the angle of seawall front slope measured from horizontal (Chapters 3 &amp; 5; Appendix D2)</td>
</tr>
<tr>
<td></td>
<td>Tangent of the angle of the armour slope measured from horizontal (Chapter 3)</td>
</tr>
<tr>
<td>tCrit</td>
<td>Value of the t statistic corresponding to the two-tailed Student's t distribution with N-p-1 degrees of freedom, for a specific level of significance α</td>
</tr>
<tr>
<td>T</td>
<td>Wave period</td>
</tr>
<tr>
<td>TL</td>
<td>Toe level above datum</td>
</tr>
<tr>
<td>TR</td>
<td>Target value for each FORM calculation</td>
</tr>
<tr>
<td>TSS</td>
<td>Total sum of squares</td>
</tr>
<tr>
<td>TSurch</td>
<td>Total surcharge on erosion area C which is the sum of the surcharges SurchEros and SurchLongT</td>
</tr>
<tr>
<td>T1</td>
<td>Y-coordinate of the intersection point between the nourished profile and the surge level</td>
</tr>
<tr>
<td>T2</td>
<td>Y-coordinate of the intersection point between the nourished profile and the gradient 1:mt of Vellinga’s profile</td>
</tr>
<tr>
<td>T3</td>
<td>Y-coordinate of the intersection point between the nourished profile and the gradient 1:md of Vellinga’s profile</td>
</tr>
<tr>
<td>T4</td>
<td>Y-coordinate of the intersection point between the nourished profile and the surcharge gradient, 1:md</td>
</tr>
<tr>
<td>T9</td>
<td>Y-coordinate of the point where the parabolic part of Vellinga’s profile finishes</td>
</tr>
<tr>
<td>Tm</td>
<td>Mean zero-crossing wave period</td>
</tr>
<tr>
<td>Tp</td>
<td>Wave period corresponding to peak spectral density</td>
</tr>
<tr>
<td>Tr</td>
<td>Return period</td>
</tr>
<tr>
<td>Tref</td>
<td>Design life or reference period of a structure</td>
</tr>
<tr>
<td>Ts</td>
<td>Significant wave period</td>
</tr>
</tbody>
</table>
| u                | Variable in the definition of the incomplete Beta function; 
|                  | u=(X-x₁)/(x₂-x₁) |
| U                | Normalised variable |

Table 1: Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Matrix of eigenvectors of $C_{xx}[X]$</td>
</tr>
<tr>
<td>Var[X]</td>
<td>Variance of $X$ ($\sigma^2_X$)</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Peak volume of water in an individual wave</td>
</tr>
<tr>
<td>$w$</td>
<td>Sediment fall velocity for a given water temperature</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight of armour stones</td>
</tr>
<tr>
<td>$x$</td>
<td>Particular value of the variable $X$</td>
</tr>
<tr>
<td>$x_1$, $x_2$</td>
<td>Respectively, the lower and upper limits on $X$ for a Beta distribution</td>
</tr>
<tr>
<td>$X$</td>
<td></td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Basic variable (Chapter 2 &amp; Appendix D)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$X$-coordinate (Chapters 4, 5 &amp; 6; Appendix C5)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>New unsmoothed value of $X$ (Chapter 5)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Independent variable or $X$-coordinate of an observed data point in a sample (Appendix A)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Variable (Appendix C3)</td>
</tr>
<tr>
<td>$XB$</td>
<td>$X$-coordinate of the intersection point between the changed profile and the surge level</td>
</tr>
<tr>
<td>$XD_{End}$</td>
<td>$X$-coordinate of the end point of a depression</td>
</tr>
<tr>
<td>$XD_{Start}$</td>
<td>$X$-coordinate of the starting point of a depression</td>
</tr>
<tr>
<td>$XH_{End}$</td>
<td>$X$-coordinate of the end point of a hump</td>
</tr>
<tr>
<td>$XH_{Start}$</td>
<td>$X$-coordinate of the starting point of a hump</td>
</tr>
<tr>
<td>$XM$</td>
<td>$X$-coordinate of the most seaward point at the nourishment top level</td>
</tr>
<tr>
<td>$X_{Max}$</td>
<td>Maximum value of variable $X$</td>
</tr>
<tr>
<td>$X_{Min}$</td>
<td>Minimum value of variable $X$</td>
</tr>
<tr>
<td>$XN$</td>
<td>$X$-coordinate of the intersection point between the changed profile and the nourishment top level</td>
</tr>
<tr>
<td>$X_0$</td>
<td></td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Point of truncation of a probability distribution, if the distribution is truncated (Chapter 5)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Particular value of $X$ (Appendix A)</td>
</tr>
<tr>
<td>$XP$</td>
<td>Initially, the $X$-coordinate of the points defining the initial profile; then, the $X$-coordinate of the points defining the changed profile; finally, the $X$-coordinate of the points defining the nourished profile</td>
</tr>
<tr>
<td>$XPV$</td>
<td>$X$-coordinate of the points defining Vellinga's profile</td>
</tr>
<tr>
<td>$XQ$</td>
<td>$X$-coordinate of the intersection point between the nourishment slope $1:mnour$ and the changed profile</td>
</tr>
<tr>
<td>$X_{max}$</td>
<td>Seaward $X$-limit of Vellinga's parabolic profile</td>
</tr>
<tr>
<td>$X_q$</td>
<td>Quantile of the input data cumulative distribution function</td>
</tr>
<tr>
<td>$X_{New}$</td>
<td>New smoothed value of $X$</td>
</tr>
<tr>
<td>$X_{Old}$</td>
<td>Value of $X$ in the previous iteration</td>
</tr>
<tr>
<td>$X_R$</td>
<td></td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Resistance variable (Chapter 2)</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Distance from the origin to the most landward position to which the dune profile has been eroded during a storm surge (Chapter 4)</td>
</tr>
<tr>
<td>$X_{R_{ch}}$</td>
<td>Characteristic value of the resistance variable, $X_R$</td>
</tr>
<tr>
<td>$X_S$</td>
<td>Load variable</td>
</tr>
</tbody>
</table>

**Table 1:** Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{S\text{in}}$</td>
<td>Characteristic value of the load variable, $X_S$</td>
</tr>
<tr>
<td>$X_T$</td>
<td>Target X-coordinate</td>
</tr>
<tr>
<td>$X^*$</td>
<td>Linearization point used in the Level II methods</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>Mean of the X values of the data points in a sample</td>
</tr>
<tr>
<td>$X_{\text{New}}^*$</td>
<td>New linearization point</td>
</tr>
<tr>
<td>$X_q^F$</td>
<td>Quantile of a fitted cumulative distribution function</td>
</tr>
</tbody>
</table>
| $Y$              | • Non-correlated variable (Chapter 2 & Appendix D)  
                   • Y-coordinate (Chapters 4, 5 & 6; Appendix C5)  
                   • Dependent variable or Y-coordinate of an observed data point in a sample (Appendix A) |
| $Y_{\text{B}}$   | Y-coordinate of the intersection point between the changed profile and the surge level |
| $Y_{\text{End}}$ | Y-coordinate of the end point of a depression |
| $Y_{\text{Start}}$ | Y-coordinate of the starting point of a depression |
| $Y_{\text{HEnd}}$ | Y-coordinate of the end point of a hump |
| $Y_{\text{HStart}}$ | Y-coordinate of the starting point of a hump |
| $Y_{\text{M}}$   | Y-coordinate of the most seaward point at the nourishment top level |
| $Y_{\text{N}}$   | Y-coordinate of the intersection point between the changed profile and the nourishment top level |
| $Y_{\text{P}}$   | Initially, the Y-coordinate of the points defining the initial profile; then, the Y-coordinate of the points defining the changed profile; finally, the Y-coordinate of the points defining the nourished profile |
| $Y_{\text{PT9}}$ | Y-coordinate of the point in the nourished profile which has $X=S9$ |
| $Y_{\text{PV}}$  | Y-coordinate of the points defining Vellinga’s profile |
| $Y_{\text{Q}}$   | Y-coordinate of the intersection point between the nourishment slope 1:mnour and the changed profile |
| $Y_{\text{max}}$ | Seaward Y-limit of Vellinga’s parabolic profile |
| $Y_{\text{med}}$ | Median of the Y values of the data points in a sample |
| $\hat{Y}$        | Predictive value of Y |
| $\bar{Y}$        | Mean of the Y values of the data points in a sample |
| $z$              | z statistic |
| $z_{\text{Crit}}$ | Value of the z statistic corresponding to the two-tailed standard Normal distribution, for a specific level of significance $\alpha$ |
| $Z$              | Failure function or limit state function |
| $Z_i$            | Value of the fitted cumulative distribution function evaluated at $x_i$, $F_X^F(x_i)$ |
| $Z_{\text{Resid}}$ | Standardised residuals |
| $Z^*$            | Value of Z evaluated at the point $X^*$ |

Table 1: Notation (continued).
<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
</table>
| $\alpha$         | • Sensitivity factor (Chapters 2, 5 & 6; Appendices D2 & D4)  
|                   | • Angle of the seawall front slope measured from the horizontal (Chapters 3 & 5)  
|                   | • Armour slope (Chapter 3)  
|                   | • Significance level (Appendix A) |
| $\beta$          | • Reliability index (Chapter 2)  
|                   | • Angle of wave approach measured from the normal to the seawall (Chapter 3)  
|                   | • Parameter of a model, $\beta_0$ or $\beta_1$ (Appendix A) |
| $\beta(\zeta, \lambda)$ | Beta function; $\beta(\zeta, \lambda) = \Gamma(\zeta) \Gamma(\lambda) / \Gamma(\zeta + \lambda)$ |
| $\beta_0(\zeta, \lambda) / \beta(\zeta, \lambda)$ | Incomplete Beta function |
| $\beta_{\text{new}}$ | Reliability index of the current iteration |
| $\beta_{\text{old}}$ | Reliability index of the previous iteration |
| $\beta_0$ | Unknown parameter of a model (also called the intercept) |
| $\beta_1$ | Unknown parameter of a model (also called the slope) |
| $\gamma$         | • Partial coefficient (Chapter 2)  
|                   | • Reduction factor to account for influences of berms, roughness, shallow water and oblique wave attack on wave run-up and overtopping (Chapter 3) |
| $\Gamma(\zeta)$ | Gamma function; if $\zeta$ is an integer value, $\Gamma(\zeta) = (\zeta - 1)!$ |
| $\Gamma(\zeta, \lambda) / \Gamma(\zeta)$ | Incomplete Gamma function |
| $\Delta h$       | A short-term increase in the water level due to gust bumps and squall oscillations |
| $\Delta C$       | Increase in the volume of erosion due to $\Delta h$ |
| $\Delta SH$      | Increased amount of erosion due to a smoothed hydrograph with a maximum $\Delta h$ higher |
| $\Delta X$       | Finite discrete integration step on $X$ |
| $\epsilon$       | Random error term which takes into account the fact that a model does not exactly describe reality |
| $\zeta$          | Parameter of probability distributions |
| $\eta$           | • Water surface elevation above still-water-level at the seawall (Chapter 3)  
|                   | • Parameter of probability distributions (Appendix C) |
| $\partial Z / \partial X^*$ | Partial derivative of $Z$ with respect to $X$, evaluated at the point $X^*$ |
| $\lambda$        | Parameter of probability distributions |
| $\mu$            | Mean value |
| $\mu_f$          | Coefficient of friction between the armour stones (in Iribarren’s equation) |
| $\mu_{X_0}$      | Mean value of the approximate Normal distribution according to the Rackwitz & Fiessler (1978) approximation |

Table 1: Notation (continued).
### List Of Notation

<table>
<thead>
<tr>
<th>NOTATION IN TEXT</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_p$</td>
<td>Surf similarity parameter calculated using the period of peak spectral density ($= \tan \alpha / \sqrt{H_s / L_{op}}$)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>3.14159....</td>
</tr>
</tbody>
</table>
| $\rho$           | - Correlation coefficient (Chapters 2 & 6; Appendices C6 & D)  
                   - Water density (Chapter 3) |
| $\rho_S$         | Density of armour stones |
| $\sigma$         | Standard deviation |
| $\sigma_N$       | Standard deviation of the approximate Normal distribution according to the Rackwitz & Fiessler (1978) approximation |
| $\tau_i$         | Length of the elementary time interval for the time-varying action, $X_i$ |
| $\varphi$        | Probability density function for the standard Normal distribution |
| $\phi$           | Natural angle of repose of the armour slope material |
| $\Phi$           | Cumulative distribution function for the standard Normal distribution |
| $\Phi^{-1}$      | Inverse function of $\Phi$ |
| $\chi^2$         | Chi-square statistic |
| $\chi^2_{\text{Crit}}$ | Value of the $\chi^2$ statistic corresponding to the one-tailed $\chi^2$ distribution with N-p-1 degrees of freedom, for a specific level of significance $\alpha$ |

**Table 1:** Notation (continued).
# LIST OF ABBREVIATIONS

## ABBREVIATIONS | DEFINITION
--- | ---
A-D | Anderson-Darling
AIME | American Institution Of Mechanical Engineers
ANOLAD | Analysis Of Least Absolute Deviations
ANOVA | Analysis Of Variance
ASCE | American Society Of Civil Engineers
ASME | American Society Of Mechanical Engineers
BS | British Standard
BSI | British Standards Institution
CDF | Cumulative Distribution Function
CERC | Coastal Engineering Research Center
CIAD | Association For Computer Applications In Applied Engineering
CIRIA | Construction Industry Research And Information Association
CUR | Centre For Civil Engineering Research And Codes / Centre For Civil Engineering Research Codes And Specifications
DUNE | Name of Dutch computer program
DUNEPROB | Name of Dutch computer program
FOMVA | First Order Mean Value Approach
FORM | First Order Reliability Method
FORTRAN | FORmula TRANslation
HR | Hydraulics Research
H&R | Hedges And Reis
IABSE | International Association For Bridge And Structural Engineering
IAHR | International Association Of Hydraulic Research
ICCE | International Conference On Coastal Engineering
ICE | Institution Of Civil Engineers
ICOSSAR | International Conference On Structural Safety And Reliability
ICTM | Institute For Marine Science And Technologies
IEEE | Institute Of Electrical And Electronic Engineers
IFIP | International Federation For Information Processing
IML | Interactive Matrix Language
IOS | Institute Of Oceanographic Sciences
IWEM | Institution Of Water And Environmental Management

Table 2: Abbreviations.
<table>
<thead>
<tr>
<th>ABBREVIATIONS</th>
<th>DEFINITION</th>
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<tbody>
<tr>
<td>jpdf</td>
<td>joint probability density function</td>
</tr>
<tr>
<td>K-S</td>
<td>Kolmogorov-Smirnov</td>
</tr>
<tr>
<td>LAD</td>
<td>Least Absolute Deviations</td>
</tr>
<tr>
<td>LBD</td>
<td>Liverpool Bay Datum</td>
</tr>
<tr>
<td>LHS</td>
<td>Latin Hypercube Sampling</td>
</tr>
<tr>
<td>LNEC</td>
<td>National Laboratory Of Civil Engineering</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
</tr>
<tr>
<td>MAFF</td>
<td>Ministry Of Agriculture Fisheries And Food</td>
</tr>
<tr>
<td>MLEs</td>
<td>Maximum Likelihood Estimators</td>
</tr>
<tr>
<td>MSL</td>
<td>Mean-Sea-Level</td>
</tr>
<tr>
<td>NAG</td>
<td>Numerical Algorithms Group Limited</td>
</tr>
<tr>
<td>NAP</td>
<td>Standard Amsterdam Datum</td>
</tr>
<tr>
<td>NATO</td>
<td>North Atlantic Treaty Organisation</td>
</tr>
<tr>
<td>NERC</td>
<td>Natural Environment Research Council</td>
</tr>
<tr>
<td>NRA</td>
<td>National Rivers Authority</td>
</tr>
<tr>
<td>OD</td>
<td>Ordnance Datum</td>
</tr>
<tr>
<td>OTC</td>
<td>Offshore Technology Conference</td>
</tr>
<tr>
<td>PARASODE</td>
<td>Probabilistic Assessment Of Risks Associated With Seawall Overtopping And Dune Erosion</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PIANC</td>
<td>Permanent International Association Of Navigation Congresses</td>
</tr>
<tr>
<td>POL</td>
<td>Proudman Oceanographic Laboratory</td>
</tr>
<tr>
<td>P-P</td>
<td>Probability-Probability</td>
</tr>
<tr>
<td>Q-Q</td>
<td>Quantile-Quantile</td>
</tr>
<tr>
<td>SAS</td>
<td>Statistical Analysis System</td>
</tr>
<tr>
<td>SI</td>
<td>Système International D'Unités</td>
</tr>
<tr>
<td>SORM</td>
<td>Second Order Reliability Method</td>
</tr>
<tr>
<td>SPE</td>
<td>Society Of Petroleum Engineers</td>
</tr>
<tr>
<td>SPSS</td>
<td>Statistical Program For Social Sciences</td>
</tr>
<tr>
<td>SSL</td>
<td>Storm Surge Level</td>
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<tr>
<td>SWL</td>
<td>Still-Water-Level</td>
</tr>
<tr>
<td>TACPI</td>
<td>Technical Advisory Committee On Protection Against Inundation</td>
</tr>
<tr>
<td>TAW</td>
<td>Technical Advisory Committee On Water Defences</td>
</tr>
<tr>
<td>2D</td>
<td>Two-Dimensional</td>
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<tr>
<td>3D</td>
<td>Three-Dimensional</td>
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Table 2: Abbreviations (continued).
1 INTRODUCTION

1.1 Importance Of The Research And Its Main Objectives

A significant proportion of the world's population lives in areas at risk from coastal erosion and inundation by the sea. For example, the coastline of England and Wales is approximately 4500 km long (Birks, 1993; MAFF, 1993c). Yet, despite having significant lengths of high rocky cliffs in the west, there are large areas in the south and east below the highest sea levels. A survey (NRA, 1991) showed a total of nearly 1300 km of coastal structures protecting low-lying areas. About a quarter of the total coast has been developed for housing, industry, or some other purpose (MAFF, 1993a). Over 5% of the population and of the nation's industry is in areas below the mean annual maximum water level (approximately the 5 m Ordnance Survey contour) and many of these areas are protected by structures (Birks, 1993). Over 50% of agricultural land is also below this level and is dependent on drainage and/or flood defence in some way to maintain its productivity. A number of cities, including London, have significant defences against river or tidal flooding, and coastal towns such as Blackpool are defended against flooding by the sea (MAFF, 1993c). Without defences, urban areas with key infrastructure, businesses, homes, and agricultural and recreational land would all be vulnerable to flooding and coastal erosion. Also, a number of historic sites and buildings, some of which are protected by statute, are at risk, together with environmentally valuable areas such as Sites of Special Scientific Interest.

CIRIA (1986b) stated that the replacement value of existing coastal structures in the UK was about £4000 million, with an average cost of £2.5 million per km for the country as a whole. Many structures are more than 100 years old and in need of replacement. Maintenance costs vary from about £500 to £2400 per km (CIRIA, 1986a). Although these values relate to the 1980s, they still indicate the considerable investment involved in the provision of coastal structures.

In The Netherlands, the River Meuse floods of 1993 and 1995 caused economic damage of 250 and 400 million Dutch guilders, respectively (Dirkson, 1996). In 1993, 8000 inhabitants had to leave their homes because
of the high water levels. In 1995, the stability of the dikes along the River Rhine could no longer be guaranteed and 0.2 million people had to be evacuated. Fortunately, none of the dikes failed.

Climatic changes over the next few decades may cause a rise in sea levels and increased storminess (Doornkamp, 1990; Vrijling, 1990; Stive et al, 1991; Townend, 1994b; Samuels & Brampton, 1996). As pointed out by Hedges (1993), substantial erosion of beaches may be expected, together with an increased flow of water over coastal structures (wave overtopping) and damage to these structures. Climatic changes may also increase vulnerability to inland flooding if there are alterations to the frequency or intensity of rainfall patterns combined with greater difficulties in river discharge to the sea if the sea level has risen (MAFF, 1993c). It is important to ensure that coastal structures are designed and managed within a framework which accounts for likely future climate changes as these changes will occur within the lifetime of current coastal defence schemes (Sorensen, 1991; Naden et al, 1996; Simm et al, 1996).

Clearly, there is a need to minimise risk to life and protect natural and man-made assets by providing defences against inundation and erosion. Furthermore, it is important that coastal structures are planned in the most environmentally sensitive manner whilst also providing an appropriate level of protection. Complete safety against flooding or erosion is unattainable. A balance has to be struck between costs and benefits to a nation as a whole. It is important that coastal defence policy and practices contribute to wider social, economic and environmental objectives. Unfortunately, anyone whose work is related to coastal structures cannot fail to be aware of the severe damage which has been inflicted on some large structures during the last few decades (Harlow, 1980; Sorensen & Jensen, 1985; Burcharth, 1987). An example is the failure of the Sines breakwater, Portugal. Questions then arise concerning the effectiveness of existing structures and of the design methods which have been commonly applied. The traditional approach to design may be inadequate and unsatisfactory, not only from an engineering stand-point but also from an economic point of view (Mol et al, 1983). That is why the probabilistic methods used in other areas of engineering have been applied in assessing the levels of safety provided by existing and new coastal structures. These methods can help the designer achieve a balanced approach in which most effort is put into addressing those parameters that
make the greatest contribution to the total probability of failure. In other words, if limited finance is available for reducing the overall failure probability of a structure, it should be invested in those components which show the largest reduction in overall failure probability for the given amount of money (Mol et al, 1984).

The recent shift towards probabilistic design in coastal engineering (Thompson & Scheffner, 1996) has been pioneered by the Dutch. They have applied the methods to natural and man-made structures, such as breakwaters, dikes and dunes. The scale and importance of coastal defence in The Netherlands has encouraged this new approach. But practice in other countries is also moving in the same direction. In this connection, the main objective of the present research is to assess the safety of coastal structures by means of probabilistic methods, with particular reference to wave overtopping of seawalls and to dune erosion. A secondary objective is to increase interest within the maritime community in the use of probabilistic techniques. Note that there is already clear cooperation in this field, evident in such multi-national publications as PIANC (1992).

1.2 Structure Of The Thesis

The structure of the present research is shown in Figure 1.1. Chapter 2 contains a review of concepts and methods of probabilistic analysis and of the state of the art in their application to the design of coastal structures. At present, applications are limited and available knowledge on coastal structures is insufficient to enable the probabilistic analysis of a whole structure to be carried out in full. Therefore, it was decided to concentrate this study on methods for the probabilistic assessment of single failure modes. Special attention is given to the particular methods used in developing a computer program, PARASODE, and in validating the results of the program using a commercial software package, @Risk. PARASODE is used to study the failure mechanisms of wave overtopping of seawalls and dune erosion.

Chapter 3 is generally concerned with wave overtopping of seawalls. It starts with a brief review of the subject, including the models currently used in predicting overtopping and the permissible levels of overtopping. Then, an
alternative model is presented; this is the H&R model. This new model is conceived through theoretical considerations and special care has been taken to consider the appropriate physical boundary conditions. A re-analysis of Owen's data (Hydraulics Research Station, 1980; Owen, 1982a) for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, is also presented. The H&R and Owen models are used in this re-analysis and represent part of the input to PARASODE. The implications of the new model for seawall freeboards are then discussed. Finally, the reliabilities of the two models are assessed.

Chapter 4 focuses upon the Dutch experience in the probabilistic assessment of dune erosion during a storm surge. Firstly, a review of some key references is presented. Secondly, the current application of Vellinga's and Steetzel's models is discussed. Since Vellinga's model is currently the one used in The Netherlands for probabilistic calculations, the two Dutch computer programs (DUNEPROB and DUNE) which rely on its use are then presented. Finally, the applicability of these programs in the British context is examined.

The outcome of the literature review on probabilistic methods (Chapter 2) and the formulation of the failure modes of wave overtopping of seawalls (Chapter 3) and dune erosion (Chapter 4) have been assembled in the form of a program, PARASODE, to undertake probabilistic calculations. The program, its input and output, are described in Chapter 5. For the failure mode of wave overtopping of seawalls, the results obtained from PARASODE have been validated using the software package @Risk which is also briefly introduced in Chapter 5.

Chapter 6 illustrates the application of PARASODE: the first set of examples relates to wave overtopping of seawalls; the second relates to dune erosion. The results obtained from PARASODE are discussed and, for overtopping, they are validated using @Risk.

The most important conclusions arising out of this research are presented in Chapter 7. This chapter also provides the recommendations for further research.
Figure 1.1: Flowchart of the structure of the thesis.
2 PROBABILISTIC METHODS FOR DESIGN AND ASSESSMENT

2.1 Introduction

Conventional practice for the design of coastal structures is essentially deterministic (Mol et al., 1983; Melchers, 1987; Burcharth, 1992): the resistance of a structure should exceed the load by an appropriate margin which is an indication of the level of safety (Townend, 1994a). This margin is required to counter lack of knowledge and uncertainties with respect to resistance, load and other factors (Lee & Mays, 1983). It is based mainly on experience rather than on quantification of the unknowns and uncertainties. As a result, some structures may be designed to unwarranted levels of conservatism but, conversely, other structures will be subject to unacceptably high risks of failure. Using conventional design practice, it is not possible to determine the extent of under-design or over-design relative to an acceptable level of risk (Meadowcroft et al., 1996).

The design load is usually defined on a probabilistic basis. For example, it might be the value \( x \) of the variable load, \( X \), which is, on average, exceeded once during some specified period (e.g. 100 years). This period, \( T_r \), is the return period of \( x \). It is given by \[ T_r = 1 / [m(1 - F_X(x))] \] where \( F_X(x) \) represents the probability of \( X \leq x \) in any one time interval. Since this time interval need not be in years, but \( T_r \) is normally expressed in years in engineering applications, the total number of time intervals per year, \( m \), needs to be calculated. Examples are wave data collected at 3-hourly intervals, resulting in \( m = \frac{365 \times 24}{3} = 2920 \) and high water levels collected at intervals of 12h25min which gives \( m = \frac{365 \times 24}{12.42} = 705 \). The return period for the extreme load may itself be chosen with regard to the value of the encounter probability, \[ E_{enc} = 1 - [1 - \frac{1}{T_r}]^{T_{ref}} \] where the probability that the \( T_r \)-year return load will be exceeded at least once during the reference period, \( T_{ref} \) (Burcharth, 1987, 1990; Casciati & Natale, 1992; PIANC, 1992; Carvalho, 1992a). However, there is often little consideration given to the uncertainties involved in establishing the probabilities (Burcharth, 1992).

In most cases, the resistance is defined in terms of the load which causes a certain degree of damage to the structure and is not given as an ultimate
Probabilistic Methods For Design And Assessment

force or deformation. As mentioned by Burcharth (1992), this is because most of the available formulae only give the relationship between wave characteristics and structural response (e.g. in terms of run-up, overtopping, armour layer damage). Almost all such design formulae are semi-empirical relationships (Burcharth, 1987), being based mainly on experience, engineering skill and central fitting to model test results (using fitting methods like least squares regression). The test results do not all fall on the line represented by a particular formula; there is often considerable scatter around the line which is not incorporated into the design process in any systematic fashion (Burcharth, 1992). Consequently, the applied characteristic value of the resistance is the mean value and not a lower fractile as is usually the case in other engineering fields (e.g. in the manufacturing and aeronautical industries).

Recent experience of well-publicised severe damage to some large coastal structures (Harlow, 1980; Burcharth, 1987, 1990) has led to the conclusion that damage was caused by a combination of aspects and that the safety levels for these structures were far too small. In other words, taking into account all stochastic variables influencing load and resistance, the probability of failure was too high, resulting in a high encounter probability of a severe damage in the years after completion of the structures (Mol et al, 1983). The deterministic methods used, which do not explicitly consider the reliability of a proposed design through the incorporation of information on the uncertainties involved in the load and resistance variables, together with other sources of uncertainties, do not allow an accurate assessment of the degree of safety in terms of the probability of failure (CIRIA, 1984; Burcharth, 1987, 1990). These deterministic methods, which disregard the fundamental stochastic nature of the problem, are inadequate (Burcharth, 1985).

Instead of the above simplistic approach which requires relatively little input data, a probabilistic approach is preferred. The latter has a number of advantages (Van der Meer, 1987; CUR-TAW, 1990; CIRIA/CUR, 1991; Lamberti, 1992) including: i) the structure under study can be analysed and described as a whole; ii) the uncertainties are rationally incorporated in the assessment of the safety of the structure; iii) it is possible to obtain a better insight into the sensitivity of the structure's failure probability to the various uncertainties: this enables a more balanced design in which priority for
further research is given to those parameters that make the greatest contribution to the total probability of failure; and iv) it is possible to assess explicitly the cost of improving the structure and of damage or loss.

This chapter continues by reviewing general probabilistic concepts and their application in design and assessment of the safety of coastal structures (Section 2.2). Section 2.3 introduces the probabilistic methods which are particularly relevant to the present work. The chapter is not intended to be an exhaustive review of the subject; a large number of references has been published and it is difficult to select among them, particularly for a brief review. The reader is referred to Shinozuka (1983), Ferry Borges & Castanheta (1983), Ditlevsen & Bjerager (1986) and Bjerager (1991), for historic reviews of probabilistic methods. The chapter illustrates the types of concepts and methods which have been used in probabilistic design and assessment of coastal structures, together with their strengths and limitations. In addition to literature related to coastal structures, a range of literature on probabilistic methods applied in other areas of engineering has also proved useful in preparing this chapter, including: Alonso (1976), D'Andrea & Sangrey (1982) and Nguyen & Chowdhury (1985) - geotechnical engineering; Flint & Baker (1976), Schueller & Choi (1977), Fjeld (1977), Jensen et al (1990), Ronold (1990), Potts (1993) and Duggal & Niedzwecki (1994) - offshore structures; Ferry Borges & Castanheta (1983) and Casciati & Faravelli (1985) - structural engineering; Plate & Duckstein (1988) - levees on a river; Yen (1989) - culvert flooding; Helton & Breeding (1993) - nuclear power plants; Jang et al (1994) - ground water flow and contaminate transport; Lumbers & Cook (1993) - water supply systems; Cullen (1990), Lafitte (1993) and Kreuzer (1994) - dams; and Melchers & Stewart (1993) and Frangopol et al (1996) - general engineering.

2.2 General Concepts And Their Application To Coastal Structures

In order to judge whether a man-made or natural structure (e.g. a seawall or a dune) satisfies the requirements that users and society apply with regard to safety and economy, it is possible to use risk analysis methods (CUR-TAW, 1990). The term risk can be defined in different ways, but for the purposes of this research, risk is defined (BSI, 1991a; Royal Society, 1992) as the combination of the probability, during a reference period of time, of an
undesirable event (e.g. a storm, which is a combination of waves and water levels resulting in extreme loading on a structure) and the consequences of its occurrence (e.g. economic loss, casualties, impact on flora and fauna). The method of combination is generally to multiply the probability and consequences. Risk analysis may then be understood as the whole set of activities aimed at quantifying the probability of occurrence, during a reference period of time, of an undesirable event (probability of failure) and its consequences. Calculation of the probability of failure alone is useful; but it fits particularly well into an analysis in which consequences of failure are also considered. This is because it is not only the probability of failure that is important: an event which has a major impact will generally be accepted less readily (i.e. it should have a lower probability of happening) than one which has only minor consequences. For example, it is clear that an accident which does not involve loss of a single life is more acceptable than one in which a thousand lives are lost. In its most general sense, reliability, R, is the probability that the structure will fulfil its design purpose during the reference period (CIRIA, 1984; Thoft-Christensen & Murotsu, 1986). Note that risk analysis and, consequently, the associated concepts (such as probability of failure, reliability, etc.) should always be referred to an interval of time (Joint Committee on Structural Safety, 1978; Ferry Borges & Castanheta, 1983). This interval of time may be taken as the lifetime of the structure (the time of undisturbed functioning) or as a standard time adopted as reference. Even if, for simplicity, this fact is omitted in the following discourse, the reader should always keep it in mind.

The three main elements of a risk analysis (Van der Meer, 1987; CUR-TAW, 1990) are hazards, failure mechanisms and consequences (Figure 2.1).
A risk analysis begins with the preparation of an inventory of the hazards which can be defined as anything which may occur during the lifetime of the structure that can potentially cause harm or loss (Godfrey, 1994). The ways in which the structure responds to hazards are called the failure mechanisms or failure modes. Note that in assessing the safety of coastal structures, it is very important to consider the structure as a whole system. This is known (Burcharth, 1992) as probabilistic analysis of failure mode systems. Some structures are rather complex and, for simplicity, they are considered as series systems, parallel systems, or a combination of both (CIAD, 1985; CIRIA/CUR, 1991). Techniques also exist to provide a logical description of the many hazards and mechanisms resulting in failure of a structure. These include the so-called event trees, which relate to consequences, and fault trees, which relate to causes (see, for example, Paté-Cornell, 1984; Melchers, 1987; Van der Meer, 1987; Yen, 1989; Andrews & Moss, 1993). Fault trees and event trees can be very complex in practice (e.g. Cullen,

The event tree is a deductive logical diagram (Paté-Cornell, 1984). Starting from an undesirable initiating event leading to a consequence for the state of the structure, it gives all possible sequences of following events (both wanted and unwanted) and determines the outcome of each considered sequence (Figure 2.2). Typical examples of initiating events include intense wave action, high water levels, strong currents, earthquakes, ice damage, collision and vandalism. Each branch of the event tree is unique and represents a distinct series of events possibly leading to failure. The assessment of an event cannot be made in isolation; it must be considered as part of a sequence of events and changes to the structure that have lead to its occurrence. The event tree methodology is useful in the analysis of the consequences of an initiating event and provides a means of identifying top events for fault trees (CIAD, 1985). A top event is a particular failure mode. In the case of a seawall, for example, it might represent the fact that the crest of the wall is built too low.

![Figure 2.2: Event tree of parallel-series system $S_1$ (modified after CIAD, 1985).](image)
A fault tree can be constructed starting at the top event and describing, by means of inductive logic, the possible initiating causes leading to the particular failure (Figure 2.3). It is related to component events and basic events by means of logical AND and/or OR gates. If one event alone can cause the top event, the occurrence in the fault tree is represented by an OR gate. If all related events are required to cause the top event, this occurrence is represented by an AND gate. The development stops if the related inputs arise from basic events only, which are generally independent of one another (Lafitte, 1993). When developing fault trees, it is important to keep in mind that each pathway up through the tree forms a unique sequence of events describing a failure, starting from a basic event, and it describes all the unwanted events leading to the unwanted top event. The quantitative analysis of the fault tree involves calculating the probability of a unique undesirable top event from the probabilities of occurrence of the basic events (Lafitte, 1993). So one proceeds systematically from the base towards the top of the tree, the probability obtained at one level being used for the calculation at the level immediately above. Each fault tree considers only one of the many possible system failure modes. Consequently, more than one fault tree may be constructed during the assessment of any system.

Figure 2.3: Fault tree of parallel-series system $S_1$ (modified after CIAD, 1985).
Those combinations of components which, if they all fail, cause system failure are called cut sets (Andrews & Moss, 1993). A minimal cut set is a cut set such that if any component is removed from the combination, the system no longer fails (Paté-Cornell, 1984). The calculation of the system probability of failure can only be performed after the minimal cut sets have been determined. For example, two minimal cut sets exist in Figure 2.3: one contains $S_{11}$ and the other contains $S_{12}$ and $S_{13}$.

The drawback of event trees and fault trees is that they are rather strictly regulated (CUR-TAW, 1990; Burcharth, 1992): in an event tree, it is not possible to combine branches, and no dividing of branches is possible in a fault tree. Furthermore, the system is essentially binary in character: an event occurs or it does not. In coastal engineering, problems of a more continuous character occur.

Event trees and fault trees have been constructed and presented for some coastal structures (CIAD, 1985; Van der Meer, 1987; CUR-TAW, 1990; CIRIA/CUR, 1991), but they have not been applied in full as logic diagrams, except for individual structures which are the subject of very detailed study. They have almost always served as schematic representations of failure modes rather than as strict logical representations of failures. Some authors (Meadowcroft et al, 1994) have found that fault trees and event trees, as used in the electronics and chemical industries, may be suitable for systems of binary components that either fail or do not fail, but they are not sufficient on their own to represent failure of seawalls and related structures which exhibit complex failure modes with interactions between them. For example, overtopping and geotechnical failure of the landward face of a seawall may not, individually, pose a high risk, but the damage due to the geotechnical failure will make erosion due to overtopping much more likely: the modes interact. Furthermore, the quantity of water overtopping is of great importance in determining the consequences: it is not possible to say that the system either "fails" or "works", since a spectrum of outcomes can result.

A useful alternative to event and fault trees is provided by the so-called cause-consequence charts. They overcome many of the drawbacks of event and fault trees outlined above. An example is given in Figure 2.4.
The concepts outlined above concern static event trees: the corresponding probability of failure applies to the final state, assuming a rapid propagation of damage once a section fails. In general, the use of dynamic event trees, which cover time-dependent aspects, should be considered. It is possible to assess dynamic problems using model simulations which include time-dependent effects. However, this approach requires complex methods which are not further discussed here (see, for example, Cumo & Naviglio, 1987; Casciati & Faravelli, 1991; Andrews & Moss, 1993).

While preparing the inventory of hazards and identifying the possible failure modes of a structure, it is obvious that from all possible failure modes, only a few are of real importance (Van der Meer, 1987). Others have such a low probability of occurrence that they may be disregarded, provided that they are independent with respect to other failure modes. However, in principle,
all failure modes must be identified and studied as far as is necessary to establish what degree of risk they pose (Meadowcroft et al., 1994). Neglect of an important failure mode will bias the estimation of the safety of the structure (Burchardt, 1990, 1992; Lamberti, 1992). Furthermore, it is important to investigate the way failure modes combine or interact, as indicated, for example, in a cause-consequence chart. Consideration must be given to two particular factors (PIANC, 1992): physical correlation (e.g. the failure of one mode triggers the failure of another) and correlation through common parameters (e.g. the same parameter triggers two different failure modes). It is not generally known how to quantify the first of these factors, even if physical correlation can be identified. Consequently, only the second can be dealt with in a quantitative way.

After identifying the failure modes and their mutual relationships, assessment of the safety of a structure depends fundamentally on the description of the individual failure modes (Thoft-Christensen & Murotsu, 1986). For each failure mode, a theoretical model may exist. Failure modes for which no mathematical-physical description is available, or for which the model is rather poor, become apparent (CIAD, 1985; CIRIA/CUR, 1991). This situation arises mainly because the load and/or the structural behaviour is complex and is not fully understood. If no theoretical models are available for a failure mechanism, simple empirical formulae can be used to describe the physical process (CIAD, 1985). When no theoretical models or empirical formulae are available, it is necessary to work on the basis of engineering judgement (Van der Meer, 1987).

Theoretical models or empirical formulae may be applied to define what is called a failure function or limit state function, $Z$ (Figure 2.5). This is a function of the basic variables, $X_i$, $i=1,...,N$, of the problem (e.g. water level, wave conditions, structure dimensions, material properties), which are the fundamental quantities that the designer has to consider and which may influence the reliability of the structure with respect to a particular failure mode (CIRIA, 1984). Note that it may not be possible to express $Z$ as an explicit function of the basic variables. Provided the function is continuous, this is of no consequence. However, if the function is discontinuous (e.g. the Van der Meer expressions for the stability of rock armour on breakwaters), it must be examined as a series of continuous functions. In any case, the simultaneous values of the variables $X_i$ must stay within certain limits in order
that the structure behaves as it is intended to do for the failure mode under study. For a given structure and failure mode, these limits may be described in terms of the failure function which divides the N-dimensional space of the \( X \)-variables into two sets (Figure 2.5): \( Z = f(X_1, \ldots, X_N) \) denotes safe states if \( Z > 0 \) and failure states if \( Z < 0 \). \( Z = 0 \) defines the failure surface. As noted by Madsen et al (1986), it is often convenient to include the failure surface in the failure states (i.e. \( Z \leq 0 \) defines failure) and it is this definition of failure states which is used in the present research. In other words, the probability of failure, \( P_f \), is defined as \( P_f = P(Z \leq 0) \), whilst the reliability, \( R \), is defined as \( R = 1 - P_f \).

![Figure 2.5: Definition of the failure surface Z=0 for the case of two basic variables, X_1 and X_2.](image)

Note that \( Z \) defines what is generally called a limit state, which is a limiting condition beyond which a structure is assumed to become unfit for its purpose (CIRIA, 1984). A distinction has to be drawn between ultimate limit states and serviceability limit states. The former refer to conditions in which the structure is unable to fulfil its principal functions (e.g. the failure of a seawall in preventing extensive flooding or erosion of the backshore). Serviceability limit states, on the other hand, exist when damage of considerable magnitude to the structure has occurred but it remains possible to rely on the structure for its main function. Under these conditions,
some disturbance related to normal use and durability of the structure is expected (BSI, 1991a).

After describing the relevant failure modes, calculation of the probability of failure associated with each of them can be performed using a single failure mode probability analysis (Burcharth, 1992), as described in Section 2.3. These probabilities are then used to determine upper and lower bounds for the probability of failure of the whole structure (Ditlevsen, 1979b; CUR-TAW, 1990). For more detailed discussions of this subject, see Thoft-Christensen & Baker (1982), Hohenbichler & Rackwitz (1983), Ang & Tang (1984), CIAD (1985), Madsen et al (1986), Thoft-Christensen & Murotsu (1986), Ditlevsen & Bjerager (1986), Melchers (1987), CUR-TAW (1990) and Bjerager (1991).

Finally, the consequences of failure must be considered. Whilst there are many events for which the consequences are obvious, there are others for which the outcomes are less easy to predict (Lumbers & Cook, 1993). An example of consequences which are obvious might be the overtopping of a seawall by a relatively small volume of water causing inconvenience and/or injury to pedestrians, but not affecting the safety of the structure. The outcome would be less easy to predict where the volume of water causes significant flooding. In this case, the consequences would depend on a number of factors such as the time of day during which flooding occurs, the storm warning service, the efficiency of people's evacuation of the expected flooded area, etc.

The probability of failure multiplied by the consequences constitutes the risk (Figure 2.1). Risk has the units of the consequences (Meadowcroft et al, 1995; Simm et al, 1996): for risk to an individual, the units of expression may be in terms of fatalities per hour or per year of the individual's activity; for risk to society, it may be expressed in units of the expected number of people to be affected to a specified degree per year; economic risk expresses the expected loss in monetary terms.

In civil engineering, there is no such concept as total safety, but there are higher or lower risks of failure (Pate, 1981; CIRIA, 1984; Vasco Costa, 1990; Pita, 1992; Vrijling, 1993). The risk of failure can only be minimised or, more realistically, the safety of the structures can be optimised to a degree.
consistent with available information and with justified socio-economic investment. The acceptable risk depends on the structural characteristics and on the consequences of failure. For a breakwater, the acceptable risk during the expected lifetime of the structure can vary from a large value (e.g. $10^{-1}$) if the consequences are insignificant, to a very small number (e.g. $10^{-4}$) if the failure of the breakwater would result in significant damage (Burcharth, 1991b). For offshore structures, values in the range of $10^{-6}$ to $10^{-8}$ are recommended (Potts, 1993). Optimal design is, essentially, weighing the risks against the costs of providing higher levels of safety (BSI, 1991a; MAFF, 1993c). If there are no intangible damages (damages which cannot be expressed or evaluated in monetary terms) then the design probability of failure or acceptable level of risk may be chosen by a process of optimisation using methods like cost-benefit analysis (CIAD, 1985; CIRIA/CUR, 1991; PIANC, 1992; Lafitte, 1993; Parker, 1993). Otherwise, the total damage (tangible and intangible) must be quantified in some way; this is a complex problem which gives rise to many discussions and ethical objections (Pate, 1981; Madsen et al, 1986; Green & Penning-Rowsell, 1989; CUR-TAW, 1990; Penning-Rowsell et al, 1992). Discussion of design optimisation is beyond the scope of the present research. Reference is made to Dover & Bea (1979), Brennan & Stickland (1981), Pate (1981), Allen (1981), Nielsen & Burcharth (1983), CIRIA (1984), CIAD (1985), Bruun (1985), Casciati & Faravelli (1985), CIRIA (1986a), Smith (1987), Parker et al (1987), CUR-TAW (1990), Vrijling (1990), De Haan (1991), Ryu et al (1992), Thomas & Hall (1992), Barber (1993), MAFF (1993b), and Ruiz & Quirós (1994).

In risk analysis, it is important to have a good overview both of the uncertainties involved and of the related consequences. Without such knowledge, it is impossible to evaluate the safety of a structure, a situation that is unacceptable for a professional engineer (Burcharth, 1985). To investigate the influence of uncertainty on safety evaluation, it is first necessary to identify and define what is not known (Kreuzer, 1994). Engineering systems inevitably involve many uncertainties in their planning, design and operation (Yen, 1989). It is important to acknowledge these uncertainties, even though they are obviously difficult to quantify (Burcharth, 1985; Melchers, 1987). It is a huge step forward simply to identify them and quantify them approximately.
Probabilistic methods are used to evaluate engineering safety. At the same time, they must account for the uncertainties in the various contributing factors and evaluate their implications for engineering design. Each basic variable, $X_i$, in the failure function, $Z$, is a potential contributor to these uncertainties. Moreover, a random variable, $X_i$, might not be a directly measurable physical quantity; it can itself represent the uncertainty in a specific factor. It can be an error term included as a variable in the failure function (Manners, 1990; Der Kiureghian, 1990).

Uncertainties in the study of a single failure mode may include (Yen, 1989; Burcharth, 1992):

- **Uncertainties related to failure mode formulae**
  Whether the formula used to describe the "real" behaviour of the structure is based on theoretical considerations or physical model tests, simplifications and idealisations are made during its development which give rise to uncertainties. In certain cases, the uncertainties associated with a failure mode formula may be much more significant than the uncertainties associated with the basic variables in the problem (Ang & Tang, 1975; Thoft-Christensen & Murotsu, 1986; Burcharth, 1992). This is clearly seen from the many diagrams presenting a formula as a smooth curve shrouded in a widely scattered cloud of data points (usually from physical model tests) which are the basis for the curve fitting. Coefficients of variation of 15-20% or even larger are quite normal. The range of validity and the related coefficient of variation should always be considered when using a formula.

- **Uncertainties related to environmental parameters**
  The specification of environmental criteria is one of the crucial steps in maritime engineering. In various coastal areas, the engineer is confronted with waves, currents, winds, storm surges, etc. The ideal situation, where both short-term and long-term wave statistics can be established from on-site measurements, almost never exists (Burcharth, 1985). According to PIANC (1992), uncertainties related to environmental parameters arise, mainly, due to: i) errors in instrument response or visual observation; ii) variability and errors due to different and imperfect calculation methods; iii) statistical uncertainties related both to short-term randomness of the variables and to extrapolation from small sets of data to events of low probability of occurrence; and iv) choice of theoretical distributions as representatives of the unknown long-term distributions.
• Uncertainties related to structural parameters

The uncertainties related to material parameters (e.g. density), to geometrical parameters (e.g. slope angle, size of structural elements) and fracture strength (e.g. of concrete blocks) are generally much smaller than the uncertainties related to the environmental parameters and to the design formulae.

Note that in a considerable number of failures, human factors have been the predominant overall component (Blockley, 1981; Madsen et al, 1986; Thoft-Christensen & Murotsu, 1986; Melchers, 1987; Burcharth, 1987; Manners, 1990; Melchers, 1993). They may be mistakes in design, analysis, construction, maintenance, or use of the structure (Townend, 1994a). Therefore, an estimate of its reliability is incomplete without considering human factors. Ways of reducing accidents caused by human errors include quality assurance techniques (see for example, PIANC, 1988; BSI, 1990, 1991c; CIRIA/CUR, 1991; Lafitte, 1993) which have developed considerably in recent years.

It is beyond the scope of this contribution to discuss in any more detail the many uncertainties related to the study of a single failure mode. However, further information may be found in Ang & Tang (1975, 1984), Blockley (1981), Thoft-Christensen & Baker (1982), Burcharth (1985), Ditlevsen & Bjerager (1986), Melchers (1987), Manners (1990), Der Kiureghian (1990), Burcharth (1992) and HR Wallingford/Sir William Halcrow & Partners Ltd (1993).

Some existing studies on coastal structures have aimed at being very broad, covering various failure modes and consequences (CIAD, 1984; CUR/TAW, 1990; HR Wallingford/Sir William Halcrow & Partners Ltd, 1993, 1995). Others have focused mainly on the study of a specific failure mode (CIAD, 1985; PIANC, 1992; Allsop & Meadowcroft, 1995). This demonstrates the difficulty in achieving a satisfactory compromise between engineering and mathematical accuracy, and developing a procedure which can be applied to a wide range of coastal structures.

This research does not consider a full risk assessment procedure for coastal structures. It has been obvious during the current review of literature that, at the moment, there is insufficient knowledge to enable such an analysis to be carried out (Burcharth, 1990). This study concentrates on the probabilistic
assessment of individual failure modes, particularly seawall wave overtopping and dune erosion. For these failure modes, it concentrates on calculating the probability of failure during a specified reference period.

2.3 Single Failure Mode Probability Analysis

2.3.1 Introduction

This section discusses techniques to quantify the probability of occurrence of a particular failure mode represented by the failure function \( Z = f(X_1,...,X_N) \) where \( X_i \) are the basic variables of the problem. For most practical applications, each basic variable, \( X_i \), is random with a probability density function \( f_{X_i} \). The failure function, \( Z \), is generally a non-linear function of the basic variables.

Now, assume that \( Z \) is a function of only two random variables, \( X_1 \) and \( X_2 \), i.e. \( Z = f(X_1,X_2) \). Given \( f_{X_1,X_2} \) as the joint probability density function (jpdf) of \( X_1 \) and \( X_2 \), then the probability of failure, \( P_f \), during a specified reference period, can be expressed as:

\[
P_f = P(Z \leq 0) = \int \int_{Z \leq 0} f_{X_1,X_2} \, dX_1 \, dX_2
\]

(2.1)

If, and only if, the variables can be assumed statistically independent, the jpdf is determined from the product of the probability density functions \( f_{X_1} \) and \( f_{X_2} \). The above equation may then be replaced by:

\[
P_f = \int \int_{Z \leq 0} f_{X_1} f_{X_2} \, dX_1 \, dX_2
\]

(2.2)

In this case, the integral is equivalent to the volume enclosed between the horizontal plane \( f_{X_1,X_2} = 0 \) and the jpdf \( f_{X_1,X_2} \), in the space where the condition \( Z \leq 0 \) is fulfilled. With two variables only, the jpdf can be shown as a surface represented by contours and the failure boundary can be drawn as a line (Figure 2.6). Figure 2.6 also shows the so-called design point which is the point on the failure surface where the jpdf attains its maximum value, i.e. the most probable point of failure.
Generally, Z is a function of more than two random variables. In this case, it is not possible to describe the jpdf as a surface but it requires an imaginary multi-dimensional space (CIRIA/CUR, 1991; Burcharth, 1992). The probability of failure is then written as follows:

\[
P_f = P(Z \leq 0) = \int_{Z \leq 0} \cdots \int f_{X_1, \ldots, X_N} dX_1 \cdots dX_N
\]

(2.3)

where, again, if \(X_1, \ldots, X_N\) are statistically independent:

\[
P_f = \int_{Z \leq 0} \cdots \int f_{X_1} \cdots f_{X_N} dX_1 \cdots dX_N
\]

= \int_{Z \leq 0} \cdots \prod_{i=1}^{N} f_{X_i} dX_i

(2.4)

These equations form the mathematical basis for probabilistic analysis. Except for some special cases, the above integrations cannot be performed analytically and have to be approximated in some way (Ferry Borges & Castanheta, 1983; Melchers, 1987; Hohenbichler et al, 1987). This is the aim of the various probabilistic methods. They are classified on the basis of the types of calculations performed and of the approximations made. In essence, the designer must choose to what degree of sophistication he wants to formulate failure. In general, three common levels are distinguished.
in the literature (Ferry Borges & Castanheta, 1983; Mol et al; 1983; CIAD, 1985; CUR-TAW, 1990; Burcharth, 1992; PIANC, 1992). They are listed in order of decreasing accuracy and complexity as follows:

- **Level III - Full distribution approach**
  This method provides an "exact" probabilistic analysis for whole structural systems, or structural elements, using full joint probability density functions including the correlations among the variables. The probability content of the entire failure region is evaluated (as opposed to Level II methods which comprise a check at only a single point on the failure surface).

- **Level II - Semi-probabilistic approach**
  Approximation methods are applied in which the generally correlated and non-Normal variables are transformed into uncorrelated and Normal variables. Reliability indices are used as measures of the structure reliability. Non-linear failure functions are approximated using a tangent hyperplane at some point (First Order Methods), using a quadratic approximation (Second Order Methods) or even higher order approximations\(^1\). Since the second and higher order methods complicate considerably the computations and, in many cases, the First Order Methods give very good approximations (Ditlevsen & Bjerager, 1986), only the First Order Methods are described in this thesis. In these methods, the failure function is linearized at a specific point in order to determine the actual probability of failure. If linearization is performed about the expected mean values of the variables involved, the method is known as the First Order Mean Value Approach, FOMVA. If the failure function is linearized about the point in the failure surface having the highest joint probability density (design-point) then the method is called a First Order Reliability Method, FORM (Burcharth, 1990, 1992). This approach requires an iterative procedure in the case of non-linear failure functions.

- **Level I - Limit state approach**
  This level comprises calculations based on characteristic values and partial load and resistance factors. The factors represent, for example, the ratio of load at failure to permissible working load. This creates a desired margin between the characteristic values of

---

resistance and working load. In this approach, a characteristic load is established (for example, a wave height with a certain return period) for which hardly any damage should occur. Strictly speaking, a calculation at Level I does not allow the determination of the reliability (or the failure probability) of the design. Consequently, it is neither possible to optimise nor to avoid over-design of a structure. It does, however, provide a method of checking whether a defined level of safety is satisfied.

A less common fourth level of probabilistic approach has also been advanced (Madsen et al, 1986; Plate & Duckstein, 1988; Casciati & Natale, 1992). This level accounts for the principles of engineering economic analysis under uncertainty, considering costs and benefits of construction, maintenance, repair, consequences of failure, etc.

The above classification of reliability methods is not exhaustive, but it has proved to be very useful in practical discourse on reliability methods. For example, this classification does not refer to any combination of the above methods using the advantages of each (see, for example, Super-Software, 1994). In the next few sections, special attention is paid to the FORM method since it is the basis of a FORTRAN program, PARASODE, developed as part of this research. However, for a fuller treatment of the method, the reader is referred to Thoft-Christensen & Baker (1982) and Madsen et al (1986). Numerical methods which rely on sampling are also briefly outlined since they are the methods applied by the software package @RISK which has been used in this research to validate the results obtained from PARASODE. Finally, Level I methods are introduced for completeness (for more detailed information, see CIRIA, 1984).
2.3.2 Level III Methods

2.3.2.1 Numerical Integration

The N-fold integral in eq. (2.3) is solved by full numerical integration, i.e. substitution of the infinitely small integration steps \( dX_i \) by finite discrete integration steps \( \Delta X_i \), and substitution of the integrals by summations. Each summation must be done using a discrete number of steps and the integration area for each random variable must be evaluated between a starting value and an end value instead of running from \(-\infty\) to \(+\infty\) (see, for example, Melchers, 1987). The method would be exact if an unlimited number of integration steps were used. Of course this is not possible. International research seems to focus on Level II methods and traditional sampling variations, but rarely on numerical integration, although this is the most correct method (Super-Software, 1994). Little literature on this topic is available. No general formulae have been found in the literature for solving a multi-dimensional integral.

2.3.2.2 Numerical Methods Which Rely On Sampling

Introduction

As the complexity of an engineering system increases, the required analytical model may become extremely difficult to formulate mathematically unless gross idealization and simplifications are invoked; moreover, in some cases even if a formulation is possible, the required solution may be analytically intractable. In these instances, a probabilistic solution may be obtained through sampling which is the process by which values are randomly drawn from the input probability distributions. In this case, the N-fold integral in eq. (2.3) is solved by letting the computer generate values for the limit state function, using a random number generator. Basically, this means that the computer needs an algorithm which generates random numbers between zero and one from a uniform distribution. Several techniques for drawing random samples are available (Ang & Tang, 1984). For work that requires the use of digital computers, it is convenient to compute a sequence of numbers by a systematic procedure. Such procedures are devised so that reasonable statistical tests do not detect any
significant deviation from randomness. Such a sequence of numbers can be duplicated exactly (e.g. for checking purposes) and therefore, strictly speaking, are really not random; for this reason, they are called "pseudo" random numbers (Hammersley & Handscomb, 1964; Haugen, 1968; Rubinstein, 1981; Cope et al, 1982; Ang & Tang, 1984; Melchers, 1987). For most practical purposes, a sequence of numbers generated by a suitable "pseudo" random number generator is indistinguishable from a sequence of strictly true random numbers (Rubinstein, 1981). The generated "pseudo" random numbers are cyclic, that is, they are repeated with a given period. To insure reasonable randomness, the period should be as long as possible. When evaluating the techniques for drawing samples, the most important factor to consider is the number of iterations required to accurately recreate an input distribution. Choosing a sampling method affects both the quality of the results and the length of time necessary for simulation. The two sampling methods briefly described in this section are those used by the software package @RISK which has been applied in this research to validate the results obtained from the Level II calculations: traditional sampling (often called Monte Carlo Sampling) and Latin Hypercube Sampling (LHS).

Once the standard uniformly distributed numbers have been obtained, random numbers with a prescribed distribution may be generated through (Rubinstein, 1981; Ang & Tang, 1984; Law & Kelton, 1991): i) direct methods (e.g. the inverse transform method, the composition method, and the convolution method); or ii) indirect methods (e.g. acceptance-rejection method). The particular method used depends on the distribution from which one wishes to generate. The reader is referred to the literature for more details on these methods.

Sampling is done repetitively, with one sample being drawn every iteration from each input probability distribution. With enough iterations, the sampled values become distributed in a manner which approximates the known input probability distributions.

Clearly, the probability distributions of the governing parameters must first be specified. An equation is then used to link the parameters to the outcome. The output process is sampled by choosing a random value for each input parameter. The outcome of the event is then recorded. The procedure is
repeated for a sufficient number of events to accurately determine the probability distribution of the outcomes.

*Traditional Sampling*

The term "Monte Carlo" was introduced by von Neumann and Ulam during World War II, as a code word for the secret work at Los Alamos; it was suggested by the gambling casinos in the city of Monte Carlo in Monaco (Rubinstein, 1981).

Monte Carlo Sampling is the traditional technique for using random numbers to draw samples from a probability distribution. Any given sample may fall anywhere within the range of the input distribution (Figure 2.7). Samples are, of course, more likely to be drawn from areas of the distribution associated with the higher probabilities of occurrence.

![Monte Carlo Sampling](image)

*Figure 2.7:* Five iterations of Monte Carlo Sampling with clustering (modified after Palisade Corporation, 1994).
When a small number of iterations is performed, a problem of clustering may arise. This causes particular difficulties when a distribution includes low probability outcomes which could have a major impact on the results (Haugen, 1968; Ang & Tang, 1984; Startzman & Wattenbarger, 1985; Ditlevsen & Bjerager, 1986; Wen & Chen, 1987). These outcomes have to be sampled. But if their probability is very low, a small number of iterations may not provide sufficient of these outcomes to accurately represent their probability. However, an increase in the number of iterations is not always computationally convenient. This problem has led to the development of techniques which reduce the sampling error without increasing the sample size. These techniques are known as variance reduction techniques and include (Hammersley & Handscomb, 1964; Halton, 1970; Rubinstein, 1981; Ang & Tang, 1984; Morgan, 1984; Smith & Buckee, 1985; Melchers, 1987; Bjerager, 1991; Law & Kelton, 1991): i) Control-Variate Sampling or Correlated Sampling; ii) Antithetic Variate Sampling; iii) Importance Sampling; iv) Stratified Sampling or Systematic Sampling (e.g. LHS); v) Implicit Multicorrelated Sampling or the E-Z-H method after Ermakov, Zolotukhin and Handscomb; and vi) Conditional Expectation Sampling. Note that comparison of earlier references indicates some differences in nomenclature for similar techniques.

In general, the reason why people specify the traditional method is because of the length of time that it has been around. Other sampling techniques such as LHS have not been as widely implemented.

Latin Hypercube Sampling

The LHS method is designed to accurately recreate the input distribution, preserving the randomness of the traditional method while using fewer samples. Typically, LHS requires about one third of the traditional method's iterations to get equal or better results (Palisade Corporation, 1994; Murtha, 1995). The key to this process is stratification of the input probability distribution (McKay et al, 1979; Startzman & Wattenbarger, 1985). Firstly, the cumulative distribution is divided using equal intervals on the probability scale of 0 to 1 (Figure 2.8). Then a sample is randomly taken from each part of the input distribution. Thus, sampling is forced to represent values in each
of the divisions, thus avoiding clustering of values and more accurately reflecting the input probability distribution.

![Latin Hypercube Sampling](image)

**Figure 2.8**: Five iterations of Latin Hypercube Sampling (modified after Palisade Corporation, 1994).

The technique being used during LHS is so-called sampling without replacement: the number of divisions of the cumulative distribution is equal to the number of iterations performed. Hence, a sample is taken randomly from each division and once a sample is taken, the division is not sampled again. Note that if a LHS simulation is stopped prior to the execution of the specified number of iterations, the results are still valid. However, they do not reflect all the benefit of the stratified sampling since not all the input strata have been filled. Since the strata which have been sampled from have been randomly selected from across a distribution, the results are at least as good as the equivalent results produced from the same number of Monte Carlo iterations, but not as good as a complete LHS simulation of the same number of samples.
2.3.3 Level II Methods

2.3.3.1 Basic Features Of The First Order Reliability Method

Suppose that the failure function, \( Z \), can be expressed as follows:

\[
Z = a_0 + a_1 X_1 + a_2 X_2 + \ldots + a_N X_N = a_0 + \sum_{i=1}^{N} a_i X_i \tag{2.5}
\]

where \( a=(a_0, \ldots, a_N) \) are constants and \( X=(X_1, \ldots, X_N) \) are mutually independent Normal basic variables with known means \( \mu_X = (\mu_{X_1}, \ldots, \mu_{X_N}) \) and standard deviations \( \sigma_X = (\sigma_{X_1}, \ldots, \sigma_{X_N}) \). It can be shown (Smith, 1986) that provided the variables, \( X \), follow the Normal distribution and are mutually independent, \( Z \) will also follow a Normal distribution having mean, \( \mu_Z \), and variance, \( \sigma_Z^2 \):

\[
\mu_Z = a_0 + a_1 \mu_{X_1} + \ldots + a_N \mu_{X_N} = a_0 + \sum_{i=1}^{N} a_i \mu_{X_i} \tag{2.6}
\]

\[
\sigma_Z^2 = a_1^2 \sigma_{X_1}^2 + \ldots + a_N^2 \sigma_{X_N}^2 = \sum_{i=1}^{N} (a_i \sigma_{X_i})^2 \tag{2.7}
\]

The probability of \( Z \) being less than or equal to zero follows from the Normal distribution, with known mean and standard deviation:

\[
P_1 = P(Z \leq 0) = \int_{-\infty}^{0} f_z \, dZ = \Phi(-\beta) \tag{2.8}
\]

where \( f_z \) is the probability density function of \( Z \), \( \Phi \) is the cumulative distribution function of the standard Normal distribution (tabulated in statistical books such as Abramowitz & Stegun, 1964) and \( \beta \) is the reliability index:

\[
\beta = \frac{\mu_Z}{\sigma_Z} \tag{2.9}
\]

Note that \( \beta \) is the inverse of the coefficient of variation of \( Z \) and is the distance from the mean value of \( Z \), \( \mu_Z \), to the failure surface, \( Z=0 \), in terms of the number of standard deviations (Figure 2.9).
If the basic variables are Normal but correlated then the expression for $\mu_Z$ still holds but $\sigma_Z^2$ is given by:

$$\sigma_Z^2 = \sigma_X^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{X_i X_j} \sigma_X \sigma_X$$

(2.10)

where the last term accounts for correlation between any pair of basic variables and $\rho_{X_i X_j}$ denotes the correlation coefficients:

$$\rho_{X_i X_j} = \frac{\text{Cov}[X_i, X_j]}{\sigma_X \sigma_X} = \frac{\mathbb{E}[(X_i - \mu_X)(X_j - \mu_X)]}{\sigma_X \sigma_X} = \frac{\mathbb{E}[X_i X_j] - \mu_{X_i} \mu_{X_j}}{\sigma_X \sigma_X}$$

(2.11)

$\mathbb{E} [...]$ is the expected value operator and represents the expected value of its argument. $\text{Cov}[X_i, X_j]$ is the covariance of $X_i$ and $X_j$; $X_i$ and $X_j$ are said to be uncorrelated if $\text{Cov}[X_i, X_j]=0$, i.e. $\rho_{X_i X_j} = 0$.

Note that eq. (2.8) gives an "exact" probability of failure only if the failure function is linear in $X$ and if all the basic variables are normally distributed.
2.3.3.2 Non-Linear Failure Functions

*Normal Random Variables*

If the failure function $Z$ is non-linear then approximate values of $\mu_Z$ and $\sigma_Z$ can be obtained by using a linearized failure function (Ferry Borges & Castanheta, 1983; CIRIA, 1984; Melchers, 1987; Burcharth, 1992). Linearization is generally performed by a truncated Taylor series expansion around some point, $X^*$, retaining only the linear terms. This results in the following approximation for $Z$:

$$ Z \approx Z^* + \sum_{i=1}^{N} (X_i - X_i^*) \left( \frac{\partial Z}{\partial X_i} \right)^* $$

(2.12)

where $Z^*$ is the value of the function $Z$ at the point $X^*$ under consideration, $\left( \frac{\partial Z}{\partial X_i} \right)^*$ is the partial derivative with respect to $X_i$, likewise evaluated at the point $X^*$. The mean value and the variance of $Z$ are, respectively:

$$ \mu_Z = Z^* + \sum_{i=1}^{N} (\mu_{X_i} - X_i^*) \left( \frac{\partial Z}{\partial X_i} \right)^* $$

(2.13)

$$ \sigma_Z^2 = \sum_{i=1}^{N} \left[ \left( \frac{\partial Z}{\partial X_i} \right)^* \sigma_{X_i} \right]^2 $$

(2.14)

The probability of failure and the reliability index are again expressed by eq. (2.8) and eq. (2.9), respectively.

When linearization is performed around the expected mean values, i.e. $X^* = (\mu_{X_1}, ..., \mu_{X_n})$, the method is often called a First Order Mean Value Approach, FOMVA (Burcharth, 1992). For non-linear failure functions, the errors incurred by neglecting second-order and higher terms in the Taylor expansion increase with increasing distance from the linearization point. Since the mean point $X = (\mu_{X_1}, ..., \mu_{X_n})$ is likely to be well within the safe region and not on the failure surface, there are likely to be considerable errors in approximating the failure surface if FOMVA is used (CIRIA, 1984).
Another problem associated with FOMVA is that the values of \( \mu_z \) and \( \sigma_z \), and thereby also the value of \( \beta \), depend on the choice of the linearization point (Thoft-Christensen & Murotsu, 1986). Moreover, the value of \( \beta \) will change when different but equivalent non-linear failure functions are used (Melchers, 1987; Burcharth, 1992). However, there is no arbitrariness due to the choice of failure function if only information about the failure surface is used, i.e. if the linearization point is selected as a point on the failure surface (Madsen et al, 1986).

The first step to obtain invariability of \( \beta \) is to apply the transformation proposed by Hasofer & Lind (1974) in which the basic variables, \( X_i \), are transformed into a new set of normalised variables, \( U_i \). For uncorrelated normally-distributed basic variables, \( X_i \), the transformation is:

\[
U_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}
\]

in which case \( \mu_{U_i} = 0 \) and \( \sigma_{U_i} = 1 \). Using this linear transformation, the failure surface \( Z=0 \) in the \( X \)-coordinate system is mapped into a failure surface in the \( U \)-coordinate system which also divides the space into a safe region and a failure region (Figure 2.10). Due to the zero mean and the unit standard deviation, the new \( U \)-coordinate system has an important characteristic, namely a rotational symmetry with respect to the standard deviations (Ditlevsen, 1979a; Melchers, 1987). Note that the origin of the normalised \( U \)-coordinate system corresponds to the mean value of the initial variables and will usually be within the safe region.

Figure 2.10 introduces the Hasofer and Lind reliability index (often called the first order second-moment reliability index) which is defined as the distance from the origin to the nearest point, \( D \), of the failure surface in the \( U \)-coordinate system (Hasofer & Lind, 1974). \( D \) is called the design point and is the point on the failure surface at which the linear approximation to the failure surface is made. For normally-distributed variables, the coordinates of the design point in the original \( X \)-coordinate system are the most probable values of the variables at failure i.e. the design point is that point on the failure surface where the probability density attains a maximum (see CIRIA, 1984, for proof). These coordinates are given as follows:
\[ X_i^* = \mu_{X_i} + \alpha_i \beta \sigma_{X_i} \]  

(2.16)

where \( \alpha_i \) are the so-called sensitivity factors defined as follows:

\[ \alpha_i = \left( -\frac{\sigma_{X_i}}{\sigma_Z} \right) \left( \frac{\partial Z}{\partial X_i} \right) \]

(2.17)

**Figure 2.10:** Joint probability density function for two random variables and for two reduced variables.

The special feature of the Hasofer and Lind reliability index is that it is related to the failure surface \( Z(U)=0 \) which is invariant to the failure function because equivalent failure functions result in the same failure surface (Thoft-Christensen & Murotsu, 1986). The two reliability indices coincide when the failure surfaces are linear (Ditlevsen, 1979a; Madsen et al, 1986). Obviously, this is also the case if non-linear failure functions are linearized by Taylor series expansion around the design point.

Note that linearization about the mean or the design point leads to different results, depending on the shape of the failure function \( Z \). In general, linearization around the design point is very much to be preferred (Melchers, 1987; CUR-TAW, 1990) because the design point is the most probable point of failure. Linearization around mean values can lead to quite erroneous results, but due to the simplicity of the method it might be used to get a first estimate of the failure probability (Burcharth, 1992).
How well a linear function approximates a non-linear function in terms of the resulting probability of failure, $P_f$, obviously depends on the shape of the non-linear function (Casciati & Faravelli, 1991). If it is concave towards the origin, $P_f$ is under-estimated by the hyperplane approximation. Similarly, a convex function towards the origin implies over-estimation, as in Figure 2.10. Note that all failure surfaces that are tangential to each other at the design point have the same reliability index (Ditlevsen & Bjerager, 1986); for example, if the curved and flat surfaces of Figure 2.10 are considered as failure surfaces of two different structures, the reliability indices for the two structures are the same suggesting equal reliability, whereas the structure with the curved surface is clearly more reliable than the one with the flat surface. It would be useful to have a measure of comparativeness of the reliability indices with respect to the implied reliability or probability content (Melchers, 1987; Casciati & Faravelli, 1991). For this purpose, Ditlevsen (1979a) introduced a reliability index which is known as the generalised reliability index. However, the lack of comparativeness is not critical for single failure mode probability analysis (Ditlevsen & Bjerager, 1986) and this is why the simple Hasofer and Lind reliability index is usually used for this purpose. Hence, details of the generalised reliability index and methods consistent with its definition (e.g. Der Kiureghian & Liu, 1986) are beyond the scope of this work.

The method in which linearization is performed around the design point is often called a First Order Reliability Method, FORM. Since the design point is not known a priori, and in most cases cannot be determined directly (except if $Z$ is linear), finding the shortest distance, $\beta$, in U-space, subject to $Z(U)=0$ is strictly a minimisation problem (Rackwitz, 1976; Flint et al, 1981; Shinozuka, 1983; Casciati & Faravelli, 1991; Liu & Der Kiureghian, 1991). There are several ways in which a solution may be found (Melchers, 1987) such as by direct minimisation using a Lagrangian multiplier (Schittkowski, 1985; Burcharth, 1990), by a numerical approach (Melchers, 1987) or by an iterative procedure. In this study, the latter is used; other methods are beyond the scope of this research.

Several iteration schemes exist (Thoft-Christensen & Baker, 1982; CIRIA, 1984; Smith, 1986; CUR-TAW, 1990; Ahammed & Melchers, 1993). In the following, the simple scheme suggested, for example, in CUR-TAW (1990)
and Ahammed & Melchers (1993), is introduced. It is used in the Level II program, PARASODE, developed as part of this research (see Chapter 5):

1) Set the initial design values of the basic variables (e.g. $X_i^* = \mu_{X_i}$)
2) Compute $Z$ and the partial derivatives $\frac{\partial Z}{\partial X_i}$ at the point $X_i^*$
3) Compute $\mu_Z$ and $\sigma_Z$
4) Compute $\beta = \frac{\mu_Z}{\sigma_Z}$
5) Compute $\alpha_i$
6) Compute new $X_i^*$
7) Repeat steps 2 to 6 until convergence is achieved within specified limits
8) Check that $Z^* = 0$, within specified limits
9) Compute the probability of failure from $P_i = \Phi(-\beta)$

The way this iteration procedure is set-up allows the safety of a structure to be assessed, i.e. it allows a calculation of the probability of failure for a given design parameter (e.g. the crest level of a seawall). However, this iteration procedure, slightly adjusted, and Level II methods in general, may also be used for design i.e. a probability of failure is first specified and one design parameter is modified until the target reliability is achieved (see Chapter 5 and Appendices C and D for further details).

Note that the iteration procedure can fail in certain circumstances (Rackwitz, 1976; Madsen et al, 1986; Melchers, 1987; CUR-TAW, 1990; Super-Software, 1994). One case is for a highly non-linear function for which it is possible to alternate between successive approximation points $i$ and $i+1$ (Fiessler, 1979). This difficulty can be overcome by starting the new iteration using a point between $i$ and $i+1$ (see Section 5.2.2.5). A second breakdown case is when the trial initial design point, $X_i^*$, lies close to a stationary point which is not a minimum; this is because the iteration procedure can only search for local stationary points and cannot distinguish between maxima, minima or saddle points (Ditlevsen & Madsen, 1980). The problem can only be overcome by selecting different starting points (see Section 5.2.2.1) and common sense appraisal of results.

A useful by-product of FORM is its ability to quantify the sensitivity of the reliability index to inaccuracies in the value of $X_i$ at the design point, i.e. to
determine the contribution to the spread of $Z$ made by each random variable. Eq. (2.14) can be rewritten as:

$$
\sum_{i=1}^{N} \left[ \left( \frac{\partial Z}{\partial X_i} \right) \frac{\sigma_{X_i}}{\sigma_Z} \right]^2 = \sum_{i=1}^{N} \alpha_i^2 = 1
$$

Thus, $\alpha_i^2$ represents the contribution to $\sigma_Z^2$ due to $\sigma_{X_i}^2$. If $\alpha_i^2$ is small, $X_i$ might be modelled as a deterministic quantity equal to its mean value. Typically, the acceptable values of the probability of failure are very small (CIRIA, 1984; CUR-TAW, 1990). This fact makes the reliability evaluations quite sensitive to the choice of the distributions of some variables and, in particular, to the choice of the tails of the distributions (which, of course, are the most difficult parts to verify by data). Thus $\alpha_i^2$ gives a powerful means of examining which variables are most important and which make a negligible contribution to the variance of $Z$. Knowing $\alpha_i^2$, one might focus attention only on the most important variables (Ditlevsen & Bjerager, 1986).

If basic variables are not independent, three possible courses of action are open (Joint Committee on Structural Safety, 1978; CIRIA, 1984):

- if the variables are strongly correlated (say, correlation coefficients greater than 0.8), the variables may be conservatively assumed to be exactly dependent, in which case the effective number of basic random variables is reduced;
- if the variables are weakly correlated (say, correlation coefficients less than 0.2), the variables may be assumed to be independent;
- for all other cases it is necessary to use a method which deals with correlated variables.

In these latter cases, correlation between variables can be dealt with using different methods, with different levels of performance and complexity. Which method should be used depends upon the problem under study, the accuracy required for the answer, and the data available.

To transform non-Normal correlated variables to independent Normal variables the Rosenblatt transformation (Rosenblatt, 1952) is usually
recommended (Ang & Tang, 1984; Madsen et al, 1986; Thoft-Christensen & Murotsu, 1986; Melchers, 1987; Casciati & Natale, 1992). However, there are other simpler approaches. For example, correlation can be accounted for by allowing the distribution of one random variable to be expressed as a function of another random variable (Van de Graaff, 1986; Burcharth, 1992; Townend, 1994a; Super-Software, 1994). Another alternative is outlined below. It transforms a set of correlated variables, \( X_i \), into a set of non-correlated variables, \( Y_i \), where \( Y_i \) are linear functions of \( X_i, i=1,...,N \) (Thoft-Christensen & Baker, 1982; Ang & Tang, 1984; CIRIA, 1984; Smith, 1986; Melchers, 1987; Burcharth, 1992):

- Compute the covariance matrix \( \text{C}_{Ov}[X] \) from
  \[
  \text{C}_{Ov}[X] = \begin{bmatrix}
  \text{Var}[X_1] & \text{C}_{Ov}[X_1,X_2] & \cdots & \text{C}_{Ov}[X_1,X_N] \\
  \text{C}_{Ov}[X_2,X_1] & \text{Var}[X_2] & \cdots & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  \text{C}_{Ov}[X_N,X_1] & \cdots & \cdots & \text{Var}[X_N]
  \end{bmatrix}
  \tag{2.19}
  \]

- Compute the matrix of eigenvectors, \( V \), and the vector of eigenvalues, \( D \), of \( \text{C}_{Ov}[X] \)

- Compute \( \mu_Y = V^T \mu_X \) where \( V^T \) is the transpose matrix of \( V \)

- Compute \( \sigma_Y^2 = V^T C_{Ov}[X] V = D \)

- Compute \( Y=V^T X \)

Note that \( C_{Ov}[Y] \) is a diagonal matrix as follows:

\[
\text{C}_{Ov}[Y] = \begin{bmatrix}
  \text{Var}[Y_1] & \cdots & \cdots & 0 \\
  \vdots & \text{Var}[Y_2] & \cdots & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & \cdots & \cdots & \text{Var}[Y_N]
  \end{bmatrix} = \begin{bmatrix}
  \sigma_{Y_1}^2 & 0 \\
  \vdots & \sigma_{Y_2}^2 & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & \cdots & \cdots & \sigma_{Y_N}^2
  \end{bmatrix}
  \tag{2.20}
  \]

So, no correlation between any pair of random variables \( Y \) exists, as expected.
Non-Normal Random Variables

It is not always reasonable to consider random variables to be normally-distributed. For example, significant wave height, $H_s$, is likely to follow other distributions (e.g. Gumbel and Weibull) quite different from the Normal distribution, and it cannot be described solely by its mean value and standard deviation. For such cases of non-Normal variables, the Rosenblatt transformation (mentioned above) could be used. On the other hand, it is also still possible to use the reliability index concept but an extra transformation of the non-Normal basic variables into Normal basic variables must be performed. A commonly-used transformation (Thoht-Christensen & Baker, 1982; Ferry Borges & Castanheta, 1983; Ang & Tang, 1984; CIRIA, 1984; CUR-TAW, 1990) is that of Rackwitz & Fiessler (1978). It is based on substitution of the non-Normal distribution of the variable $X_i$ by a Normal distribution in such a way that the original probability density and cumulative distribution functions ($f_{X_i}$ and $F_{X_i}$, respectively) are equal to the corresponding values of the probability density and the cumulative distribution functions for a normally-distributed variable ($\phi$ and $\Phi$, respectively) at the design point $X^*$ (Figure 2.11):

![Figure 2.11: Rackwitz and Fiessler approximation for non-Normal variables (modified after Van der Meer, 1987).](image)
where $\mu_{X_i}^N$ and $\sigma_{X_i}^N$ are the mean and standard deviation of the approximate (fitted) Normal distribution. Solving the above equations for $\mu_{X_i}^N$ and $\sigma_{X_i}^N$:

$$
\mu_{X_i}^N = X_i^* - \Phi^{-1}[F_X(X_i^*)] \sigma_{X_i}^N \tag{2.22}
$$

$$
\sigma_{X_i}^N = \frac{\varphi[\Phi^{-1}[F_X(X_i^*)]]}{f_X(X_i^*)}
$$

The iterative method presented earlier can still be used if the values of $\mu_{X_i}^N$ and $\sigma_{X_i}^N$ are calculated during each iterative loop. Eq. (2.16) for calculation of the design point is applied using the values of $\mu_{X_i}^N$ and $\sigma_{X_i}^N$. In this case of non-Normal variables, the point determined by iteration does not correspond exactly to the point of maximum probability of failure density (see CIRIA, 1984, for further details). In general terms, the point may be considered to be a close approximation to the set of values of the basic variables most likely to cause failure.

### 2.3.3.3 Combinations Of Actions

In nature, many actions (or loads) vary with time$^1$. If a structure is subject to only one significant time-varying action, it is necessary to consider only the distribution of the maximum action during the anticipated life of the structure or the reference period, $T_{\text{ref}}$, for which the risk of failure is being assessed. However, if the structure is subjected to the effects of more than one time-varying action (e.g. waves and surges), then it is extremely unlikely that all of the actions will reach their peak lifetime values at the same moment (Turkstra, 1970; Der Kiureghian, 1980; Thoft-Christensen & Baker, 1982; CIRIA, 1984; Smith, 1986). Some benefit can be gained, in terms of reduced structural capacity, if this fact is taken into account, i.e. a structure can be

---

$^1$ Note that resistance also changes with time (e.g. Vasco Costa, 1987) and it can also be dealt with in a probabilistic manner (e.g. Nielsen & Burchar, 1983). This aspect is beyond the scope of the present research.
designed for a total action less than the sum of the peak actions. This fact has long been recognised (Turkstra, 1970; Ferry Borges & Castanheta, 1971, 1983).

Although complex stochastic models may be used with Level III methods (CIRIA, 1984), some simplifications are required at Level II. A popular model for treating combinations of actions at Level II is that due to Ferry Borges & Castanheta (1971, 1983), which is a development of an earlier proposal by Turkstra (1970). Several meetings have been held between Mr. Castanheta, a Research Engineer at the National Laboratory of Civil Engineering (LNEC), Lisbon, Portugal, and the author. They have stimulated very useful discussions about the implementation of combinations of actions at Level II.

Ferry Borges and Castanheta’s model has generally been very well accepted in the field (Turkstra & Madsen, 1980; Ditlevsen & Madsen, 1981; Bjerager & Skov, 1982; Thoft-Christensen & Baker, 1982; Melchers, 1987). Applications of the method are given in Ferry Borges & Castanheta (1971), CIRIA (1984) and Allsop & Meadowcroft (1995). It is described here and subsequently applied in the development of PARASODE. Note that some approximations in its basic concepts have been proposed by authors such as Paloheimo (1975), Ditlevsen (1976) and Rackwitz & Fiessler (1978), and other models have also been developed (Der Kiureghian, 1980; Madsen & Tvedt, 1990).

In the combinations of actions with which engineers are normally concerned, some actions change in intensity very much more rapidly than others (see Figure 2.12). If one action reaches its extreme value at some time during the design life, the combination of this action with the simultaneous values of the other actions may give the worst loading case (Turkstra, 1970).
In the Ferry Borges and Castanheta model, it is first assumed that for each time-varying action, $X_i$, the design life or reference period, $T_{ref}$, is sub-divided into a number, $r_i$, of elementary time intervals of equal length, $\tau_i$, such that $r_i = T_{ref} / \tau_i$ and $\tau_i = \ell_i \tau_{i+1}$, in which $\ell_i$ is an integer. The three following requirements should also be satisfied (CIRIA, 1984):

- for each interval, the occurrence or non-occurrence of the action corresponds to repeated independent trials with a probability $p_i$ of occurrence;
- for the duration of the time interval, the load $X_i$ remains at a constant intensity (or zero);
- the intensities of the action in different time intervals are independent.

This model is represented in Figure 2.13 for three time-varying actions.
For each time-varying action, $X_i$, it is then necessary to define the cumulative distribution function of the intensity of the action, $F_{X_i}$, corresponding to the basic time interval, $\tau_i$. The cumulative distribution function of the maximum intensity of action $X_i$ within the reference period, $F_{X_{i,r}}$, may then, in most cases, be approximated by (CIRIA, 1984):

$$F_{X_{i,r}} \approx F_{X_i}^{m_i}$$  \hspace{1cm} (2.23)

where $m_i = p_i r_i$. For any other shorter period $t < T_{ref}$, corresponding to, say, $s_i$ intervals of duration $\tau_i$, the cumulative distribution function of the maximum intensity becomes:

$$F_{X_{i,s_i}} \approx F_{X_i}^{s_i}$$  \hspace{1cm} (2.24)
The model then requires the ranking of the actions in increasing order of the individual number of possible repetitions \( r_i \) during the reference period such that \( r_1 \leq r_2 \leq \ldots \leq r_k \). As Ferry Borges & Castanheta (1983) demonstrate, an approximation to the maximum combined action may then be obtained by considering \( 2^{k-1} \) combinations of actions. For example, for \( k=3 \) time-varying actions:

<table>
<thead>
<tr>
<th>Comb.</th>
<th>( F_{X_1}^{r_1} )</th>
<th>( F_{X_2}^{r_2/r_1} )</th>
<th>( F_{X_3}^{r_3/r_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comb. 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comb. 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comb. 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comb. 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In practice, and consequently in certain literature (Ferry Borges & Castanheta, 1974; CIRIA, 1984; Allsop & Meadowcroft, 1995), some of the \( 2^{k-1} \) combinations of actions are neglected and only \( k \) combinations are considered. In this case (i.e. when the number of combinations taken into account is equal to the number of time-varying actions), the combination rule can easily be generalised as follows: when \( k \) time-varying actions are combined in \( k \) combinations of actions, each combination includes only one extreme distribution in the design life \( F_{X_i}^s \); actions having a number of repetitions less than \( r_i \) are idealised by their basic distributions \( F_{X_i} \); actions having a number of repetitions exceeding \( r_i \) are idealised by the reduced distributions \( F_{X_i}^{r_i/r_1} \) (see Table 2.1).

Referring to the example of the three time-varying actions shown in Figure 2.13, application of the combination rule (or Table 2.1) would require consideration of the three first combinations listed above, i.e.:

- combination of the distribution of the maximum value of the action \( X_1 \) during the reference period \( T_{ref} \) (\( NR_{11}=2 \); \( NR_{ji} \) is the power to which each basic distribution, \( F_{X_i} \), should be raised, for each time-varying action \( X_i, i=1,\ldots,k \), and for each combination \( j, j=1,\ldots,k \)) with the distribution of the maximum value of \( X_2 \) during a period \( \tau_1 \) (\( NR_{12}=6/2=3 \)), and with the distribution of the maximum value of \( X_3 \) during a period \( \tau_2 \) (\( NR_{13}=24/6=4 \));

- combination of the distribution of the action \( X_1 \) based on its elementary time interval \( \tau_1 \) (\( NR_{21}=1 \)) with the distribution of the maximum value of \( X_2 \) during the reference period \( T_{ref} \) (\( NR_{22}=6 \)), and with the distribution of the maximum value of \( X_3 \) during a period \( \tau_2 \) (\( NR_{23}=24/6=4 \));
• combination of the distribution of the action $X_1$ based on its elementary time interval $\tau_1$ (NR$_{31}=1$) with the distribution of the action $X_2$ based on its elementary time interval $\tau_2$ (NR$_{32}=1$), and with the distribution of the maximum value of $X_3$ during the reference period $T_{ref}$ (NR$_{33}=24$).

<table>
<thead>
<tr>
<th>Combination $j$</th>
<th>Action, $X_i$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>...</th>
<th>$X_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Power to which each basic distribution, $F_{X_i}$, should be raised, $NR_{ji}$</td>
<td>$r_1$</td>
<td>$r_2$</td>
<td>$r_3$</td>
<td>...</td>
<td>$r_k$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>NR$_{11}=r_1$</td>
<td>NR$_{12}=r_2/r_1$</td>
<td>NR$_{13}=r_3/r_2$</td>
<td>...</td>
<td>NR$<em>{1k}=r_k/r</em>{k-1}$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>NR$_{21}=1$</td>
<td>NR$_{22}=r_2$</td>
<td>NR$_{23}=r_3/r_2$</td>
<td>...</td>
<td>NR$<em>{2k}=r_k/r</em>{k-1}$</td>
</tr>
<tr>
<td>k</td>
<td></td>
<td>NR$_{k1}=1$</td>
<td>NR$_{k2}=1$</td>
<td>NR$_{k3}=1$</td>
<td>...</td>
<td>NR$_{kk}=r_k$</td>
</tr>
</tbody>
</table>

**Table 2.1**: Values of NR$_{ji}$ (modified after Rackwitz, 1976).

The distributions used in the combinations are referred to here as the modified distributions (note that, depending on the combination considered, these modified distributions may include basic, extreme and reduced distributions). To apply this model, either the modified distributions are known or they can be obtained if the basic distributions are available together with the number of repetitions of each action in the reference period. In the latter case, care should be taken in determining the number of repetitions to be adopted. As an example, Ferry Borges & Castanheta (1971, 1983) have shown that, for a reference period of fifty years, probability distributions of mean wind velocities with respect to elementary time intervals of one hour are transformed into distributions of yearly maxima by considering a fictitious number of repetitions of $r=50 \times 1000 = 50000$, considerably less than the total number of elementary intervals $r=24 \times 365 \times 50 = 438000$. In this case, the difference is due to the strong correlation between the successive hourly mean velocities, i.e. hourly mean values are not independent. When there are insufficient observations to determine the number of repetitions, the values adopted should be based on experience (Joint Committee on Structural Safety, 1978).

Finally, to determine the probability of failure of a structure associated with a specific failure mode and subjected to more than one time-varying action, the
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Level II methods described in Section 2.3.3.2 are applied for each of the $k$ combinations using the appropriate modified distributions of the time-varying actions. This yields $k$ values of the reliability index, $\beta$, which may then be combined to estimate the total failure probability using the relationship:

$$P_f \approx \sum_{j=1}^{k} \Phi(-\beta_j) \quad (2.25)$$

2.3.4 Level I Methods

As noted earlier, each failure mode must be described by a formula. Level I methods involve introducing partial coefficients (or partial safety factors) into the formula. The coefficients are chosen to give an acceptable margin between the characteristic resistance and the design load. By this means, the probability of failure is kept to a suitably low level. Safety factors were traditionally selected largely on the basis of intuition and experience (Committee of the Institution of Structural Engineers, 1955). However, the availability of Level II probability methods has made it possible to relate probabilistic measurements of safety, such as $P_f$ or $\beta$, to the partial coefficients in Level I methods. Hence, the safety factors can now be determined on a more scientific and rational basis (Yen, 1989).

The partial coefficients, $\gamma$, to be applied to a failure mode formula are usually larger than or equal to one. Consequently, if one splits the formula into either load variables $X_{S,i}$ or resistance variables $X_{R,i}$ then the related partial coefficients should be applied as follows to obtain the corresponding design values, $X_{S,j}$ and $X_{R,j}$:

$$X_{R,j}^* = \frac{X_{R,j}}{\gamma_{X_{R,i}}}$$
$$X_{S,j}^* = \gamma_{X_{S,i}} X_{S,in}$$

$X_{R,in}$ and $X_{S,in}$ are characteristic values of the resistance and load variables, respectively. These equations represent the key to the relationship between Level I and Level II methods: the so-called design point (Melchers, 1987; Van der Meer, 1987).
The term characteristic value was introduced in the late 1950s at the time when probabilistic concepts were first being introduced into structural codes of practice, and when it was recognised that few basic variables have clearly-defined upper and lower limits that can sensibly be used in design. For example, in civil engineering codes of practice, the characteristic values of resistance parameters, $X_{R_{ch}}$, might be chosen as values below which not more than 5% of test results may be expected to lie (the 5% fractile). Similarly, characteristic values of loads, $X_{S_{ch}}$, might be defined as the loads with a 5% probability of being exceeded (the 95% fractile) during the lifetime of the structure. Alternatively, they may be chosen to be the mean values. Other definitions may also be used (Madsen et al, 1986). Note, however, that the values of the partial coefficients are uniquely related to the chosen definitions of the characteristic values.

The magnitude of $\gamma_i$ reflects both the uncertainty in evaluating the related parameter $X_i$ and the relative importance of $X_i$ in the failure function (Burcharth, 1990). It is to be stressed that the magnitude of $\gamma_i$ is not, in a mathematical sense, a rigorous measure of the sensitivity of the failure probability to the parameter $X_i$ (PIANC, 1992).

When the partial coefficients are applied to the characteristic values of the parameters in a failure function, a design equation is developed. For the basic case of one resistance variable, $X_R$, and one load variable, $X_S$, the minimum requirement applied to a structure at Level I is that the following condition is satisfied:

$$Z = \frac{X_{R_{ch}}}{\gamma_{X_R}} - \gamma_{X_S} < 0$$  \hspace{1cm} (2.27)

$\gamma_{X_R}$ and $\gamma_{X_S}$ are partial safety factors of the resistance and load, respectively. Assuming a Normal distribution for $X_R$ and $X_S$, as adopted in the Level II analysis, the characteristic values are defined as:

$$X_{R_{ch}} = \mu_X - K_{x_R} \sigma_X$$

$$X_{S_{ch}} = \mu_X + K_{x_S} \sigma_X$$  \hspace{1cm} (2.28)
where $K_{X_R}$ and $K_{X_S}$ are coefficients defining the fractile which corresponds to the characteristic value (Madsen et al, 1986). This is illustrated in Figure 2.14 for the special case where $K_{X_R} = K_{X_S} = 1.96$.

![Figure 2.14: Definition of characteristic resistance, $X_{R,ch}$, and characteristic load, $X_{S,ch}$, when $X_R$ and $X_S$ are both normally distributed (modified after Melchers, 1987).](image)

Considering the general multi-variate basic variable problem and comparing eqs. (2.28) with eq. (2.16) for the design point, $X'_i$, a relationship between the partial coefficients in eqs. (2.26), the reliability index, $\beta$, and the sensitivity factors, $\alpha_i$, can be obtained (Ferry Borges & Castanheta, 1983; PIANC, 1992):

$$
\gamma_{X_R,i} = \frac{1 - K_{X_R,i} \frac{\sigma_{X_R,i}}{\mu_{X_R,i}}}{1 + \alpha_i \beta \frac{\sigma_{X_R,i}}{\mu_{X_R,i}}} \\
\gamma_{X_S,i} = \frac{1 + \alpha_i \beta \frac{\sigma_{X_S,i}}{\mu_{X_S,i}}}{1 + K_{X_S,i} \frac{\sigma_{X_S,i}}{\mu_{X_S,i}}} \quad (2.29)
$$

The partial coefficients can be related either to each parameter or to combinations of the parameters (as overall coefficients). They can also be tuned to ensure equal contributions from the various failure modes to the failure probability of the structure (Burcharth, 1990). Clearly, it is desirable to have a system which is as simple as possible, i.e. with as few partial coefficients as possible, but without invalidating the accuracy of the design equation beyond acceptable limits. Fortunately, it is very often possible to use overall coefficients without losing significant accuracy within the realistic
range of combinations of parameter values. This is the case for the system of partial safety factors developed for rubble-mound breakwaters proposed by PIANC (1992) where only two partial coefficients are used in each design formula. For full details, the reader should consult references such as Burchar (1991a) and PIANC (1992).

The accuracy of the Level I methods depends on several factors. The most important have been summarised by Allsop & Meadowcroft (1995) as follows:

- the range of applicability of the design procedure (the wider the area of application the less accurate are the safety factors);
- the complexity of the partial safety factor system (a system with many safety factors will easily match the target reliability over the area of application, but will be more complex to develop and apply);
- the underlying probabilistic methods (the reliability of sample designs is assessed using Level III or Level II methods; hence, the partial factors developed depend on the accuracy of those methods);
- the underlying failure functions (the reliability with which the failure functions represent the failure modes affects the accuracy of the safety factors);
- the source data (if the data used are not very reliable, the safety factors cannot be relied upon either).

Level I safety-checking methods are the basis for codes of practice, although the safety factors used are assessed by Level II and/or Level III methods (CIRIA, 1984). Codes of practice exist for offshore structures (see, for example, Fjeld, 1977) and nearly all types of land-based structures (Burchart, 1987). The question of whether coastal structures need such codes of practice is often raised (CUR-TAW, 1990) but has not yet been fully answered.

2.3.5 Comparison Of The Methods

The last few sections illustrate the features of different probabilistic methods. All of them have their advantages and disadvantages (Yen, 1989). There are
various reasons why one method might be used instead of another. The issues are discussed below.

In the numerical integration method, the calculation of an N-fold integral may be extremely time-consuming and it usually requires a considerable computational effort. Even with modern computer facilities, an enormous number of calculations is involved if the number of variables exceeds 5 or 6 and if the failure function is a complex one (Hohenbichler et al, 1987; Bjerager, 1991).

Traditional sampling is an acceptable alternative when dealing with simple failure functions and failure probabilities which are not very low. However, it suffers from the fact that if an "accurate" answer is desired for extreme conditions associated with relatively low probabilities of failure (approximately $10^{-4} < P_f < 10^{-8}$, according to Bjerager, 1991), many simulations are required. This is a drawback that recent methods, like Latin Hypercube sampling, may address to some extent by reducing the required number of simulations. In other cases, difficulties can be overcome by using Level II methods like FORM. The main practical advantages of this approach are that it is less time-consuming than Level III methods, the computational effort is independent of the probability level, it provides a rational basis for evaluating partial safety factors and it also provides an automatic procedure for determining the sensitivity of the computed failure probability to each of the basic design variables. This latter characteristic allows the designer to focus his attention on the parameters which are of greatest significance and shows where effort to reduce uncertainty should be concentrated. Due to their simplicity, these methods have become very popular, particularly in calibration work for codes of practice (Melchers, 1987). Note that unlike Level III methods which can be used only for reliability analysis (safety checking), Level II procedures may be used also for design (i.e. design for a specified reliability level). However, these procedures also have their limitations. Amongst others, the main reason for discrepancy between a Level II and a Level III method is that the failure function is usually non-linear. The stronger the non-linearity, the greater is the chance that the Level II results will differ considerably from the "exact" answer. However, the FORM results can be improved through a second or higher order approximation (Ang & Tang, 1984; Madsen et al, 1986; Jang et al, 1994), as mentioned briefly in Section 2.3.1. But computational complications are
increased considerably. Therefore, at present, these methods are seldom used. It is more common to use the Level III methods, especially simulation, to validate the Level II results. Although a FORM method can provide an answer to a problem, it is never known how accurate the answer is unless a check is done using numerical integration or simulation techniques. Nevertheless, the FORM method is one of the most important tools in probabilistic design because one can rarely afford to make a million Level III calculations during preliminary design.

Besides the calculations at Level III and Level II, there are those at Level I. Level I calculations are particularly suitable for everyday design (where a large body of previous experience of similar systems is available), although the determination of the partial coefficients must be based upon higher level results. Level I calculations are the basis of codes of practice.

If probabilistic methods are used with foresight and understanding, they are powerful and can provide reliable results. For example, comparison of design alternatives using these methods is a promising way in which to apply them (Dover & Bea, 1979; Townend & Fleming, 1991; Melchers, 1993). One can use these methods to decide what is the difference in probability of failure of a structure designed with strategy A compared with one designed with strategy B and what are the associated projected consequences. Meaningful decisions can be based on such comparisons. However, application of probabilistic methods leads to the question of whether or not the calculated probability of failure corresponds to reality. It is often argued that a probabilistic analysis is meaningful only if there is a complete understanding of the physics and if the analysis is based on accurate computational models and on sufficient statistical data (Burcharth, 1983, 1985; CUR-TAW, 1990). However, in practice, these requirements are seldom fulfilled. Analysis has to address an idealised system founded upon assumptions, simplifications and the collective judgement of a number of individuals, so that the real system becomes manageable. As CIRIA (1984) emphasise, probabilistic methods should be viewed as an aid to the application of humanity, experience and judgement, not as their replacement. In this connection, the calculated probability of failure is related to the idealised system (and not directly to the real system) and should be interpreted as a measure of the confidence in a particular design. It is a notional probability of failure, instead of the frequency of cases of failure (Veneziano, 1976; CIRIA, 1984; Van der Meer,
1987, CUR-TAW, 1990). In other words, the numerical and graphical results produced by the probabilistic methods should be taken as illustrating orders of magnitude and trends, rather than describing reality (Fjeld, 1977; Burcharth, 1985). Hence, the purpose of a reliability analysis is not so much to calculate the exact failure frequency as to produce as good and balanced a design as possible with the available information. In fact, the engineer is expected to develop a model of the phenomenon under study which embodies its salient features and which can be used to make optimal decisions using the data available (Turkstra, 1970; Burcharth, 1985; Ditlevsen & Bjerager, 1986; Melchers, 1993). He is not expected to produce a perfect image of reality which is an impossible task (Lamberti, 1992). In this pragmatic sense, even simplistic models, approximate probabilities, notional reliabilities, etc., may be satisfactory (Veneziano, 1976). It is also worthwhile noting that the less one knows, the more important it is to try to assess the reliability of a structure (Burcharth, 1983, 1990). The probabilistic approach is the only one which gives information on the risk of failure with due consideration to the uncertainties of the various parameters involved. As Harlow (1985) stated:

The art and science of civil engineering deals with applying the materials of nature to the use of mankind, but it does not presuppose complete understanding of all facts. Civil engineers have always worked with incomplete information and probably they always will.

The fact that there is neither a complete understanding of the physics nor accurate computational models and sufficient statistical data to make the best use of probabilistic methods, does not mean that they should be discarded (Yen, 1989). On the contrary, effort should be devoted to describing the physical processes and establishing the appropriate models and data sets required for their full implementation. Thus, much more work is required if complete and objective risk assessments are to be reached. For instance, there is a great need for detailed monitoring of existing coastal structures, including recording the wave conditions to which they are subjected (Ouellet, 1974; MAFF, 1993c). Furthermore, it is important to incorporate as much experience as possible from failures. Although unacceptable and costly, failures test the limits of our knowledge and, in some way, are the price of progress (Eberhardt, 1979; Sorensen & Jensen, 1985). Failures are of such great importance to the engineering community that their details should be widely published. For example, a team from HR Wallingford/Sir William Halcrow & Partners Ltd created, as part of
research carried out for the National Rivers Authority, a database of flooding instances, including information such as details of the location and type of structure, a description of the failure mode, and an assessment of consequences (Meadowcroft et al, 1996). This sort of initiative should be encouraged.

2.4 Closing Remarks

Risk analysis provides a powerful framework for the design of coastal structures, accounting for the probability and consequences of failure as well as coping, to some degree, with variability and uncertainty. However, when assessing structural safety using probabilistic methods, it must be stressed that the process involves knowledge about the individual structure. Therefore, confidence in the calculated value of the probability of failure must change with the amount and quality of the information used for its calculation. With this philosophy in mind, risk analysis may be seen simply as a design tool based on scientific methods which can facilitate good engineering decisions, but not a process which will necessarily provide a precise assessment of safety.

In recent years, much has been learned by coastal engineers about risk analysis, but progress in formulating methods and gaining confidence in new design procedures is inevitably slow. At present, there is insufficient knowledge about coastal structures to enable a probability analysis of failure mode systems to be carried out in full. However, instead of abandoning this “new” approach to design, efforts should be made to better identify the specific physical processes with which coastal engineers must deal, to better communicate their data requirements to researchers, to subsequently collect the required data sets, and to establish the appropriate models necessary for the complete implementation of the methods.

Some existing studies have aimed at being very broad, covering failure modes, consequences and costs. Others have focused on particular failure modes. This demonstrates the difficulty in achieving a satisfactory compromise between theory and practice, and developing an approach applicable to a wide range of coastal structures.
Diagrams like event trees, fault trees and cause-consequence charts have been presented for some coastal structures. However, such techniques have still almost always served essentially as schematic representations or research tools rather than as strict logical analyses of failure. Information on failures tend to concentrate on the consequences rather than on the causes of failure.

Assessment of the safety of coastal structures depends fundamentally on assessment of individual failure modes. The single failure mode probabilistic methods relevant to this research have been presented in Section 2.3. All of them have their advantages and disadvantages (see Section 2.3.5). The important issue is to be aware of the characteristics of the methods, their applicability and their limitations, otherwise wrong conclusions can be drawn, incorrect decisions can be made and unsound action may be taken.
3 WAVE OVERTOPPING OF SEAWALLS

3.1 Introduction

Seawalls are expensive, and fixing a seawall freeboard at too large a value has both a financial penalty and is unnecessarily damaging to the natural environment owing to the increased impact of the structure on its surroundings. On the other hand, if the crest of a seawall is set too low, then there are problems with structural safety and potential social problems associated with flooding from wave overtopping. Hence, it is important to strike the correct balance between satisfying the structural and functional requirements of the project, avoiding unnecessary expense, and having undesirable impacts on the surrounding environment.

Recent damage caused by excessive wave overtopping (Jensen, 1984), concerns over global warming, the allied rise in mean-sea-level and increased storminess have all drawn attention to the fitness of existing coastal structures. There is a need to assess their capability in withstanding both higher water levels and increased wave activity. Effective evaluation depends upon having an adequate theoretical framework for predicting wave overtopping and suitable data for validating the theory, for evaluating the associated empirical coefficients and for undertaking risk analyses.

Overtopping may occur for relatively few waves under the design event, and low overtopping rates may often be accepted without severe consequences for the seawall or the area protected by it. On the other hand, some structures are designed to have quite severe overtopping under design conditions. Other structures, such as breakwaters, may be so low that they are overtopped daily. The acceptable overtopping discharge depends upon the activities normally performed in the lee of the structure, the need to prevent erosion of its rear face, and the socio-economic consequences of flooding.

Under random waves, the overtopping discharge varies greatly from wave to wave. There are very few data sets available to quantify this variation. It may be described by (Allsop, 1994):
• the percentage of waves passing over the crest, $N_{WO\%}$;
• the mean overtopping discharge per unit length of structure, $Q$ ($m^3/s/m$);
• the peak volume in an individual wave, $V_p$.

Each of these responses depends on wave and structure parameters, including: the seawall freeboard, the crest geometry, the seaward slope, the significant wave height, and the mean or peak wave period, the angle of wave attack measured from the normal to the structure, the water depth at the toe of the seawall, and the seabed slope. In many cases, for example for determining the required drainage capacity behind seawalls and the depth of flooding in the hinterland, it is sufficient to use the mean discharge, $Q$ (Jensen & Sorensen, 1979; Owen, 1982b; Jensen & Juhl, 1987; Kobayashi & Wurjanto, 1989; CIRIA/CUR, 1991; Van der Meer, 1993). The calculation of the mean overtopping discharge for a particular structure geometry, water level and wave condition is mainly based on empirical equations fitted to hydraulic model test results (Ward et al, 1994). A well-known data set applies to flat-topped embankments fronted in some cases by a flat berm (Hydraulics Research Station, 1980; Owen, 1982a). These tests were aimed at establishing the impact on overtopping discharge of the wave climate (including the angle of wave attack and the wave steepness), the seawall slope, the crest and berm elevations, and the berm width.

In the remainder of this chapter, a brief review of wave overtopping of seawalls is first given, including the models currently used for prediction and the permissible limits of overtopping. This brief review has the objectives of highlighting the deficiencies in current knowledge which are relevant to the work developed as part of this research, and to direct the reader towards sources of more detailed information. Then, a new model is presented: this is the H&R model. This model is conceived from theoretical considerations: the purpose has been to construct a model which accounts for the physical boundary conditions. The H&R model and the Owen model are used in a re-analysis of Owen’s data for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, subjected to random waves approaching normal to the slope. Owen’s data were chosen for re-analysis because of their extensive nature and ready availability from HR Wallingford Ltd. There were two main difficulties in processing these data: first, the database contained a considerable number of errors; and, second, some information was missing.
The problems were overcome by consulting the original data sheets at HR Wallingford. The implications of the H&R model for seawall freeboards are also discussed and an illustrative example is given. Finally, the reliability of the two models is assessed by considering the randomness of possible model parameters. The software package BestFit is used to support the choice of probability distributions to describe the model parameters.

3.2 Literature Review

3.2.1 Historical Perspective

Until about 1980, the crest levels of UK seawalls were generally set in relation to an extreme wave run-up level (Allsop et al, 1985a; Allsop, 1986). In The Netherlands, the level for coastal embankments was an extreme water level with an allowance for the 2% run-up, $R_{2\%}$ (TACPI, 1974; CUR-TAW, 1990). This fact implied acceptance that about 2% of waves might overtop under the design event. Random wave run-up levels are described in the CIRIA/CUR (1991) manual. Methods from the USA and Japan predicted wave run-up levels based on regular waves only (U.S. Army Corps of Engineers, Coastal Engineering Research Center, 1984; Goda, 1985; Douglass, 1986).

Since the late 1970s, there have been major advances in the prediction of wave overtopping, particularly from laboratory work in the UK, The Netherlands and Italy (Allsop, 1994). These studies concentrated on the prediction of mean overtopping discharge over unit length of seawall. In the late 1970s, studies using random waves were analysed by Owen. He developed a design equation relating the mean overtopping discharge to incident wave conditions, described by the significant wave height and the mean zero-crossing wave period, and the seawall's freeboard (Hydraulics Research Station, 1980; Owen, 1982a). The results covered simple and bermed seawalls, but were then extended to include slopes with crown or recurved walls and rough or armoured slopes (Bradbury & Allsop, 1988; Bradbury et al, 1988; Aminti & Franco, 1988; Owen & Steele, 1991; Pederssen & Burcharth, 1992; Allsop & Franco, 1992; De Waal & Van der Meer, 1992; Besley et al, 1993; Herbert et al, 1994; Monso et al, 1996). Methods to predict overtopping of vertical seawalls were derived by
Goda (1971), Goda et al (1975) and Goda (1985), and were extended by random wave tests in the UK and The Netherlands (Herbert, 1993; De Waal, 1993; Herbert et al, 1994).

Recent studies in Italy, The Netherlands and the UK have quantified not only the mean overtopping discharge but also the distribution of overtopping volumes per wave and, particularly, the maximum individual overtopping volume (Smith et al, 1994; Franco et al, 1994; Van der Meer & Janssen, 1995). A relationship may then be used to relate overtopping volume and mean overtopping discharge, but this relationship varies with the structure geometry and wave conditions.

### 3.2.2 Prediction Methods

In general, the mean overtopping discharge per unit length of seawall, \( Q \), depends upon the wave motion, the seawall profile, the foreshore characteristics, and the water properties:

\[
Q = f(H_s, T_m, \beta, R_c, \alpha, d_s, g, \ldots)
\]  

(3.1)

\( H_s \) is the significant height of the incident waves; \( T_m \) is the mean zero-crossing wave period; \( \beta \) is the angle of wave approach measured from the normal to the seawall; \( R_c \) is the seawall’s freeboard (the height of the crest of the structure above the still-water-level); \( \alpha \) is the angle of the seawall front slope measured from the horizontal; \( d_s \) is the still-water-depth at the toe of the structure; and \( g \) is the acceleration due to gravity.

Figure 3.1 shows this notation. In the figure, \( T_p \) is the wave period corresponding to the peak spectral density, \( CL \) denotes crest level, \( TL \) denotes toe level and \( SWL \) is the still-water-level above datum.
Eq. (3.1) may be rewritten in the form of dimensionless groups:

$$\frac{Q}{\sqrt{gH_s^3}} = f\left(\frac{R_c}{H_s}, \frac{H_s}{gT_m^2}, \frac{d_s}{H_s}, \alpha, \beta, \ldots\right)$$

(3.2)

or

$$\frac{Q}{g^2 T_m^3} = f\left(\frac{R_c}{H_s}, \frac{H_s}{gT_m^2}, \frac{d_s}{H_s}, \alpha, \beta, \ldots\right)$$

(3.3)

$H_s/gT_m^2$ is a measure of the incident wave steepness. Owen combined this group both with $Q / \sqrt{gH_s^3}$ (or $Q / g^2 T_m^3$ ) and with $R_c/H_s$, to write:

$$\frac{Q}{T_m gH_s} = f\left(\frac{R_c}{T_m gH_s}, \frac{H_s}{gT_m^2}, \frac{d_s}{H_s}, \alpha, \beta, \ldots\right)$$

(3.4)

He then suggested that:

$$\frac{Q}{T_m gH_s} = A \exp\left(-B \frac{R_c}{T_m gH_s}\right)$$

(3.5)

A and B are best-fit coefficients determined from experimental data. However, other arrangements are possible (Aminti & Franco, 1988; Ahrens &
Wave Overtopping Of Seawalls

Bender, 1991), including use of the wave period of peak spectral density, $T_p$, rather than the mean zero-crossing period, $T_m$. Table 3.1 summarises some of the options for dimensionless discharge, $Q^*$, and dimensionless freeboard, $R^*$, used in studies of the overtopping of seawalls and other structures.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Dimensionless Discharge, $Q^*$</th>
<th>Dimensionless Freeboard, $R^*$</th>
<th>Overtopping Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulics Research Station (1980); Owen (1982a)</td>
<td>$Q \over T_m g H_s = Q \over \sqrt{2 \pi g H_s^3}$</td>
<td>$R_c \over T_m \sqrt{g H_s} = \overline{R_c} \over H_s \sqrt{2 \pi}$</td>
<td>$Q = A \exp(-BR)$</td>
</tr>
<tr>
<td>Bradbury &amp; Allsop (1988)</td>
<td>$Q \over T_m g H_s$</td>
<td>$R_c \over T_m \sqrt{g H_s^3}$</td>
<td>$Q = A(R)^{-B}$</td>
</tr>
<tr>
<td>Ahrens &amp; Heimbaugh (1988)</td>
<td>$Q \over \sqrt{g H_s^3}$</td>
<td>$R_c \over (H_s^3 L_p)^{1/3}$</td>
<td>$Q = A \exp(-BR)$</td>
</tr>
<tr>
<td>Sawaragi et al (1988)</td>
<td>$Q \over \sqrt{g L_s H_s^3}$</td>
<td>$R_c \over H_s$</td>
<td>$Q_A R_B^*(\cdot) = -\exp(-\alpha)$</td>
</tr>
<tr>
<td>Aminti &amp; Franco (1988)</td>
<td>$Q \over T_m g H_s$</td>
<td>$R_c \over H_s$</td>
<td>$Q = A(R)^{-B}$</td>
</tr>
<tr>
<td>Ward (1992)</td>
<td>$Q \over \sqrt{g H_s^3}$</td>
<td>$R_c \over (H_s^3 L_p)^{1/3}$</td>
<td>$Q = A \exp(-BR) \exp(C \cot \alpha)$</td>
</tr>
<tr>
<td>Pedersen &amp; Burcharthy (1992)</td>
<td>$Q \over T_m L_m^2$</td>
<td>$R_c \over H_s$</td>
<td>$Q = A(R)^{-B}$</td>
</tr>
<tr>
<td>De Waal &amp; Van der Meer (1992)</td>
<td>$Q \over \sqrt{g H_s^3}$</td>
<td>$R_c - R_{2%} \over H_s$</td>
<td>$Q = A \exp(-BR)$</td>
</tr>
<tr>
<td>Van der Meer (1993); Smith et al (1994); Van der Meer &amp; Janssen (1995)</td>
<td>$Q \over \sqrt{g H_s^3}$ for $\xi_p &lt; 2$</td>
<td>$R_c - R_{2%} \over H_s$ for $\xi_p &lt; 2$</td>
<td>$Q = A \exp(-BR)$</td>
</tr>
<tr>
<td>Franco et al (1994)</td>
<td>$Q \over \sqrt{g H_s^3}$ for $\xi_p &gt; 2$</td>
<td>$R_c - R_{2%} \over H_s$ for $\xi_p &gt; 2$</td>
<td>$Q = A \exp(-BR)$</td>
</tr>
</tbody>
</table>

**Table 3.1:** Some options for dimensionless discharge, dimensionless freeboard and overtopping model.

Note that $H_s$ has been used to denote the significant wave height calculated either as the mean height of the highest one third of the waves in a record or estimated from the zeroth moment of the surface elevation spectrum (IAHR, 1989). $L_s$ is the Airy wavelength calculated using the water depth at the toe of the structure and the significant wave period, $T_s$. $L_m$ and $L_p$ are the
corresponding wavelengths calculated using $T_m$ and $T_p$. $s_m = 2\pi H_s / g T_m^2$ is the deep water wave steepness calculated using $T_m$. $R_{2\%}$ is the run-up exceeded by only 2% of the incident waves. $\gamma$ is a reduction factor to account for influences of berms, roughness, shallow water and oblique wave attack on wave run-up and overtopping. $\xi_p$ is the surf similarity parameter calculated using the wave period of peak spectral density ($\xi_p = \tan \alpha / \sqrt{H_s / L_{op}} = \tan \alpha / \sqrt{s_p}$) in which $L_{op} = g T_p^2 / 2\pi$ and $s_p = 2\pi H_s / g T_p^2$. $C$, like $A$ and $B$, is a coefficient. Finally, note that $R^*$ for De Waal & Van der Meer (1992) is not strictly a dimensionless freeboard but the dimensionless excess of the crest level above the 2% run-up level.

Table 3.1 shows that the two most common overtopping models relating the dimensionless groups are:

$$Q^* = A \exp(-BR^*) \quad (3.6)$$

$$Q^* = A(R^*)^B \quad (3.7)$$

where, again, $A$ and $B$ are best-fit coefficients determined from the experimental data. Clearly, coefficients $A$ and $B$ must account for all influences on $Q^*$ other than $R^*$. Note that the $\gamma$ reduction factor used by some authors (Van der Meer, 1993; Van der Meer & Janssen, 1995) to account for influences of berms, roughness, shallow water and oblique wave attack on wave run-up and overtopping can similarly be incorporated in any other empirical model.

Dimensional analysis provides no means for determining which sets of dimensionless groups may be especially informative or helpful in dealing with a particular data set. A possible problem in using many of the pairings in Table 3.1 is the potential for spurious correlation. A spurious correlation may arise when dimensionless groups plotted against one another contain a common variable (Massey, 1971). There is nothing wrong with the presence of a common variable, but care must be taken in interpreting such plots. Scatter in the data may be suppressed simply by the presence of this variable.
3.2.3 Numerical And Physical Modelling

Much of the existing literature on overtopping reports the results of physical model tests. Note, in particular, that:

- It is not yet possible to predict wave overtopping from an entirely theoretical basis, even for regular waves.
- The empirical methods (Table 3.1) were derived by means of measurements in random wave physical models. Such models can be constructed in any well-equipped wave basin or flume. Test procedures used by the major laboratories are well-established, and overtopping measurements made using these procedures are generally comparable. However, measurements can be subject to scale effects and this fact has to be taken into consideration (Thomas & Hall, 1992).
- The empirical methods (Table 3.1) are valid only for the ranges of structure configurations and of wave conditions tested, but many practical situations fall outside of these ranges. Uncertainties in a simple empirical method may not be acceptable for particular design problems. In such instances, a short series of physical model tests may provide the most accurate and economic design tool (CIRIA/CUR, 1991; Allsop, 1994).
- The physical model results may be used to calibrate numerical models.

Numerical modelling of wave overtopping is still under development. The description of wave hydrodynamics at structures is very complex which makes the task very difficult (Allsop, 1994). However, knowing the wave hydrodynamics (such as the water velocity and depth of the water overtopping the crest of the structure) would enable, for example, a more sensible prediction of the severity of the damage caused by wave overtopping (Lording & Scott, 1971; Jensen, 1983; Kobayashi & Raichle, 1994). In addition, it would help in filling the gap between empirical formulae (valid only for the ranges of structure configurations tested) and the many practical situations which fall outside of the conditions tested (Kobayashi & Wurjanto, 1989). Two types of numerical models are relevant for overtopping (Van der Meer, 1994): i) the simplest one uses a depth-averaged wave formulation, running a bore-like wave up a (shallow) slope; ii) an alternative method uses the "Volume of Fluid" method to describe the full fluid motions in two or, possibly, three dimensions; this method can calculate fluid motions...
beyond the point of wave breaking, but requires super-computer power for even relatively few waves.

Prototype measurements of wave overtopping quantities are scarce. To the author's knowledge, only two sets of measurements are available from literature. Fukuda et al (1974) made direct observations of the rate of wave overtopping at the wave absorbing revetment of Niigata East Port, facing the Sea of Japan, in the winter of 1971 and 1972. The Danish Hydraulic Institute made field measurements on a small Danish breakwater in the Port of Hundested in 1977 (Jensen & Juhl, 1987).

3.2.4 Permissible Limits Of Overtopping

The definition of tolerable limits for overtopping is still an open question, given the high irregularity of the phenomenon and the difficulty of measuring it and its consequences (Franco et al, 1994). Remarkably little information is available on the effects of wave overtopping on the defence structure itself, or on activities behind the structure (Allsop, 1994). Obviously the overtopping criteria for design depend upon the structure’s function and degree of protection required, and upon the associated risks, taking into account the joint probability of wave heights and water levels. In fact, relatively large overtopping might be allowed during extreme storms if pedestrians and vehicle movements on the structure are prohibited.

Until very recently, the permissible limits on the mean overtopping discharge were set by three criteria (Allsop, 1994):

- storage volume available behind the structure for overtopping during high water;
- potential damage to the crest or rear slope of the structure;
- danger or inconvenience to people or vehicle traffic, or damage to buildings.

The limits set by storage volume are specific to the individual site; they are not amenable to general guidance. An analysis of wave overtopping of seawalls in Japan suggested limiting values of the mean overtopping discharge to ensure the safety of vehicles and people in the immediate
vicinity of the seawall, and to prevent damage to buildings in its lee and to the seawall itself. Based on the impressions of experts observing prototype overtopping, the values can be seen in Figure 3.2. They are included in design manuals/standards such as the CIRIA/CUR (1991) manual and BS 6349 (BSI, 1991b). A further design manual is being planned which will describe in detail the overtopping performance of seawalls and the standards to which they should be designed (Herbert et al, 1994). Although anticipated to be available in draft form in late 1995, it has not been released to date.

Figure 3.2: Critical mean overtopping discharges for use in design (modified after CIRIA/CUR, 1991).

Figure 3.2 shows that full-scale discharges greater than about $0.001 \times 10^{-3} \text{m}^3/\text{s/m}$ will be unsafe for vehicles at high speed and may cause minor damage to the fittings of buildings. Conditions become dangerous for pedestrians when the discharge exceeds $0.03 \times 10^{-3} \text{m}^3/\text{s/m}$. Discharges greater than about $2 \times 10^{-3} \text{m}^3/\text{s/m}$ may damage embankment seawalls, whilst $50 \times 10^{-3} \text{m}^3/\text{s/m}$ is approximately the critical discharge for seawalls without back slopes. Research in Italy using model cars and people (De Gerloni et al, 1991; Franco et al, 1994) suggests that higher limits than those shown in Figure 3.2 might be appropriate in some circumstances. However, these limits take no account of the psychological effects of sudden wetting, shock, and related factors (Allsop, 1994).
The main point to note from Figure 3.2 studies is that the range of critical mean discharges runs from as little as \(10^{-6}\) m\(^3\)/s/m to about \(2\times10^{-1}\) m\(^3\)/s/m. Higher overtopping rates are of little interest in the design of seawalls, though they may be of concern to the designers of breakwaters. These apparently low figures account for the fact that danger levels are actually determined by the single largest overtopping wave which, due to the high irregularity of the physical phenomenon, can produce peak intensities much greater than the average intensity (Aminti & Franco, 1988).

According to Allsop (1994), continuing research in Italy and in the UK suggests that the volume in the largest individual overtopping wave may be about 6 to 10 times the average volume in an overtopping wave. Assuming that this quantity is discharged over 1/4 of a wave period, it may be shown that the peak overtopping discharge, \(Q_p\), is then given by:

\[
Q_p \approx 4000 \frac{Q}{N_{WO\%}}
\]  

(3.8)

Now, consider a low level of overtopping, say \(N_{WO\%} = 0.5\%\), and \(Q = 0.03 \times 10^{-3}\) m\(^3\)/s/m. Under this condition, the worst wave could well project about 50 litres at significant speed, equivalent to an instantaneous discharge of \(240 \times 10^{-3}\) m\(^3\)/s/m, probably with relatively little warning (Allsop, 1994). The impact of such a volume of cold water could cause anyone walking or running to fall. In this connection, another factor which is worth considering is the intensity of water falling as a function of the horizontal distance behind the structure. Work has already begun on this subject related to breakwaters (Jensen & Sorensen, 1979; Jensen & Juhl, 1987). Similar research for seawalls would also be beneficial to designers.

Whether considering mean overtopping discharges or volumes of overtopping, attention must be paid to the influence which the wind has both on overtopping quantity and distance of travel (TACPI, 1974; Jensen & Sorensen, 1979; Gadd et al, 1984; Allsop, 1986; Thomas & Hall, 1992). The most severe incident wave conditions are likely to be associated with strong onshore winds which increase the overtopping rate. As a guide, the overtopping discharges calculated using the empirical models (Table 3.1) may be multiplied by the wind correction factor quoted in the Shore Protection Manual (U.S. Army Corps of Engineers, Coastal Engineering

3-11
Research Center, 1984). The factor ranges from a value of unity for a very flat slope with a low freeboard or when there is no wind, to over three for a vertical wall with a high freeboard and with very strong onshore winds. However, Jensen & Juhl (1987) show physical model results which suggest that the effects of the wind may be even greater if the seawall surface throws water into the air, such as may occur when the front slope is armoured with natural stone or concrete units with large voids. Unfortunately, spray is not correctly simulated in hydraulic model tests owing to the influence of surface tension. Research on the effects of wind on wave run-up and overtopping is currently being conducted (Ward et al, 1994, 1996; De Waal et al, 1996), including laboratory and prototype measurements. Laboratory results from research undertaken by Ward et al (1996), which have not yet been published but have been made available to the author, suggest that model wind speeds of up to 6.5m/s have little effect on run-up and overtopping of smooth and rough slopes of 1:1.5, 1:3 and 1:5. Only strong winds of 12m/s and 16m/s have greatly increased both run-up and overtopping. The difficulty remains of scaling up the effects of wind in the model to prototype values. More information from the above studies will be very useful in improving understanding of the phenomenon, with a view to further development of the models used to predict overtopping. The studies can also help in defining the content of further research on the effects of wind.

### 3.3 Regression Analysis

Once experimental data have been collected, they may be used to confirm the validity of some theory or, where no satisfactory theory exists, they may be used to construct regression models. However, it is always useful to have some theoretical basis for choosing amongst the possible models. Furthermore, there are many techniques available for fitting regression models (Gunst & Mason, 1980). Which ones are appropriate for a particular study depend upon its objectives. For example, it may be possible to develop a model which is good at predicting values of the response variable but which, nevertheless, is incorrectly specified (i.e. the model does not include all relevant predictor variables or it has an incorrect functional form). In describing a regression model, care should be taken to emphasise the range of conditions over which there are data to support its use. Unfortunately, it is sometimes impossible to collect data on the dependent or response variable.
(in this instance, overtopping) over the entire range of interest of the independent or predictor variables (wave height, structure profile, etc).

Consider the following example. The ability of armour stones to remain in place on a slope under the action of waves may be characterised in terms of their stability number, \( N_s \):

\[
N_s = \frac{H}{\left( \frac{\rho_s}{\rho} - 1 \right) \left( \frac{W}{\rho_s g} \right)^{1/3}}
\]  

(3.9)

In this expression, \( H \) is the height of regular waves for which armour stones of weight \( W \) and density \( \rho_s \) are just stable; \( \rho \) is water density and \( g \) is gravitational acceleration. The influence of the armour slope, \( \alpha \), on the value of \( N_s \) has been investigated in many studies including those of Iribarren (1938) and Hudson (1959). They proposed the following relationships:

Iribarren: \[ N_s = \frac{\left( \mu_f \cos \alpha - \sin \alpha \right)}{K^{1/3}} \]  

(3.10)

and

Hudson: \[ N_s = \left( K_D \cos \alpha \right)^{1/3} \]  

(3.11)

\( K \) and \( K_D \) are armour stability coefficients and \( \mu_f \) is the coefficient of friction between the armour stones. Note that Iribarren’s equation correctly predicts that \( N_s \) will be zero when the armour slope is at its natural angle of repose, \( \phi \). In this state, \( \mu_f = \tan \phi = \tan \alpha \). Under these conditions, there is a balance between the shear resistance along potential failure planes within the mass of stones and the downslope force induced by gravity (Hedges, 1983, 1985). Any small wave height would disturb this balance and, consequently, \( N_s \) must be zero. However, Hudson’s relationship gives \( N_s \) as zero only when \( \alpha = 90^\circ \) (assuming that \( K_D \) is not zero). Note, also, that Hudson’s formula suggests that the armour will be infinitely stable when \( \alpha = 0^\circ \) (suggesting that even particles of sand would not move on a horizontal seabed), whilst Iribarren’s formula predicts a finite level of stability. Figure 3.3 shows the relationships between \( N_s \) and \( \alpha \) provided by the two empirical expressions.
Both Iribarren's and Hudson's formulae may be used to fit experimental data over the normal range of slopes upon which armour is placed, for the types of armouring and wave conditions for which they have been developed, and so on. In other words, they may be good for predicting responses within the limitations imposed by the range of conditions for which they have been tested. However, there could be considerable errors if these formulae were used for other purposes (for example, to assess the relative importance of individual predictor variables to armour failure). Note that neither formula explicitly includes wave period as a predictor variable (though its influence could be contained within the values of the coefficients $K$ and $K_D$).

The above example suggests that whilst Hudson's formula may fit data within the normal range of slopes, it does not fit the obvious physical boundary conditions which require that $N_s=0$ when $\alpha = \phi$ and $N_s$ remains finite when $\alpha = 0^\circ$. If this formula was to be applied near the boundaries, it would result in considerable errors. This experience can be used to provide a more physically reasonable regression model for wave overtopping data rather than merely providing an empirical fit to the data. As well as ensuring that all
relevant predictor variables are identified, it seems important to address the physical boundary conditions which should be satisfied. As a start, consider the physical boundary conditions to be met in addressing wave overtopping:

i) when the embankment has a large freeboard (i.e. when its crest elevation is well above the level of wave uprush), the predicted overtopping discharge should be zero (assuming that the effects of wind-blown spray are ignored);

ii) when the embankment has zero freeboard (i.e. when still-water-level is at the crest level of the embankment) then the predicted overtopping discharge may be large but should still remain finite.

As mentioned in Section 3.2.2, eqs. (3.6) and (3.7) represent two of the more common functions used to predict wave overtopping. However, when $R^*$ is large, both expressions suggest that the discharge will be finite rather than zero (though it is small provided that $A$ is not very large and provided also that $B>1$). When $R^*$ is zero, the first of these expressions gives $Q^* = A$, a finite quantity, whilst the second expression gives $Q^* = \infty$. Thus neither expression satisfies both boundary conditions, with the second of them satisfying neither. Since most seawalls are designed to permit only relatively small overtopping discharges (see Section 3.2.4), it is especially important to satisfy the first of the two boundary conditions.

In addition to considering the boundary conditions, there is also the need to establish the line of “best fit” to the observed data. There are many criteria for defining the best fit. One possibility is to minimise the sum of the squared deviations of the observations from the values predicted from the expression. But real data usually do not completely satisfy the classical assumptions for a least-squares (LS) fitting (Rousseeuw & Leroy, 1987). For example, the deviations may not follow a Normal distribution. Reliable inferences may be drawn from regression models fitted by the LS method only if the assumptions are valid (Draper & Smith, 1981; Rousseeuw & Leroy, 1987). Furthermore, an LS fitting has the disadvantage that the result is not “robust”: it is sensitive to outlying data points. Whilst “outliers” could be removed, such a procedure should only be considered if there is reason to doubt their validity. Such data must not be removed merely because they do not support the regression model: it may be the model which is wrong.
Performing a least-absolute-deviations (LAD) fitting, involves minimising the sum of the absolute deviations rather than the sum of the squared deviations. It does not rely upon the Normal assumption and allows outliers to be retained but prevents these points from exerting a disproportionate influence on the values of the regression coefficients. If the deviations are assumed to follow a Double Exponential distribution, which has thicker tails than the Normal distribution, then the parameter values are maximum likelihood estimates. In this research, it was decided to fit the regression lines using both the LS and LAD methods. The results of the two approaches could then be compared.

### 3.4 A New Regression Model

#### 3.4.1 A Simple Overtopping Theory For Regular Waves

Stepping back from the complications of random waves, consider the simpler case of regular waves of height $H$ approaching normal to a seawall. It will be assumed that the instantaneous discharge of water over unit length of the seawall, $q$, is given by the weir formula (Streeter & Wylie, 1979):

$$q = \frac{2}{3} C_d \sqrt{2g (\eta - R_c)^{3/2}} \quad \text{for } \eta > R_c \quad (3.12)$$

in which $\eta$ is the water surface elevation above still-water-level at the seawall (a periodic function of time); $C_d$ is a discharge coefficient. Obviously, overtopping occurs only when the water surface is above the structure’s crest.

Assume that:

$$\eta = k HF(t) \quad (3.13)$$

$F(t)$ denotes a function of time, $t$. For simple, sinusoidal, progressive waves, $k=0.5$ and $F(t) = \cos(2\pi t / T)$, where $T$ is the wave period. However, following Kikkawa et al (1968), an even simpler form for $F(t)$ will be adopted (see Figure 3.4); $k$ remains a coefficient determined by the particular wave and wall details.
The mean discharge, $Q$, is determined as follows:

$$Q = \frac{2}{3} C_d \sqrt{2g} \frac{1}{T} \int_{t_1}^{t_2} (kH(t) - R_c)^{3/2} \, dt$$  \hspace{1cm} (3.14)$$

in which $t_1 < t < t_2$ corresponds to the interval during each wave period for which $kH(t) > R_c$. Using the form for $F(t)$ given in Figure 3.4 then yields:

$$Q = \frac{2\sqrt{2}}{15} C_d \left\{ 1 - \frac{R_c}{kH} \right\}^{5/2}$$  \hspace{1cm} (3.15)$$

Note that overtopping occurs only when $R_c < kH$. In other words, $kH$ represents the run-up on the face of the seawall. Since wave run-up is a function of the incident wave height and steepness, and of the seawall slope, the overtopping discharge can be expected also to depend upon these parameters.
3.4.2 The Hedges & Reis Overtopping Model

The above theory suggests a regression equation for the random overtopping data of the following form:

\[
Q_r = A(1 - R_r)^B \quad \text{for } 0 \leq R_r < 1
\]
\[
= 0 \quad \text{for } R_r \geq 1
\]

(3.16)

in which

\[
Q_r = \frac{Q}{\sqrt{gR_{max}^3}} = \frac{Q}{\sqrt{g(CH_s)^3}}
\]

(3.17)

and

\[
R_r = \frac{R_c}{R_{max}} = \frac{R_c}{CH_s}
\]

(3.18)

Eq. (3.16) is the Hedges & Reis overtopping model (H&R model). Coefficient k in the expression for regular waves has been replaced by C in this regression model for random waves characterised by H_s. Note that CH_s represents R_{max}, the maximum run-up induced by the random waves, not the run-up induced by a wave of height H_s. Consequently, C will depend upon the duration of the incident wave conditions unless the wave heights in front of the wall are limited by the available water depth. Unless the maximum run-up, R_{max}, exceeds the freeboard, R_c, there is no overtopping (apart from wind-blown spray). It is also clear that coefficient B is related, in the case of regular waves, to the shape of the function F(t) which describes the water surface variation on the seaward face of the wall. There will be a similar dependence on the detailed behaviour of the water surface at the face of the wall in the case of random waves. Finally, coefficient A represents the dimensionless discharge over the seawall when the freeboard is zero. All three coefficients will be influenced by the seaward profile of the structure.

The above model for overtopping has the advantage that Q_r=0 when R_r \geq 1 and that Q_r=A when R_r=0, in accordance with the required boundary conditions. Figure 3.5 shows the influences of coefficients A, B and C in the new overtopping model.
The value of C (=R_{max}/H_s) to be adopted would best be determined from experimental data. Unfortunately, Owen’s data set (and others) do not provide an adequate number of cases involving zero or very small discharges. Consequently, its value has been estimated from run-up measurements for random waves acting on slopes for which there is no overtopping. Although not ideal for the determination of C, these additional data on run-up complement Owen's overtopping results, allowing the new model to be applied outside the range of his experimental data. This option has been adopted rather than including C alongside A and B as a regression coefficient.

A number of equations describing random wave run-up are available (CIRIA/CUR, 1991; Van der Meer & Janssen, 1995). For example, the CIRIA/CUR (1991) manual gives two equations for evaluating the significant wave run-up, R_s, on smooth slopes without overtopping. It notes that these equations, based upon Ahrens' data (Ahrens, 1981), are probably conservative and that data from Allsop et al (1985b) give values 20 to 30% lower. Rewritten in the present notation and allowing for a printing error, the expressions are:
\[ \frac{R_s}{H_s} = 1.35 \xi_p \quad \text{for} \quad 0 < \xi_p < 2 \]  
\[ \frac{R_s}{H_s} = 3.00 - 0.15 \xi_p \quad \text{for} \quad 2 < \xi_p < 20 \]  

Here, \( \xi_p \) is the surf similarity parameter calculated using \( T_p \) which was estimated for Owen’s data using the relationships between \( H_s \), \( T_m \) and \( T_p \) provided by Isherwood (1987).

Adopting the common assumption that run-up may be described by a Rayleigh distribution (Battjes, 1974; Ahrens, 1977; Bruun, 1985; CIRIA/CUR, 1991; Kobayashi & Raichle, 1994; Van der Meer & Janssen, 1995), then the \( p\% \) confidence value of maximum run-up (defining a level below which \( p\% \) of the cases should lie) is related to the significant wave run-up by (Hogben, 1990):

\[
(R_{\text{max}})_{p\%} = \left[ \frac{1}{2} \left( \ln N - \ln \left( -\ln \left( \frac{p}{100} \right) \right) \right) \right]^{1/2} R_s
\]  

(3.20)

\( N \) is the number of run-up values, here taken conservatively to be equal to the number of incident waves.

Owen recorded his overtopping discharges during tests involving sets of five different runs, each of 100 waves, characterised by the same significant wave height. The most probable maximum run-up during each run (the value not exceeded in 37% of the cases for a Rayleigh distribution of run-ups) is then:

\[
(R_{\text{max}})_{37\%} = \sqrt{\ln 100 / 2} R_s = 1.52 R_s
\]  

(3.21)

In none of Owen’s cases were there overtoppings for freeboards greater than \( (R_{\text{max}})_{37\%} \) if \( R_s \) was evaluated using eqs. (3.19). In fact, all nine reported cases of zero overtopping were for freeboards of less than this value. Hence, setting \( C = (R_{\text{max}})_{37\%}/H_s \) is conservative in this instance and the following expressions for \( C \) then arise from eqs. (3.19) and (3.21):

\[
C = 1.52(1.35 \xi_p) \quad \text{for} \quad 0 < \xi_p < 2
\]
\[
C = 1.52(3.00 - 0.15 \xi_p) \quad \text{for} \quad 2 < \xi_p < 20
\]  

(3.22)
The fact that these expressions for $C$ are conservative may be a result either of the conservative nature of eqs. (3.19) or of deficiencies in the assumptions relating to the distribution of run-ups. However, setting $C=(R_{\text{max}})_{37}\%/H_S$ may not always be appropriate. Note that the value of $C$ to be adopted in the regression model depends both upon the level of confidence associated with the prediction of $R_{\text{max}}$ and upon the duration of the incident wave conditions. If $C$ is changed then there will be corresponding changes in the values of $A$ and $B$. The implications for seawall freeboards of adopting different levels of confidence in $R_{\text{max}}$ are considered later.

### 3.5 Results Of Regression Analysis

Appendices A1 to A3 describe in detail how Owen's data have been used in carrying out regression analyses for the three embankment slopes of 1:1, 1:2 and 1:4. The H&R model (employing both $(R_{\text{max}})_{37}\%$ and $(R_{\text{max}})_{99}\%$ in defining the value of $C$) and Owen's model have each been considered. Regression analysis started by applying the LS method. The presence of both outlier data points and violation of the Normal error LS assumption lead to the subsequent use of the LAD technique (Gentle, 1977; Narula & Wellington, 1977; Sposito et al, 1977; Rousseeuw & Leroy, 1987). The LAD results appear more reliable than those of the LS method. Consequently, the LAD estimates of the regression coefficients are recommended for further use in the two models.

Figure 3.6 shows an example of the overtopping data collected by Owen: the results for a simple seawall with a uniform front slope of 1:2. The data are plotted in the formats required for fitting regression equations using both the H&R and the Owen models. Figure 3.6(a) shows the best-fit lines established using LS and LAD procedures for the H&R model. Comparison of the regression coefficients shows the influence which the outlying data points have on the LS values. For example, the magnitude of $B$ obtained from the LAD fitting is about 92% of the LS result. Similar comments may be made about the regression lines obtained for Owen’s model. Note, that the values of $A$ and $B$ reported in Figure 3.6(b) for Owen’s model are not those which Owen himself recommended.
Figure 3.6: Wave overtopping data for slope 1:2 plotted in the formats required for fitting regression equations, (a) using the H&R model, \((R_{max})^{37\%}\), and (b) using Owen’s model.

Table 3.2 gives the regression coefficients for all three slopes which have been obtained for the H&R model and for Owen’s model, using both LS and LAD fitting. Also included for reference are Owen’s recommended values.
Wave Overtopping Of Seawalls

<table>
<thead>
<tr>
<th>Slope</th>
<th>H&amp;R MODEL (C given by (R max) 37%)</th>
<th>H&amp;R MODEL (C given by (R max) 99%)</th>
<th>OWEN’S MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LAD</td>
<td>LS</td>
<td>LAD</td>
</tr>
<tr>
<td>1:1</td>
<td>A 0.00703</td>
<td>0.00581</td>
<td>0.00515</td>
</tr>
<tr>
<td></td>
<td>B 3.42</td>
<td>3.22</td>
<td>6.06</td>
</tr>
<tr>
<td>1:2</td>
<td>A 0.00753</td>
<td>0.00790</td>
<td>0.00542</td>
</tr>
<tr>
<td></td>
<td>B 4.17</td>
<td>4.55</td>
<td>7.16</td>
</tr>
<tr>
<td>1:4</td>
<td>A 0.0104</td>
<td>0.00792</td>
<td>0.00922</td>
</tr>
<tr>
<td></td>
<td>B 6.27</td>
<td>5.94</td>
<td>10.96</td>
</tr>
</tbody>
</table>

Table 3.2: Regression coefficients obtained for use in the H&R model and Owen’s model. Also included for reference are Owen’s recommended (Rec.) values.

Owen restricted his analysis to a particular set of conditions whilst, in this analysis, all available data were included apart from eleven of the 110 results for the 1:4 slope. Of these eleven, nine had Q=0. Figure 3.5 shows that there will be many values of R* for which Q*=0 and data points with Q=0 must be excluded from a regression analysis, otherwise a regression line (if it could be fitted) would pass through these data rather than defining their lower limit. The other two excluded values (for which Q was not zero) were from a set of five runs with the same dimensionless freeboard, three of which had zero overtopping discharge. Including only two of these five data points would have severely biased the positions of the regression lines. Furthermore, the validity of these two data points is doubtful since a full set of five runs at a smaller dimensionless freeboard all had Q recorded as zero. Although removed for the purposes of regression analysis, the eleven points were reinstated for inclusion in Figure 3.7 (see later). The full data set fell within the following ranges:

\[
0 < \frac{Q}{\sqrt{g(CH_s)^3}} < 0.0056, \quad 0.14 < \frac{R_c}{CH_s} < 0.90
\]

\[
0 < \frac{Q}{T_m gH_s} < 0.0039, \quad 0.053 < \frac{R_c}{T_m \sqrt{gH_s}} < 0.239
\]

\[
0.0053 < \frac{H_s}{gT_m^2} < 0.0095, \quad 1.65 < \frac{d_s}{H_s} < 5.20
\]

Earlier, the problem of spurious correlation was mentioned. Along with most other overtopping models (see Table 3.1), the H&R model employs a
dimensionless discharge and a dimensionless freeboard which contain a
common variable ($R_{\text{max}}$ or $C_{\text{H}}$). The presence of this common variable may
reduce the apparent scatter in the data. Consequently, Figure 3.7 shows
directly the level of agreement between Owen’s measured values of $Q$
(converted by Owen to full-scale discharges for a seawall in 4m water depth)
and the predicted values, $Q_{\text{PRED}}$. Of course, the scatter in the relationship
between $Q$ and $Q_{\text{PRED}}$ could have been disguised by plotting against
logarithmic scales (Massey, 1971). But such an attempt is both misleading
and unnecessary. Under random wave conditions, overtopping will be
dominated by the few waves with large run-ups: most waves will contribute
no overtopping if the seawall has a substantial freeboard (Jensen & Juhl,
1987; Aminti & Franco, 1988). Thus, particularly for short runs of random
waves, as in Owen’s tests, some variability in the measured values of $Q$ can
be expected. Indeed, one of the purposes of Owen’s tests was to show this
inherent variability.

In Figure 3.7, most data points lie within a range for $Q/Q_{\text{PRED}}$ of 3/4 to 4/3,
whichever model is adopted. It is not obvious from the figure which model
best fits the data, nor is it obvious from the plots for simple seawalls with 1:1
and 1:4 front slopes. Consequently, the data points for discharges in the
ranges of practical interest (see Figure 3.2) were looked at in more detail.
For these purposes, the H&R model appeared generally better than Owen’s
model owing to its ability to predict zero overtopping at finite values of
freeboard. Furthermore, the next section shows that it tends to give lower
required crest levels than Owen’s model for small permissible discharges,
offering lower environmental impact and potential cost savings.
Figure 3.7: Wave overtopping data for slope 1:2, showing the level of agreement between $Q$ and $Q_{PRED}$, (a) using the H&R model, $(R_{max})_{37\%}$, and (b) using Owen’s model.

3.6 Some Implications Of The New Model For Seawall Freeboards

According to Owen’s model, the freeboard, $R_c$, necessary to limit overtopping to a specified value, $Q$, is given by:

$$R_c = \frac{T_m \sqrt{gH_s}}{B} \ln \left\{ \frac{AT_m gH_s}{Q} \right\}$$

(3.24)
The H&R model gives:

\[
R_c = CH_s \left(1 - \frac{Q}{A \sqrt{g(CH_s)^3}} \right)^{1/3}
\]  

(3.25)

Note that eq. (3.24) incorporates the mean zero-crossing wave period, \(T_m\), whilst eq. (3.25) involves coefficient \(C\) which has been described in terms of the period of peak spectral density, \(T_p\). In order to compare the output from the two expressions, it has been assumed that the incident waves conform to the Pierson-Moskowitz spectrum. In this case (for \(H_s\) in metres, with \(T_m\) and \(T_p\) in seconds):

\[
T_m = 3.55 \sqrt{H_s} \\
T_p = 5.00 \sqrt{H_s}
\]  

(3.26)

Figure 3.8 provides a comparison between the freeboards predicted using eqs. (3.24) and (3.25) for embankments with uniform front slopes of 1:2 subject to random waves with a significant height of 2m. Similar figures could be prepared for embankments with seaward slopes of 1:1 and 1:4, for additional incident significant wave heights and for different values of the confidence level associated with \(R_{max}\).

Owen stated (Hydraulics Research Station, 1980) that his empirical coefficients \(A\) and \(B\) were determined only for particular ranges of the dimensionless groupings given in eq. (3.4). The conditions included the following: \(10^{-6} < Q / T_m gH_s < 10^{-2}\) and \(0.05 < R_c / T_m \sqrt{gH_s} < 0.30\). Many of the discharges shown in Figure 3.8 have \(Q / T_m gH_s < 10^{-6}\). For the conditions of Figure 3.8, this limit is approximately equivalent to \(Q < 10^{-4} m^3/s/m\). Nevertheless, Owen suggested that it was possible to use his equation to extrapolate results when the dimensionless freeboard was such that the dimensionless discharge fell below \(10^{-6}\). Thus, for a typical seawall in 4m water depth, it is possible to compare the minimum necessary freeboards predicted by the H&R model with those predicted by Owen's expression if overtopping is to be limited to specified values.
Two points are worth noting:

- There is reasonable agreement between the H&R model and Owen's model for overtopping discharges in the range of $10^{-2} \text{m}^3/\text{s}/\text{m}$ to $2 \times 10^{-1} \text{m}^3/\text{s}/\text{m}$. This is irrespective of the confidence level (37% or 99%) assigned in the evaluation of $R_{\text{max}}$. However, it is in the range where there are significant differences between the two models that most seawalls are designed (see Figure 3.2).

- As the confidence level in $R_{\text{max}}$ is increased, the freeboards predicted by the H&R model approach those values obtained from Owen's model. Nevertheless, even using $(R_{\text{max}})_{99\%}$ there remain significant differences. This observation has important implications for seawall design. For example, for an expected overtopping discharge of $10^{-4} \text{m}^3/\text{s}/\text{m}$, the difference amounts to about 1.9m. It is even greater both for the lower expected overtopping rates associated with functional safety requirements (Figure 3.2) and for higher $H_s$ values. Owen's model suggests that the freeboard must continue to increase in order to reduce the overtopping rate even when the crest of the seawall is clearly above any possible run-up level induced by the random waves.
3.7 Model Reliability

Neither the H&R model nor Owen’s model provide a perfect description of the overtopping data. There is some scatter about the line of perfect agreement between predicted and measured values (see Figure 3.7). This scatter can be described by interpreting the coefficient $A$ in the models as a random variable for a given coefficient $B$ (Allsop & Meadowcroft, 1995) or by interpreting the coefficient $B$ as a random variable for a given value of $A$ (Van der Meer, 1993; Franco et al, 1994; Van der Meer & Janssen, 1995). Alternatively, the degrees of variability in $A$ and $B$ may be represented by parameters $e_A$ and $e_B$, respectively:

H&R Model: \[ Q = e_A A (1 - R.)^B \] or \[ Q = A (1 - R.)^{e_B} \]

leading to

\[ e_A = \frac{Q}{A (1 - R.)^B} \] and \[ e_B = \frac{\ln Q - \ln A}{BLn(1 - R.)} \] (3.27)

Owen’s Model: \[ Q = e_A A \exp(-BR.) \] or \[ Q = A \exp(-e_BBR.) \]

leading to

\[ e_A = \frac{Q}{A \exp(-BR.)} \] and \[ e_B = \frac{\ln A - \ln Q}{BR}. \] (3.28)

If the models were perfect representations of reality, $e_A$ and $e_B$ would both be equal to one. Otherwise they are random variables which may be described by a probability distribution.

The variability of $e_A$ and $e_B$ as functions of $R.$ have been investigated for the three front slopes of 1:1, 1:2 and 1:4. The H&R model (employing both $(R_{\text{max}})^{37\%}$ and $(R_{\text{max}})^{99\%}$ in defining the value of $C$) and Owen’s model have each been considered. Similar results were obtained in all cases.
Figure 3.9: Variability of $e_A$ and $e_B$ as functions of $R_*$. 

Figure 3.9 shows an example of the results for slope 1:1 using the H&R model with $(R_{\text{max}})_{37\%}$. The variability of $e_A$ as a function of $R_*$ is not constant: the figure suggests an increasing variability in $e_A$ as $R_*$ increases. The degree of variability in $e_B$ as a function of $R_*$ appears more constant. This, in addition to the smaller spread in the data for $e_B$, suggests that $e_B$ is more appropriate than $e_A$ for representing the reliability of the models. Hence the probability distribution of $e_B$ for each model has been further investigated using the software package BestFit (see Appendix A4). Details of the analyses are contained in Appendix A5.

First, the $e_B$ values were assessed using the summary statistics provided by BestFit and by plotting histograms of the data. General guidance on the
choice of distribution was obtained from these sources. Then, all the statistical distributions available both in PARASODE and BestFit were considered and full optimisation was adopted for calculation of their parameters. Finally, the adequacy of these distributions was determined using the three goodness-of-fit tests available in BestFit, together with histograms, P-P and Q-Q plots. Based on the numerical and graphical results, the Log-Normal, Maxima Type I (Gumbel) and Gamma distributions were chosen as possible candidates for describing the randomness of $e_B$. Thus, these three are used in the probability calculations performed with PARASODE and @Risk. Table 3.3 shows the means and the standard deviations of the input data for $e_B$. Table 3.4 shows the corresponding means and standard deviations of the fitted distributions.
<table>
<thead>
<tr>
<th>INPUT</th>
<th>H&amp;R MODEL</th>
<th>OWEN'S MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Rmax)37%</td>
<td>(Rmax)99%</td>
</tr>
<tr>
<td>Slope 1:1</td>
<td>Slope 1:2</td>
<td>Slope 1:4</td>
</tr>
<tr>
<td>Slope 1:1</td>
<td>Slope 1:2</td>
<td>Slope 1:4</td>
</tr>
<tr>
<td>Slope 1:1</td>
<td>Slope 1:2</td>
<td>Slope 1:4</td>
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<td>Slope 1:2</td>
<td>Slope 1:4</td>
</tr>
<tr>
<td>e_B</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td></td>
<td>1.030</td>
<td>0.346</td>
</tr>
<tr>
<td></td>
<td>1.020</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
<td>1.044</td>
<td>0.236</td>
</tr>
<tr>
<td></td>
<td>1.044</td>
<td>0.236</td>
</tr>
</tbody>
</table>

**Table 3.3:** Mean, μ, and standard deviation, σ, of the input data for e_B.

<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
<th>H&amp;R MODEL</th>
<th>OWEN'S MODEL</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(Rmax)37%</td>
<td>(Rmax)99%</td>
</tr>
<tr>
<td></td>
<td>Slope 1:1</td>
<td>Slope 1:2</td>
</tr>
<tr>
<td></td>
<td>Slope 1:1</td>
<td>Slope 1:2</td>
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<td></td>
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<td>Slope 1:2</td>
</tr>
<tr>
<td>LOG-NORMAL</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td>0.214</td>
</tr>
<tr>
<td>MAXIMA TYPE I</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td>(GUMBEL)</td>
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</tr>
<tr>
<td></td>
<td>1.039</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>1.044</td>
<td>0.236</td>
</tr>
<tr>
<td>GAMMA</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td></td>
<td>1.030</td>
<td>0.324</td>
</tr>
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<tr>
<td></td>
<td>1.044</td>
<td>0.217</td>
</tr>
</tbody>
</table>

**Table 3.4:** Mean, μ, and standard deviation, σ, of the three distributions chosen for e_B.
3.8 Summary

Wave overtopping of seawalls has been the subject of many studies. Nevertheless, field measurements are scarce and numerical modelling of wave overtopping is not yet well developed. The calculation of overtopping discharge is based mainly on equations which have been obtained from empirical fitting to hydraulic model test results. These equations have not been based upon any overtopping theory and no account has generally been taken of the physical boundary conditions.

As part of this research, a new regression model has been presented for describing wave overtopping data. Part of the motivation in deriving this new model was to improve the methods available to the designers of seawalls by developing a model closely related to the physics of wave overtopping. The important features of the model are as follows:

- Unlike existing expressions, it satisfies the relevant physical boundary conditions, a feature which is especially important when the model is used near these boundaries.
- It explicitly recognises (through its foundations in a simple theoretical model for regular waves) that regression coefficient A depends upon the shape of the structure since the shape, particularly at its crest, affects the discharge coefficient; coefficient A represents the dimensionless discharge when the dimensionless freeboard is zero.
- Coefficient B depends upon the detailed behaviour of the water surface on the seaward face of the structure; it increases as front slopes become flatter.
- Coefficient C relates the maximum run-up to the significant height of the incident waves and may be chosen to allow for the influences of the seawall slope, the surface roughness and porosity, and the incident wave steepness. Coefficient C can also account for storm duration in influencing $R_{\text{max}}$ (though the regression coefficients in the present study have been established only for short sequences of 100 random waves). Finally, it may be chosen so that there is a specified confidence level associated with $R_{\text{max}}$. In this thesis, the most probable value of $R_{\text{max}}$, $(R_{\text{max}})_{37\%}$, has been adopted in order to establish the expressions for C (eqs. (3.22)) since they were conservative in this instance, but the use of $(R_{\text{max}})_{99\%}$ has also been illustrated. Note, that if the confidence level associated with $R_{\text{max}}$ changes, the expressions for C change and there are corresponding changes in the values of A and B (see Table 3.2).
It is suggested that the regression coefficients contained within the model should be established using a robust regression technique. Examples are given of the differences between the LS and the LAD fitting methods in analysing overtopping data collected by Owen (Hydraulics Research Station, 1980; Owen, 1982a). The LAD regression coefficients are recommended for use both in the H&R and Owen models.

For the present test results, the H&R model is little different from Owen’s model in its ability to represent the data, except for small discharges for which the H&R model is better suited. An example is given of the application of the H&R and Owen models in predicting the freeboards necessary to limit overtopping to specified values. This example shows that, for the small allowable discharges associated with normal design conditions, the H&R model predicts seawall crest elevations which may be several metres lower than values from Owen's model. Such differences may have very significant financial and environmental consequences and are worthy of further investigation.

The reliabilities of the H&R model and of Owen’s model have been assessed by introducing a multiplying parameter to coefficient B of the models. In order to determine possible distributions for this model parameter, a software package called BestFit has been applied. The Log-Normal, Maxima Type I (Gumbel) and Gamma distributions are seen to be acceptable for use in the probabilistic calculations performed in this study.

Whilst it is possible to use Owen’s data to show the validity of the approach adopted in developing the new wave overtopping model, the data are far from ideal for evaluating the empirical coefficients A, B and C. Owen collected his data for short runs of waves and the bulk of his data are for typical full-scale conditions which produce overtopping discharges well in excess of the allowable values shown in Figure 3.2. This second deficiency also applies to more recent data sets (Van der Meer & Janssen, 1995). Thus, the data available at present are not good for evaluating coefficient C which fixes the lowest value of R* for which Q* is zero. Neither are they good for evaluating coefficient A, the value of Q* when R* is zero. Owen performed no tests with zero freeboard. In the absence of information to locate the extremes of the curve shown in Figure 3.5, the values of coefficient B also remain in doubt. However, the author is aware that overtopping and run-up
tests now being carried out at the Technical University of Braunschweig, Germany (Schuttrumpf, 1997), include measurements when there is zero freeboard and when the freeboard is sufficiently great to prevent any overtopping. The latter condition will allow the direct measurement of $R_{\text{max}}$, obviating the need to estimate it.

In addition to considering the deficiencies of existing data for the above purposes, it is worth emphasising that approaches to coastal engineering design are shifting towards probabilistic rather than deterministic procedures. The variability in overtopping discharge must then be considered: it is necessary not only to predict the expected mean value of $Q$, but also the probability distribution of $Q$ about this value. As a consequence, much larger data sets are needed. Furthermore, some attention must be paid to the horizontal distribution of the total overtopping volume and the influence which the wind has on overtopping discharges (De Waal et al, 1996; Ward et al, 1996).
4 Dune Erosion During A Storm Surge

4.1 Introduction

Dunes erode in two main ways: a gradual erosion (structural or long-term erosion) and a fast, sudden erosion/recession during a storm surge (short-term erosion). Long-term erosion of beaches and dunes can be very inconvenient but future losses can be foreseen and, in most cases, can be predicted quantitatively. In contrast, high water levels and high waves during a storm surge erode huge quantities of dune material in a short time. Thus, short-term erosion is a condition which must be analysed very carefully. This review considers only the short-term erosion due to a severe storm surge.

Dunes occur naturally in many parts of the world. In their most natural state, they are associated with exposed dry sand being transported by the wind. In this state, they can migrate with the wind, sometimes invading and disrupting the works of man (e.g. to cover highways and railroads and to destroy productive agricultural land). On the other hand, dunes act as the primary protection against the sea in some regions of the world (e.g. in parts of the United Kingdom and The Netherlands). In these cases, effort must be put into preserving or enhancing the dunes (Thomas & Hall, 1992; Simm et al, 1996). In some regions, a high investment has been made in property which is very close to the edge of the sea. Figure 4.1 shows two schematic examples: (a) a low-lying area behind a narrow line of dunes; and (b) buildings close to the sea. Apart from a possible gradual overall erosion of the coast, both cases are safe under "normal" conditions. During a storm surge, however, the mean sea level rises considerably above normal high water level, higher waves than usual approach the shore and offshore transport occurs, especially of material from the dunes (Van de Graaff, 1986; Van de Graaff & Bijker, 1988). Figure 4.1 shows schematically what happens: sand eroded from the dunes is transported towards deeper water and settles there. The new beach profile develops at a more elevated level and the overall slope becomes less steep than the original. Consequently, the rate of erosion of the dunes slows down with time. After the storm, the water level and the wave heights go to "normal" conditions again, the dune

---

1 Surge is the difference between the measured water level and the predicted (astronomical) tide level (Pugh, 1987).
erosion process is stopped, and a retreat distance, RD, can be observed. For safety reasons, the likely position of point R after the surge has to be known. In case (a), erosion of the dunes will cause flooding and damage to property with possible loss of life. In case (b), no serious flooding occurs but there may be destruction of buildings. Hence, the problem of managing a dune system is not so much to prevent erosion but to know in detail the rate of erosion that can be expected in order to judge dunes as safe or unsafe.

Figure 4.1: Schematic dune erosion situations (modified after Van de Graaff, 1986).

If an existing dune/beach system is unsafe, then consideration may be given to the possibility of providing nourishment. Although not appropriate for all locations, nourishment has proved to be a cost-effective, flexible and environmentally sensitive "soft" engineering strategy (when compared to other methods such as the use of groins, offshore breakwaters, etc.). It has been used in many places including the United Kingdom (Fleming, 1990; Townend & Fleming, 1991; Motyka & Brampton, 1994; HR Wallingford, 1994), The Netherlands (Vellinga, 1986; Van de Graaff & Bijker, 1988; Van
Raalte & Loman, 1993), and the USA (Dean, 1976; Housley, 1996). Nourishment usually does not create detrimental side effects in adjacent coastal areas (Van de Graaff et al, 1991). Consequently, there is a clear interest in nourishment as a strategy for controlling dune/beach erosion evident in such multi-national publications as Technical University of Braunschweig/SOGREAH Ingenierie/Centro de Estudios y Experimentacion de Obras Publicas (1997).

It must be stressed that a long-term commitment to nourishment is required to ensure that the benefits anticipated in the design will actually occur (Housley, 1996). Nourishment may seem expensive and the need for repetition may discourage coastal managers. However, repetition should be seen as regular maintenance, as conservation of valuable investments, just as for other structures (Van de Graaff et al, 1991). For example, in The Netherlands, dune/beach replenishment is generally expected to provide a buffer for more than five years (Van de Graaff et al, 1991; Van Raalte & Loman, 1993) and in the UK, "lives" of ten years or more are the norm (Motyka & Brampton, 1994). Careful consideration of capital and maintenance costs frequently proves that nourishment is the optimal solution. An added advantage is that the recreational function of the dune/beach system is preserved.

The type and volume of sediment required, possible borrow areas (e.g. inland sources or material dredged from navigation channels), the transportation system, the precise nourishment location and ease of placement, and socio-economic aspects have all to be considered. Possible nourishment areas include the land side of the dune, the seaward dune face, and the foreshore and inshore zones (CUR et al, 1987; Van de Graaff & Koster, 1990; Van de Graaff et al, 1991; D'Angremond, 1992; Van Raalte & Loman, 1993; Liverpool/Thessaloniki Network, 1996). The Manual on Artificial Beach Nourishment (CUR et al, 1987), the Beach Management Manual (Simm et al, 1996) and Beach Recharge Material - Demand and Resources (CIRIA, 1996) all provide useful further information.

The main approaches to modelling dune erosion may be categorised according to the type of integration (Steetzel, 1993):
• Space integration (a fixed shape for the cross-shore profile is assumed).

• Time integration (the net effect of the complete storm surge is accounted for).

Using these two types of integration, three main categories of dune erosion models can be distinguished (Steetzel, 1993):

• **Space and time integrated concept** - This approach results in a prediction of the erosion profile which is supposed to be present after a specific storm event. It can be characterised by so-called *equilibrium or erosion profile models* (e.g. Bruun´s model - Bruun, 1954, 1962; Dean´s model - Dean, 1977, 1982, 1987, 1991; the DUROS model - Vellinga, 1986).

• **Space integrated, instantaneous concept** - The development of the shape of the cross-shore profile is described during the storm event using a time-dependent shape function. Typically, this approach results in a negative exponential development of the profile (for constant hydraulic conditions). The term *quasi-equilibrium models* can be used for this category (e.g. Swart, 1974; the SBEACH model - Larson, 1988, Larson & Kraus, 1989; the DUIN model - Roelvink & Stive, 1989).

• **Local and instantaneous concept** - The local transport rate must be known at every position on the cross-shore profile in order to compute its development during a storm surge. An expression for the local transport rate has to be derived. The expressions available differ, mainly, as a result of the approach used in assessing the local transport rate (e.g. Kriebel & Dean, 1985; Kriebel, 1990; the DUROSTA model - Steetzel, 1990, 1993; Watanabe et al, 1994).

Obviously, the equilibrium and the quasi-equilibrium models are not suitable for the assessment of the effects of arbitrary hydraulic conditions on a cross-shore profile. However, they have the advantage of simplicity. Applications of Dean´s equilibrium model have been presented recently (see, for example, Kriebel, 1990; Dean, 1991). Vellinga´s model is still used in The Netherlands to check the safety of the Dutch dunes.

Coastal engineers are still becoming acquainted with the use of probabilistic methods for the design and assessment of coastal structures. A simple equilibrium model is a good starting point for combining probabilistic methods and dune erosion (Van de Graaff, 1986; Dong & Riddell, 1996). The use of simple erosion models allows the engineer to concentrate on understanding
the application of the probabilistic methods whilst also saving the considerable computational time needed in running the more sophisticated alternatives. Implementation of these sophisticated models in a probabilistic approach is possible but represents such a complex task that, as far the author knows, it has not yet been put into practice, even in pioneering countries like The Netherlands. Furthermore, some of the simpler models, in particular Vellinga’s approach, have proved to be generally conservative when compared to Steetzel's more sophisticated formulation (Van de Graaff, 1995). This may cause problems in relation to the public perception of the safety in applying Steetzel's model.

In the following sections, especial attention is given to the research carried out in The Netherlands. Vellinga's cross-shore erosion model (Vellinga, 1986) is studied in detail, not only because it has been the basis for the calculation of the safety of the dunes throughout The Netherlands but also because probabilistic methods have been applied in conjunction with it. Steetzel's model (Steetzel, 1993) is referred to only because it is the latest Dutch model; however, no probabilistic calculations have been undertaken with it so far. Next, the computer programs which are currently used in The Netherlands are presented briefly. Finally, the applicability of these programs in the British context is examined.

### 4.2 Vellinga’s Model

#### 4.2.1 Prior Research

Edelman (1968) was the first to present a method for the prediction of dune erosion in The Netherlands. His method was based on the assumption that during a storm surge a normal beach slope develops but at a higher level than before. Edelman used a straight beach slope of 1:50 in his computations. His method was improved by Van de Graaff (1977) who employed a realistic concave-upward erosion profile based on field observations. This profile is used in what is known as the provisional computational model (Vellinga, 1983, 1986).

Van der Meulen & Gourlay (1968) were the first in The Netherlands to investigate the process of dune erosion in small scale movable bed models.
The tests were mainly carried out in a basin with monochromatic waves. The tests provided qualitative answers to the question of how dune erosion is influenced by dune height, initial beach profile, wave height, wave period, sea level, and grain size characteristics. However, Hulsbergen (1974) found that such tests suffered from the effects of secondary waves.

In 1972, a research project started on the erosion of coastal dunes during storm surges. The aims of the research were (Vellinga, 1983, 1986): i) to increase insight into the phenomenon, and to develop a general model for the computation of the erosion quantity as a function of the hydraulic conditions and the coastal profile; and ii) to use this model to check the safety of existing beaches and dunes as primary coast protection and to determine the required reinforcement. Engineers in charge of coast protection were conscious that there was insufficient theoretical background and that a firmer basis for decisions was urgently needed. Hence, it was decided that the research would consist mainly of extensive experimental tests on small and large scale models, together with prototype measurements involving waves up to 2.0m significant height. Table 4.1 summarises how this research evolved. Vellinga (1982, 1983, 1986) provides details about the various stages of the project.
PRIOR RESEARCH

1972
Provisional computational model - provisional guideline for the computation of dune erosion during a storm surge (TAW, 1972)

1974-1975
2D scale
Idealised coastal profile
4 depth scales ($n_d=26, 47, 84, 150$)
2 sand grain sizes ($D_{50}=150, 225\mu m$)
Design Storm Surge Conditions:
- water level=5m MSL
- $H_s=7.6m$; $T_p=12s$

1976-1978
2D series
Idealised coastal profile
3 depth scales ($n_d=26, 47, 84$)
4 sand grain sizes ($D_{50}=95, 130, 150, 225\mu m$)
Design Storm Surge Conditions:
- water level=5m MSL
- $H_s=7.6m$; $T_p=12s$

Analysis and evaluation of scale relationships:
1. Froude scale for hydraulic conditions: $n_H=n_L=n_T^2$;
2. morphological time scale: $n_t=n_d^{0.5}$;
3. model distortion: $n_l/n_d=(n_d/n_w)^{0.28}$


1979-1980
Verification of the 2D approach by means of 3D movable bed small scale tests: these tests confirmed that a 2D approach was fully acceptable for relatively straight beaches.

1981-1982
Parametric small scale model investigations to define the effect of dune erosion parameters on the rate of erosion: i) water level during storm surge, wave height and particle diameter were determining parameters: an increase in the water level during storm surge produced an increase in the dune recession and erosion quantity; the wave height had the same effect but to a much lesser extent (the angle of wave incidence did not have a significant effect on the erosion quantity); the finer the dune material the less erosion quantity and distance expected; ii) wave period and shape of the energy spectrum of the incoming waves did not have a significant effect; iii) steeper dune fronts had the highest amounts of erosion and the lowest erosion distances; iv) as the dune height increased the erosion quantity also increased while the dune recession decreased.

Table 4.1: Research prior to the development of Vellinga’s model.

4.2.2 Formulation

Based on the above research project, a dune erosion model was developed by Vellinga. It is known as the DUROS model (see Figure 4.2):

\[ n - \text{ratio of the prototype value to the model value; } n_H - \text{wave height scale; } n_L - \text{wavelength scale; } n_d - \text{depth scale (for beach profile and hydraulic conditions); } n_T - \text{wave period scale; } n_t - \text{time scale; } n_l - \text{length scale for beach profile; } n_w - \text{scale for the fall velocity of the sand.} \]
The input parameters required by the model are (Vellinga, 1983):

i) coordinates of the initial profile \((X_i, Y_i)\);

ii) significant deep water wave height, \(H_S\) (significant wave height at depth \(d>0.5L_{n0}\), where \(L_{n0}\) is the significant deep water wavelength);

iii) median grain size diameter of dune sand, \(D_{50}\) (50% of the weight being finer) and its corresponding fall velocity, \(w\), for a given water temperature;

iv) maximum water level during storm surge, \(h\).

During a storm surge, an erosion profile develops (Figure 4.3). The level of the profile is determined by the maximum water level during the storm surge. Its shape, perpendicular to the coast, is determined by the wave height and the fall velocity of the bed material and can be described by the following equation (Vellinga, 1986):

\[
Y = 0.47 \left( \frac{7.6}{H_S} \right)^{1.28} \left( \frac{w}{0.0268} \right)^{0.56} X + 18 \right]^{0.5} - 2.00 \tag{4.1}
\]

\(X\) is the distance (in metres, positive seawards) from the new dune foot and \(Y\) is the depth (in metres) below maximum water level during storm surge.
Figure 4.3: Representation of Vellinga’s profile after a storm surge (modified after Van de Graaff & Koster, 1990).

- Eq. (4.1) is valid up to a seaward limit \((X_{\text{max}}, Y_{\text{max}})\) given by:

\[
X_{\text{max}} = 250 \left( \frac{H_s}{7.6} \right)^{1.28} \left( \frac{0.0268}{w} \right)^{0.56} \\
Y_{\text{max}} = 5.72 \left( \frac{H_s}{7.6} \right) = 0.75 H_s 
\] (4.2)

Seaward from this point, the profile continues as a straight line with a gradient of 1:12.5 (fixed in agreement with model tests) until it intersects the initial profile. The slope of the dune face \((X<0)\) is at 1:1 (consistent with field observations and large scale tests).

- After determining the shape of the erosion profile for a given set of parameters, this profile must be shifted in relation to the initial profile (the profile before the storm) in such a way that erosion is in balance with accretion. Transport of sand is in the seaward direction and there is no provision for handling longshore gains or losses to the profile.

- The erosion quantity is determined by the difference between the initial profile and Vellinga’s erosion profile.

- The outputs from the model are (Vellinga, 1983): i) the recession of the dune front; ii) the erosion quantity above storm surge level; and iii) the beach profile after the storm surge.
Vellinga’s model accounts for the following factors in predicting dune erosion (Van de Graaff, 1983, 1986):

- maximum water level during storm surge;
- significant deep water wave height during the maximum of the surge;
- particle diameter of dune material;
- shape of initial profile (including dune height);
- storm surge duration;
- gust bumps and squall oscillations;
- accuracy of the computation method.

The model disregards the influences of:

- temperature of the sea water (which affects $w$ in eq. (4.1));
- irregularities in cross-sections over small distances alongshore;
- storm surge direction;
- groins;
- dune vegetation.

### 4.2.3 Verification From Laboratory And Field Data

The computational model was verified using both large scale tests and tests with a depth scale $n_d=30$. The effects of storm surge level, significant wave height, significant wave period, shape of the spectrum, dune height and initial profile were all checked. It was concluded that (Vellinga, 1983):

- The computational model accounts adequately for the impacts of storm surge level, wave height and profile shape.
- The model is valid for typical North Sea storm surge conditions (Figure 4.4) involving waves with steepness $H_s/L_{OS} \geq 0.02$. Erosion during such an event is equivalent to the erosion which takes place during a period of about 5 hours when the water level is held constant at the maximum surge level (Van de Graaff, 1983, 1986; Vellinga, 1986). Storm surges with different hydrographs may have different equivalent durations. A correction factor for a
A different hydraulic regime has been determined on the basis of the model tests with constant water level. The erosion quantity above storm surge level, \( C \) (see Figure 4.10 later), should be increased by 5% to 10% for each additional hour of storm duration (maximum addition not to exceed 50% of \( C \)). Storm duration is defined as the amount of time in hours that the surge level is within 1 m of the maximum level (5 hours for the North Sea hydrograph). For example, if the storm duration is 9 hours and assuming a conservative 10% additional erosion per additional hour, then a total of \( 10(9-5) = 40\% \) additional erosion should be considered. Since this is less than the maximum recommended addition of 50%, the value of \( C \) adjusted to account for the duration of the storm can be computed as \( 1.4C \) (Sargent & Birkemeier, 1985).

Figure 4.4: Example of the standard North Sea storm surge hydrograph (modified after Vellinga, 1986).

- The prediction model is somewhat conservative for beaches with large bars and troughs, but it should be considered reasonably accurate for a large range of hydraulic conditions and initial profiles normally found in the field (Vellinga, 1986).

The computational model has also been verified using field measurements from the 1953 and 1976 storm surges in The Netherlands and from Hurricane Eloise in Walton County, Florida in 1975 (Hughes & Chiu, 1981). In addition, Sargent & Birkemeier (1985) verified the model for a number of storm surges along the USA East coast and the Gulf coast.
4.2.4 Accuracy

Vellinga's model is a relatively simple way of schematising a complicated natural process. Its accuracy was established in the following way (Vellinga, 1983). First, all measurements were considered 100% true. Next, the distribution of the differences between the measured and computed quantities was evaluated. It was concluded that the accuracy of the computational model could be described by a Normal distribution with a mean of zero and a standard deviation \( \sigma = 0.10C + 20 \text{ m}^3/\text{m} \), in which \( C \) is the computed erosion above storm surge level in \( \text{m}^3/\text{m} \) and \( \sigma \) is the standard deviation of the differences between the computed and measured dune erosion quantities (Figure 4.5). This relationship describes the accuracy of the prediction model for given input parameters (such as initial profile, maximum surge level, wave height and particle size diameter). The inaccuracy of these input parameters is an additional source of errors. The effects of gust bumps, squall oscillations and the duration of the storm are not included in the model, although Vellinga identified these factors as important (see Section 4.2.2) and addressed the problem to some degree (see Sections 4.5.2 and 4.5.3). Gradients in the longshore transport are also ignored in the model.

![Figure 4.5](image)

**Figure 4.5:** Comparison between measurements and computations (modified after Vellinga, 1983).
4.2.5 Application And Limitations

Vellinga’s model has been applied to check the safety of existing dunes in The Netherlands and to determine the required nourishment. However, the model is based upon certain assumptions and has limitations of which the user should be aware. The assumptions have been described in Section 4.2.2. The model's limitations are described here.

According to Vellinga (1986), the model is only valid formally for waves with $H_s/L_{OS}=0.034$ and for conditions with a constant water level for 5 hours. However, in practice, the model has been applied to conditions with $0.02 \leq H_s / L_{OS} \leq 0.04$ and for a realistic North Sea storm surge hydrograph as shown in Figure 4.4. This hydrograph is characterised by its large height and short duration. However, with minor adjustments for the effect of storm surge duration, the model can also be used for the prediction of beach and dune erosion in other parts of the world (Vellinga, 1983).

The model also has some limitations related to cross-shore profiles. The model results have not always proved to be reliable (Steetzel, 1993) for an initial profile with a nearshore bank or a very steep bottom slope (owing, perhaps, to the presence of a tidal gully). Furthermore, Vellinga's erosion profile does not deal with beaches which have a wide range of grain sizes such as occur when there is a mixture of sand and shingle. Such cases exist, for example, along parts of the UK coastline (Simm et al, 1996). One should also note that the model is only applicable in situations with relatively straight homogeneous coastlines, i.e. where a two-dimensional idealisation of the dune erosion process is possible (Vellinga, 1986). Extra dune erosion should be expected where the shoreline is curved, to compensate for possible losses of sand due to gradients in the longshore transport rate. Furthermore, some coastal problems cannot adequately be solved using this kind of dune erosion model (Table 4.2).
Dune Erosion During A Storm Surge

<table>
<thead>
<tr>
<th>TYPE OF PROBLEM</th>
<th>RELATED EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-standard coastal profiles or hydraulic conditions</td>
<td>- nearshore banks</td>
</tr>
<tr>
<td></td>
<td>- tidal gullies</td>
</tr>
<tr>
<td></td>
<td>- non-standard surge conditions</td>
</tr>
<tr>
<td>interference by structures</td>
<td>- dune revetments</td>
</tr>
<tr>
<td></td>
<td>- offshore breakwaters</td>
</tr>
<tr>
<td>effect of longshore transport gradients</td>
<td>- shoreline curvature</td>
</tr>
<tr>
<td></td>
<td>- tidal gradients</td>
</tr>
</tbody>
</table>

Table 4.2: Limitations of Vellinga’s model (modified after Vellinga, 1986).

4.3 Steetzel’s Time-Dependent Model

Although Vellinga’s model provides a fair estimate of the amount of dune erosion (Steetzel, 1993), a number of problems remain (see Section 4.2.5). To overcome some of the limitations, the Technical Advisory Committee on Water Defences (TAW) had a more sophisticated model developed. It is based on the vertical water velocity and sediment concentration profiles. Erosion is then calculated as a function of time. The main goal of the new model was to quantify the amount of dune erosion for arbitrary profiles (e.g. with bars and tidal gullies) and arbitrary hydraulic conditions. This model, developed by Steetzel, is known as the DUROSTA model (Delft Hydraulics, 1991). Existing data on storm-induced profile changes were re-analysed and some additional investigations were carried out. The overall performance of the model was good (Steetzel, 1993).

4.4 Current Application Of Vellinga’s And Steetzel’s Models

Currently, the safety of the Dutch dunes is determined by the use of a design method (seen as Level I) which is based on Vellinga’s equilibrium profile model (CUR-TAW, 1989). Probabilistic calculations at higher levels are only made on a few occasions when there is some particular need for them.

Steetzel’s time-dependent model is more sophisticated. At present, it is applied by specialists only. Nevertheless, it is expected eventually to replace
the current procedures. But, owing to its complexity, much still remains to be done before it can be successfully combined with probabilistic calculations. Such a task is beyond the scope of the present study.

4.5 Two Computer Programs Which Use Vellinga's Model

4.5.1 Introduction

Many parameters are important in the design of dune nourishment. Furthermore, the precise values of these parameters, both before and during a surge, are uncertain and can vary in time and space. Thus, a probabilistic design approach seems appropriate.

The occurrence of future storm surges accompanied by dune erosion is a highly stochastic process (Van de Graaff, 1986). At best, one is able to predict the probability of occurrence of a certain set of extreme conditions during a certain storm season or in a certain year. Such predictions are based on long-term observations and/or simulations of the process. Using probabilistic methods and taking into account the transfer function between the surge conditions and the amount of dune erosion, one is eventually able to determine the probability of exceedance per year of particular dune retreats (Figure 4.6).

![Erosion as a function of frequency of exceedance](image)

**Figure 4.6:** Erosion as a function of frequency of exceedance (modified after Van de Graaff, 1986).
An acceptable risk of failure of the dunes has to be established. The level of acceptability depends mainly on the likely consequences. For example, a very low chance of failure is demanded in The Netherlands, due to the high importance associated with the low-lying hinterland. There, a probability of failure of no more than $10^{-5}$/year (a return period of 100000 years) is expected (Van de Graaff, 1983, 1986). That is, a dune system is assumed safe in The Netherlands only if it is wide enough to withstand erosion with a chance of exceedance of no more than $10^{-5}$/year. However, a situation like that shown in Figure 4.1(b) might permit the allowable probability of a retreat beyond point R to be far larger ($10^{-2}$/year or $10^{-3}$/year) due to the relatively minor importance of the threatened area.

Predicted dune erosion during a surge depends on the values of the seven determining parameters in Vellinga's model (see Section 4.2.2). Knowing the density functions of the parameters, the probability of occurrence of a set of particular values can be computed using a Level III approach. A probability of exceedance curve of properties such as erosion distance, RD, can then be found by integration (Van de Graaff, 1986). However, the number of computations is enormous when seven parameters are involved. Due to the mass balancing procedure required in calculating the erosion volume, the computation time for even one calculation is very long. Thus, a large number of computations is not an attractive proposition. In The Netherlands, a Level III method has been applied only for test purposes. In these tests, large integration steps were adopted.

Another problem with Level III methods is that it is impossible to gain insight into the relative importance of the parameters involved. This shortcoming, together with the number of computations needed, is the main reason why Level III methods have not been used frequently. Furthermore, the results obtained hardly differ from those of a Level II approach (Van de Graaff, 1983). Probabilistic calculations on a personal computer then become possible.

In The Netherlands, Vellinga’s model has been used as the basis for the development of two main computer programs:

- a Level II probabilistic program (DUNEPROB);
- a Level I simplified calculation program (DUNE).
The two programs are introduced here. Appendix B contains an example of their use and a comparison of results.

### 4.5.2 DUNEPROB

DUNEPROB was developed by Koster Engineering. It calculates the probability that erosion will exceed a certain distance, assuming that the eroded sand is transported only in a seaward direction. For this purpose, the probabilistic FORM is used. In other words, given a certain target $X$-coordinate, $X_T$ (e.g. the location of a building close to the sea), the program calculates the probability that erosion occurs such that $X_R$ in Figure 4.7 is landward of $X_T$.

![Figure 4.7: Schematic representation of the dune erosion problem (modified after Van de Graaff & Koster, 1990).](image)

As noted earlier, a reliability function $Z$ is defined in FORM calculations such that $Z \leq 0$ represents failure of the system. In this case, the following reliability function meets this requirement:

$$Z = X_R - X_T$$  \hspace{1cm} (4.3)
\( X_R \) is the X-coordinate of point R and depends on the seven parameters affecting the erosion profile according to Vellinga. The main characteristics of these seven parameters, as used in DUNEPROB, are described next.

### 4.5.2.1 Maximum Water Level During Storm Surge

The total water level reached during a storm depends mainly upon (Van de Graaff, 1983, 1986; Pugh, 1987; Thomas & Hall, 1992): (i) the astronomical tide; and (ii) the wind and wave setup and low atmospheric pressure associated with meteorological forces (see Figure 4.8). Accounting for these phenomena, it is possible to derive the resulting frequency of exceedance curve of an arbitrary maximum water level during storm surge (Vrijling & Bruinsma, 1980). The Delta Committee (1960) presented similar frequency of exceedance curves, based largely on extrapolation of historical data, for locations along the Dutch coast. As the Vrijling & Bruinsma curves do not differ essentially from the Delta Committee curves, the latter are used here.

The probability of exceedance curve for the maximum water level during storm surge in metres above datum (NAP under Dutch conditions) can be described by an Exponential distribution as follows (Van de Graaff & Koster, 1990):

\[
1 - F_h = \exp \left( \frac{h - \zeta}{\lambda} \right)
\]  

(4.4)

\( \zeta \) and \( \lambda \) are parameters depending on site location along the Dutch coast. DUNEPROB assumes this distribution for \( h \) and requires \( \zeta \) and \( \lambda \) as input parameters.

### 4.5.2.2 Significant Wave Height During The Storm Surge

Water levels and wave heights along a coast are related. Figure 4.8 shows why. Wind blowing over water exerts a shear stress which may pile water against the coast. Waves induced by the wind add a further setup in the water surface as they break. The effects are enhanced by low atmospheric
pressure causing a general rise in the sea surface. Water levels and wave heights may be strongly or weakly related depending upon the location. Wind speeds, directions and durations are all important determining factors, as is the tidal range.

Vrijling & Bruinsma (1980) studied the joint distribution of water levels and wave heights in establishing the boundary conditions for the Oosterschelde storm surge barrier in The Netherlands. Van Aalst (1983) derived the maximum water level during storm surge versus significant wave height relationships for various locations along the Dutch coast (Figure 4.9). The given significant wave height represents the mean value, \( \mu \). For each location, a standard deviation, \( \sigma \), is also shown. DUNEPROB uses the following expressions to describe the statistical distribution of the deep water significant wave height as a function of the water level during storm surge:

\[
\mu_{H_S|h} = ah^{b} \tag{4.5}
\]

where \( \mu_{H_S|h} \) is the mean value of \( H_S \) given a value for \( h \); \( a \) and \( b \) are coefficients that depend on the location and

\[
H_S|h = \mu_{H_S|h} + H_S \ln \text{acc} \tag{4.6}
\]
H$_{S,\text{Inacc}}$ is a Normal random variable having a zero mean and a standard deviation, $\sigma_{H_{S,\text{Inacc}}}$, which is usually set to 0.6 or 0.75 for The Netherlands, depending on the site location (Van de Graaff, 1986).

![Diagram](image)

**Figure 4.9:** Expected value of $H_S$ as a function of $h$ at locations along the Dutch coast (modified after Van de Graaff, 1986).

There are situations where the expected value for $H_S$, according to eq. (4.5), might be too high if the wave conditions are depth limited. For this reason, an upper limit, $H_{\text{wavemax}}$, can be introduced in the program.

### 4.5.2.3 Particle Diameter Of Dune Material

As noted earlier, the amount of dune erosion depends on the grain size of the sand (via its fall velocity $w$). According to CUR-TAW (1989), the fall velocity for the period of the year during which storm surges can be expected
Dune Erosion During A Storm Surge

in The Netherlands, should be calculated for a salt water temperature of 5°C. It can be approximated by:

$$\log \left( \frac{1}{w} \right) = 0.476 (\log D_{50})^2 + 2.18 (\log D_{50}) + 3.226$$

(4.7)

where $D_{50}$ and $w$ are expressed in SI units.

Kohsiek (1984) analysed samples taken from different locations along the coast and determined for each location a mean value $\mu_{D_{50}}$ and a standard deviation for $D_{50}$ (Normal distribution). The ratio $\sigma_{D_{50}} / \mu_{D_{50}}$ varied from location to location with values from 0.01 to 0.15.

4.5.2.4 Shape Of Initial Profile

Two different cases have to be considered when analysing the influence of the shape of the initial beach profile on the resulting dune erosion (Van de Graaff, 1983): (i) there is a stable profile; and (ii) there is an unstable (long-term eroding) profile. Even a stable profile varies from day to day and from season to season. One single profile measurement represents only one sample from some distribution. Assuming a Normal distribution, the mean profile, $\mu_{DP}$, and the standard deviation of the profile, $\sigma_{DP}$, can be calculated when a series of measurements is available. Note that $\mu_{DP}$ and $\sigma_{DP}$ are expressed in terms of units of m$^3$/m above a datum and not in metres as is DP in PARASODE.

4.5.2.5 Storm Surge Duration

Storm surge duration is not a parameter in eq. (4.1). In reality, dune erosion is time-dependent and the time during which the water level is near the maximum surge level is one of the main factors determining the amount of erosion (Van de Graaff, 1983, 1986). Vellinga carried out two sets of tests to investigate the matter: a) tests with a constant maximum surge level; and b) tests with an actual hydrograph (see Figure 4.4). Note that the shape of this hydrograph was only one possibility out of a number of alternatives, all with the same maximum water level (Van de Graaff, 1986). From the tests, it was concluded that the normal result of an erosion computation (volume C
above surge level) can be assumed valid for a surge duration of 5 hours. Variations in the storm surge duration around this value can be represented as an additional erosion above storm surge level having a Normal distribution and the following characterising parameters: $\mu = 0$ and $\sigma = 0.1C$ (Van de Graaff, 1983, 1984, 1986).

4.5.2.6 Gust Bumps And Squall Oscillations

Squall oscillations are periodic, local variations in the water level during a storm surge with an amplitude of the order of 0.20m and a period of the order of 45min (Van de Graaff, 1986). Gust bumps are short-term increases in water level caused by the passage of a front or a heavy shower. In contrast to squall oscillations, gust bumps can be traced over a large area. Their amplitude is about 0.40m and they have a period of about 60min (Van de Graaff, 1986). The occurrence of both phenomena is highly variable. In recording storm surges, the effects of these irregularities, which mostly increase maximum water level, are neglected in Dutch practice. Actual hydrographs are smoothed and the maximum values are stored. Equations such as eq. (4.4) represent the smoothed curves. However, the amount of dune erosion is highly dependent on the maximum water level and any rise, even for a short time, results in an increase in the volume of erosion (Van de Graaff, 1983).

A short-term increase, $\Delta h$, in water level above the smoothed peak will lead to an increase, $\Delta C$, in the volume of erosion. Van de Graaff (1984) argued that $\Delta C$ is approximately $0.5 \Delta SH$, $\Delta SH$ being the increased amount of erosion due to a smoothed hydrograph with a maximum $\Delta h$ higher:

$$\Delta C = 0.05C \left[ \frac{\Delta h}{0.40} \right]$$  \hspace{1cm} (4.8)

where $C$ is the eroded volume above the maximum water level which is calculated ignoring gust bumps and squall oscillations. Generally, the effects of these phenomena can be represented as an additional erosion above storm surge level having a Normal distribution and the following parameters: $\mu = 0.05C$ and $\sigma = 0.0125C$ (Van de Graaff, 1984).
4.5.2.7 Accuracy Of The Computation Method

The accuracy of the computation method is taken into account as explained in Section 4.2.4 (assuming a Normal distribution with a zero mean and a standard deviation, $\sigma = 0.10C + 20\text{m}^3/\text{m}$).

4.5.2.8 Surcharge

Variations in the last three parameters listed above can be combined to form one new single variable, Surcharge, having a Normal distribution with the following characteristics: $\mu_{\text{Surcharge}} = 0.05C$, $\sigma_{\text{Surcharge}} = \sqrt{(0.1C)^2 + (0.0125C)^2 + (0.10C + 20)^2}$. The effect of this surcharge is expressed in an additional recession of the dune front corresponding to an amount of erosion referred to as SurchEros (Figure 4.10).

![Figure 4.10: Surcharge on erosion area C above surge level.](image)

Note that in DUNEPROB, the variable Surcharge has a slightly different meaning from that described here: it is a surcharge coefficient. This coefficient follows a Normal distribution and its mean and standard deviation are often approximated by $\mu = 0$ and $\sigma = 1$, respectively.
4.5.3 DUNE

At present, the probabilistic approach to dune erosion computations is still thought of as rather complicated for "everyday" computations by coastal managers. That is the reason why a simplified calculation method has been introduced in The Netherlands, based on the results of extended probabilistic calculations. This method can be seen as a Level I approach. It is presented in the "Guide to the Assessment of the Safety of Dunes as a Sea Defence", CUR-TAW (1989). In the Level I approach, it is necessary to account only for possible variations in $D_{50}$ and in the initial profile for a specified surge level and significant wave height. Calculations are based on the assumption that the eroded sand is transported only in a seaward direction (CUR-TAW, 1989). Table 4.3 gives the set of characteristic values which yield, within 2% accuracy, the retreat distance expected with a $10^{-5}$/year frequency of exceedance. If one needs to consider a higher chance of failure (e.g. $10^{-4}$/year or $10^{-3}$/year), adopting a maximum water level during surge, occurring with a probability of exceedance which is a factor 2.15 times the acceptable chance of failure, and maintaining the other parameters according to Table 4.3, a rather accurate approximation is found (Van de Graaff, 1986).

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>CHARACTERISTIC VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum water level during surge</td>
<td>Value with a frequency of exceedance of 2.15x10^{-5}/year</td>
</tr>
<tr>
<td>Significant deep water wave height</td>
<td>Expected value for the given water level (Figure 4.9)</td>
</tr>
<tr>
<td>Diameter</td>
<td>$D_{50} = \mu_{D_{50}} \left(1 - 5\sigma_{D_{50}}^2 / \mu_{D_{50}}^2 \right)$. Holds at least for $0% &lt; \sigma_{D_{50}} / \mu &lt; 12.5%$</td>
</tr>
<tr>
<td>Change in the initial profile</td>
<td>Profile containing $\sigma_{DP}^2 / 275 m^3/m$ &quot;less sand&quot; than the average profile. Holds at least for $0 &lt; \sigma_{DP} &lt; 150 m^3/m$ ($\sigma_{DP}$ is the standard deviation of the volume about the mean initial profile)</td>
</tr>
<tr>
<td>Surge duration</td>
<td>0.1Cm$^3$/m addition (C being the volume of sand eroded above the maximum water level during surge, using the parameters defined as above)</td>
</tr>
<tr>
<td>Gust bumps and squall oscillations</td>
<td>0.05Cm$^3$/m addition</td>
</tr>
<tr>
<td>Accuracy of computation method</td>
<td>$(0.1C+20)m^3$/m addition</td>
</tr>
</tbody>
</table>

Table 4.3: Characteristic parameter values (modified after Van de Graaff, 1986).
The above simplified calculation method has been implemented in a computer program called DUNE. This program has also been developed by Koster Engineering. Appendix B includes an example of its use.

Note that wave period is not an explicit parameter in the above table or in Vellinga’s model. However, DUNE uses the peak period of the wave spectrum to calculate a limiting profile defined by the minimum necessary dune crest level and the minimum width at this level, together with an acceptable maximum backslope. The erosion profile must remain seaward of this limiting profile (CUR-TAW, 1989).

DUNE may also account for situations in which there is net loss of sand from the profile owing to a gradient in the longshore transport rate. The loss, \( G \) (m\(^3\)/m), for coastal sections of low or moderate curvature may be calculated as follows (CUR-TAW, 1989):

\[
G = \frac{(C + \text{SurchEros})}{300} \left( \frac{H_b}{7.6} \right)^{0.72} \left( \frac{w}{0.0268} \right)^{0.56} G_o
\]  

where \( G_o \) is a reference value for \( G \) (m\(^3\)/m) and is tabulated below:

<table>
<thead>
<tr>
<th>Curvature (degrees/1000 m)</th>
<th>( G_o ) (m(^3)/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 6</td>
<td>0</td>
</tr>
<tr>
<td>6 - 12</td>
<td>50</td>
</tr>
<tr>
<td>12 - 18</td>
<td>75</td>
</tr>
<tr>
<td>18 - 24</td>
<td>100</td>
</tr>
<tr>
<td>&gt; 24</td>
<td>further investigation</td>
</tr>
</tbody>
</table>

Table 4.4: \( G_o \) values (modified after CUR-TAW, 1989).

The result of calculating \( G \) may be expressed as an additional recession of the dune front above maximum water level.
4.5.4 Applicability Of The Programs In The British Context

The programs DUNE and DUNEPROB are based on the dune erosion profile determined by Vellinga, combined with probabilistic calculations. The program DUNE calculates the erosion profile after a design storm corresponding to a probability of exceedance of approximately $10^{-5}$/year. This extremely low probability is justified for Dutch conditions (see Section 4.5.1) but it is not always appropriate. This drawback can be overcome as described in Section 4.5.3 or by using DUNEPROB.

Both programs relate the significant wave height to the maximum water level during storm surge (see Figure 4.9). If the expected values of $H_S$ as a function of $h$ do not follow a similar trend, then the direct application of DUNE or DUNEPROB will produce erroneous results. This is the case on the Sefton coast, UK (see Section 6.2.2), where there is little correlation between extreme wave heights and extreme water levels (Hawkes & Hague, 1994). For this reason, it was decided to introduce Vellinga's model and some of the features of DUNEPROB and DUNE into the Level II computer program, PARASODE, developed as part of this research (Chapter 5). Probabilistic calculations can be performed which allow for appropriate combinations of wave heights and storm surge levels. Some examples of the application of PARASODE to dune erosion are presented in Section 6.2 of this thesis.

4.6 Summary

Dutch experience with regard to the probabilistic design of dunes has been examined. The computational model currently used throughout The Netherlands is based on Vellinga's equilibrium profile model. The more sophisticated time-dependent model developed by Steetzel is not yet used as the basis for probabilistic calculations.

The Dutch programs are not directly applicable to conditions along coasts such as that in Sefton, UK, where there is a much weaker correlation than in The Netherlands between wave heights and water levels. Consequently, it was decided to introduce Vellinga's model and some features of the Dutch programs into PARASODE, and to carry out new probabilistic calculations.
5 SOFTWARE APPLIED IN THIS RESEARCH FOR UNDERTAKING PROBABILISTIC CALCULATIONS

5.1 Introduction

This chapter describes the software used in this research to perform probabilistic calculations on wave overtopping of seawalls and dune erosion. The main element of software, called PARASODE, has been developed as part of this work. It is based upon the Level II probabilistic methods reviewed in Chapter 2 and the formulation of the failure modes provided in Chapters 3 and 4.

Section 5.2.1 gives a general description of PARASODE. Section 5.2.2 explains the parameters in the program which control each FORM calculation. Section 5.2.3 illustrates how truncation of probability distributions has been implemented in PARASODE. Sections 5.2.4 and 5.2.5 describe, respectively, the incorporation of wave overtopping of seawalls and dune erosion into the program. Finally, Sections 5.2.6 and 5.2.7 refer to the input and the output of PARASODE, respectively.

To validate the results from PARASODE, various Level III calculations have been performed using the commercial software package @Risk, acquired from PALISADE Corporation. This program is briefly introduced in Section 5.3. For further details about the program, the reader is referred to its manual (Palisade Corporation, 1994).

5.2 PARASODE

5.2.1 General Description

PARASODE (Probabilistic Assessment of Risks Associated with Seawall Overtopping and Dune Erosion) has been developed for assessing the safety of coastal structures. In particular, as the name suggests, it concentrates on the potential failure mechanisms associated with wave overtopping of seawalls and dune erosion. The amount of wave overtopping is calculated by both the H&R equation and Owen’s formula. Dune erosion is calculated
using Vellinga´s model. Note that incorporation of dune erosion into PARASODE presented a more complex task than including the overtopping models. Additional problems arise because dune erosion cannot be described by an explicit failure function of the basic variables. Details of dune erosion calculations are given in Appendix C5.

Although the program incorporates two specific failure mechanisms, the majority of the code is generic and can be applied with minor adjustments to other types of failure.

PARASODE operates in two ways (see Figure. 5.1):

- **MODE 1**, the analysis mode, in which the failure probability is calculated for a given value of the design parameter, e.g. the crest level of a seawall.
- **MODE 2**, the design mode, in which the value of a specific design parameter is calculated for a target probability of failure.

![Figure 5.1: Illustration of Mode 1 and Mode 2 for failures resulting from overtopping.](image)

Mode 1 allows for combinations of time-varying actions using the method of Ferry Borges & Castanheta (1983). This method could also have been implemented in Mode 2. However, implementation is complex and computational time would be considerable. Instead, Mode 2 results involving
combinations of actions are best provided by running the program in Mode 1 for several different values of the design parameter, producing a design curve as shown in Figure 5.1. The answers required from Mode 2 calculations can then be obtained from the curve. Appendix C1 shows simplified flowcharts of the program for the analysis and design modes, respectively.

The program uses the Level II First Order Reliability Method - FORM (see Sections 2.3.3 and 5.2.2). It incorporates routines for transforming the correlated variables to a set of non-correlated variables and for mapping non-Normal distributions to equivalent Normal distributions. There are ten continuous pre-defined statistical distributions programmed in PARASODE. Each distribution may be truncated either on the left or on the right side (see Section 5.2.3). Details of these distributions (Law & Kelton, 1991; Evans et al, 1993) are tabulated in Appendix C3. In PARASODE, the user can also add his or her own distributions (see Appendices C4 and C6 for more details). This facility is of particular help when the distribution of a variable, X, is the result of measurements which are not easily fitted by a pre-defined distribution. At present, PARASODE has three user-defined distributions: i) observed water levels at Liverpool; ii) observed extreme water levels at Liverpool; and iii) predicted tide levels at Liverpool (see Appendix C4). They are used in the examples in Chapter 6.

PARASODE has been written in FORTRAN 77. General references on the FORTRAN language are Koffman & Friedman (1987), Davis & Hoffman (1988) and Etter (1992). Pre-defined subroutines have been used in some cases. These subroutines have been extracted from Press et al (1992) and NAG (1993).

Appendix C2 shows the various subroutines used in PARASODE. The program listing is provided in Appendix C7. It contains a description of the subroutines, the variables used in each of them are listed and described, and there are guiding comments throughout the code. SI units are used within the program, except if otherwise specified.

Appendix C6 provides a detailed list of the input files required and their contents. Examples of input and output files are provided in Appendices D1 and D3.
Before applying PARASODE to wave overtopping of seawalls and dune erosion, it was tested using examples reported in literature such as Thoft-Christensen & Baker (1982), Ang & Tang (1984), Thoft-Christensen & Murotsu (1986), Madsen et al (1986), Smith (1986), Van der Meer (1987), CUR-TAW (1990), Pilarczyk (1990), and Burcharth (1992). Examples run with other computer programs like PROBA2 (Delft Hydraulics, undated) and Super-Risk (Super-Software, 1994) were also reproduced. The results from PARASODE were always highly satisfactory.

5.2.2 FORM In PARASODE

The parameters controlling each FORM calculation have to be specified in the input file *form.dad*. These parameters are described below.

5.2.2.1 Starting Point

The iteration process needs starting values for the design point. It is common to choose the mean value of each variable: in other words, iteration starts by using the mean value approach. Sometimes, there are reasons to start computations at another point. For example, a solution might be found only by specifying another starting point. Also, in a case where there is more than one solution to the problem, one might find other solutions by trying other starting points (see, for example, Wen & Chen, 1987).

When no combinations of actions are involved, or combinations of actions are considered and the modified distributions are provided (see Section 2.3.3.3), PARASODE uses the mean value starting point, unless otherwise specified by the user. If combinations of actions are considered and the basic distributions are given, then the starting values for the design point are obtained as follows:

- If the power, NR, to which the distribution of the variable is raised is 1 then the starting value is the mean value of the variable.
- If the power, NR, to which the distribution of the variable is raised is not 1, then the starting value corresponds to an extreme cumulative distribution function value of 0.5 (\( X = F_X^{-1}(0.5^{1/NR}) \) where \( F_X^{-1} \) is the inverse of \( F_X \) evaluated at \( 0.5^{1/NR} \)).
5.2.2.2 Minimum And Maximum Values

Variables which have a physical meaning may be restricted within particular limits during the iteration process. Also, the form of the failure function may prevent variables taking certain values (e.g. a variable which is raised to the power 0.5 cannot take negative values). In PARASODE the user can either accept the default minimum and maximum values (XMin=-1E25; XMax=1E25) or specify required values. However, if the user's limits exceed the boundaries defined previously by the variable's distribution, the program adopts the more limiting boundaries. So, for example, if a variable follows a Log-Normal distribution, then X>0; if the user inputs XMin=-10, the program neglects this latter value and adopts XMin=1E-25.

During the iteration process, if a variable X lies outside the boundaries (XMin, XMax), then the program continues calculations using a new value of X between the last computed value and the boundary which was exceeded. This procedure gives final results where otherwise the program would fail.

5.2.2.3 Number Of Iterations

In a FORM calculation, the design point can only be found by iteration. The number of iterations required depends on the failure function (the more linear the function is, the faster the iteration procedure converges), on the point used to start the iteration process, on the required relative accuracy of the reliability index, on the iteration smoothing process, and on the required accuracy of Z being zero.

The maximum number of iterations in a FORM calculation is designated in PARASODE by MaxIter. It can be set to any positive value less than 200. If no solution is found within 200 iterations then it is likely that some error has occurred (e.g. the calculation may be in a loop). After MaxIter iterations, the program stops its present calculation and either continues with the next one (specified in the input file form.dad) or ends (if no more calculations are required).
5.2.2.4 Accuracy Of The Reliability Index

Accuracy of the reliability index, $\beta$, is essential to insure a corresponding accuracy of the calculated probability of failure, $P_f$. It is important that the difference in the reliability index between the last two iterations does not correspond to a significant difference in the probabilities of failure. Suppose that the value of $\beta$ in the last iteration resulted in $P_f=10^{-1}$. This answer would be unreliable if the next iteration gave $P_f=10^{-5}$.

One way of controlling the relative accuracy of the solution is by calculating a parameter, BetaAcc

$$\text{BetaAcc} = 100 \left| \frac{\beta_{\text{New}} - \beta_{\text{Old}}}{\beta_{\text{New}}} \right|$$

(5.1)

such that the program does not stop iterating while BetaAcc is greater than the required accuracy, ReqBetaAcc. Of course, the required number of iterations increases for decreasing values of ReqBetaAcc. The default value in PARASODE for ReqBetaAcc is 1 which guarantees a relative accuracy of within 1%, which is usually sufficient. The program requires a value for ReqBetaAcc within the bounds 0 and 1.

5.2.2.5 Smoothing Of The Iteration Process

There are cases where the iteration process does not converge owing to instability: the new calculated design point differs considerably from the calculated design point of the preceding iteration. This difficulty can lead either to divergence of the process, or to values of the random variables which cause problems in the failure function. In such cases, "smoothing" of the iteration process may help. Smoothing is applied in the following manner:

$$X_{\text{New}}(i) = (1 - \text{Smooth}) \cdot X(i) + \text{Smooth} \cdot X_{\text{Old}}(i)$$

(5.2)

where $X_{\text{New}}(i)$ is the new smoothed value of $X$, $X(i)$ is the new unsmoothed value, $X_{\text{Old}}(i)$ is the preceding value and Smooth is the smoothing coefficient for the iteration process which has a value between 0 and 1. Setting Smooth=0 means that no smoothing of the iteration process is performed.
Setting Smooth=0.5 provides an average between the old and the new value of $X_i$.

### 5.2.2.6 Accuracy Of The Failure Function

Just as a relative accuracy for $\beta$ has been defined, it is possible to specify an accuracy for the failure function, $Z$, being zero. A dimensionless measure of accuracy is used in PARASODE (after Super-Software, 1994). The basis of this formulation is that if the standard deviation of the failure function, $\sigma_Z$, is small, the accuracy of $Z$ being zero is more important than in the case where $\sigma_Z$ is large. Hence, the program does not stop iterating while

$$|Z| > \frac{\text{ReqZAcc}}{100} \sigma_Z$$  \hspace{1cm} (5.3)

where ReqZAcc is the required accuracy of $Z$. ReqZAcc has to be defined within the bounds of 0 and 1. If ReqZAcc=1, then the value of $Z$ at the design point is less than 1% of the calculated standard deviation for $Z$ away from zero. The default value of 1 is sufficient for most cases. Setting ReqZAcc to lower values means a higher accuracy for the answer but requires more iterations.

### 5.2.3 Truncation In PARASODE

Sometimes it is necessary to truncate a theoretical distribution of a random variable in order that it conforms to measurements or to known physical constraints. The truncation is said to be to the right of $X=X_o$ if all values of $X$ above $X_o$ are discarded, and is said to be to the left of $X=X_o$ if all values of $X$ below $X_o$ are discarded. Since the area beneath the probability density function must remain 1, it is necessary to scale the original non-truncated values of the probability density function over the truncated range.

Scaling can be performed in a number of ways, depending on factors such as the physical meaning of the variable. In this study, truncation has been performed as follows (Beaumont, 1986):
The probability density function for significant wave height, \( H_S \), provides an example of the need for truncation. \( H_S \) may be limited by the available water depth. In this case, several approaches for truncation to the right of \( X_0 \) are possible, depending on the cut-off technique and the definition of the point of truncation, \( X_0 \) (Thornton & Guza, 1983; Allsop & Meadowcroft, 1995). In this study, for the failure mode of overtopping, truncation of the distribution describing \( H_S \) has been performed according to Table 5.1 and Figure 5.2. To determine the point of truncation, \( X_0 \), it is assumed that the heights of individual waves (broken and unbroken) are described by the Rayleigh distribution and that \( H_{\text{rms}} = 0.42d_s \) (Thornton & Guza, 1983). Furthermore, taking into account that for a Rayleigh distribution \( H_S \approx 1.42H_{\text{rms}} \), it follows that a sensible approximation for \( X_0 \) is:

\[
X_0 = 0.6d_s = 0.6(\text{SWL} - \text{TL})
\]  

(5.4)
where the variables $d_s$, SWL and TL are defined as in Chapter 3 (see Figure 3.1). Hence, the point of truncation is considered simply proportional to the local water depth. For simplicity, factors such as the bottom slope are not considered to influence $X_o$. Note that in PARASODE, SWL is forced to be always greater than TL.

This approach to truncating the distribution of $H_S$ means that $H_S$ is assumed to have some value smaller than $X_o$ after wave breaking has been initiated and that the depth-limited values are redistributed across the range of significant heights in proportion to the unlimited values at each significant height.

### 5.2.4 Overtopping in PARASODE

The failure mode of overtopping is implemented in PARASODE using both the H&R model and Owen’s model. The basic variables in these models are:

<table>
<thead>
<tr>
<th><strong>H&amp;R Model</strong></th>
<th><strong>Owen’s Model</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Peak wave period, $T_p$</td>
<td>• Mean wave period, $T_m$</td>
</tr>
<tr>
<td>• Significant wave height, $H_S$</td>
<td>• Significant wave height, $H_S$</td>
</tr>
<tr>
<td>• H&amp;R parameter, $A$</td>
<td>• Owen parameter, $A$</td>
</tr>
<tr>
<td>• H&amp;R parameter, $B$</td>
<td>• Owen parameter, $B$</td>
</tr>
<tr>
<td>• Still-water-level, SWL</td>
<td>• Still-water-level, SWL</td>
</tr>
<tr>
<td>• Tangent of the seawall slope, $\tan \alpha$</td>
<td>• Roughness, $r$</td>
</tr>
<tr>
<td>• Roughness, $r$</td>
<td>• Model parameter, $e_B$</td>
</tr>
</tbody>
</table>

The program allows SWL to be specified either as a variable in its own right or as the sum of two variables: i) tide level (Tide); and ii) surge (Surge).

Model parameter $r$, describing the roughness of the seawall front slope, is the ratio of the run-up on a rough slope to that on the corresponding smooth slope. Values range from about 0.5 to 1.0 (CIRIA/CUR, 1991). It is readily incorporated in both the H&R model and Owen’s model.

The two failure functions may then be written as follows:
H&R Model: \[ Z = TR - A \sqrt[3]{g(CH_s)} \left[ 1 - \frac{(CL - SWL)}{rCH_s} \right]^{\alpha_0B} \] for \( 0 \leq R < 1 \)
\[ Z = TR \] for \( R \geq 1 \)

(5.5)

Owen’s Model: \[ Z = TR - A \left[ \frac{gH_s^3}{s_m} \right] \exp \left[ -e_0B \frac{\sqrt{s_m}(CL - SWL)}{rH_s \sqrt{2\pi}} \right] \]

where TR is the discharge allowed for a specific FORM calculation (the target value).

The program can run for as many as ten different values of TR. Plots can then be produced of the probability of failure as a function of the design parameter, seawall crest level, for different allowable discharges (see Figure 5.1). These plots are a valuable tool in the preliminary design of seawalls using probabilistic analysis. They can be used to make a cost optimisation for the structure during the reference period or design life (Van der Meer & Pilarczyk, 1987).

For the H&R model, the user has to choose if the coefficient C is calculated using a 37% or 99% confidence level for the maximum run-up. Depending on the choice made by the user, the program uses a constant of 1.52 or 2.15, respectively, in eq. (3.22). The value 2.15 arises from substitution of N=100 and p=99 in eq. (3.20). At present only these two alternatives are available. This is due to the fact that a change in C implies corresponding changes in the values of A and B.

For the failure mode of overtopping, the first partial derivatives of the failure function required to perform the FORM calculations can be calculated either by using their expressions, provided in the code, or by using the subroutine EO4XAF (NAG, 1993), called by PARASODE, which computes finite-difference approximations to the first derivatives for a given failure function.
5.2.5 Dune Erosion In PARASODE

The failure mode of dune erosion is implemented in PARASODE using Vellinga’s model. In Mode 1, the program calculates the probability of failure associated with a prescribed value of the nourishment width. In Mode 2, the nourishment width is calculated for a target probability of failure.

The basic variables in the program are:

- Significant wave height, $H_S$
- Median sediment size, $D_{50}$
- Change in the initial profile, $\Delta P$
- Surge duration, $SD$
- Gust bumps, $GB$
- Accuracy of the computation, $Ac$
- Maximum water level during surge, $h$

The program allows the maximum water level during surge, $h$, to be specified either as a variable in its own right or as the sum of two variables: i) tide level (Tide); and ii) surge (Surge).

Dune erosion is not an explicit function of the basic variables. Consequently, it is not possible to express the failure function as a simple equation. Thus, the first partial derivatives of the failure function required to perform the FORM calculations must be evaluated using the subroutine EO4XAF (NAG, 1993), called by PARASODE.

In PARASODE, the eroded sand can either be assumed to be transported only seaward (as in DUNEPROB and DUNE) or, alternatively, it can be assumed to move both landward and seaward during a storm surge. The common assumption, that during the short period of a storm surge the cross-shore sediment transport will principally be in an offshore direction, is conservative. Allowing the user to choose between movements only seaward or also in the landward direction, provides two answers which give an idea of the range of erosion to be expected.
Figure 5.3 illustrates the main erosion situations which can be studied using PARASODE (modified after CUR-TAW, 1989).

Figure 5.3 illustrates the main erosion situations which can be studied using PARASODE. Case (a) is the situation which normally occurs during high storm surges. Case (b) may occur for coastal profiles with flat slopes; after the storm surge, Vellinga's profile will be partly below the original profile. If
movements of sand are allowed only seaward, the original profile is raised by sand from the dune only. If movements of sand are allowed both seaward and landward, then the depression in the foreshore can be filled both from the dune and from the seaward part of the bed which lies above Vellinga’s profile, resulting in a smaller amount of dune erosion. Case (c) is similar to case (b). If movements of sand are allowed only seaward, the sand movements on the seaward side of the bank are of no importance to the recession of the dune (since the sand between the bank and Vellinga’s profile is sufficient to raise the seaward part of the original profile). If movements of sand are allowed both seaward and landward, then the depression can be filled both from the dune and from the bank, resulting again in a smaller amount of dune erosion. In case (d), the bank is fully eroded to raise the original bed towards Vellinga’s profile, and the amount of sand further required for the development of Vellinga’s profile is eroded from the dune. Finally, in case (e), Vellinga’s profile is entirely below the original profile. This situation occurs frequently during storm surges at low tide levels. According to PARASODE, no dune erosion will take place (CUR-TAW, 1989). In practice, however, a minor amount of dune erosion may be expected owing to wave run-up, particularly if the dune face is steep.

The general calculation procedure is described here. For further details, the reader is referred to Appendices C5 and C7. Figures 5.4 to 5.7, presented at the end of this section, illustrate the procedure used and the notation applied in the FORTRAN code. Some of the notation in these figures is not mentioned in the main text but can be found in the program listing.

The calculation procedure differs depending on the direction chosen for the sand movements. In any case, for a given initial profile with NPD points, the program starts by establishing a changed profile. The latter is obtained by changing the Y-coordinate, \( Y_P \), of some points in the initial profile (see Figure 5.4(a)). The number of points changed is \( N_{P_{\text{ch}}} \) and the change is \( D_P \). The purpose of making these changes is to represent the possible error in the initial profile immediately before the storm surge. These errors arise as a consequence of measurement inaccuracies and changes in the profile between the time of measurement and occurrence of the storm surge.

Next, a new profile is defined based on the nourishment characteristics (nourishment top level, nourtlev, and gradient of the nourished face,
1:mnour) and on the value of the design parameter (nourishment width at top level, nourwidt). The new profile is referred to as the nourished profile (see Figure 5.4(b)). The point of intersection of the nourished profile and the surge level, h, is (S1,T1). Note that if the parameter nourwidt is set to zero, PARASODE assumes that there is no nourishment. Consequently, if nourishment is to be provided without a berm, nourwidt must be set at a small positive value. Note also that the gradient of the nourished face depends not only upon the method chosen for placing material\(^1\) but also upon other factors such as the grain size (CUR et al, 1987). Table 5.2 provides some guidance on expected gradients.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>GRAIN SIZE (µm)</th>
<th>DRY FILL ABOVE WATER (usually for dune nourishment)</th>
<th>HYDRAULIC FILL (usually for beach nourishment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine Sand</td>
<td>60-200</td>
<td>up to the</td>
<td>Above Water Smooth Sea Below Water Rough Sea</td>
</tr>
<tr>
<td>Medium Sand</td>
<td>200-600</td>
<td>natural angle</td>
<td>1.50 - 1:100</td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>600-2000</td>
<td>of repose</td>
<td>1:10 - 1:25</td>
</tr>
<tr>
<td>Gravel</td>
<td>&gt;2000</td>
<td>(1:1 - 1:2)</td>
<td>1:5 - 1:10</td>
</tr>
</tbody>
</table>

| Table 5.2: Expected gradients of nourished dune/beach face (modified after CUR et al, 1987). |

In its present version, PARASODE allows consideration of a simplified form of nourishment to the seaward face of the dune and/or to the beach (see Figure 5.4(b)). However, it would be relatively straightforward to modify PARASODE to deal with more complicated nourishment profiles. Note that it is assumed that once nourishment has taken place, the material stays where it has been deposited until a storm surge occurs. The material used for nourishment is also assumed, for simplicity, to be of the same type and size as the native sediment; usually, the preferred grain size of borrow material is equal to or larger than that of the native sediment (Hedges, 1977; CUR et al, 1987; Simm et al, 1996).

After the nourished profile is defined, the shape of Vellinga's parabolic

---

\(^1\) In broad outline, the methods can be distinguished as follows (CUR et al, 1987):

i) dry fill - transportation of dry sand to the site by trucks, etc;

ii) hydraulic fill - transportation of a sand-water mixture via a pipeline.
post-storm profile is calculated according to eq. (4.1). As a first approximation, it is assumed that the parabolic part of the profile starts at point (S1,T1) (see Figures 5.5(a) and 5.6(a)). The X-coordinate of this starting point is always designated as S8. The position of the offshore point, (S9,T9), where the parabolic part of the profile terminates is also calculated: the length of the profile is Le and the depth is Depth. If point (S9,T9) is located above the nourished profile (Figure 5.5), Vellinga's profile continues seaward as a straight line with a gradient of 1:mt until it intersects the seabed. Otherwise (Figure 5.6), a vertical line is drawn until intersection with the nourished profile occurs. The point of intersection is (S2,T2). Landward of X=S8, the gradient of the eroded dune face is 1:md. The point of intersection of the eroded dune face and the nourished profile is (S3,T3). Note that in PARASODE, 1:md is not a constant of 1:1 as in Vellinga's original model. The user is free to define this slope. Likewise, the gradient of the toe of the post-storm profile, 1:mt, need not be taken as 1:12.5.

After Vellinga's profile is defined, it has to be located in such a way with regard to the nourished profile that the total area of eroded sand is equal to the area of accretion. In order to achieve this required final position, Vellinga's profile is moved along the X-axis, the corresponding areas of erosion and accretion are calculated and the balance tested. The methods used to calculate the areas of erosion and accretion and to test the required balance between these areas depend on the direction of the sand movements. Details are provided in Appendix C5.

Finally, the failure function is calculated as follows (Figure 5.7):

$$Z = TR + S4$$

(5.6)

TR (the target value) is the allowable erosion distance measured from the reference line X=0 and S4 is the X-coordinate of the most landward point to which the profile has been eroded. Note that S4 is the estimate provided by PARASODE of the position R, in Figure 4.1. The program can be run for as many as ten different values of TR. Plots can then be produced of the probability of failure as a function of the design parameter, nourishment width, for the different allowable erosion distances. Such plots are a valuable tool in the preliminary design of dune nourishment using probabilistic analysis.
Figure 5.4: Definition of initial, changed and nourished profiles.
Figure 5.5: Definition of Vellinga’s post-storm profile - example 1.
Figure 5.6: Definition of Vellinga's post-storm profile - example 2.
5.2.6 Input

The program runs simply by executing the command PARASODE. The input data can be read either from the computer screen or from input data files. If the data are to be read from the screen, the user only has to answer the questions asked and choose between alternatives. If the input is provided by data files, then the following four files have to be prepared by the user, no matter which failure mode is studied:

- `general.dad`
- `form.dad`
- `meandev.dad`
- `coefcor.dad`

A fifth data file is required if the failure mode under study is dune erosion. This file is called `perfil.dad`.
Description of the input files is given in Appendix C6 and examples are provided in Appendices D1 and D3.

5.2.7 Output

Two output files are created when running PARASODE:

- summary.dat - A file which contains the input data and the most important final numerical results only.
- results.dat - A text file which contains the input data and the most important numerical results for all iterations.

Examples of the output file summary.dat are given in Appendices D1 and D3.

5.3 Validating PARASODE Using @RISK

To validate the results of a Level II program like PARASODE, Level III calculations have to be carried out (Ang & Tang, 1984; Van der Meer, 1987).

The normal way to implement Level III methods is to write a computer program. Such a program would consist of random number generation (normally a built-in function), solving the appropriate inverse distribution functions for the parameters which have been defined, calculating the result, and repeating for another set of random numbers. After the required number of samples, the results are summarised in terms of a probability distribution, or simply the proportion of results corresponding to failure.

A simpler way to carry out a Level III analysis is to use existing software packages. A search was made for software suitable for this task. Programs such as PREDICT (Risk Decisions Ltd., Oxfordshire, UK), @RISK (Palisade, New York), STRUREL (Reliability Consulting Programs, Munich, Germany) and SUPER-RISK (Super-Software, Heemstede, The Netherlands) were considered. Some programs included probabilistic methods other than Level III; however these packages were expensive. Since the main objective was to obtain a package solely to carry out Level III calculations, @RISK was chosen.
@RISK is sold as an add-in for Microsoft Excel or Lotus 123. It uses simulation to combine all the uncertainties identified in the modelling. The options available for controlling and executing a simulation in @RISK are quite powerful. They include: i) Traditional and Latin Hypercube Sampling (see section 2.3.2.2); ii) unlimited number of iterations per simulation; iii) multiple simulations in a single analysis; iv) continuing a simulation after viewing results and performing more iterations if necessary; and v) seeding the random number generator.

The random number generator used in @Risk is a portable random number generator based on a subtractive method, not linear congruential (for more details see, for example, Law & Kelton, 1991). The cycle time is long enough that it has no effect on the simulations (Palisade Corporation, 1994). The period of the generator is effectively infinite. The seed or starting value, if not set manually, is clock dependent, not machine dependent. The results of a simulation are reproducible from run to run. If the seed is set to zero it means that the sequence of random numbers will start at a random value. The result will differ each time a run is made (using the same input). If however the seed is set to any positive number, it means the random generated numbers will start at a specific place in the sequence. This allows @Risk to give reproducible results of a simulation from run to run, because each time a run is made the same sequence of random numbers will be used.

The way the program works is appealing because it conforms to the way that many engineers now carry out calculations: formulae are entered into the spreadsheet as usual, but any data item in a cell (or range of cells) can be specified as a probability distribution instead of as a single value. The software provides a library of about 30 different distributions, including the distributions available in PARASODE. The user issues the command to carry out a simulation and the software automatically carries out the task using the prescribed probability distributions, recording each interim result. Simulation results generated by @Risk include statistics and data reports for both input and output variables. The probability distribution of the results for each output cell is then displayed graphically. @ RISK graphs include: i) relative frequency distributions and cumulative probability curves; ii) summary graphs for multiple distributions across cell ranges; iii) statistical reports on generated distributions; and iv) probability of occurrence of target values in a
distribution. @Risk results and graphs can be placed directly in the Microsoft Excel or Lotus 123 spreadsheet for reporting purposes.

The software also has modelling techniques to deal with dependencies. This is very important because in many practical engineering analyses, random variables are often statistically and physically dependent. Furthermore, actual distribution types for the random variables involved can be a mixture of different theoretical distributions. To properly replicate such systems, simulation should be able to preserve the correlation relationship among the stochastic parameters and their distributions (Iman & Conover, 1980, 1982; Chang et al, 1994). In @Risk, to allow for correlation, one can build a correlation matrix for the input variables. This matrix forms the basis for the correlated sampling of the input variables during simulation. The facility is especially useful when pre-existing correlation coefficients are available and one wants sampling to be governed by those coefficients.

The main advantage of this software is its flexibility and ease of use for anyone familiar with spreadsheets. However, because of its user-friendly characteristics, there are dangers in the use of @Risk (and similar computer programs) unless the user is fully aware of issues such as the importance of formulating the correct relationships between input and output variables, the selection of the probability distributions, and the choice of sample size as it affects the stability of estimates of the output variables, including their extreme values.

5.4 Summary

A FORTRAN Level II program, PARASODE, has been developed. In particular, the program concentrates on the failure modes of random wave overtopping of simple seawalls and dune erosion. The quantity of wave overtopping is calculated using both the H&R formula and Owen's formula. Dune erosion is calculated using Vellinga's model. However, much of the program is generic and can be adapted to other failure modes without undue difficulty. A Level III software package, @Risk, has been used to validate the output from PARASODE with regard to wave overtopping.
6 APPLICATION OF PARASODE

6.1 Examples - Wave Overtopping Of Seawalls

6.1.1 Input To PARASODE

Section 6.1 illustrates the use of PARASODE and the differences between the H&R overtopping model and Owen's formulation. Note that only some features of PARASODE are shown here. For example, due to lack of data, Ferry Borges & Castanheta's method of combinations of time-varying actions (see Section 2.3.3.3) is not illustrated, although PARASODE is fully developed to allow its use.

The geometry of the simple seawall used in the case studies (see Figure 3.1) is as follows:

- impermeable slope of 1:2 with a relatively smooth surface;
- toe level of 0m OD (OD denotes Ordnance Datum);
- crest level of between 8m OD and 16m OD.

Such a seawall is typical of potential developments around Liverpool Bay in the south-eastern corner of the Irish Sea. The allowable discharges considered in the examples lie in the range $10^{-1}$ to $10^6$ m$^3$/s/m (see Figure 3.2).

The main statistical characteristics of the basic variables in the H&R and Owen models (see Section 5.2.4) are described next. For each basic random variable, it is necessary to define a mean value, the corresponding standard deviation and to postulate a type of distribution. Depending on the distribution type, other statistical values may also have to be provided (e.g. the lower limit of a Weibull distributed variable or the lower and upper limits of a Beta distribution).

Examples of input files are provided in Appendix D1 as well as the corresponding summary.dat output files.
6.1.1.1 Distributions Of The Sea State Parameters

The scatter diagram describing the long-term distributions of wave heights and periods at the Mersey Bar in Liverpool Bay can be found in Salih (1989). Using the method of moments for one year's data (from September 1965 to September 1966) recorded at three-hourly intervals (Draper & Blakey, 1969), Salih fitted three-parameter Weibull distributions to the significant wave heights and mean zero-crossing wave periods. The following approximate statistical parameters are derived from his results:

<table>
<thead>
<tr>
<th>( H_s ) (m)</th>
<th>( T_m ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>1.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 6.1: Means, standard deviations and lower limits for the Weibull distributions of \( H_s \) and \( T_m \).

Since the H&R model uses \( T_p \) instead of \( T_m \) in its formulation, a mean JONSWAP relationship between \( T_m \) and \( T_p \) was assumed as follows (Hogben, 1990): \( T_p = 1.28 T_m \). Hence, a three-parameter Weibull distribution was adopted for \( T_p \) with \( \mu = 6.4s \); \( \sigma = 1.152s \) and lower limit = 4.224s.

Salih also reported a linear correlation coefficient between \( H_S \) and \( T_m \) of approximately \( \rho = 0.6 \). This same correlation has been assumed between \( H_S \) and \( T_p \).

6.1.1.2 Distributions Of Water Levels

In this study, two main situations have been evaluated, bearing in mind that the design conditions which are critical for one type of problem might not be critical for others (CIRIA/CUR, 1991; Havno et al, 1996):

1) **Performance under normal conditions** - The performance of the structure for any possible value of total water level (tide plus surge) is evaluated.
2) **Performance under extreme conditions** - The ability of the structure to survive extreme total water levels is checked.

The first condition is relevant, for example, to design of retention and drainage systems and for checking the safety of people and vehicles. The second provides peak values which are important for structural safety.

PARASODE allows the water level to be specified either as a variable in its own right or as the sum of the tide and surge components (see Chapter 5). All the information on water levels at Liverpool made available to the author was provided by the Proudman Oceanographic Laboratory (POL), UK. However, only data on total water levels, both for normal and extreme conditions, were initially available. These data were applied to specify user-defined distributions for Liverpool (see Appendix C4). Calculations with PARASODE were then performed using this information. Later, data on the separate tide and surge components for normal conditions were also acquired. With the new data, a user-defined distribution for the predicted tide levels at Liverpool was established (see Appendix C4), and a Gumbel distribution ($\mu=0.019m$; $\sigma=0.192m$) was fitted to represent the surge component by applying the software package BestFit. Calculations using PARASODE were then repeated using these distributions. As expected, the results obtained were identical. Hence, for normal conditions, only the results of the calculations which used the separate tide and surge components are reported in this thesis in Appendix D2. In this same appendix, the results for extreme conditions using total water levels are also tabulated. The tide and surge components have been assumed independent, based on information also provided by POL. Note that tide-surge interaction may be important in very shallow water regions (see Alcock & Carter, 1985).

### 6.1.1.3 Interrelationship Between Sea State And Water Level

Overtopping of a seawall does not depend solely on the individual sea state or the individual water level but on their combination (HR Wallingford, 1989; Thomas & Hall; 1992). Even a very severe storm, leading to massive waves, may pass virtually unnoticed if water levels are low. At such times, because of the generally shallow beach slope below mean water level, waves break rather harmlessly by spilling rather than plunging, and there is considerable loss of energy due to friction over the beach. These effects, combined with
the higher freeboard, mean that no overtopping or damage is likely to occur. In contrast, at very high water levels, even quite modest waves can cause problems. The increased water depth allows waves to break by plunging on the seawall, and also reduces the effect of friction. With the reduction in freeboard caused by the high water level, waves can overtop the structure. Hence, in design of coastal structures, combinations of large waves and high water levels are of particular interest (Simm et al, 1996). In most cases, correlation between large waves and high water levels should be considered. However, the scope of each correlation assessment should be decided on its own merits, in terms of the input data available, the intended end use, and the potential benefits to be derived (Hawkes & Hague, 1994). It is beyond the scope of the present study to carry out a detailed analysis of the correlation between waves and total water levels but the interested reader can refer to work such as Hague (1992) and Hawkes & Hague (1994).

The main problem when trying to account for the correlation between sea state and total water levels is the fact that it is necessary either to assign a correlation coefficient directly between the sea state and the total water level or, alternatively, between the sea state and surge and between surge and tide level (see Figure 4.8). Unfortunately, information necessary to accurately determine the correlation coefficients is often unavailable. Suppose some correlation between waves and total water levels is expected, but the correlation coefficient is not known. Confidence in accepting a particular seawall configuration can be reinforced by examining a pessimistic view of the suspected correlation. If the results are still acceptable, then confidence in the seawall has been justified. Conversely, if a seawall configuration appears unacceptable, one may be reassured that rejection of the design is justified by examining an optimistic view of the effect of the suspected correlation.

In the Liverpool Bay area of the Irish Sea, the tidal range is around 10m. This very big tidal range masks the correlation between waves and surge (Hawkes & Hague, 1994). As a consequence, sea states and total water levels are not completely independent, but the correlation is very weak. The question remained of what degree of correlation to consider in the present study. Hawkes & Hague (1994) suggested a positive though weak correlation between waves and water levels for North Wales whereas, according to Alcock (1984), Hydraulics Research Station assumed that SWL and wave
heights were independent for North Wales and for Fleetwood and Cleveleys. Since Liverpool Bay falls between these locations (Figure 6.1), a similar assumption might reasonably be made in the absence of further data.

In this study, no correlation has been assumed either between waves and total water level or between waves and surge. Note that PARASODE allows the user to consider correlation between any two variables by providing a non-zero correlation coefficient between them. Likewise, independence between two variables can be ensured by adopting a value of zero for the correlation coefficient.

![Figure 6.1: Location of Liverpool Bay in relation to North Wales, Fleetwood and Cleveleys.](image)

### 6.1.1.4 Distribution Of The Tangent Of The Seawall Slope

The angle at which the seawall front slope is constructed will never be exactly as specified in its design. Therefore, this parameter has been introduced as a random variable having a Normal distribution with a mean $\mu=0.5$ and a standard deviation of 10% of the mean value, i.e. $\sigma=0.05$. One would expect the angle of the seawall slope to be formed with the same
tolerance above and below the design angle. Hence in a narrow band about
the mean, the Normal distribution is expected to fit well. Obviously, the tailing
off of the distribution to infinity is not representative of the slope angle.
However, the behaviour of the tails is not important where there is a small
standard deviation associated with a high mean as is the case in this
example.

6.1.1.5 Distribution Of The Roughness Of The Seawall Slope

The slope roughness, \( r \), for different types of cover layer can be found, for
example, in CIRIA/CUR (1991) or Van der Meer & Janssen (1995). As for the
angle of the seawall slope, the roughness of the relatively smooth
impermeable slope has been considered as a random variable. A Beta
distribution has been chosen, with \( \mu = 0.95, \sigma = 0.01 \) and lower and upper limits
\( x_1 = 0.9 \) and \( x_2 = 1 \). The use of the Beta distribution addressed the fact that \( r \)
can never be greater than 1 and it has a specific range of values depending
on the type of cover layer. A Rectangular distribution might have been used
instead, particularly since it is simpler than the Beta distribution.

6.1.1.6 Distributions Of The Parameters Of The Models

In the example, the values of \( A \) and \( B \) for both the H&R and Owen models
have been set to fixed values according to Table 3.2 of Chapter 3:

- H&R model, \((R_{\text{max}})^{37\%}\): \( A = 0.00753 \) and \( B = 4.17 \)
- H&R model, \((R_{\text{max}})^{99\%}\): \( A = 0.00542 \) and \( B = 7.16 \)
- Owen's model: \( A = 0.0117 \) and \( B = 21.71 \)

Following the recommendations of Chapter 3 (Section 3.7), parameter \( e_B \) has
been considered as a Log-Normal distributed variable, with mean and
standard deviation as shown in Table 3.4 of that chapter:

- H&R model, \((R_{\text{max}})^{37\%}\): \( \mu = 1.049 \) and \( \sigma = 0.241 \)
- H&R model, \((R_{\text{max}})^{99\%}\): \( \mu = 1.044 \) and \( \sigma = 0.200 \)
- Owen's model: \( \mu = 1.027 \) and \( \sigma = 0.150 \)
Note that $e_B$ could equally have been chosen as Gumbel or Gamma distributed (see Section 3.7).

### 6.1.2 PARASODE Results And Discussion

#### 6.1.2.1 Normal Conditions

The results produced by PARASODE for wave overtopping of seawalls under normal conditions are presented both in tabular form in Appendix D2 and graphically in this section.

Figure 6.2 shows the probabilities of failure, $P_f$ (%/year), versus the crest level, CL (m OD), for different values of the allowable discharge, for the H&R model and for Owen's model.

Figures 6.3 and 6.4 display the sensitivity of the probability of failure to inaccuracies in the values of the H&R model basic variables at the design point, as a function of the allowable discharge and the seawall crest level, respectively. Figures 6.5 and 6.6 give the same results for Owen's model.

Figures 6.7 to 6.14 show parameter values at the design point as a function of the seawall crest level and the allowable discharge, for the H&R model and for Owen's model. Only the values of the variables which were found to have a major contribution to the probability of failure have been plotted, i.e. sea state parameters (wave height and period), tide and model parameter $e_B$.

Note that the results for the H&R model, $(R_{\text{max}})_{99\%}$, are tabulated in Appendix D2 but are not plotted here since the observations which could be made are essentially identical to those for $(R_{\text{max}})_{37\%}$.

From the tables in Appendix D2 and the figures mentioned above, the following observations may be made:

- **Figure 6.2** - As expected, $P_f$ (%/year) decreases as the crest level of the seawall increases. Likewise, for the same value of the crest level, $P_f$ decreases as the allowable discharge increases. For the
same values of the crest level and the allowable discharge, Owen's model predicts higher probabilities of failure than the H&R model. For the H&R model the probabilities of failure are higher for $(R_{\text{max}})_{99\%}$ than for $(R_{\text{max}})_{37\%}$. Note that these observations are consistent with the comments made in Section 3.6. Acceptable probabilities of failure for coastal structures are given in Section 4.5.1 as generally between $10^{-2}$ and $10^{-5}$ (i.e. 1 to 0.001%/year). Consequently, for the input conditions considered, a crest level greater than about 10m would be required to satisfy structural safety. A level of at least 12m would be required to satisfy functional safety according to the H&R model whilst Owen's model would demand a crest level greater than 16m.

As will be shown later in Section 6.2.1.2, the probability of failure over the lifetime of the structure may be determined quite simply from knowledge of the probability of failure in a year, provided that statistical independence of each year is assumed.

**Figures 6.3 and 6.4** - For the H&R model, the sensitivity parameters, $\alpha^2 (%)$, show that the main influence on the probability of failure is generally provided by the uncertainty in the sea state, i.e. $H_S$ and $T_p$ (up to 58%). The tide also has a major contribution (up to 47%), although it is never as large as the contribution of the sea state. For the biggest allowable discharges, the model parameter, $e_B$, occasionally plays the strongest role (up to 49%). The effect of the surge is much less important (up to about 10%), and the angle of the seawall front slope and its roughness have only minor influences on the resulting variance. For each value of the seawall crest level, the importance of the sea state and the tide tends to increase as the allowable discharge decreases, while the effect of $e_B$ decreases. For each allowable discharge considered, the contributions of the sea state and the tide decrease as the crest level increases, while the contribution of $e_B$ increases.

**Figures 6.5 and 6.6** - As with the H&R model, the sensitivity parameters for Owen's model show that the most important factor is generally the sea state, i.e. $H_S$ and $T_m$ (up to 66%). The tide also has a major contribution (up to 72%), and in some cases, it is even more important than the contribution of the sea state. Model parameter $e_B$ is, in some instances, more significant than the tide (up to 34%); as with the H&R model, the effect of $e_B$ is greatest for the largest allowable discharges and highest crest levels. For Owen's model, the sensitivity to variability in the surge is even less important (up to only 4%) than for the H&R model, and again the roughness of the seawall front slope makes only a minor contribution to the resulting variance. Unlike for the H&R model, the influence of the sea state does not show any obvious pattern with crest level or allowable discharge. However, as for the H&R model, the importance of the tide increases as the allowable discharge decreases, for each value of the seawall crest level; and for each allowable discharge, the tide's contribution decreases as the crest level increases.
- The above discussion of Figures 6.3 to 6.6 highlights how crucial it is to obtain reliable and sufficient data on wave, tide and $e_B$ to enable the probability distributions of these variables to be determined accurately.

- Figures 6.7 to 6.14 - The value of $T_p$ at the design point for the H&R model does not follow any special trend with the crest level or the allowable discharge. $T_m$ for Owen's model increases as the crest level and the allowable discharge increase. The values of $H_S$ and Tide at the design point increase with increasing values of the crest level and of the allowable discharge for both overtopping models. Also for the two models, the value of $e_B$ at the design point decreases as the crest level and the allowable discharge increase. The values of $H_S$ and Tide at the design point are higher for the H&R model than for Owen's model despite the generally higher probabilities of failure associated with Owen's model (see Figure 6.2).

- From all figures, it can be seen that the choice of overtopping model is very important in the probability assessment of the safety of seawalls exposed to normal conditions. The two models lead to quite different results. Use of Owen’s model in design would be more conservative than use of the H&R model. However, the conservative nature of Owen’s model also implies that its use in design will result in more expensive structures than those designed using the H&R formulation.

Note that the results for an individual case study cannot be adopted in a general sense. Each situation has particular characteristics which make it unique.
Figure 6.2: Probability of failure versus crest level for different values of the allowable discharge, for the H&R model and for Owen's model.
Figure 6.3: Sensitivity of the probability of failure to inaccuracies in the values of the H&R model basic variables at the design point as a function of the allowable discharge.
Figure 6.3: continued.
Figure 6.3: continued.
Figure 6.4: Sensitivity of the probability of failure to inaccuracies in the values of the H&R model basic variables at the design point as a function of the seawall crest level.
Figure 6.4: continued.
Figure 6.4: continued.
Figure 6.5: Sensitivity of the probability of failure to inaccuracies in the values of Owen's model basic variables at the design point as a function of the allowable discharge.
Figure 6.5: continued.
Figure 6.5: continued.
Figure 6.6: Sensitivity of the probability of failure to inaccuracies in the values of Owen's model basic variables at the design point as a function of the seawall crest level.
Figure 6.6: continued.
Figure 6.6: continued.
Figure 6.7: Value of $T_p$ at the design point as a function of the seawall crest level and the allowable discharge, for the H& R model.

Figure 6.8: Value of $H_S$ at the design point as a function of the seawall crest level and the allowable discharge, for the H& R model.
**Figure 6.9:** Value of Tide at the design point as a function of the seawall crest level and the allowable discharge, for the H&R model.

**Figure 6.10:** Value of $e_B$ at the design point as a function of the seawall crest level and the allowable discharge, for the H&R model.
Figure 6.11: Value of $T_m$ at the design point as a function of the seawall crest level and the allowable discharge, for Owen's model.

Figure 6.12: Value of $H_S$ at the design point as a function of the seawall crest level and the allowable discharge, for Owen's model.
**Figure 6.13:** Value of Tide at the design point as a function of the seawall crest level and the allowable discharge, for Owen's model.

**Figure 6.14:** Value of $e_B$ at the design point as a function of the seawall crest level and the allowable discharge, for Owen's model.
6.1.2.2 Extreme Conditions

The results produced by PARASODE for wave overtopping under extreme conditions are presented in tabular form in Appendix D2. These results could also have been presented graphically, as in Section 6.1.2.1. However, for simplicity, only the main conclusions are drawn here.

From the tables in Appendix D2, the following observations may be made:

- As anticipated, the probabilities of failure associated with extreme conditions are much higher than those for normal conditions. For the same values of the crest level and the allowable discharge, Owen's model still predicts higher probabilities of failure than the H&R model. For the H&R model the probabilities of failure are generally higher for $(R_{\text{max}})_{99\%}$ than for $(R_{\text{max}})_{37\%}$. Consequently, for the extreme conditions, a crest level greater than 14m would be required to satisfy structural safety according to the H&R model whilst Owen's model would demand a crest level greater than 16m. As for normal conditions, $P_f$ ($\%/\text{year}$) decreases as the crest level of the seawall increases. Likewise, for the same value of the crest level, $P_f$ decreases as the allowable discharge increases.

- For the H&R model, the sensitivity parameters, $\alpha^2$ (%), show again that the major influence on the probability of failure is generally provided by the uncertainty in the sea state (up to 99%). Unlike for normal conditions, the water level is much less important (up to 10%), whilst for the biggest allowable discharges, the model parameter, $e_B$, still occasionally plays a major role (up to 49%). The effect of the other variables is negligible. For each value of the seawall crest level, the importance of the sea state tends to increase as the allowable discharge decreases, while the effect of $e_B$ decreases. For each allowable discharge, the behaviour of the sea state and $e_B$ show no obvious relationship to the crest level.

- Once more, the sensitivity parameters for Owen's model show that the most important factor is the sea state (up to 95%). Model parameter $e_B$ represents, in some instances, the most significant contribution (up to 58%); the effect of $e_B$ is greatest for the smallest allowable discharges. The other variables make only minor contributions to the resulting variance. The influence of the sea state shows no obvious relationship to the crest level or allowable discharge.

- The behaviours at the design point of the values of the most important variables in relation to the crest level and allowable discharge are identical to the behaviours described for normal conditions.
As for normal conditions, it can be seen that the choice of overtopping model is very important in the probability assessment of the safety of seawalls subjected to extreme conditions. The two models lead to quite different results. Once again, use of Owen's model in design would be more conservative than use of the H&R model.

6.1.3 Validating PARASODE Results Using @Risk

The accuracy of the Level II (FORM) reliability algorithms used in PARASODE has been evaluated by comparison with the results provided by the Level III method of Latin Hypercube Sampling (LHS) available in @Risk. This evaluation has been carried out for the Level II results of the H&R model, \( (R_{\text{max}})^{37\%} \), and Owen's model, for normal conditions only. The results of the Level III method are used as a benchmark, since their only limitation is the computer time needed to perform a sufficiently large number of iterations (Jang et al, 1994).

In simulation, with more iterations, output distributions become increasingly stable as the statistics describing each input distribution change less with additional samples. It is important to run enough iterations so that the output statistics are reliable. However, there comes a point when the time spent on additional iterations is unnecessary because the output statistics are not significantly changed. The number of iterations required to generate stable output distributions varies depending on the model used in the simulation and the distribution functions in the model.

Like other research (Startzman & Wattenbarger, 1985; Super-Software, 1994), this work concentrates on the variability of the following statistics as measures of simulation convergence (Figure 6.15):

- mean;
- standard deviation;
- coefficient of variation.
Figure 6.15: Example of the convergence of the mean, standard deviation and coefficient of variation of Q using Latin Hypercube Sampling for the H&R model and for Owen's model.
Other statistics, such as the skewness and the kurtosis, have also been analysed. The results are not shown here because they behaved in similar ways to the above statistics.

Figure 6.15 shows that estimates of the mean, standard deviation and coefficient of variation are sufficiently accurate after about 30000 samples for Owen's model, while the H&R model does not approach reasonably converged statistics until about 60000 samples are used. Simulating more samples would not introduce a noticeable improvement on the calculations, and would only require more time and computer memory. Several other LHS simulations were performed using random seeds and similar results were obtained. Therefore, it was decided to carry out the simulations for the H&R model and Owen's model using 60000 and 30000 samples, respectively. In practice, Owen's model required only about 35% of the computer time required by the H&R model, for the input conditions considered. However, even 30000 samples is a considerable number, especially when LHS has been used instead of the Traditional Sampling method.

Many other statistical properties may be determined by simulation including confidence limits. Often, the entire cumulative distribution of the result is required. However, there is no reason to believe that the convergence of the above statistics would not be accompanied by the corresponding convergence of other properties (Startzman & Wattenbarger, 1985; Law & Kelton, 1991).

The convergence of the probability of failure for different allowable discharges has also been analysed. Figure 6.16 suggests that 60000 samples for the H&R model and 30000 for Owen's model are more than sufficient.

The distributions of the basic variables obtained during the simulation were checked against the input distributions. Agreement was excellent, as expected with the large number of samples involved in each simulation for the H&R model and for Owen's model.

Figures 6.17 and 6.18 show the probability of failure obtained using the Level II and Level III methods plotted against the crest level of the seawall for particular values of the allowable discharge, both for the H&R model,
(R_{max})^{37\%} and for Owen's model. The two methods give comparable results. However, as the crest level decreases, the FORM results diverge from the LHS results, generally overestimating the probability of failure. The differences in the probability of failure between FORM and LHS results suggest that the failure surfaces for both models are curved near the design point. In such cases, a Second Order Reliability Method (SORM) would be expected to account for the non-linearity of the failure surface and to remain accurate, giving results consistent with the LHS method, while still providing sensitivity factors and other details which do not depend on the magnitude of the probability of failure. Note that in the present work the FORM calculations took only between about 5\% and 20\% of the computer time required for the LHS computations.

![Figure 6.16: Example of the convergence of the probability of failure for different allowable discharges using Latin Hypercube Sampling for the H&R model and for Owen's model.](image-url)
Figure 6.17: Comparison of the probability of failure obtained using Level II (FORM) and Level III (LHS) methods for particular values of the allowable discharge, for the H&R model, \((R_{\text{max}})^{37\%}\).
Figure 6.17: continued.
Figure 6.18: Comparison of the probability of failure obtained using Level II (FORM) and Level III (LHS) methods for particular values of the allowable discharge, for Owen’s model.
Figure 6.18: continued.
6.2 Examples - Dune Erosion

6.2.1 General Cases Of Dune Erosion

6.2.1.1 Input To PARASODE

Table 6.2 summarises the dune erosion examples studied using PARASODE. In each example, only one parameter (e.g. the initial profile or the allowable erosion distance) has been changed as shown in the table. The remaining input data is common to all examples. The input data files used in example 1 are provided in Appendix D3 as well as the corresponding output file summary.dat.

<table>
<thead>
<tr>
<th>EXAMPLE</th>
<th>PROFILE</th>
<th>NOURISHMENT</th>
<th>ALLOWABLE EROSION DISTANCE, TR (m)</th>
<th>MODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
<td>No</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>B1</td>
<td>No</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>C1</td>
<td>No</td>
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<td>D1</td>
<td>No</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>A1</td>
<td>Yes nourw=75m nourtlev=6m 1:mnour=1:1.5</td>
<td>90</td>
<td>1</td>
</tr>
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<td>No</td>
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<td>1</td>
</tr>
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<tr>
<td>12</td>
<td>A1</td>
<td>No</td>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>A1</td>
<td>No</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>A1</td>
<td>Yes nourtlev=6m 1:mnour=1:1.5</td>
<td>90</td>
<td>2 P(%)=0.001011</td>
</tr>
</tbody>
</table>

Table 6.2: Dune erosion examples (note that movements of sand have been allowed only seaward in these examples).

Profiles A1, B1, C1 and D1 (Figure 6.19) have been used to illustrate the main erosion situations which can be studied using PARASODE (see Section 5.2.5).
Figure 6.19: Initial profiles used in dune erosion examples 1 to 14.
The adopted characteristics of the random variables are shown in Table 6.3 and are typical of Dutch conditions (see Chapter 4 and Appendix D3).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>Normal</td>
<td>0</td>
<td>0.6</td>
<td>------------</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>Normal</td>
<td>225E-6</td>
<td>225E-7</td>
<td>------------</td>
</tr>
<tr>
<td>DP</td>
<td>Normal</td>
<td>0</td>
<td>0.6</td>
<td>------------</td>
</tr>
<tr>
<td>SD</td>
<td>Normal</td>
<td>0</td>
<td>1</td>
<td>------------</td>
</tr>
<tr>
<td>GB</td>
<td>Normal</td>
<td>0.4</td>
<td>0.1</td>
<td>------------</td>
</tr>
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<td>Normal</td>
<td>0</td>
<td>1</td>
<td>------------</td>
</tr>
<tr>
<td>h</td>
<td>Weibull</td>
<td>2.52</td>
<td>0.33</td>
<td>2.19</td>
</tr>
</tbody>
</table>

Table 6.3: Characteristics of the random variables adopted for examples 1 to 14.

Correlation between $h$ and $H_S$ has been considered as follows (Van de Graaff, 1986):

$$
\mu_{H_S|h} = 4.82 + 0.6h - 0.0063(7 - h)^{3.13} \quad 3 \leq h < 7 \\
\mu_{H_S|h} = 4.82 + 0.6h \quad h > 7
$$

where $\mu_{H_S|h}$ is the mean value of $H_S$ given a value for $h$. Hence, $H_S$ has been modelled as the sum of $\mu_{H_S|h}$ and the variability of $H_S$ about its mean value (see Section 4.5.2). This variability has been considered as Normal distributed with a mean of zero and a standard deviation of 0.6m.

Note that PARASODE also allows the user to consider correlation between $h$ and $H_S$ by providing a non-zero correlation coefficient between the two variables. Unlike DUNEPROB and DUNE, independence between $h$ and $H_S$ can be ensured by adopting a value of zero for the correlation coefficient.
6.2.1.2 Results And Discussion

The results from PARASODE for dune erosion are shown in tabular form in Appendix D4. This section provides a graphical illustration of some of the results.

Examples 1 to 4 show how different initial profiles can affect the final results (see Tables D4.1 to D4.4). From the sensitivity parameters, $\alpha^2(\%)$, the most important contribution to the resulting variance is given by the maximum water level during surge, $h$ (Figure 6.20). Values for $\alpha^2(\%)$ of about 80% to 90% indicate that this variable is by far the most important one. The sediment size, $D_{50}$, and the accuracy of the computation, $Ac$, also make some significant contribution to the resulting variance. The contribution of the surge duration is less important and the significant wave height, $H_s$, the change in the initial profile, $DP$, and the gust bumps, $GB$, make only minor contributions. It might be expected that $H_s$ would play a strong role in the effects of erosion. Note, however, that due to the relationship between $h$ and $H_s$, the contribution of $H_s$ shown in the tables represents only the effect of the variability in $H_s$ about its expected value.

![Profiles A1 to D1](image-url)
**Figure 6.20:** Sensitivity of the probability of failure to inaccuracies in the values of the erosion basic variables at the design point (examples 1 to 4).

Example 5 illustrates how nourishment can be used to decrease the dune’s failure probability. The nourishment characteristics applied to profile A1 are: i) width at top level = 75m; ii) top level = 6m; and iii) gradient of the nourished face = 1:1.5 (Table 6.2). If nourishment is not provided (example 1), the probability of failure is approximately 0.06%/year (Table D4.1). This probability has been reduced to about 0.001%/year by nourishment. Example 14 represents the same conditions as example 5 but PARASODE has been run in mode 2; i.e. a target probability of failure of about 0.001%/year has been input and a corresponding nourishment width has been computed (74.87m). Examples 5 and 14 demonstrate the converse nature of modes 1 and 2 and show consistency between results obtained in running PARASODE in both modes (Tables D4.5 and D4.14).

Examples 1 and 6 to 13 illustrate how changes in the allowable erosion distance, TR, affect the results. Values of TR between 60m and 140m have been considered since they provide probabilities of failure between about $P_f=2.6\%$/year and $P_f=0.0006\%$/year, respectively. This range of probabilities covers all likely normal design cases, i.e. $10^{-2} \leq P_f/\text{year} \leq 10^{-5}$ (see Section 4.5.1). A plot has been produced of the probability of failure as a function of TR (Figure 6.21). For instance, the probability of failure per year associated with an allowable erosion distance of 100m is equal to about 0.02%/year. Hence, for Dutch conditions (see Chapter 4), if a $10^{-5}$/year probability of exceedance is needed, TR=100m is unacceptable. However, if one considers a higher chance of failure, e.g. $10^{-3}$/year, then TR=100m is acceptable. Note that a probability of failure associated with a value of TR represents a probability of exceedance of this TR value in one year.

Assuming statistical independence of each year, the probability of failure for a $T_{ref}$-year period can be obtained using (Van der Meer & Pilarczyk, 1987; Van der Meer, 1990; Van der Meer et al, 1994):

$$P(Z \leq 0; T_{ref}\text{ years}) = 1 - [1 - P(Z \leq 0; 1\text{ year})]^{T_{ref}}$$  \hspace{1cm} (6.2)

Results derived from Figure 6.21 using eq. (6.2) are shown in Figure 6.22. Curves are drawn for three lifetimes: 20, 50 and 100 years. From this figure it follows, for example, that the allowable erosion distance TR=100m will be...
exceeded with a probability $0.5 \leq P_f(\%) \leq 1$ during a lifetime of 20 years, whilst for lifetimes of 50 and 100 years, $1 \leq P_f(\%) \leq 5$.

**Figure 6.21:** Probability of failure in one year of profile A1 as a function of allowable erosion distance (examples 1 and 6 to 13).

**Figure 6.22:** Lifetime probability of failure of profile A1 as a function of allowable erosion distance.

Plots like Figures 6.21 and 6.22 are valuable tools in preliminary design.
Tables D4.1 to D4.14 show that, for all examples, by far the most important contribution to the resulting variance is given by uncertainty in the maximum water level during surge followed by much smaller contributions from Ac, D_{50} and SD. The remaining variables make only minor contributions to the resulting variance. The dominant contribution of the surge is in accordance with Dutch studies (Van de Graaff, 1986; Van de Graaff, 1995). This fact is not surprising since the adopted characteristics of the random variables are typical of Dutch conditions.

Note that between 6 and 20 iterations have been required to run each of the above examples. The number does not depend on the probability of failure.

Finally, it is important to appreciate that the results of a numerical study are specific to the set of parameters used.

6.2.2 Particular Case Of Dune Erosion: The Sefton Coast, UK

6.2.2.1 Introduction

In The Netherlands, the narrow stretch of sandy beaches and dunes (in some places, the dunes are less than 200m wide) has to be maintained in order to protect people and property from damage. On the British Sefton coast, with a dune frontage up to 2km wide, the same problem does not arise. However, in view of increasing concern about the possibility that dune erosion may spread or accelerate in response to sea level rise or an increase in storminess associated with greenhouse warming, studies are in progress to achieve a better understanding of beach-dune interaction in this area (Pye, 1991). As mentioned by Pye & Neal (1994), erosion poses a significant management problem for the authorities responsible for the coast. Sefton Metropolitan Borough Council has statutory obligations to defend property from erosion and flooding, but it is also interested in preserving the natural character of the coast in order to maximise the recreational and nature conservation benefits. Large areas of the Sefton dune system lie within designated National and Local Nature Reserves, or are owned by the National Trust, while other areas are owned by private landowners and are used as caravan parks, golf courses, or for residential purposes.
It was of interest to evaluate the Dutch erosion prediction methods for use on this section of the British coast. The Dutch methods allow the retreat distance associated with a storm surge to be determined. Subsequently, a convenient position for erecting sand-trapping fences along the backshore to encourage foredune accretion (Thomas & Hall, 1992; Simm et al, 1996) may be fixed in such a way that the predicted erosion distance does not reach the fence line. Obviously, the methods' inherent limitations and assumptions must be borne in mind when applying the models. Furthermore, the statistical characteristics of the basic variables of the problem and the associated failure criteria used for Dutch conditions have to be analysed. They should not, in any circumstances, be adopted blindly for UK use.

6.2.2.2 General Description Of The Sefton Coastline

Figure 6.23 shows the Sefton coast, situated on the edge of Liverpool Bay between the Ribble and Mersey estuaries in the south-eastern corner of the Irish Sea.

The Irish Sea is almost completely enclosed, with two relatively narrow passages to oceanic waters: the North Channel between Scotland and Northern Ireland and St. George's Channel between Wales and Eire. There is a slow overall drift of water from south to north through the Irish Sea. As far as the coast is concerned, tidal currents, wave action and local sea-bed drifts are of much greater importance, together with fresh-water inputs from the main rivers. The Sefton coast is shielded from direct oceanic waves by the mainly enclosed nature of the Irish Sea and the additional blocking effects of the Isles of Anglesey and Man (see Figure 6.1). However, perhaps once or twice a year, during locally calm conditions, long-crested waves, or swells, are apparent and these have most likely been generated in the Atlantic Ocean.

Due to its natural state (unconsolidated sands, silts and perhaps a little clay with some outcrops of peat), the coast has very little strength. It has been moulded by the prevailing environment over many years to an overall form approaching equilibrium. This equilibrium is easily upset by very slight changes in the environment, leading on the one hand to accretion or equally on the other to erosion.
The area is notable for its high tidal range, in excess of 10m at maximum springs. It is occasionally subject to large meteorological surge contributions to high-water which when combined with strong wave activity cause severe erosion to the dune coast and some structural damage to coastal defence works. Very damaging storm tides occur on average every 5 or 6 years.
6.2.2.3 The Sefton Coast Dune System

The Sefton coastal dune complex is the largest in the British Isles and is of major significance in a European context (Doody, 1989; Atkinson & Houston, 1993; MAFF, 1993a; Pye & Neal, 1994). On the Sefton coast, dunes up to 2km wide are important not only for nature conservation and recreation, but also in terms of flood defence since they act as a natural barrier which prevents tidal inundation of a large area of West Lancashire and North Merseyside.

Following a period of rapid accretion in the second half of the last century, the dune frontage at Formby Point has been eroding since about 1906 at an average rate of up to 3m/year (Figure 6.24). The southern limit of erosion has remained roughly stable in the area of Lifeboat Road, but the northern erosion limit has gradually extended northwards. The rate of recovery of the frontal dunes following a storm has generally been insufficient to prevent a long-term net erosional trend between Lifeboat Road and Fisherman's Path. However, further north, between Ainsdale and Southport, and on the south side of Formby Point between Alexandra Road and the mouth of the River Alt, erosion during storms is generally less severe and the rate of dune recovery has been sufficiently rapid to maintain a net accretion throughout this century (Pye, 1991).

Foredune erosion at Formby Point was accelerated between 1900 and the mid 1970s by the abandonment of dune and foreshore management which had been extensively practised during the late 19th century, and by a significant increase in recreational pressure, sand mining and military activities (Pye & Neal, 1994). Since the establishment of a Coastal Management Scheme in 1977, damage to the dunes from these causes has been greatly reduced, but the dune protection and restoration works employed have had little effect on the problem of beach and foredune erosion by waves, especially during storm surges.

According to Pye & Neal (1994), possible factors contributing to the change from accretion to erosion around 1906 include:

- effects of dredging and training wall construction on sediment transport and wave regime;
• reduction or exhaustion of sand supply;
• effects of changes in bathymetry on wave climate;
• change in wind/wave climate;
• abandonment of beach and foredune management practices.

Figure 6.24: Plans showing the growth of Formby Point 1845-1906 and subsequent erosion in the period 1906-1990 (after Pye, 1991).
6.2.2.4 Storm Surge Of February 1990

In 1990, a major storm surge struck the Sefton coast. The storm surge was associated with the passage of a vigorous depression across northern Scotland and the North Sea during the period of 25-27 February. According to Pye (1991), on 26 February the mean hourly wind speed at Squiresgate Airport, Blackpool, increased from 33km/h to 73km/h, while the direction shifted from south-westerly to westerly. The strong onshore winds produced a surge which raised the height of predicted high water at 12.00hrs by approximately 1m along the coast between Morecombe Bay and North Wales. Strong westerly winds continued during the period 27-28 February, although a slight reduction in wind velocity and a reduction in wave height meant that the very severe conditions experienced on the morning of the 26 February were not repeated.

Structural damage was caused along the promenade at Southport and Crosby, and coastal defences were breached at Towyn in North Wales, causing flooding to several thousand homes. The storm surge and wave action also eroded large sections of the natural dune belt between Hightown and Southport (Pye, 1991). The greatest erosion was at Wick's Lane and Victoria Road (Table 6.4).

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>EROSION DISTANCE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albert Road</td>
<td>6.0</td>
</tr>
<tr>
<td>Lifeboat Road</td>
<td>8.3</td>
</tr>
<tr>
<td>Wick's Lane</td>
<td>11.1</td>
</tr>
<tr>
<td>Victoria Road</td>
<td>13.6</td>
</tr>
<tr>
<td>Fisherman's Path</td>
<td>7.5</td>
</tr>
<tr>
<td>Ainsdale-Southport</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table 6.4: Erosion between Hightown and Southport due to the 1990 storm surge (modified after Pye, 1991).

At Massam's Slack, where previous erosion had truncated the ends of dune ridges created artificially during the 1920s, waves overtopped the frontal ridge and flooded the slack behind.
6.2.2.5 Input To PARASODE

Due to the lack of data on the basic variables of dune erosion, PARASODE has been applied in a deterministic fashion to evaluate the erosion expected at Wick’s Lane and Victoria Road due to the storm surge of February 1990. The data collected and used for calculations are presented in the following sections.

Selection Of Initial Profiles

Pye & Neal (1994) present a number of beach profiles, between Hightown and Southport, surveyed by the Sefton Borough Council Engineer and Surveyor's Department in August 1979. They also provide some information on dune heights. More detailed foreshore cross-sections for 1981 were obtained from the Metropolitan Borough of Sefton, Department of Technical Services. Unfortunately, no surveys were made shortly before or after the storm surge of 1990. The 1981 cross-sections are the latest available for the site and the ones used here for dune profile definition. Accretion and erosion records for the sand dune front at Formby Point are the only existing sources of information about the erosion caused by the storm surge (see Table 6.4). This information was also obtained from the Metropolitan Borough of Sefton.

The Metropolitan Borough of Sefton also provided maps from an aerial survey of July 1982, undertaken by Meridian Airmaps Limited. These maps provided information on the slopes and crest elevations of the foredunes. A considerable alongshore variation in the dune characteristics is evident in these maps. This variability caused difficulties in fixing a representative slope and crest elevation for the initial dune cross-sections.

A decision was finally made to study only the Wick’s Lane and Victoria Road profiles. This selection was based, mainly, upon two factors:

- Vellinga’s model, can be applied only to parts of the coast which are not strongly curved in plan (see Section 4.2.5).
- These two locations experienced the maximum erosion during the 1990 storm surge (see Table 6.4).

The two profiles are shown in Figure 6.25.
Sand Grain Size

In the light of measurements reported by Pye (1991), the value adopted for the median sand diameter was $D_{50} = 215 \, \mu m$.

As stated earlier, the amount of dune erosion depends on the particle diameter of the dune material via the fall velocity, $w$, which is calculated for a specific salt water temperature. In the case of the Sefton coast, the fall velocity for the entire period during which storm surges can be expected is calculated for a salt water temperature of 5° Celsius using eq. (4.7).

Sea State

A significant wave height $H_s=5.85m$ and an associated mean wave period, $T_m=7.55s$, were employed. These data were provided by Dr. Xiaoming Wu of
the Proudman Oceanographic Laboratory and are based on the computer model WAM. While these assumed wave conditions may not be exactly what occurred, they are considered a reasonable estimate.

**Total Water Level**

The maximum water level during surge is the sum of the tide level and surge. The mean of the values at Liverpool, 6.3m OD, and at Heysham and Southport, 6.4m OD (according to Pye, 1991) has been adopted: \( h = 6.35 \text{ m OD} \).

**Interrelationship Between Sea State And Total Water Level**

The occurrence of waves is likely to be at least partially correlated with the SWL, since both waves and storm surges are generated by meteorological conditions (Alcock, 1984), as shown in Figure 4.8. On the Sefton coast there is strong correlation between surges and waves. However, if the tidal range is very big, as it is on the Sefton coast (around 10m), the tide masks the correlation between waves and surge (Hawkes & Hague, 1994). Complete independence between sea states and total water levels is not expected, but the anticipated correlation is very weak. As part of Liverpool Bay (see Section 6.1), the Sefton coast falls between North Wales and Fleetwood and Cleveleys (see Figure 6.1). In the absence of data and following the comments in Section 6.1, it was assumed in this study that SWL and wave heights were independent. Note that PARASODE allows the user to consider correlation between \( h \) and \( H_S \) by providing a non-zero correlation coefficient between the two variables. Likewise, independence between \( h \) and \( H_S \) can be ensured by adopting a value of zero for the correlation coefficient.

**6.2.2.6 Results And Discussion**

According to Table 6.5, there is no difference in the computed erosion distance at Wick’s Lane whether movements of sand are allowed only seaward or in both directions. In contrast, at Victoria Road the difference
amounts to about 22% of the value when movements are allowed seaward only.

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>EROSION DISTANCE FROM PARASODE (m)</th>
<th>MEASURED EROSION DISTANCE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Movements Of Sand Only Seaward</td>
<td>Movements Of Sand In Both Directions</td>
</tr>
<tr>
<td>Wick’s Lane</td>
<td>12.7</td>
<td>12.7</td>
</tr>
<tr>
<td>Victoria Road</td>
<td>6.8</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Table 6.5: Computed and measured erosion distances at Wick’s Lane and Victoria Road.

In the case of Wick’s Lane, agreement between computed and measured erosion distances is very satisfactory. Agreement is much less satisfactory for Victoria Road. Two main reasons can be offered in order to explain the differences:

- The initial profiles used were unreliable because they had been taken nearly ten years before the storm surge of February 1990. Unfortunately, they were the latest profiles available.
- The measured erosion distance (the difference between the locations of the dune toe between measurements) did not relate solely to the surge event; the initial location of the dune toe had been measured eleven days before the surge and the location after the surge was taken three days later. Consequently, it would be expected that measurements would exceed computed values. This is the case for Victoria Road.

Given the potential errors, Vellinga’s model can be regarded as having performed satisfactorily.

Finally, note that if the required data on the Sefton coast were available, then PARASODE could be run to produce probabilistic results for dune erosion due to the storm surge of February 1990. Unlike DUNE and DUNEPROB, PARASODE allows any degree of correlation between variables to be considered.
6.3 Summary

Chapter 6 illustrates the use of some features of PARASODE.

Firstly, the program has been used in a study of wave overtopping of a seawall to show the differences between the results of the H&R overtopping model and Owen's formulation. In this study, two main situations have been evaluated: i) the performance of the seawall for any possible value of total water level (tide plus surge); and ii) the ability of the structure to survive extreme total water levels. The first condition is relevant to design of retention and drainage systems, and for checking the safety of people and vehicles. The second provides peak values which are important for structural safety. Probabilities of failure per year versus the seawall crest level for different allowable discharges have been calculated. Sensitivity parameters have also been analysed, and the value of the design point has been examined as a function of the seawall crest level and the allowable discharge. Two main points are worth noting:

- The choice of overtopping model is very important in the probability assessment of the safety of seawalls. The two models lead to quite different results. Use of Owen's model in design would be more conservative than use of the H&R model. However, the conservative nature of Owen's model also implies that its use in design will result in more expensive structures than those designed using the H&R formulation.
- For both overtopping models, the sensitivity parameters show that the main influence on the variability of the probability of failure is generally provided by the uncertainty in the sea state.

Secondly, the accuracy of the Level II (FORM) reliability algorithms used in PARASODE has been evaluated by comparison with the results provided by the Level III method of Latin Hypercube Sampling (LHS) available in @Risk. This evaluation has been carried out for the Level II results of the H&R model, \( R_{\text{max}} \approx 37\% \), and Owen's model, for normal design conditions only. The two methods give comparable results. However, as the seawall crest level decreases, the FORM results diverge from the LHS results, generally overestimating the probability of failure. The differences in the probabilities of failure suggest that the failure surfaces for both overtopping models are curved near the design point.
Thirdly, PARASODE has been used to study general examples of dune erosion during a storm surge. In these examples, movements of sand have been allowed only seaward. Four profiles have been considered for illustration of the main erosion situations which can be studied using PARASODE. Some examples illustrate how nourishment can be used to decrease the dune’s failure probability caused by erosion due to a storm surge. The adopted characteristics of the random variables are typical of Dutch conditions. PARASODE has been run both in mode 1 (a nourishment width has been chosen and the corresponding probability of failure calculated) and in mode 2 (the computed probability of failure for mode 1 has been input and a corresponding nourishment width has been computed). These tests demonstrate the converse nature of modes 1 and 2 and show consistency between the results obtained.

Fourthly, an attempt has been made to apply PARASODE in a study of dune erosion on the Sefton coast, UK, due to the storm surge of February 1990. The study allowed the retreat distance associated with this storm surge to be estimated. However, due to lack of data, PARASODE could be applied only in a deterministic fashion to determine the erosion expected at Wick’s Lane and Victoria Road. These two sections of the Sefton coast were selected because Vellinga’s model can be applied only to parts of the coast which are not strongly curved in plan, and because these two locations experienced the maximum erosion during the 1990 storm surge. PARASODE results suggest that there is no difference in the computed erosion distance at Wick’s Lane whether movements of sand are allowed only seaward or in both directions. In contrast, at Victoria Road the difference amounts to about 22% of the value when movements are allowed seaward only. In the case of Wick’s Lane, agreement between computed and measured erosion distances is very satisfactory. Agreement is much less satisfactory for Victoria Road. Two main reasons can be offered in order to explain the differences: i) the initial profiles used were unreliable because they had been taken nearly ten years before the storm surge of February 1990; and ii) the measured erosion distance did not relate solely to the surge event. Given the potential for errors, Vellinga’s model can be regarded as having performed satisfactorily.
7 CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The main objective of the present research was to assess the safety of coastal structures by means of probabilistic methods, with particular reference to wave overtopping of seawalls and to dune erosion. This chapter lists the principal conclusions of the research and, where appropriate, provides recommendations for further work.

**Probabilistic methods**

- Probabilistic methods provide a powerful framework for the design of coastal structures, accounting for the probability and consequences of failure as well as coping, to some degree, with variability and uncertainty. However, when assessing structural safety using probabilistic methods, it must be stressed that the process involves detailed knowledge about the individual structure. Therefore, confidence in the calculated value of the probability of failure must change with the amount and quality of the information used for its calculation. With these facts in mind, probabilistic methods may be seen simply as a design tool based on scientific methods which can facilitate good engineering decisions, but not a process which will necessarily provide a precise assessment of safety.

- Probabilistic approaches are increasingly being applied in engineering practice. This fact is apparent in civil engineering from the use of Level II calculations for determining the partial safety factors applied in standards and codes for the design of structures. Direct probabilistic approaches have increasingly become the rule in connection with the assessment of special structures (e.g. nuclear power stations and storage tanks for hazardous substances).

- In recent years, much has been learned by coastal engineers about probabilistic methods, but progress in formulating methods and gaining confidence in new design procedures is inevitably slow. At present, there is insufficient knowledge about coastal structures to enable a probability
Conclusions And Recommendations For Further Research

analysis of failure mode systems to be carried out in full. However, instead of abandoning this “new” approach to design, efforts should be made to better identify the specific physical processes with which coastal engineers must deal, to better communicate their data requirements to researchers, to subsequently collect the required data sets, and to establish appropriate models for the complete implementation of the methods. Furthermore, it is important to incorporate as much experience as possible from failures.

- Diagrams like event trees, fault trees and cause-consequence charts have been presented for some coastal structures. However, such techniques have still almost always served essentially as schematic representations or research tools rather than as strict logical analyses of failure. Information on failures tends to concentrate on the consequences rather than on the causes of failure.

- Assessment of the safety of coastal structures depends fundamentally on assessment of individual failure modes. All single failure mode probabilistic methods have their advantages and disadvantages:

  - In the numerical integration method, the calculation of an N-fold integral may be extremely time-consuming and it usually requires a considerable computational effort, even with modern computer facilities. Traditional sampling is an acceptable alternative when dealing with simple failure functions and failure probabilities which are not very low. However, it suffers from the fact that if an “accurate” answer is desired for extreme conditions associated with relatively low probabilities of failure, many simulations are required. This is a drawback that recent methods, like Latin Hypercube sampling, may address to some extent by reducing the required number of simulations. In other cases, difficulties can be overcome by using Level II methods like FORM.

  - The main practical advantages of the FORM approach are that it is less time-consuming than Level III methods, the computational effort is independent of the probability level, it provides a rational basis for evaluating partial safety factors and it also provides an automatic procedure for determining the sensitivity of the computed failure probability to each of the basic design variables. This latter characteristic allows the designer to focus his attention on the parameters which are of greatest significance and shows where effort to reduce uncertainty should be concentrated. Due to their simplicity, these methods have become very popular, particularly in calibration work for codes of practice. However, these procedures also have their limitations. Amongst others, the main reason for
discrepancy between a Level II and a Level III method is that the failure function is usually non-linear. The stronger the non-linearity, the greater is the chance that the Level II results will differ considerably from the "exact" answer. However, the FORM results can be improved through a second or higher order approximation, but computational complications are increased considerably. It is more common to use the Level III methods, especially simulation, to validate the Level II results. Although a FORM method can provide an answer to a problem, it is never known how accurate the answer is unless a check is done using numerical integration or simulation techniques. Nevertheless, the FORM method is one of the most important tools in probabilistic design because one can rarely afford to make a million Level III calculations during preliminary design.

- Besides the calculations at Level III and Level II, there are those at Level I. Level I calculations are particularly suitable for everyday design (where a large body of previous experience of similar systems is available), although the determination of the partial coefficients must be based upon higher level results. Level I calculations are the basis of codes of practice.

- It is important to be aware of the characteristics of the various probabilistic methods, their applicability and their limitations, otherwise wrong conclusions can be drawn, incorrect decisions can be made and unsound action may be taken. If probabilistic methods are used with foresight and understanding, they are powerful and can provide reliable results. For example, comparison of design alternatives using these methods is a promising way in which to apply them.

**Wave overtopping of seawalls**

- Seawalls are expensive, and fixing a seawall freeboard at too large a value has both a financial penalty and is unnecessarily damaging to the natural environment owing to the increased impact of the structure on its surroundings. On the other hand, if the crest of a seawall is set too low, then there are problems with structural safety and potential social problems with flooding and with people’s protection.

- Wave overtopping of seawalls has been the subject of many studies. Nevertheless, field measurements are scarce and numerical modelling of wave overtopping is not yet well developed. The calculation of overtopping discharge is based mainly on equations which have been
obtained from empirical fitting to hydraulic model test results. These equations have not been based upon any overtopping theory and no account has generally been taken of the physical boundary conditions.

- As part of this research, a new regression model (the H&R model) has been presented for describing wave overtopping data. Part of the motivation in deriving this new model was to improve the methods available to the designers of seawalls by developing a model closely related to the physics of wave overtopping. The main feature of the model is the fact that it satisfies the relevant physical boundary conditions, a feature which is especially important when the model is used near these boundaries.

- The H&R and Owen models have been used in a re-analysis of Owen’s data for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, subjected to random waves approaching normal to the slope. Both models represent part of the input to a FORTRAN Level II program, developed as part of this research, PARASODE. It is suggested that the regression coefficients contained within the models should be established using a robust regression technique such as the Least Absolute Deviations (LAD) method. The LAD regression coefficients are recommended for use both in the H&R and Owen models.

- For Owen’s test results, the H&R model is little different from Owen’s model in its ability to represent the data, except for small discharges for which the H&R model is better suited. An example of the application of the two models in predicting the freeboards necessary to limit overtopping to specified values shows that, for the small allowable discharges associated with normal design conditions, the H&R model predicts seawall crest elevations which may be several metres lower than values from Owen’s model. Such differences may have very significant financial and environmental consequences and are worthy of further investigation.

- Whilst it is possible to use Owen’s data to show the validity of the approach adopted in developing the new wave overtopping model, the data are far from ideal. Consequently, it is recommended that the present study on overtopping is extended:
Conclusions And Recommendations For Further Research

- to encompass both the very small allowable discharges associated with normal design conditions and to permit proper evaluation of the empirical coefficients in the equations used to describe overtopping;
- to allow evaluation both of the probability distributions of the parameters involved in overtopping and of the horizontal distribution of the total overtopping volume;
- to collect data on the effects of wind on wave overtopping with the objective of permitting further development of overtopping equations.

- It is believed that the environmental, social and economic benefits likely to derive from implementing these recommendations would provide a very valuable return for the investment of the time, effort and costs involved, particularly for those countries with an exposed coast such as the United Kingdom, The Netherlands and Portugal. An important socio-economic justification for the work is the possibility of including information on the variability of overtopping volumes and their horizontal distribution not only in detailed design but also in political decision-making relating to urban planning issues such as the location of housing and basic infrastructure.

Dune erosion during a surge

- Dutch experience with regard to the probabilistic design of dunes has been examined. The computational model currently used throughout The Netherlands is based on Vellinga's equilibrium profile model. The more sophisticated time-dependent model developed by Steetzel is not yet used as the basis for probabilistic calculations.

- The Dutch programs are not directly applicable to conditions along coasts such as that in Sefton, UK, where there is a much weaker correlation than in The Netherlands between wave heights and water levels. Consequently, it was decided to introduce Vellinga's model and some features of the Dutch programs into PARASODE, and to carry out new probabilistic calculations.
PARASODE

- PARASODE concentrates on the failure modes of random wave overtopping of simple seawalls and dune erosion. The quantity of wave overtopping is calculated using both the H&R formula and Owen's formula. Dune erosion is calculated using Vellinga's model. However, much of the program is generic and can be adapted to other failure modes without undue difficulty. PARASODE has been applied to several different examples in order to illustrate the use of some of its features.

- The program has been used in a study of wave overtopping of a seawall to show the differences between the results of the H&R overtopping model and Owen's formulation. Two main points are worth noting:
  
  - The choice of overtopping model is very important in the probability assessment of the safety of seawalls. The two models lead to quite different results. Use of Owen's model in design would be more conservative than use of the H&R model. However, the conservative nature of Owen's model also implies that its use in design would result in more expensive structures than those designed using the H&R formulation.
  
  - For both overtopping models, the sensitivity parameters show that the main influence on the variability of the probability of failure is generally provided by the uncertainty in the sea state.

- The accuracy of the Level II (FORM) reliability algorithms used in PARASODE has been evaluated by comparison with the results provided by the Level III method of Latin Hypercube Sampling (LHS) available in the commercial software package @Risk. The two methods give comparable results. However, as the seawall crest level decreases, the FORM results diverge from the LHS results, generally overestimating the probability of failure. The differences in the probabilities of failure suggest that the failure surfaces for both overtopping models are curved near the design point.

- PARASODE has been used to study general examples of dune erosion during a storm surge. In these examples, movements of sand have been allowed only seaward. Four profiles have been considered for illustration of the main erosion situations which can be studied using PARASODE. Some examples illustrate how nourishment can be used to decrease the
Conclusions And Recommendations For Further Research

dune's failure probability caused by erosion during a storm surge. The adopted characteristics of the random variables are typical of Dutch conditions. PARASODE has been run both in mode 1 (a nourishment width has been chosen and the corresponding probability of failure calculated) and in mode 2 (a probability of failure has been input and a corresponding nourishment width has been computed). These tests demonstrate the converse nature of modes 1 and 2 and show consistency between the results obtained.

- An attempt has been made to apply PARASODE in a study of dune erosion on the Sefton coast, UK, due to the storm surge of February 1990. The study allowed the retreat distance associated with this storm surge to be estimated. However, due to lack of data, PARASODE could be applied only in a deterministic fashion.

The above conclusions demonstrate that the principal objective of the research has been accomplished. Arrangements are already in hand to extend the work described here through international collaboration involving the University of Liverpool, the National Laboratory of Civil Engineering (LNEC), Portugal, and other partners.
REFERENCES


References


References


References


APPENDIX A

Analysis Of Overtopping Data
APPENDIX A1 - Regression Theory


The linear model with one independent variable, X, can be written as follows (Gunst & Mason, 1980):

\[ Y = \beta_0 + \beta_1 X + \varepsilon \]  

(A1.1)

where \( Y \) is the dependent variable, and \( \beta_0 \) and \( \beta_1 \) are unknown parameters of the model. Sometimes \( X \) is called the predictor or regressor variable, \( Y \) the predicted or response variable, and \( \beta_0 \) and \( \beta_1 \) the intercept and the slope, respectively. \( \beta_0 + \beta_1 X \) represents the systematic component of the variability of \( Y \), and \( \varepsilon \) is a random error term which takes into account the fact that the model does not exactly describe the behaviour of the response. The linear model can be expressed as follows:

\[ Y = b_0 + b_1 X + e \]  

(A1.2)

where, based on a sample of \( N \) observed \((X,Y)\) values, \( b_0 \) and \( b_1 \) are estimates of parameters \( \beta_0 \) and \( \beta_1 \). These estimates are used to construct the fitted model (also called the predictive or prediction equation, or the linear regression line for \( Y \) on \( X \)) as follows:

\[ \hat{Y} = b_0 + b_1 X \]  

(A1.3)

where \( \hat{Y} \) is the predictive value of \( Y \). From eqs. (A1.2) and (A1.3), the residual from the regression is defined by \( e = Y - \hat{Y} \) (see Figure A1.1).
A1.1 Least-Squares (LS) Method

A1.1.1 Basic LS Principle
There are many criteria for determining the regression line which best fits the observed data, i.e. for determining estimates \( b_0 \) and \( b_1 \) (Draper & Smith, 1981; Dodge, 1987). The most common criterion, the basis of the least-squares method, minimises the sum of the squares of the residuals. \( b_0 \) and \( b_1 \) are then calculated as follows (Gunst & Mason, 1980):

\[
b_1 = \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{N} (X_i - \bar{X})^2} \quad \quad b_0 = \bar{Y} - b_1 \bar{X} \quad \quad \text{(A1.4)}
\]

where

\[
\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \quad \text{and} \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i \quad \quad \text{(A1.5)}
\]

are the mean values of X and Y, respectively. \( b_0 \) and \( b_1 \) are called the LS estimates. The slope estimate, \( b_1 \), represents the estimated change in Y
associated with a unit change in $X$. The intercept, $b_0$, estimates the value of $Y$ when $X$ equals zero. Substituting $b_0$ into eq. (A1.3):

$$\hat{Y} = \bar{Y} + b_1 (X - \bar{X})$$  \hspace{1cm} (A1.6)

Note that when $X = \bar{X}$, $\hat{Y} = \bar{Y}$ so that $(\bar{X}, \bar{Y})$ lies on the fitted line.

In determining the LS estimates, $b_0$ and $b_1$, the following assumptions have to be made (Lewis-Beck, 1980):

1) No specification error:
   i. the relationship between $X$ and $Y$ is linear
   ii. no relevant independent variables have been excluded
   iii. no irrelevant independent variables have been included

2) No measurement error, i.e. $X$ and $Y$ are accurately measured.

3) For each observation, the expected value of the error term is zero ($E[\varepsilon_i] = 0$).

4) The variance of the error term is constant for all values of $X$ ($E[\varepsilon_i^2] = \sigma^2$, known as homoscedasticity).

5) The error terms for different observations are uncorrelated ($E[\varepsilon_i \varepsilon_j] = 0$ for $i \neq j$).

6) The independent variable is uncorrelated with the error term ($E[\varepsilon_i X_i] = 0$).

Assumption 1 asserts that the theoretical model embodied in the equation is correct, or almost so, over the range of observed values; that is, the functional form of the relationship is actually a straight line and no variables have been improperly included or excluded. The need for assumption 2 is self-evident: if the measurements are inaccurate, then the estimates could be inaccurate. If only the dependent variable is measured with error, then the LS estimates may remain unbiased\(^1\), provided the error is random (Lewis-Beck, 1980). If the independent variable is measured with any error, then the LS estimates will be biased. However, if the variance of the error is small compared to the variance of the true $X$ values, this error is usually ignored (Draper & Smith, 1981) and this policy is adopted in this study. Assumption 3 is not restrictive for regression models containing an intercept,

\(^1\) An unbiased estimator correctly estimates the population parameter on average, i.e. $E[b] = \beta$ (Beaumont, 1986).
because if the intercept term is included in the equation, the mean of the residuals is always zero so that the mean of the residuals provides no information about the mean of the errors (Gunst & Mason, 1980; SPSS, 1993). If this assumption is not met and the model includes the intercept, the intercept is biased but the slope estimate is unchanged. If assumption 4 is violated, the LS estimates remain unbiased, but they do not have minimum variance and the significance tests and confidence intervals, usually given in statistical packages, are invalid (Lewis-Beck, 1980). Assumption 5 requires that there is no correlation between errors. If correlation is present, the LS estimates are still unbiased but the significance tests and confidence intervals are invalid (Lewis-Beck, 1980). However, if the number of observations is large when compared to the number of independent variables, the dependency can be ignored for practical purposes (SPSS, 1993) and this is the case for Owen's data set. Note that since the sum of the residuals is constrained to be zero, the residuals cannot be strictly independent. If assumption 6 does not hold, the LS estimates are biased.

In general terms, when assumptions 1 to 6 are met, desirable estimates of the population parameters $\beta_0$ and $\beta_1$ are obtained, i.e. they are unbiased and, of all the estimates that are unbiased, they have minimum variance (Lewis-Beck, 1980; Everitt & Dunn, 1991). Furthermore, if the error term, $\varepsilon_i$, is normally distributed, or almost so, the estimates are maximum likelihood estimates. In this case, significance tests carried out in order to determine how likely it is that the population parameter values differ from zero, are reliable. Confidence intervals can also be relied upon. If the error term is not normally distributed, significance tests and confidence intervals should not be interpreted in the usual fashion (Lewis-Beck, 1980; SPSS, 1993). Rarely are assumptions not violated in one way or another in regression analyses (SPSS, 1993). However, ignoring the assumptions can lead to results which are improperly used and interpreted. One should not rush into using statistics lightly without proper understanding and without verifying the underlying assumptions (Carvalho, 1982).

Knowing the estimates of the parameters, $Y$ can be predicted for a given $X$ value. It is important to know how good is a prediction of $Y$ provided by the prediction equation, i.e. how well does the equation account for variations in the dependent variable. The total observed variability in the dependent variable from its mean value, $\overline{Y}$ (Figure A1.2), can be divided conveniently
into two components (Lewis-Beck, 1980; Freund & Littell, 1991; SPSS, 1993):

i) \( \hat{Y}_i - \bar{Y} \), the variability accounted for by the regression line;

ii) \( Y_i - \hat{Y}_i \), the variability unexplained by regression.

\[ \begin{align*}
\text{Total Variability in } Y & = \sum_{i=1}^{N} (Y_i - \bar{Y})^2 \\
\text{Residual Variability} & = \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \\
\text{Regression Variability} & = \sum_{i=1}^{N} (\hat{Y}_i - \bar{Y})^2 \\
\end{align*} \]  

\[ (A1.7) \]

Figure A1.2: Components of the variability in the dependent variable, \( Y \), for the LS method (modified after SPSS, 1993).

It can be shown (Gunst & Mason, 1980; Draper & Smith, 1981) that:

\[ \sum_{i=1}^{N} (Y_i - \bar{Y})^2 = \sum_{i=1}^{N} (\hat{Y}_i - \bar{Y})^2 + \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \]  

(A1.7)

The quantity before the equals sign is the total sum of squares, TSS, and it is invariant for a given set of data \((X_i, Y_i)\). The first quantity following the equals sign is the regression sum of squares, SS\(_{\text{Reg}}\), and the second quantity is the residual sum of squares, SS\(_{\text{Res}}\). It is recalled here that the LS method guarantees that SS\(_{\text{Res}}\) is at its minimum. Clearly, SS\(_{\text{Reg}}\) should be large relative to SS\(_{\text{Res}}\), since then the regression line is explaining the majority of the variability in \( Y \) about its mean. The sum of squares has associated with it the degrees of freedom, DF. DF indicates how many independent pieces of information involving the \( N \) independent numbers, \( Y_1, \ldots, Y_N \), are needed to
compile the sum of squares (Draper & Smith, 1981). These definitions form the basis of some of the tests used to establish the goodness of fit of the regression line.

A1.1.2 Goodness Of Fit

Summary Statistics For The Regression Line
A commonly used measure of the goodness of fit of the regression line (Gunst & Mason, 1980; Draper & Smith, 1981; Groebner & Shannon, 1993) is the coefficient of determination, $R^2$:

$$R^2 = \frac{SS_{\text{Reg}}}{TSS} = \frac{\sum_{i=1}^{N} (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{N} (Y_i - \bar{Y})^2} \quad (A1.8)$$

$R^2$ measures the proportion of total variation about the mean of $Y$ explained by the regression line. The possible values of $R^2$ range from 0 to 1. If $R^2=1$, all observations fall on the regression line, so that $Y$ can be predicted from $X$ without error. If $R^2=0$, there is no linear relationship between $X$ and $Y$. The closer $R^2$ is to unity, the better the fit of the regression line to the data points.

$R^2$ tends to be an optimistic estimate of how well the model fits the population (SPSS, 1993). The model usually does not fit the population as well as it fits the sample from which it is derived. The adjusted statistic $R_a^2$ attempts to correct $R^2$ to more closely reflect the goodness of fit of the model in the population (Freund & Littell, 1991; SPSS, 1993):

$$R_a^2 = R^2 - \frac{p(1-R^2)}{N-p-1} \quad (A1.9)$$

where $p$ is the number of independent variables in the model.

The square root of $R^2$, $R$, equals the correlation coefficient between $X$ and $Y$, $\rho_{XY}$, and that between $Y$ and $\hat{Y}$, $\rho_{Y\hat{Y}}$. The sign of the square root is the sign of $b_1$. 
Another commonly used measure of the goodness of fit of the regression line is the sample standard error of the estimate, $S$:

$$S = \sqrt{\frac{SS_{\text{Res}}}{N-p-1}} = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{N-p-1}}$$ (A1.10)

The population variance of the residuals, $\sigma^2$, is generally not known and $S^2$ is its usual estimate (SPSS, 1993). $S$ is the measure of deviation of the observed $Y$ values around the regression line and so the smaller $S$ is, the more reliable are the predictions (Gunst & Mason, 1980). $S^2$ is also known as the mean square residual, $MS_{\text{Res}}$ (see Table A1.1).

**Analysis Of Variance - ANOVA Table**

One method of testing if the regression line is statistically significant (i.e. that predicting $Y$ based on $X$ and using the LS regression line is preferable to just using the overall mean of $Y$) is to use analysis of variance of $Y$ from its mean value, as shown in Table A1.1.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>$p$</td>
<td>$SS_{\text{Reg}} = \sum (\hat{Y}_i - \bar{Y})^2$</td>
<td>$MS_{\text{Reg}} = \frac{SS_{\text{Reg}}}{p}$</td>
<td>$\frac{MS_{\text{Reg}}}{S^2}$</td>
</tr>
<tr>
<td>Residual</td>
<td>$N-p-1$</td>
<td>$SS_{\text{Res}} = \sum (Y_i - \hat{Y}_i)^2$</td>
<td>$MS_{\text{Res}} = S^2 = \frac{SS_{\text{Res}}}{N-p-1}$</td>
<td>—</td>
</tr>
</tbody>
</table>

*Table A1.1:* ANOVA table (modified after Draper & Smith, 1981).

If the regression assumptions are met, the ratio of the mean square regression to the mean square residual is distributed as a one tailed $F$ statistic with $p$ and $N-p-1$ degrees of freedom (SPSS, 1993). Generally, $F$ is a statistic used to test the null hypothesis (Figure A1.3) $H_0$: the regression model does not explain any of the total variation in the dependent variable (Groebner & Shannon, 1993). In the case of a single independent variable, it tests the null hypothesis $H_0: \beta_1 = 0$ (SPSS, 1993). If $F = MS_{\text{Reg}} / S^2 > F_{\text{Crit}} = F(p, N-p-1; 1-\alpha)$, the null hypothesis can be rejected at
the \( \alpha \) level of significance on the basis of the data used and it can be inferred that the LS equation appears to be a suitable predictor (see Figure A1.4). Very often \( \alpha \) is chosen as 0.05 (Hutchinson, 1993), although there is no logical reason for using this convention (see Appendix A5 for more details on significance level, \( \alpha \)).

**HYPOTHESIS TESTING**

**Basic idea**
- State a particular hypothesis (known as the null hypothesis and denoted Ho).
- Assume that the hypothesis is true and determine the probability of getting a result at least as far from the hypothesised value as is the observed value.
- If this probability is very low, say less than \( \alpha \), then reject the hypothesis (\( \alpha \) is known as the significance level).

**Type I and Type II errors**
- In reality, the null hypothesis is either true or false.
- The conclusion at the end of a hypothesis test is to reject or not reject Ho.
- Type I error: Ho is rejected when it is true (the probability of doing this, given that Ho is correct, is the significance level, \( \alpha \)).
- Type II error: Ho is not rejected when it is false.
- Choosing a very low value of \( \alpha \) gives a very low probability of a Type I error, but gives a high probability of a Type II error.

*Figure A1.3:* Hypothesis testing.
Statistics Of The Parameters In The Regression Line

Since $b_0$ and $b_1$ are estimates of the population's parameters, they typically differ from the population values and vary from sample to sample (SPSS, 1993). When the assumptions of linear regression are met, the distributions of $b_0$ and $b_1$ are Normal, with means $\beta_0$ and $\beta_1$. The standard deviations of the parameters are, respectively:

$$SEb_0 = \sigma \sqrt{\frac{1}{N} + \frac{\bar{X}^2}{(N-1)S_X^2}}$$  
$$SEb_1 = \frac{\sigma}{\sqrt{(N-1)S_X^2}}$$  

where $S_X$ is the standard deviation of the X values. $SEb_0$ and $SEb_1$ are commonly referred to as the standard errors of $b_0$ and $b_1$, respectively. They are useful measures of the dispersion of the parameters' estimates (Groebner & Shannon, 1993). Since $\sigma$ is usually not known, $S$ is used instead and $SEb_0$ and $SEb_1$ are then called estimates of the standard errors of $b_0$ and $b_1$ (Figure A1.5). As $S_X^2$ is the sample variance,

$$S_X^2 = \frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N-1}$$  

(A1.12)
Another way of testing if the intercept and/or the slope of the regression line are zero, i.e. $H_0: \beta_0 = 0$ and/or $H_0: \beta_1 = 0$, is to use the following statistic (SPSS, 1993):

$$t = \frac{b_0}{SE_{b_0}} \quad \text{or} \quad t = \frac{b_1}{SE_{b_1}}$$  \hspace{1cm} (A1.13)

The distribution of the statistic, when the assumptions are met, is Student’s $t$ distribution with $N-p-1$ degrees of freedom, and is used in a two-tailed test (Lewis-Beck, 1980). So if $|t| > t_{critical} = t(N-p-1; 1 - 0.5\alpha)$, then the parameter estimate is significant and $H_0$ can be rejected at the $\alpha$ level of significance (Figure A1.6).
The values of $b_0$ and $b_1$ are point estimates of the unknown parameters. However, if the assumptions of regression are not violated, a range of values can be calculated for each parameter which, within a designated likelihood, $\alpha$, includes the population value. This range is called the confidence interval (Groebner & Shannon, 1993). A $(1-\alpha)\%$ confidence interval for the parameters is defined as follows:

$$b \pm t(N-p-1;1-0.5\alpha)SE_b$$

where $b$ is the parameter considered ($b_0$ or $b_1$) and $SE_b$ is its standard error. The most widely used interval is a two-tailed 95% confidence interval, i.e. $\alpha = 0.05$: if repeated samples are drawn from a population under the same conditions and if 95% confidence intervals are calculated from each sample, then 95% of the intervals will contain the unknown parameter, $\beta$ (Law & Kelton, 1991). As there is no logical reason for using specifically a 95% confidence interval, the 90, 95 and 99% confidence intervals are given here for completeness.

The confidence interval not only allows an interval estimate of the parameters (as opposed to a point estimate) but also provides a test of the null hypothesis $H_0: \beta = 0$, where $\beta$ can be either $\beta_0$ or $\beta_1$. If the value of zero does not fall within the interval, the null hypothesis can be rejected on the basis of the data used, and $\beta \neq 0$ is accepted (Draper & Smith, 1981).
Predicted Values And Their Standard Errors

A major use of regression is prediction (Gunst & Mason, 1980). However, the fact that a regression model explains a significant proportion of variation in the dependent variable, does not mean that it is satisfactory for prediction (Groebner & Shannon, 1993). It is important to know how useful the independent variable is for predicting \( Y \). Furthermore, it must be stressed that inferences made from a regression line only apply over the range of the data contained in the sample.

One may wish to predict the mean of \( Y \) for all cases with a given value of \( X \), \( X_0 \), or to predict the value of \( Y \) for a single case. In both instances, the predicted value is the same (Draper & Smith, 1981); what differs is the standard error (Groebner & Shannon, 1993). The standard error of the predicted mean value of \( Y \) at a specific value \( X_0 \) of \( X \) is:

\[
\text{SE} \hat{Y} = \sigma \sqrt{\frac{1}{N} + \frac{(X_0 - \bar{X})^2}{(N-1)S_x^2}} \tag{A1.15}
\]

Replacement of \( \sigma \) by \( S \) provides the corresponding estimated standard error. It can be seen that the smallest value occurs when \( X_0 = \bar{X} \). The larger the distance from the mean, the greater the standard error. Thus, the mean of \( Y \) for a given value of \( X \) is better estimated for central values of the observed \( X \)’s than for extreme values (Draper & Smith, 1981). Prediction intervals for the mean predicted \( Y \) at \( X_0 \) are calculated in the standard way:

\[
\hat{Y} \pm t(N-p-1; 1-0.5\alpha) \text{SE} \hat{Y}.
\]

The variance of the individual prediction is the variance of the mean prediction plus the variance of \( Y \), for a given \( X \) (Groebner & Shannon, 1993). Hence, the standard error of the individual prediction for a given value of \( X \), \( X_0 \), is:

\[
\text{SE}_{\text{Ind}} \hat{Y} = \sigma \sqrt{\frac{1}{N} + \frac{(X_0 - \bar{X})^2}{(N-1)S_x^2}} \tag{A1.16}
\]

with the corresponding estimated value obtained by inserting \( S \) for \( \sigma \). Prediction intervals for the value of \( Y \) at \( X_0 \) are calculated in the standard way:

\[
\hat{Y} \pm t(N-p-1; 1-0.5\alpha) \text{SE}_{\text{Ind}} \hat{Y}.
\]
Finally, note that a regression equation is determined from data which "cover" certain areas of the X-space. Suppose the point $X_0$ lies outside the region covered by the original data. While a predicted value at the point $X_0$ can be obtained mathematically, reliance on such a prediction is dangerous. It becomes increasingly so the further $X_0$ lies from the original region, unless there is additional knowledge indicating that the regression equation is valid over a wider region of the X-space.

**Searching For Violation Of Assumptions**

Testing the goodness of fit of the regression model includes detecting possible violations of the assumptions relating to the data being analysed (SPSS, 1993). The most common way to look for evidence that the necessary assumptions are violated is a search focused on residuals (Everitt & Dunn, 1991). As noted earlier, a residual is the difference between an observed value and the value predicted by the regression equation. It represents the amount which the regression equation has not been able to explain. In performing the regression analysis, certain assumptions about the errors have been made. Thus if the fitted model is correct, the residuals should exhibit tendencies confirming the assumptions made for the errors. At the very least, they should not negate these assumptions.

There are several different types of residuals (Gunst & Mason, 1980), although each is a function of the difference between the observed and predicted values. For example, the standardised residuals, $Z_{\text{Resid}}$, are the values of the residuals divided by the sample standard deviation of the residuals. They have a mean of zero and a standard deviation of 1. These residuals are used in this study for plotting purposes.

The behaviour of the residuals can be analysed numerically and/or graphically. Certain statistics provide a numerical check on some of the assumptions (see, for example, Anscombe, 1961, Anscombe & Tukey, 1963, Gunst & Mason, 1980, and Draper & Smith, 1981). However, this study generally adopts the graphical approach as it is good at revealing violations of the assumptions. The lack of emphasis on numerical tests is deliberate, since a detailed examination of the plots of the residuals is usually more informative. Plots reveal violations of the assumptions serious enough to require corrective action (Draper & Smith, 1981).
A plot of the residuals against the predicted values and against the values of the independent variable can be used to check the assumptions of homoscedasticity and the linear relationship between X and Y (SPSS, 1993). The reason the residuals are plotted against the predicted values, $\hat{Y}_i$, and not the measured values, $Y_i$, is because the residuals and the measured values are usually correlated. The residuals and the predicted values are not correlated, for prediction equations with intercepts (Gunst & Mason, 1980; Draper & Smith, 1981). A satisfactory residuals plot should give the overall impression of a horizontal "band" equally spread about the horizontal line through the zero value of the residuals. Systematic patterns in these plots indicate a changing variance or inadequate model.

In detecting homoscedasticity, it is also helpful to plot squared residuals versus $\hat{Y}_i$ and $X_i$. The advantage of plotting the squared residuals instead of the residuals is that such a plot can accentuate some types of trends existing between the residuals and $\hat{Y}_i$ or $X_i$ (Gunst & Mason, 1980).

The normality of the residuals can be checked by superimposing the probability density function (PDF) of a Normal distribution on the histogram of the residuals. A major problem of using PDF (or cumulative distribution function, CDF) plots to judge the correctness of a specific distribution is the curvature of these functions. It is easier to assess departures from a straight line. In probability plots of the data, the points lie on a straight line if the hypothesised distribution is the actual underlying distribution (D'Agostino & Stephens, 1986). There are several possible probability plots. In this study, the P-P (probability-probability) and the Q-Q (quantile-quantile) plots are used to compare the observed distribution of residuals with the expected distribution under the assumption of normality.

A P-P plot is a graph of the fitted cumulative distribution function, $F_X^F$, versus the input data cumulative distribution function, $F_X$. The basis of this type of plot is illustrated in Figure A1.7 and examples are shown in Appendices A2 and A5. If the two cumulative distribution functions are close together, then the P-P plot will be approximately linear with an intercept of 0 and a slope of 1. The P-P plot amplifies differences between the two distributions in the middle values (Law & Kelton, 1991).
Figure A1.7: Definition of P-P and Q-Q plots (modified after Law & Kelton, 1991).

A Q-Q plot is a graph of the \( X_q^F \) quantile of a fitted cumulative distribution function versus the \( X_q \) quantile of the input data cumulative distribution function. The basis of this type of graph is also illustrated in Figure A1.7 and examples are shown in Appendices A2 and A5. If the hypothesised distribution is the actual distribution of the input data and if the sample size is large, then the Q-Q plot will also be approximately linear. The Q-Q plot amplifies differences in the tails between the two distributions (Law & Kelton, 1991).

The linearity, or lack of linearity, in a P-P or Q-Q graph, can be used as a basis for determining whether the input data could reasonably have been drawn from the hypothesised distribution. The variance of the points in the tails of the P-P or Q-Q plots is generally larger than that of the points at the centre of the distribution. Thus the relative linearity of the plots near the tails of the distributions will often seem poorer than at the centre, even if the “correct” model has been chosen (Hahn & Shapiro, 1967; Gunst & Mason, 1980). As the plotted points are ordered, they are not independent, and so they are not randomly scattered about the straight line. Therefore, even if the chosen model is appropriate, the plots may consist of a series of successive points above and below the straight line (Hahn & Shapiro, 1967; D’Agostino & Stephens, 1986). Finally, it is unreasonable to expect the observed
residuals to be exactly Normal; some deviation is expected because of sampling variation (SPSS, 1993). Even if the errors are normally distributed in the population, sample residuals are only approximately Normal.

Residuals are also used to detect outliers in the Y direction. An outlier in the Y direction among residuals is one that is far greater than the rest in absolute value and lies some standard deviations from the mean of the residuals (Draper & Smith, 1981). In this study, data with absolute standardised residual values greater than 2.5 are considered outliers (Rousseeuw & Leroy, 1987; Hadi, 1992; Gilbert & Antille, 1992) although other limits, such as 3, have also been suggested (SPSS, 1993). Outliers can be spotted readily on residual plots: they are not typical of the rest of the data. The presence of these outliers can bias the estimates of the regression parameters and make the resulting analysis less useful (Daniel & Wood, 1980; D'Agostino & Stephens, 1986; Freund & Littel, 1991).

Residuals can also be used to detect points that have unusual values for the independent variable and which can have a substantial impact on the results of the analysis (SPSS, 1993). These points are known as leverage points, outliers in the X direction, or influential points.

Basically, there are two ways of dealing with outliers (Rousseeuw & Leroy, 1987). The first is to use regression diagnostics in which certain statistics are computed from the data to detect deviating points, after which the outliers are deleted or corrected. An LS analysis is carried out on the remaining data. Rules exist for rejecting outliers (D'Agostino & Stephens, 1986). However, automatic rejection is not always wise, because the outliers might provide information which other data points do not. They might arise from an unusual combination of circumstances which are of vital interest and require further investigation rather than rejection. As a general rule, outliers should be rejected only if they can be traced to causes such as recording errors or mistakes in setting up the apparatus (Anscombe, 1960). When there is only a single outlier, some of these methods work quite well by looking at the effect of deleting one point at a time. Unfortunately, it is much more difficult to diagnose outliers when there are several of them (Portnoy, 1987).
Using LS residuals is not a good way of identifying real outliers (Rousseeuw & Leroy, 1987): the LS fit is pulled considerably in the direction of the deviating points and so the real outliers may possess quite small LS residuals. The alternative approach is to use robust regression, i.e. data-fitting procedures which are less sensitive than LS to typical departures from the idealised assumptions for which the estimators are optimised, and where possible outliers among the data points are identified and given less weight (Madsen & Nielsen, 1989). Hence, when LS assumptions are not met and/or when outliers are present, a more robust technique should be applied to confirm or refute LS results. There are several robust regression techniques (Rousseeuw & Leroy, 1987).

### A1.2 Least Absolute Deviations Method (LAD)

#### A1.2.1 Basic LAD Principle

To define more robust regression alternatives, statisticians have exploited the fact that the median is a better measure of central tendency than the mean if the variable can take very large or very small values. Extreme values can greatly affect the mean even if they are very unlikely to occur; this is not the case with the median (Law & Kelton, 1991).

Suppose that errors $\epsilon_i, i = 1,...,N$ are independent and follow the Double Exponential (or Laplace) distribution:

$$f_{\epsilon_i} = \frac{1}{2\sigma} \exp \left[ -\frac{|\epsilon_i|}{\sigma} \right] \quad \text{for} \quad -\infty \leq \epsilon_i \leq +\infty \quad (A1.17)$$

rather than the Normal distribution:

$$f_{\epsilon_i} = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{\epsilon_i^2}{2\sigma^2} \right] \quad \text{for} \quad -\infty \leq \epsilon_i \leq +\infty \quad (A1.18)$$

as is assumed for the LS method. The Double Exponential probability density function has a pointed peak of height $1/2\sigma$ at $\epsilon_i = 0$ and tails off to zero as $\epsilon_i$ goes to both plus and minus infinity. Then, application of the maximum likelihood principle for estimating the parameters $\beta_0$ and $\beta_1$, assuming $\sigma$ fixed, involves minimisation of the sum of absolute errors, $\sum |\epsilon_i|, i=1,...,N,$
and not the minimisation of the sum of squares of errors, \( \sum_{i=1}^{N} e_i^2 \) (Draper & Smith, 1981). Since the errors are not squared, the influence of the outliers in the estimators is less.

The method of robust regression which minimises the sum of the absolute deviations is known as the least absolute deviations method or the \( \ell_1 \)-norm. The LAD method does not rely upon the Normal error assumption: it assumes a Double Exponential distribution of the errors which has thicker tails than the Normal distribution. Hence, the LAD method provides maximum likelihood estimates when the errors follow a Double Exponential distribution with zero mean and with mean absolute deviation \( \sigma \). This method can be seen as a generalisation of the concept of the median, because minimisation of the sum of the absolute errors defines the median of \( N \) observations of \( Y \) (Rousseeuw & Leroy, 1987).

Figure A1.8 illustrates the different results obtained from using the LS and LAD methods. Figure A1.8(a) is a plot of five points which lie almost on a straight line: the LS fit and the LAD fit are essentially the same. Figure A1.8(b) displays a situation where, for some reason, point 4 has been wrongly moved from its original position (indicated by the dashed circle). This point is an outlier in the \( Y \) direction and it has a rather strong influence on the LS line, which is quite different from the line in Figure A1.8(a). Figure A1.8(c) shows the robustness of the LAD fit with respect to such an outlier: the line remains approximately where it was when observation 4 was correct.

The LAD method is recommended when outliers exist in the \( Y \) direction but it does not accommodate outliers in the \( X \) direction (Rousseeuw & Leroy, 1987). Since, in Owen's data set, outliers in the \( Y \) direction are the most important, the LAD method is used here. There is no need to use a robust technique which is computationally more costly (Rousseeuw & Leroy, 1987).

LAD is probably the oldest of all robust methods for estimating regression coefficients\(^1\) (McKean & Schrader, 1987). However, two major difficulties have impeded its general adoption (Dodge, 1987). These are computational difficulties due, firstly, to the lack of closed form formulae for estimating the

---

\(^1\) A brief review and comprehensive bibliography of LAD estimation until 1977 is given in Gentle (1977).
parameters and, secondly, to the lack of accompanying statistical inference procedures (e.g. for testing the general linear hypothesis, for obtaining confidence intervals and for constructing analysis of variance tables). Fortunately, interest has increased recently in developing statistical methods based on LAD rather than on LS (McKean & Schrader, 1987; Gentle et al, 1987; Sposito, 1990; Armstrong & Beck, 1990; Bai et al, 1990; Ataa, 1994). The motivation for this interest has arisen largely from the recognition that many real data sets do not conform to the Normal error assumption, but also from the conceptual simplicity of the LAD estimates and their increasingly competitive computational cost (Ataa, 1994).

![Figure A1.8: LS and LAD regression lines (modified after Rousseeuw & Leroy, 1987).](image-url)
To infer accurately the values of $\beta_0$ and $\beta_1$ from the LAD values of $b_0$ and $b_1$, respectively, the six assumptions mentioned in Section A1.1.1 have to be valid. For further details, see McKean & Schrader (1987) and Koenker (1987).

**A1.2.2 Goodness Of Fit**

**Summary Statistics For The Regression Line**

The measure of the goodness of fit of the LAD regression line is the sample standard error of the estimate, $S$, defined as (McKean & Schrader, 1987):

$$S = \frac{2 \text{SAD}_{\text{Res}}}{N - p - 1} = \frac{2 \sum_{i=1}^{N} |Y_i - \hat{Y}_i|}{N - p - 1} \quad \text{(A1.19)}$$

where $\text{SAD}_{\text{Res}}$ is the sum of the absolute deviations of the residuals (see Table A1.2). Since the population variance of the residuals, $\sigma^2$, is generally not known, $S^2$ is its usual estimate.

**Analysis Of Variance - ANOLAD Table**

The statistical methods of hypothesis testing associated with the LAD method have been discussed recently by a number of authors (McKean & Shrader, 1987; Bai et al, 1990) and the suggested tests are similar to those for the LS method. For testing a general linear hypothesis, the LS minimisation in sums of squared residuals is replaced by minimisation in sums of absolute deviations. By using such a direct analogy, an analysis of variance table based on the LAD method (an ANOLAD table) can be constructed (Table A1.2). It summarises the tests of the hypothesis in a form similar to an ANOVA table. The ANOLAD table shows whether predicting $Y$ based on $X$ using the LAD regression line is preferable to just using the overall median of $Y$, $Y_{\text{med}}$. Note that in the LAD method, the median is used as the centre of location instead of the mean used in the LS method.
Table A1.2: ANOLAD table (modified after McKean & Schrader, 1987).

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
<th>Sum of Absolute Deviations</th>
<th>Mean Absolute Deviations</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>( p )</td>
<td>( SAD_{\text{Reg}} = \sum_{i=1}^{N}</td>
<td>\hat{\gamma}<em>i - Y</em>{\text{med}}</td>
<td>)</td>
</tr>
<tr>
<td>Residual</td>
<td>( N-p-1 )</td>
<td>( SAD_{\text{Res}} = \sum_{i=1}^{N}</td>
<td>Y_i - \hat{Y}_i</td>
<td>)</td>
</tr>
</tbody>
</table>

If the assumptions are met, the ratio of the mean absolute regression to the mean absolute residual is distributed as a \( \chi^2 \) statistic with \( N-p-1 \) degrees of freedom (McKean & Schrader, 1987; Koenker, 1987; Ataa, 1994). \( \chi^2 \) is a statistic used to test the null hypothesis Ho: the regression model does not explain any of the total variation in the dependent variable. For a single independent variable, it tests the null hypothesis Ho: \( \beta_1 = 0 \). If \( \chi^2 = 2\text{MAD}_{\text{Reg}} / S > \chi^2_{\text{Crit}} = \chi^2 (N-p-1; \alpha) \), the null hypothesis can be rejected at the \( \alpha \) level of significance on the basis of the data used and it can be inferred that the LAD equation appears to be a good predictor, running a risk of less than \( \alpha \) of being wrong (Figure A1.9).

![Figure A1.9: The \( \chi^2 \) distribution used as a significance test for LAD regression (modified after Bendat & Piersol, 1971).](image-url)
Statistics Of The Parameters In The Regression Line

Under the above assumptions, Bassett & Koenker (1978) showed that the distributions of $b_0$ and $b_1$ are approximately Normal, with means of $\beta_0$ and $\beta_1$ and with standard deviations (or standard errors of the estimates) as defined in eq. (A1.11), where $\sigma$ is estimated by $S$ calculated from eq. (A1.19).

Another way of testing if the intercept and/or the slope of the regression line are zero, i.e. Ho: $\beta_0 = 0$ and/or Ho: $\beta_1 = 0$, is to use the following statistic:

$$z = \frac{b_0}{SE_{b_0}} \quad \text{or} \quad z = \frac{b_1}{SE_{b_1}}$$  \hspace{1cm} (A1.20)

The distribution of the statistic, when the assumptions are met, is standard Normal. So if $|z| > z_{\text{Crit}} = z(\alpha / 2)$, then the parameter estimate is significantly non-zero at the $\alpha$ level (Figure A1.10): that is, Ho can be rejected at the $\alpha$ level of significance.

![Figure A1.10: The standard Normal distribution used as a significance test for LAD regression (modified after Bendat & Piersol, 1971).](image)

Based on the LAD estimates of $b_0$ and $b_1$, and on the underlying assumptions, a $(1 - \alpha)$% confidence interval similar to the LS counterparts can be calculated for each parameter. Here, SEb is calculated as for eq. (A1.20) and the t-critical values in eq. (A1.14) are replaced by z-critical values (Stangenhaus, 1987):
where $z(\alpha / 2)$ is the upper tailed standard Normal critical value (Figure A1.10). As for the LS method, the confidence intervals can be used to test the null hypothesis $H_0: \beta = 0$, where $\beta$ can be either $\beta_0$ or $\beta_1$. If zero does not fall within the interval, the null hypothesis can be rejected.

### Searching For Violation Of Assumptions

Analogous to plots of the LS residuals, the LAD residuals may also be plotted. If $\varepsilon$ follows a Double Exponential distribution with zero mean and mean absolute deviation $\sigma$, then the probability density function of $|\varepsilon|$ follows an Exponential distribution with a mean and standard deviation equal to $\sigma$ (Ataa, 1994):

$$f_{|\varepsilon|} = \frac{1}{\sigma} \exp \left[ -\frac{|\varepsilon|}{\sigma} \right]$$

This fact can be exploited by plotting the absolute values of the residuals against a theoretical Exponential fit, allowing the suitability of the Double Exponential distribution to be assessed visually.
APPENDIX A2 - Examples Of Regression Analysis

This appendix describes how regression equations have been fitted to Owen's data. The H&R model (employing both \((R_{\text{max}})^{37\%}\) and \((R_{\text{max}})^{99\%}\) in defining the value of C) and Owen's model have been considered. Similar inferences were made in all cases. Here, an illustrative example is given for each model using data for the embankment slope of 1:2.

The H&R and Owen models are represented by eqs. (3.16) and (3.6), respectively (Chapter 3). These equations contain the parameters A and B to be estimated by regression. They can be expressed, by suitable transformation, in the form of the standard linear regression model (see eq. (A1.3) in Appendix A1):

\[
\begin{align*}
\text{H&R Model:} & \quad \ln(Q) = \ln(A) + B \ln(1 - R) \\
\text{Owen's Model:} & \quad \ln(Q) = \ln(A) - BR. \\
\end{align*}
\]

For both models, \(\ln(Q)\) is the dependent variable, \(\ln(A)\) is the intercept and B the slope; for the H&R model, \(\ln(1 - R)\) is the independent variable and for Owen's model, \(R\) is the independent variable. Note that \(R\) and \(Q\) are defined differently in the two models (see Chapter 3).

In this study, regression analysis has started by applying the least-squares (LS) method (see Appendix A1). The following results have been obtained using the statistical software package SPSS for Windows (SPSS, 1993).

A2.1 LS Regression

A2.1.1 H&R Model

From the scatterplot in Figure A2.1, the relationship between \(\ln(Q)\) and \(\ln(1 - R)\) appears to be approximately linear and the values of \(R^2\) and \(R_a^2\) in Table A2.1 suggest that no relevant independent variable has been excluded: these values reveal that \(\ln(1 - R)\) accounts for about 81% of the variation in \(\ln(Q)\), which can be considered satisfactory. This is reflected in the ANOVA results (Table A2.1) where the mean square regression is much bigger than the mean square residual.
Summary Statistics

<table>
<thead>
<tr>
<th>Coefficient of Determination, $R^2$</th>
<th>Adjusted Coefficient of Determination, $R^2_a$</th>
<th>Standard Error of the Estimate, $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.811</td>
<td>0.809</td>
<td>0.523</td>
</tr>
</tbody>
</table>

ANOVA

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>109.239</td>
<td>109.239</td>
<td>399.588</td>
</tr>
<tr>
<td>Residual</td>
<td>93</td>
<td>25.424</td>
<td>0.273</td>
<td>------</td>
</tr>
</tbody>
</table>

Statistics of LS Parameters

<table>
<thead>
<tr>
<th>Parameter, $b$</th>
<th>Estimated Standard Error of $b$, $SE_b$</th>
<th>Confidence Interval of $b$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>0.120</td>
<td>-5.041 -4.641 -5.079 -4.603 -5.157 -4.525 -4.525 -4.525</td>
<td>-40.466</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.228</td>
<td>4.168 4.928 4.095 5.000 3.947 5.149</td>
<td>19.990</td>
</tr>
</tbody>
</table>

Table A2.1: Summary of LS statistics for the H&R model, $(R_{max})^{37\%}$, slope 1:2.
Since $F > F_{\text{Crit}}$ for $\alpha$ equal to 0.01, 0.05 and 0.1 (as tabulated in statistical books) and $|t| > t_{\text{Crit}}$ for $b_1$ for any value of $\alpha$ between 0.001 and 0.9 (as tabulated in statistical books), the slope estimate $b_1 = 4.548$ differs significantly from zero. Equivalently, the 90, 95 and 99% confidence intervals in Table A2.1 do not include zero. The same conclusions apply to the parameter $b_0$.

The above results, although pleasing, should not be accepted too readily, because the applicability of the LS method has not yet been assessed. This analysis should be an indispensable part of regression, even when $R^2$ (or $R_a^2$) is large and the statistical tests are significant (Rousseeuw & Leroy, 1987).

Figure A2.2 appears to show a tendency for the variation (or spread) in the standardised residuals to decrease as the predicted values of $\ln(Q.)$ and $\ln(1−R.)$ increase. The assumption of homoscedasticity is unlikely to be valid. Although unbiased, the estimated parameters would not have minimum variance and the confidence intervals and significance tests obtained from regression are not reliable (see Appendix A1).

Figure A.2.3, a plot of the squared standardised residuals versus the predicted values of the dependent variable and versus the independent variable, accentuates the fact that there does not appear to be a constancy of the error variances. Yet what is most evident from this figure is that three of the standardised residuals look suspiciously large in magnitude. These points are clearly outliers (see Table 2.2).
Figure A2.2: Scatterplots of the standardised residuals against the predicted values of Ln(Q*) and against Ln(1-R*): LS regression for the H&R model, (R_{max})_{37\%}, slope 1:2.
Figure A2.3: Scatterplots of the squared standardised residuals against the predicted values of Ln(Q) and against Ln(1−R): LS regression for the H&R model, \((R_{max})^{37\%}\), slope 1:2.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Predicted Dependent Variable</th>
<th>Residuals</th>
<th>Standardised Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.683</td>
<td>-10.061</td>
<td>-7.949</td>
<td>-2.113</td>
<td>-4.041</td>
</tr>
<tr>
<td>-0.683</td>
<td>-9.765</td>
<td>-7.949</td>
<td>-1.816</td>
<td>-3.473</td>
</tr>
<tr>
<td>-1.075</td>
<td>-11.200</td>
<td>-9.734</td>
<td>-1.466</td>
<td>-2.804</td>
</tr>
<tr>
<td>-0.687</td>
<td>-6.790</td>
<td>-7.966</td>
<td>1.176</td>
<td>2.249</td>
</tr>
</tbody>
</table>

Table A2.2: Details of the four highest residuals from LS regression for the H&R model, \((R_{max})^{37\%}\), slope 1:2.
From Figures A2.4 to A2.6, it seems that the Normal distribution fits the residuals poorly; the residuals appear to follow a distribution with thicker tails than the Normal distribution. The non-normality has been confirmed by the results of three goodness-of-fit tests available in the software package BestFit (see Appendices A4 and A5 for more details of BestFit and goodness-of-fit tests, respectively) which all rejected the hypothesis of the residuals being Normal distributed at any \( \alpha \) level of significance considered (these results are not presented here, but similar analyses are shown in Appendix A5). Since the assumption of normality seems to be violated, the confidence intervals and results of significance tests are unreliable. Even if the parameters are still unbiased, the violation of assumptions can be important (Draper & Smith, 1981).

Figure A2.4: Comparison of the Normal probability density function for standardised residuals with data from LS regression for the H&R model, \((R_{\text{max}})_{37\%}\), slope 1:2.
A2.1.2 Owen's Model

Since the results obtained for the Owen model are very similar to those for the H&R model, they are not discussed here. Nevertheless, the corresponding figures are presented for completeness.
Figure A2.7: Scatterplot and LS regression line for Owen’s model, slope 1:2.

### SUMMARY STATISTICS

<table>
<thead>
<tr>
<th>Coefficient of Determination, $R^2$</th>
<th>Adjusted Coefficient of Determination, $R^2_a$</th>
<th>Standard Error of the Estimate, $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.814</td>
<td>0.812</td>
<td>0.522</td>
</tr>
</tbody>
</table>

### ANOVA

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>111.198</td>
<td>111.198</td>
<td>407.613</td>
</tr>
<tr>
<td>Residual</td>
<td>93</td>
<td>25.371</td>
<td>0.273</td>
<td>----------</td>
</tr>
</tbody>
</table>

### STATISTICS OF LS PARAMETERS

<table>
<thead>
<tr>
<th>Parameter, $b$</th>
<th>Estimated Standard Error of $b$, SE$b$</th>
<th>Confidence Interval of $b$ 90%</th>
<th>Confidence Interval of $b$ 95%</th>
<th>Confidence Interval of $b$ 99%</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>-4.385</td>
<td>-4.643</td>
<td>-4.933</td>
<td>-4.793</td>
<td>-28.325</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-22.798</td>
<td>-24.685</td>
<td>-25.046</td>
<td>-25.780</td>
<td>-20.189</td>
</tr>
</tbody>
</table>

Table A2.3: Summary of LS statistics for Owen’s model, slope 1:2.
Figure A2.8: Scatterplots of the standardised residuals against the predicted values of Ln(Q*) and against R*: LS regression for Owen's model, slope 1:2.
Owen's Model  
Slope 1:2

\[ Z_{\text{Resid}}^2 \]

\[ \text{Predicted Value Of Ln}(Q_{*}) \]

Figure A2.9: Scatterplots of the squared standardised residuals against the predicted values of \( \text{Ln}(Q_{*}) \) and against \( R_{*} \): LS regression for Owen's model, slope 1:2.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Predicted Dependent Variable</th>
<th>Residuals</th>
<th>Standardised Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.177</td>
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<td>-8.425</td>
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</tr>
<tr>
<td>0.239</td>
<td>-11.530</td>
<td>-9.824</td>
<td>-1.706</td>
<td>-3.267</td>
</tr>
<tr>
<td>0.177</td>
<td>-10.107</td>
<td>-8.425</td>
<td>-1.682</td>
<td>-3.221</td>
</tr>
<tr>
<td>0.152</td>
<td>-6.666</td>
<td>-7.839</td>
<td>1.173</td>
<td>2.247</td>
</tr>
</tbody>
</table>

Table A2.4: Details of the four highest residuals from LS regression for Owen's model, slope 1:2.
Figure A2.10: Comparison of the Normal probability density function for standardised residuals with data from LS regression for Owen's model, slope 1:2.

Figure A2.11: Comparison of the Normal cumulative distribution function for standardised residuals with data from LS regression for Owen's model, slope 1:2.
Figure A2.12: Comparison of the Normal quantiles for standardised residuals with data from LS regression for Owen's model, slope 1:2.

A2.2 LAD Regression

From the above LS regression results, the H&R and Owen models appear to perform equally well. This is not surprising. The main differences between the models are expected to exist for very small discharges (see Chapter 3) which are not part of Owen’s measured conditions.

The above departures from the idealised LS assumptions and the occurrence of several outliers suggest that it might be appropriate to apply robust regression. In this study, the least absolute deviations (LAD) method has been adopted (see Appendix A1). It is more likely than the LS method to accommodate outlying data points and to provide reliable estimates of the parameters. The results for the illustrative example given here have been obtained using the statistical software package SAS for Windows (Freund & Littell, 1991) and using the program presented in Appendix A3 written in SAS/IML language (SAS Institute Inc, 1988).
A2.2.1 H&R Model

![Graph of H&R Model](image)

**Figure A2.13:** Scatterplot and LAD regression line for the H&R model, \((R_{\text{max}})_{37\%}\), slope 1:2.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
<th>Sum of Absolute Deviations</th>
<th>Mean Absolute Deviations</th>
<th>(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>68.623</td>
<td>68.623</td>
<td>189.109</td>
</tr>
<tr>
<td>Residual</td>
<td>93</td>
<td>33.747</td>
<td>0.363</td>
<td>-------</td>
</tr>
</tbody>
</table>

**Standard Error of the Estimate, S**

0.726

**ANOLAD**

<table>
<thead>
<tr>
<th>Parameter, (b)</th>
<th>Estimated Standard Error of (b), SE(b)</th>
<th>Confidence Interval of (b) 90%</th>
<th>Conf. Interval of (b) 95%</th>
<th>Conf. Interval of (b) 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_0)</td>
<td>-4.889</td>
<td>-5.162</td>
<td>-5.214</td>
<td>-5.316</td>
</tr>
<tr>
<td>(b_1)</td>
<td>4.174</td>
<td>3.854</td>
<td>3.555</td>
<td>3.361</td>
</tr>
</tbody>
</table>

**Table A2.5:** Summary of LAD statistics for the H&R model, \((R_{\text{max}})_{37\%}\), slope 1:2.
Figure A2.14: Scatterplots of the standardised residuals against the predicted values of $\ln(Q^*)$ and against $\ln(1-R^*)$: LAD regression for the H&R model, $(R_{\text{max}})^{37\%}$, slope 1:2.
Figure A2.15: Scatterplots of the squared standardised residuals against the predicted values of \( \ln(Q) \) and against \( \ln(1-R) \): LAD regression for the H&R model, \((R_{\text{max}})_{37\%}\), slope 1:2.

Table A2.6: Details of the four highest residuals from LAD regression for the H&R model, \((R_{\text{max}})_{37\%}\), slope 1:2.
Example Of Regression Analysis

Figure A2.16: Comparison of the Exponential probability density function for absolute values of the standardised residuals with data from LAD regression for the H&R model, (R_{\text{max}})^{37\%}, slope 1:2.

Figure A2.17: Comparison of the Exponential cumulative distribution function for absolute values of the standardised residuals with data from LAD regression for the H&R model, (R_{\text{max}})^{37\%}, slope 1:2.
Figure A2.18: Comparison of the Exponential quantiles for absolute values of the standardised residuals with data from LAD regression for the H&R model, (R_{max})_{37\%}, slope 1:2.

A number of points are worth noting in relation to the H&R model:

- The LAD results support a linear relationship between Ln(Q.) and Ln(1−R.) (see Figure A2.13; \( \chi^2 > \chi^2_{\text{Crit}} \) for any value of \( \alpha \) and \( \text{SAD}_{\text{Reg}} \gg \text{SAD}_{\text{Res}} \) in Table A2.5).
- The facts that \( \chi^2 > \chi^2_{\text{Crit}} \) for any value of \( \alpha \), \(|z| > z_{\text{Crit}}\) for \( b_1 \) for any value of \( \alpha \) and the 90, 95 and 99% confidence intervals for \( b_1 \) do not contain the value zero (Table A2.5), confirm that the slope estimate \( b_1 = 4.174 \) is significantly different from zero. For the same reasons, the results in Table A2.5 also confirm that \( b_0 \neq 0 \).
- The LAD estimates \( b_0 \) and \( b_1 \) (Table A2.5) differ from the corresponding LS estimates (Table A2.1), showing the influence of the outlying data points in the LS results.
- Comparison of Figure A2.2 with Figure A2.14 and of Figure A2.3 with Figure A2.15 shows that the LAD method accommodated extreme points much better than the LS method. Figure A2.14 also shows points being better balanced above and below the zero line. This is reflected in the smaller values of the standardised residuals shown in Table A2.6 compared with the corresponding values in Table A2.2.
Figures A2.16 to A2.18 indicate that the absolute errors may follow an Exponential distribution (i.e. the errors may follow a Double Exponential distribution, as hypothesised). In view of the approximately linear behaviour shown in the P-P and the Q-Q graphs, the LAD method seems suitable for fitting the H&R model to this data set. When similar graphs have been constructed as a diagnostic tool for the LS method, some systematic non-linear behaviour has clearly been identified (see Figures A2.5 and A2.6). This improvement has been confirmed by running BestFit to check the goodness of fit of the Exponential distribution.

**A2.2.2 Owen’s Model**

The remarks made with regard to the H&R model also seem to apply to Owen’s model. Indeed, the improvement obtained with the LAD method for the latter is even more evident (see Figures A2.22 to A2.24) than for the former.

As in the case of the LS analysis, the difference in performance between the H&R model and the Owen model does not seem important for the range of discharges tested, as expected.

![Figure A2.19: Scatterplot and LAD regression line for Owen’s model, slope 1:2.](image-url)
### Standard Error of the Estimate, $S$

|       | 0.718 |

### ANOLAD

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<th>Source of Variation</th>
<th>Degrees of Freedom, DF</th>
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<th>Mean Absolute Deviations</th>
<th>$\chi^2$</th>
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<td>73.742</td>
<td>73.742</td>
<td>205.350</td>
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<td>Residual</td>
<td>93</td>
<td>33.397</td>
<td>0.359</td>
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### STATISTICS OF LAD PARAMETERS

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<tr>
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<th>Confidence Interval of $b$</th>
<th>z</th>
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<tr>
<td></td>
<td></td>
<td>90%</td>
<td>95%</td>
</tr>
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<td></td>
<td>0.213</td>
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<tr>
<td></td>
<td>1.553</td>
<td>-19.152</td>
<td>-18.663</td>
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Table A2.7: Summary of LAD statistics for Owen's model, slope 1:2.
Owen's Model

Figure A2.20: Scatterplots of the standardised residuals against the predicted values of Ln(Q*) and against R*: LAD regression for Owen's model, slope 1:2.
Figure A2.21: Scatterplots of the squared standardised residuals against the predicted values of Ln(Q*) and against R*: LAD regression for Owen's model, slope 1:2.

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<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
<th>Predicted Dependent Variable</th>
<th>Residuals</th>
<th>Standardised Residuals</th>
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<td>0.177</td>
<td>-10.404</td>
<td>-8.291</td>
<td>-2.112</td>
<td>-2.910</td>
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<td>0.239</td>
<td>-11.530</td>
<td>-9.624</td>
<td>-1.906</td>
<td>-2.626</td>
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<tr>
<td>0.177</td>
<td>-10.107</td>
<td>-8.291</td>
<td>-1.816</td>
<td>-2.501</td>
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<tr>
<td>0.152</td>
<td>-6.666</td>
<td>-7.734</td>
<td>1.068</td>
<td>1.472</td>
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Table A2.8: Details of the four highest residuals from LAD regression for Owen's model, slope 1:2.
**Figure A2.22:** Comparison of the Exponential probability density function for absolute values of the standardised residuals with data from LAD regression for Owen's model, slope 1:2.

**Figure A2.23:** Comparison of the Exponential cumulative distribution function for absolute values of the standardised residuals with data from LAD regression for Owen's model, slope 1:2.
Example Of Regression Analysis

A2.3 Main Conclusions Of The Analysis

Regression equations were fitted to Owen’s data using the LS method. The presence of outliers and violation of the Normal error LS assumption lead to the subsequent use of the more robust technique of LAD. The LAD assumption that errors have a Double Exponential distribution appears much more plausible than the Normal error assumption for Owen’s data set. The results obtained from the LAD regression seem to be more sensible than those of the LS method. As expected, the parameters are less affected by the outliers identified in the diagnostic plots. Thus, the LAD estimates of the coefficients are recommended both for the H&R and Owen models.

The LAD regression results suggest that the H&R model and the Owen model perform almost equally well for the range of conditions tested. This is not surprising since the main differences between the two models are expected to exist for very small discharges in the ranges of practical interest. These conditions are not covered by Owen’s data set.
LS parameter values can be calculated explicitly from the data but the LAD parameters cannot. However, efficient algorithms have now eliminated this computational difficulty. Statistical inference procedures for LAD have recently been developed but not completely implemented in the usual statistical packages. LAD forms an attractive robust alternative to LS for data sets with outlying observations which violate the LS assumption of normality. The author hopes that this study will encourage engineers to use robust techniques in general; in particular, the LAD method is an attractive alternative to the classical LS technique.
This program uses the SAS/IML language (SAS Institute Inc, 1988) to perform regression analysis on Owen’s data, using the least-squares (LS) method and the least-absolute-deviations (LAD) method. Both the H& R model and Owen’s model are used in the analyses. For the program to work, two files are needed on the A drive:

i) file SLOPE12.POR which is the data file that has to contain the X and Y values for each regression line. Here, for the H& R model, X is called "lnkrstar" and Y is called "lnkqstar"; for the Owen model, X is called "rstarow" and Y is called "lnqstow". The file must contain the values of these four variables. The extension *.POR allows this file to be used as a data file in SAS and SPSS.

ii) file CODE.SAS which contains this program.

Also needed on the A drive are two directories called A:\SASFILES and A:\SASDESC.

When this program has been run in SAS, two files called A:\REG_RES.DBF and A:\SASDESC\EXCHANGE.SA2 have been created. The former consists of variables saved during the regression process and which are then used for plotting purposes (plots can be done, for example, in Excel). The latter is of no importance for the user. These two files cannot be overwritten, so after each run of the program, the user has to rename the file A:\REG_RES.DBF and has to delete the file A:\SASDESC\EXCHANGE.SA2.

Note that the C drive can be used instead of the A drive if the code in this program is changed accordingly.

For more details on this program, the user is referred to SAS Institute Inc (1988) and Freund & Littel (1991).

libname slope12 spss 'a:slope12.por';
libname tete1 'a:sasfiles';
libname tete2 'a:sasfiles';
libname tete3 'a:sasfiles';
libname tete5 'a:sasfiles';
libname tete6 'a:sasfiles';
libname tete4 'a:sasdesc';
data sastemp;
set slope12._f_; 
run;

rq performs the LAD calculations.

proc iml;
start rq(yname,Y,xname,X,b,predict,error,zres,q);
bound=1.0e10;
coef=X';
nc=nrow(coef);
m=ncol(coef);
r=repeat(0,m+2,1); L=repeat(q-1,1,n) || repeat(0,1,m) || -bound || -bound ; u=repeat(q,1,n) || repeat(0,1,m) || {..} ; a=(y || repeat(0,1,m) || {-1 0}) // (repeat(0,1,n) || repeat(-1,1,m) || {0 -1}) // (coef || 1(m) || repeat(0,m,2)); basis=n+m+2-(0:n+m+1); call lp(rc,p,d,a,r,,u,L,basis);

proc iml;
start rq(yname,Y,xname,X,b,predict,error,zres,q);
bound=1.0e10;
coef=X';
nc=nrow(coef);
m=ncol(coef);
r=repeat(0,m+2,1); L=repeat(q-1,1,n) || repeat(0,1,m) || -bound || -bound ; u=repeat(q,1,n) || repeat(0,1,m) || {..} ; a=(y || repeat(0,1,m) || {-1 0}) // (repeat(0,1,n) || repeat(-1,1,m) || {0 -1}) // (coef || 1(m) || repeat(0,m,2)); basis=n+m+2-(0:n+m+1); call lp(rc,p,d,a,r,,u,L,basis);

predict=X*b;
error=y-predict;
wsnum=sum(choose(error<0,(q-1)*error,q*error));
SA=2*wsnum;
MSA=SA/(n-m);
s1=2*MSA;
zres=error/s1;
OBS=X[,2]|Y|predict|error|zres;
print,...,'LEAST ABSOLUTE REGRESSION',

*END OF SAS/IML PROGRAM*
'Dependent Variable: ' yname,
'Regression Quantile: ' q,
'Number of Observations: ' n,
'Sum of Absolute Errors: ' wsum,
'Mean Absolute Deviations: ' MSA,
'Estimate of s: ' s1,
'variable b,..
'PREDICTED VALUES AND RESIDUALS',
OBS ((|colname={X Y YPred Resid Z Resid} format=9.5|));
finish;

Calls rq to perform the LAD calculations for the H& R model.

use sastemp;
read all var {'lnkrstar'} into X;
read all var {'lnkqstar'} into Y;
n=nrow(X);
m=ncol(X);
X=repeat(1,n,1) || X;
run rq('lnkqstar',Y,{'lnA' 'ln(1-R*)'},X,b,pre,res,zres,.5);

LS calculations for the H&R model.

c=inv(X`*X);
b=c*X`*Y;
dfe=n-m-1;
SSE=Y`*Y-b`*X`*Y;
MSSE=SSE/dfe;
s2=sqrt(MSSE);
L={0 1};
dfr=m;
SSR=(L*b)`*inv(L*c*L`)*(L*b);
MSR=SSR/dfr;
P=MSR/MSSE;
ProbF=1-PROBF(F#F,dfr,dfe);
SOURCE=(m||SSR||MSSR||F||ProbF)/(dfe||SSE||MSSE||{.}||{.});
seb=sqrt(vecdiag(c)#MSSE);
T=b/seb;
ProbT=1-PROBF(T#T,1,dfe);
VARIABLE=b||seb||T||ProbT;

SOURCE=(m||SSR||MSSR||F||ProbF)/(dfe||SSE||MSSE||{.}||{.});
seb=sqrt(vecdiag(c)#MSSE);
T=b/seb;
ProbT=1-PROBF(T#T,1,dfe);
VARIABLE=b||seb||T||ProbT;

Calls rq to perform the LAD calculations for the Owen model.

use sastemp;
read all var {'rstarow'} into XOW;
read all var {'lnqstow'} into YOW;
n=nrow(XOW);
m=ncol(XOW);
XOW=repeat(1,n,1) || XOW;
run rq('lnqstow',YOW,{'lnA' 'R*'},XOW,bow,resow,zresow,.5);

LS calculations for the Owen model.

cow=inv(XOW`*XOW);
bow=cow*XOW`*YOW;
DATA A:
FILE A:\REG_RES.DBF;
INPUT X Y;
CARDS;
1 2
2 3
3 4
4 5
5 6
;
APPENDIX A4 - BestFit

BestFit is a software package for fitting probability distributions to data (Palisade Corporation, 1996). Selected statistical distributions can be fitted to a maximum of 30,000 data points. Results may be displayed numerically and graphically. The package allows the user to transfer all results, including graphs, statistics and distribution functions, to other programs (e.g. Excel, @Risk) for further analysis and presentation. Examples of output which can be generated are given in Appendix A5.

BestFit goes through the following steps when attempting to find the “best fit” to the input data:

- For each distribution type selected, a first guess for its parameters is made using Maximum Likelihood Estimators - MLEs (Bury, 1975; Law & Kelton, 1991; Walpole & Myers, 1993; Palisade Corporation, 1996).

- The fit is optimised (if the user wishes) using the Levenberg-Marquardt method (Press et al, 1992; Palisade Corporation, 1996). Otherwise, the maximum likelihood estimates for the parameters are adopted. The Levenberg-Marquardt method takes an iterative approach to minimise a goodness-of-fit statistic. The goodness-of-fit test used by BestFit for optimisation is the Chi-Square test. Note that the Levenberg-Marquardt method does not necessarily find the absolute minimum for the Chi-Square value; rather, it may find a local minimum.

- The goodness of fit is measured for the distribution functions. For continuous distributions, BestFit can use the Kolmogorov-Smirnov (K-S) statistic and the Anderson-Darling (A-D) statistic, as well as the Chi-Square statistic (Bendat & Piersol, 1971; D'Agostino & Stephens, 1986; Law & Kelton, 1991; Press et al, 1992; Groebner & Shannon, 1993; Walpole & Myers, 1993; Palisade Corporation, 1996). These tests provide an idea of how well a certain distribution fits the input data.

- For continuous distributions, twenty distribution functions are compared and the one with the lowest goodness-of-fit value can be considered the “best fit” to the input data. Including continuous and discrete distributions, there are 26 statistical distributions available in BestFit (Evans et al, 1993; Palisade Corporation, 1996).
Despite its name, BestFit does not provide an absolute answer to the question of which distribution best fits the data. It identifies the likelihood that the input data is taken from a selected distribution. Before employing any fitted distribution obtained from BestFit, the user should assess the results quantitatively and qualitatively by examining the summary statistics and the results of the goodness-of-fit tests which BestFit produces, and by considering the graphs.
APPENDIX A5 - Fitting A Distribution To Data

The choice of probability distribution to describe the randomness of a variable can have a large impact on the accuracy of the results of probabilistic calculations and on the quality of the decisions made using these results (Burcharth, 1992). Consequently, the task of selecting a distribution deserves special consideration, particularly if a sensitivity analysis of the problem confirms a significant contribution of the distribution to the final results (see Chapters 2 and 6).

If it is possible to collect data on the input random variable of interest, these data can be used in one of the following two ways to specify a distribution (Law & Kelton, 1991):

- The data values themselves are used to define an empirical distribution function.
- Standard techniques of statistical inference are used to “fit” a theoretical distribution (e.g. Normal, Log-Normal) to the data, to calculate its parameters and to determine its goodness of fit.

If a theoretical distribution can be found that fits the observed data reasonably well, then this approach is preferable to using an empirical distribution (Law & Kelton, 1991): i) a theoretical distribution "smooths out" the data and may provide information on the overall underlying distribution; ii) with a fitted theoretical distribution, values outside the range of the observed data can be generated, though care should be taken when considering extrapolated values (Copeiro, 1978; Carvalho, 1992b).

This appendix investigates which distributions (if any) available in PARASODE and BestFit (Normal, Log-Normal, Gumbel, Rectangular, Gamma, Beta, Exponential, Rayleigh and Weibull) provide a reasonable model for the $e_B$ variable (Chapter 3). It discusses the three basic steps in specifying theoretical distributions on the basis of field and/or laboratory data (Law & Kelton, 1991):

Step I To hypothesise the form of the distributions.
Step II To estimate the parameters of the distributions.
Step III To test the appropriateness of the fitted distributions.
The theoretical distribution of $e_B$ has been separately analysed for the three slopes of 1:1, 1:2 and 1:4, both for the H&R model and for Owen's model. The H&R model employed both $(R_{\text{max}})_{37\%}$ and $(R_{\text{max}})_{99\%}$ in defining the value of C. Similar conclusions were drawn in all cases. In this appendix, an illustrative example is provided for the H&R model, $(R_{\text{max}})_{37\%}$, for slope 1:1.

A5.1 Hypothesise The Form Of The Distributions

In modelling the physical world, the form of the probability distribution of a random variable may be deduced theoretically on the basis of assumptions about the reality. However, the process is often difficult. Firstly, the probability model needed to describe a physical phenomenon may not be formulated readily (Ang & Tang, 1975). Secondly, the functional form of the probability distribution may not be easily derived, although under certain circumstances, the properties of the physical process may suggest the form of the distribution. Therefore, on many occasions, the required probability distribution has to be inferred empirically, that is, based entirely on available observational data. This is the case in this study in hypothesising the form of the distribution of $e_B$.

In practice, the choice of the probability distribution may also be dictated by mathematical tractability and convenience. For example, because of the mathematical simplifications possible with the Normal distribution, and the wide availability of information associated with this distribution, it is frequently used to model non-deterministic problems, even when there is no clear basis for such a choice (Ang & Tang, 1975).

The first stage in hypothesising a particular form of distribution from observed data is to decide what general characteristics the distribution is expected to have. In the second column of Table A5.1, summary statistics are presented for the input data corresponding to the H&R model, $(R_{\text{max}})_{37\%}$, slope 1:1. In Figure A5.1, a corresponding histogram based on 11 classes of width 0.23 is shown. Other graphs could have been used for general guidance (see, for example, Law & Kelton, 1991, for further details).
### Statistical Report

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<th>DISTRIBUTION</th>
<th>LOG NORMAL</th>
<th>MAXIMA TYPE I (GUMBEL)</th>
<th>GAMMA</th>
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<td>10.38084</td>
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<td>0.358773</td>
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### Critical Values

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<td>0.683968</td>
<td>0.651058</td>
<td>0.663384</td>
<td>0.642357</td>
</tr>
<tr>
<td>#1 Percentile</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>#2 Value</td>
<td>0.828544</td>
<td>0.828544</td>
<td>0.828544</td>
<td>0.828544</td>
</tr>
<tr>
<td>#2 Percentile</td>
<td>30%</td>
<td>30%</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>#3 Value</td>
<td>0.979562</td>
<td>0.974732</td>
<td>0.996358</td>
<td></td>
</tr>
<tr>
<td>#3 Percentile</td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>#4 Value</td>
<td>1.134359</td>
<td>1.157788</td>
<td>1.160769</td>
<td>1.172682</td>
</tr>
<tr>
<td>#4 Percentile</td>
<td>70%</td>
<td>70%</td>
<td>70%</td>
<td>70%</td>
</tr>
<tr>
<td>#5 Value</td>
<td>1.385293</td>
<td>1.472816</td>
<td>1.490116</td>
<td>1.497141</td>
</tr>
<tr>
<td>#5 Percentile</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>#6 Value</td>
<td>1.571693</td>
<td>1.654788</td>
<td>1.699021</td>
<td>1.71543</td>
</tr>
<tr>
<td>#6 Percentile</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
</tr>
</tbody>
</table>

**Table A5.1:** Statistical report obtained from BestFit for the H&R model, \(\left(R_{\text{max}}\right)^{37}\% \text{, slope } 1:1\).
The shape of the histogram strongly suggests that the actual underlying distribution is skewed to the right (i.e. its right tail is longer than its left tail). This fact is supported by the values in Table A5.1 which show that the mean (1.0302) is 10.6% bigger than the mode (0.9316), and the skewness is 2.56 (the skewness is zero for a symmetric distribution). These characteristics of the input data appear to rule out symmetric distributions (in this case, the Normal and the Rectangular distributions). Furthermore, the kurtosis of the input distribution is 10.42, bigger than 3, which indicates distributions with "thicker" tails than for a Normal distribution. The coefficient of variation, $\sigma/\mu = 0.336$, makes it fairly unlikely that the actual distribution could be Exponential (Law & Kelton, 1991) which has a coefficient of variation of 1. On the other hand, the Log-Normal, Gumbel, Gamma, Beta, Rayleigh and Weibull distributions can all take on shapes similar to that of the histogram shown in Figure A5.1. So, it seems that, potentially, $e_B$ may be described by any of these distributions and they are proposed here as candidate distributions.

### A5.2 Estimate The Parameters Of The Distributions

After one or more candidate distributions have been hypothesised, the values of the parameters of these distributions are estimated so that the
distributions may be used in the FORM analysis and in simulation. BestFit uses both the Maximum Likelihood Estimators (MLEs) and the Levenberg-Marquardt method to estimate the parameters of the distributions from the input data (see Appendix A4 for further details).

A5.3 Test The Appropriateness Of The Fitted Distributions

After identifying the candidate distributions and estimating their parameters, they must be examined to assess how well they represent the data. If an objective assessment is desired, statistical goodness-of-fit tests should be used in conjunction with probability plots (Ang & Tang, 1975). This approach has been used here. The Log-Normal, the Gumbel and the Gamma distributions performed better than the others in describing $e_B$. So, for convenience, the results shown below are limited to these three distributions. The fact that the analysis did not suggest one "best" distribution is not surprising: there are insufficient data to identify unambiguously one distribution as the most appropriate (Melchers, 1987).

A5.3.1 Probability Plots

First, graphs comparing the histogram of the data with the probability density functions of the fitted distributions have been plotted (Figure A5.2). In general, the agreement between the theoretical distributions and the input data seems good.

Figures A5.3 and A5.4 show the P-P and the Q-Q graphs plotted for the three distributions (see Appendix A1 for explanations of P-P and Q-Q plots). Clearly, the three hypothesised distributions appear to fit the input data equally well both in the body and at the tails. As expected (Appendix A1), the data points are not randomly scattered about the straight lines.
Figure A5.2: Comparison of theoretical probability density functions for $e_B$ with input data.
Figure A5.3: Comparison of theoretical cumulative distribution functions for $e_B$ with input data.
Figure A5.4: Comparison of theoretical quantiles for $e_B$ with input data quantiles.
A5.3.2 Goodness-Of-Fit Tests

A graphical approach for testing hypothesised distributions should always be complemented by statistical goodness-of-fit tests. A goodness-of-fit test assesses formally whether the observations $X_1, \ldots, X_N$ can be considered to be a sample from a particular distribution with cumulative distribution function $F_X$. It can be used to test the null hypothesis $H_0$: The $X_i$'s are independent random variables with cumulative distribution function $F_X$ (Law & Kelton, 1991). The acceptance or rejection of $H_0$ depends on the significance level, $\alpha$. $H_0$ is accepted at the $\alpha$ level of significance if the test value is less than or equal to the critical value associated with $\alpha$. If the test value is greater than the critical value, then the assumed distribution is rejected at the $\alpha$ level of significance. A distribution that is acceptable at one significance level may be unacceptable at another significance level.

The choice of $\alpha$ is largely a subjective matter (Ang & Tang, 1975). If the chosen $\alpha$ is very small (say, 0.001), the null hypothesis might not be rejected even if appreciable differences exist between the two distributions (type II error). If the chosen $\alpha$ is large (say, 0.20), rejection of the null hypothesis could well occur even when the two distributions are almost identical (type I error). So, there is the need to compromise between type I and type II errors when choosing a significance level (Groebner & Shannon, 1993). $\alpha$ is commonly set at 0.05 (Hutchinson, 1993), although there is no logical reason to select this particular value. Preferably, goodness-of-fit tests should be performed for several values of $\alpha$, allowing the user to determine, approximately, the highest level of significance at which the hypothesised distribution should not be rejected. In BestFit the significance level, $\alpha$, is indicated as "confidence" and the critical values for each test are indicated as "Critical Value @ $\alpha$" (see Table A5.1).

Importantly, goodness-of-fit tests may reject a model as inadequate, but they can never prove that a model is correct.

Sample size is crucial in influencing decisions based on statistical tests. Such tests are often not very powerful for small to moderate sample sizes: they might not be sensitive to subtle disagreements between the data and the fitted distribution. They should be regarded as a systematic approach for detecting fairly gross differences. On the other hand, if the sample is very large, these tests will almost always reject $H_0$ (Law & Kelton, 1991). Since
Ho is virtually never exactly true, even a minor departure from the fitted distribution will be detected for large samples.

The tests mentioned here are those used by BestFit: the Chi-Square test, the Kolmogorov-Smirnov (K-S) test and the Anderson-Darling (A-D) test. D’Agostino & Stephens (1986) describe other goodness-of-fit tests and stress that no one test is optimal for all possible deviations from the hypothesised distribution. In fact, tests might not provide a unique answer. The ranking of the distributions for one test is unlikely to be identical with that from another test due to their different characteristics:

- The Chi-Square test is the most common goodness-of-fit test due to its flexibility and ease of use; but it requires the input data to be grouped into classes and there are no firm guide-lines for selecting the number and size of these classes. Unfortunately, results depend, to some degree, on the number and size of the classes adopted. BestFit uses, as the default, Scott’s Normal approximation (Law & Kelton, 1991) to calculate the number of classes, k, from the number of data points, \( N \): 
  \[ k = (4N)^{2/5} \]
  But other formulations can be applied (see, for example, Hahn & Shapiro, 1967; Law & Kelton, 1991). For the Chi-Square test, the distribution which has the smallest value of the test statistic is ranked in first place. The Chi-Square statistic is defined as (Law & Kelton, 1991):
  \[
  \chi^2 = \sum_{j=1}^{k} \frac{(N_j - Np_j)^2}{Np_j}
  \]  
  (A5.1)
  where \( N_j \) is the number of \( X_i \)'s in the \( j \)th class (note that \( \sum_{j=1}^{k} N_j = N \)); \( p_j \) is the expected proportion of the \( X_i \)'s that would fall in the \( j \)th class if sampling was done from the fitted distribution.

- The K-S test does not require the data to be grouped into classes but is poor of detecting tail discrepancies. The K-S statistic is (Law & Kelton, 1991):
  \[
  D_n = \sup \left| F_X(x) - F_X^e(x) \right|
  \]  
  (A5.2)
  where \( F_X^e(x) \) is the hypothesised cumulative distribution function evaluated at \( X=x \); \( F_X(x) = N_X/N \) is the empirical cumulative distribution; \( N_X \) is the number of \( X_i \)'s less than or equal to \( x \), \( i=1,...,N \); "sup" means the smallest upper bound on members of the set [...]. "sup" is used here instead of the more familiar "max" since,
in some cases, the maximum may not be well defined (Law & Kelton, 1991).

- The A-D test does not require the data to be grouped into classes and places more emphasis on tail values than the K-S test. The form of the A-D statistic used for computations is (Law & Kelton, 1991):

\[
A_{n}^{2} = \sum_{i=1}^{N} \left( \frac{2(2i-1)\left( nZ_{i} + \ln(1 - Z_{N+1}) \right)}{N} \right) - N
\]  

(A5.3)

where \( Z_{i} = \frac{F_{X}(x_{i})}{n} \) and the other variables are defined as previously.

For the K-S and the A-D tests, the distribution which has the smallest value of the adjusted test statistic is ranked in first place (see Table A5.1). The adjustment performed to the actual test statistic depends on the distribution (for more details, see Law & Kelton, 1991; Palisade Corporation, 1996).

In BestFit, no account is taken of the fact that the parameters of the distributions are estimated from the input data: for the Chi-Square test, the critical value used for comparison with the actual value of the test is the upper \( \alpha \) critical point for a Chi-Square distribution with \( k-1 \) degrees of freedom, \( \chi^{2}(k-1; \alpha) \), and not \( k-m-1 \) degrees of freedom (Palisade Corporation, 1996), where \( k \) is the number of classes and \( m \) is the number of parameters of the distribution estimated from the data. Ho is rejected only if \( \chi^{2} > \chi^{2}(k-1; \alpha) \). This is often recommended (Law & Kelton, 1991) since it is conservative. Thus, the actual probability of rejecting Ho when it is true (type I error) is at least as small as the stated probability, \( \alpha \). On the other hand, the probability of a type II error (the probability of not rejecting a false Ho) is increased.

Note that, unlike the Chi-Square test, where the critical values are the same for all distributions, the K-S and the A-D tests include special cases depending on the distribution (for more details, see Palisade Corporation, 1996).

In Table A5.1, the Chi-Square test rejects the Log-Normal and the Gamma distributions at any of the usual \( \alpha \) levels. On the other hand, the values of \( \chi^{2}(10; \alpha) \) for \( \alpha < 0.07 \) for the Gumbel distribution, are not exceeded by the
test value $\chi^2 = 17$; so, Ho for the Gumbel distribution should not be rejected at the $\alpha < 0.07$ level. Thus, this test gives no reason to conclude that the input data is poorly fitted by the Gumbel(0.889, 0.270) distribution but it suggests that the probability is very low that the input data came from the Log-Normal(-0.021, 0.319) or the Gamma(10.08, 0.102) distributions.

According to the K-S test, the Log-Normal and the Gamma distributions should not be rejected for any of the levels of significance listed ($\alpha < 0.15$) and the Gumbel distribution should not be rejected for $\alpha < 0.05$. Thus this test gives no reason to conclude that the input data is poorly fitted by any of the three distributions.

The A-D test also suggests that the Log-Normal and the Gamma distributions should not be rejected at any of the levels of significance listed ($\alpha < 0.15$). The Gumbel distribution should not be rejected at the 2.5% level (since the adjusted value 0.813 is less than the critical A-D test value for $\alpha < 0.025$) but it should be rejected for larger values of $\alpha$.

A5.4 Main Conclusions Of The Analysis

This appendix provides some guidance on how a theoretical distribution should be fitted to data. An illustrative example is given for the H&R model, $(R_{\text{max}})_{37\%}$, slope 1:1.

From the statistics presented in Table A5.1 and from Figures A5.1 to A5.4, it can be inferred that the Log-Normal, Gumbel and Gamma distributions provide reasonable representations of the input data. It can also be inferred that there is little to choose between the three distributions since they perform almost equally well.

In summary, there is no reason to believe, based on the figures shown above and on the statistical tests performed, that any of the three distributions should be rejected as adequate models for describing the randomness of $e_B$. Similar conclusions may be drawn by looking at all other cases.
APPENDIX B

Example Of Input And Output For DUNEPROB And DUNE
The computer programs DUNEPROB and DUNE introduced in Chapter 4 have been run for the same data set representing conditions near Hook of Holland (Figure B.1).

Figure B.1: Exceedance curve for the maximum storm surge level, Hook of Holland (modified after Van de Graaff, 1983).

The main purpose of this appendix is to show, with an example, that the input data related to the seven main erosion parameters is strongly orientated to conditions in The Netherlands. For this reason, only the data related to these seven parameters is shown in detail. Any additional input is mentioned only briefly. Results for the current example which are relevant to the discussion are also presented and compared.

**DUNEPROB**

The program starts by asking the user for a filename (e.g. *Hook*). It uses this information to name the four required input files:

- *Hook.Pro* - contains the pairs of coordinates (X,Y) of the initial profile.
- *Hook.For* - contains the information required for a FORM calculation (e.g. maximum number of iterations, number of FORM calculations).
**Example Of Input And Output For DUNEPROB And DUNE**

*Hook.Ran* - contains the random variables and their statistical properties.

*Hook.AB* - contains the parameters required to define the distribution of the significant wave height as a function of the surge level during storm conditions.

Table B.1 shows the data provided in this example for the input file *Hook.Ran*.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>( \mu )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h ) (m)</td>
<td>Exponential</td>
<td>2.1894</td>
<td>0.3322</td>
</tr>
<tr>
<td>( D_{50} ) (m)</td>
<td>Deterministic</td>
<td>225E-6</td>
<td>0</td>
</tr>
<tr>
<td>( H_{S\text{_Inacc}} ) (m)</td>
<td>Normal</td>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>Surcharge (-)</td>
<td>Normal</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table B.1**: Input file *Hook.Ran* for the current example.

The following data have been provided for the input file *Hook.AB* (see Figure B.1 and eqs. (4.5) and (4.6) in Section 4.5.2.2 of the main text):

- \( a=3.795 \)
- \( b=0.444 \)
- \( H_{\text{wavemax}}=10\text{m} \)

Table B.2 shows the main results obtained for a retreat distance with a probability of exceedance of approximately \( 10^{-5} \)/year. The retreat distance is 118.5m. From the sensitivity parameters, \( \alpha^2 \), the most important contribution to the resulting variance is given by the surge parameter, a figure of 95\%, indicating that this parameter is by far the most important one. The minor contribution of \( H_{S\text{\_Inacc}} \) to the resulting variance seems strange. Note, however, that due to the relationship between the maximum surge level and the significant wave height, \( \alpha_{H_{S\text{\_Inacc}}}^2 = 0.01 \) represents only the effect of the variability in \( H_{S\text{\_Inacc}} \) about its expected value. In this case, \( \sigma_{H_{S\text{\_Inacc}}}=0.60\text{m} \).
Variable | Design Point | $\partial Z / \partial X$ | $\alpha^2$
---|---|---|---
h (m) | 5.483E+00 | -3.484E+01 | 0.95
$H_S_{\text{Inacc}}$ (m) | 1.652E-01 | -5.592E+00 | 0.01
Surcharge (-) | 8.083E-01 | -9.861E+00 | 0.04

Target Coordinate = -1.185E+02
$P(Z \leq 0) = 3.367E-05$

Table B.2: Relevant DUNEPROB results for the current example.

DUNEPROB can also output results in the form of figures. For example, the probability of failure can be plotted as a function of the retreat distance.

**DUNE**

To run the program DUNE for the present example, the following two input data files are required:

*Hook.Pro* - contains the pairs of coordinates (X,Y) of the initial profile.

*Hook.In* - contains the values adopted for the maximum water level during surge, significant wave height, peak wave period and sand diameter (see Section 4.5.3); it also contains the parameter $G_O$ which allows longshore transport to be considered (see Table 4.4); some settings for the dune erosion calculation are also part of this file.

For the present example (Ministerie van Verkeer en Waterstaat, 1994):

- Maximum water level during surge 5.7m
- Significant wave height 8.6m
- Peak wave period 12s
- Sand diameter 225E-06m
- $G_O$ 0m$^3$/m

The main results are presented in Table B.3.
### Erosion Quantities (m$^3$/m)

- Erosion above surge level = -219.52
- Surcharge above surge level = -74.88

### Distances (m)

- Erosion distance = 88.34
- Shift distance for the surcharge = 30.53
- Total retreat distance = 118.87

**Table B.3:** Relevant DUNE results for the current example.

DUNE gives a retreat distance of 118.87m associated with a probability of failure of $10^{-5}$/year. As expected, this value is almost the same as the one obtained with DUNEPROB (118.5m).

Note that DUNE produces more numerical output than is illustrated here. It also provides graphical output showing the initial dune profile, Vellinga’s post-storm profile and the limiting profile.
Figure C1.1: Simplified flowchart of PARASODE for Mode 1.
Figure C1.2: Simplified flowchart of PARASODE for Mode 2.
Figure C2.1: Flowchart showing the subroutines used in PARASODE.
<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
<th>CDF</th>
<th>PDF</th>
<th>INVERSE OF CDF</th>
<th>RESTRICTIONS</th>
<th>SUBROUTINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>$\Phi \left( \frac{x - \mu}{\sigma} \right)$</td>
<td>$\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right]$</td>
<td>$\zeta + \lambda \Phi^{-1}(F_x)$</td>
<td>$\lambda &gt; 0$</td>
<td>Normal: NormalDistInverse; InNormal</td>
</tr>
<tr>
<td>LOG-NORMAL</td>
<td>$\Phi \left( \frac{\ln(x) - \mu}{\sigma} \right)$</td>
<td>$\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2} \left( \frac{\ln(x) - \mu}{\sigma} \right)^2 \right]$</td>
<td>$\exp \left[ \zeta + \lambda \Phi^{-1}(F_x) \right]$</td>
<td>$X &gt; 0$, $\lambda &gt; 0$</td>
<td>LogNormal: NormalDistInverse; InNormal</td>
</tr>
<tr>
<td>MAXIMA TYPE I</td>
<td>$\exp \left[ -\exp \left( -\zeta(X - \lambda) \right) \right]$</td>
<td>$\zeta \exp \left[ -\zeta(X - \lambda) \right] F_x$</td>
<td>$\lambda - \frac{\ln \left( -\ln(F_x) \right)}{\zeta}$</td>
<td>$\zeta &gt; 0$</td>
<td>GumbelInverse</td>
</tr>
<tr>
<td>(GUMBEL)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RECTANGULAR (UNIFORM)</td>
<td>$\frac{X - \xi}{\lambda - \zeta}$</td>
<td>$\frac{1}{\lambda - \zeta}$</td>
<td>$\zeta + \lambda \Phi^{-1}(F_x)$</td>
<td>$\zeta \leq x \leq \lambda$, $\zeta &lt; \lambda$</td>
<td>RectangularInverse</td>
</tr>
<tr>
<td>GAMMA</td>
<td>$\frac{\Gamma(\xi + \lambda)}{\Gamma(\xi)}$</td>
<td>$\frac{X^{-\lambda} \exp(-\lambda X)}{\Gamma(\xi)}$</td>
<td>No explicit form</td>
<td>$X \geq 0$, $\zeta &gt; 0$, $\lambda &gt; 0$</td>
<td>Gamma: Gammp; GSEPARATOR; GOF; GammaInv; InGamma; DirectMM</td>
</tr>
<tr>
<td>BETA</td>
<td>$\frac{\beta_1 \Gamma(\xi + \lambda)}{\beta_1 \Gamma(\xi)}$</td>
<td>$\frac{(X - \xi x_1 - \frac{\lambda}{2} \xi x_2 - \lambda)^{\frac{1}{\lambda} - 1}}{\beta_1 \Gamma(\xi) x_2 - \xi x_1} X^{\frac{1}{\lambda} - 1}$</td>
<td>No explicit form</td>
<td>$x_1 &lt; x_2$, $\zeta &gt; 0$, $\lambda &gt; 0$</td>
<td>BetaDir; Beta; BetaInv; InBeta; InBetaInverse</td>
</tr>
<tr>
<td>MAXIMA TYPE II</td>
<td>$\exp \left[ -\frac{\lambda}{\xi} \right]$</td>
<td>$\frac{\zeta \left( \frac{\lambda}{\xi} \right)^{\lambda + 1}}{\lambda - \zeta} \exp \left[ -\frac{\lambda}{\xi} \right]$</td>
<td>$\lambda - \frac{\ln \left( -\ln(F_x) \right)}{\zeta}$</td>
<td>$\zeta &gt; 0$, $\lambda &gt; 0$</td>
<td>FrechetInverse</td>
</tr>
<tr>
<td>(FRÉCHET)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXPONENTIAL</td>
<td>$1 - \exp \left( -\frac{X - \xi}{\lambda} \right)$</td>
<td>$\frac{1}{\lambda} \exp \left( -\frac{X - \xi}{\lambda} \right)$</td>
<td>$\zeta - \ln \left( 1 - F_x \right)$</td>
<td>$X \geq \xi$, $\lambda &gt; 0$</td>
<td>ExponentialInverse</td>
</tr>
<tr>
<td>RAYLEIGH</td>
<td>$1 - \exp \left( -\frac{X^2}{2\sigma^2} \right)$</td>
<td>$\frac{X^2}{2\sigma^2} \exp \left( -\frac{X^2}{2\sigma^2} \right)$</td>
<td>$\zeta \sqrt{2 \ln \left( 1 - F_x \right)}$</td>
<td>$X \geq 0$, $\zeta &gt; 0$</td>
<td>RayleighInverse</td>
</tr>
<tr>
<td>MINIMA TYPE III</td>
<td>$1 - \exp \left( -\frac{X - \xi}{\lambda} \right)^n$</td>
<td>$\frac{\eta \left( X - \xi \right)^{n-1}}{\lambda} \exp \left[ -\frac{X - \xi}{\lambda} \right]^n$</td>
<td>$\zeta + \lambda \left[ -\ln \left( 1 - F_x \right) \right]^\eta$</td>
<td>$X \geq \xi$, $\lambda &gt; 0$, $\eta &gt; 0$</td>
<td>WeibullInverse</td>
</tr>
<tr>
<td>(WEIBULL)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C3.1: Cumulative distribution function, probability density function, inverse function and restrictions for the distributions available in PARASODE (the subroutines used for their calculation are also tabulated).
<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>RESTRICTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\lambda &gt; 0$</td>
</tr>
<tr>
<td>LOG-NORMAL</td>
<td>$\exp\left(\zeta + \frac{\lambda^2}{2}\right)$</td>
<td>$\exp\left(\zeta + \frac{\lambda^2}{2}\right) \sqrt{\exp(\lambda^2) - 1}$</td>
<td>$X &gt; 0$ $\lambda &gt; 0$</td>
</tr>
<tr>
<td>MAXIMA TYPE I (GUMBEL)</td>
<td>$\lambda + \frac{0.57722}{\zeta}$</td>
<td>$\frac{\pi}{\sqrt{6\zeta}}$</td>
<td>$\zeta &gt; 0$</td>
</tr>
<tr>
<td>RECTANGULAR (UNIFORM)</td>
<td>$\zeta + \lambda$</td>
<td>$\frac{\lambda - \zeta}{\sqrt{2}}$</td>
<td>$\zeta &lt; \lambda$ $\zeta \leq X \leq \lambda$</td>
</tr>
<tr>
<td>GAMMA</td>
<td>$\frac{\zeta}{\lambda}$</td>
<td>$\frac{\sqrt{\zeta}}{\lambda}$</td>
<td>$X \geq 0$ $\zeta &gt; 0$ $\lambda &gt; 0$</td>
</tr>
<tr>
<td>BETA</td>
<td>$x_1 + \frac{\zeta}{\zeta + \lambda}(x_2 - x_1)$</td>
<td>$(x_2 - x_1) \sqrt{\frac{\zeta \lambda}{(\zeta + \lambda) \sqrt{(\zeta + \lambda + 1)}}}$</td>
<td>$x_1 \leq X \leq x_2$ $x_1 &lt; x_2$ $x_2 &gt; \zeta &gt; 0$ $\lambda &gt; 0$</td>
</tr>
<tr>
<td>MAXIMA TYPE II (FRÉCHET)</td>
<td>$\lambda \Gamma\left(1 - \frac{1}{\zeta}\right)$</td>
<td>$\sqrt{\lambda^2 \Gamma\left(1 - \frac{2}{\zeta}\right) - \left(\lambda \Gamma\left(1 - \frac{1}{\zeta}\right)\right)^2}$</td>
<td>$X &gt; 0$ $\zeta &gt; 2$ $\lambda &gt; 0$</td>
</tr>
<tr>
<td>EXPONENTIAL</td>
<td>$\zeta + \lambda$</td>
<td>$\lambda$</td>
<td>$X \geq \zeta$ $\lambda &gt; 0$</td>
</tr>
<tr>
<td>RAYLEIGH</td>
<td>$\zeta \sqrt{\frac{\pi}{2}}$</td>
<td>$\zeta V\sqrt{2 - \frac{\pi}{2}}$</td>
<td>$X \geq \zeta$ $\zeta &gt; 0$</td>
</tr>
<tr>
<td>MINIMA TYPE III (WEIBULL)</td>
<td>$\lambda \Gamma\left(1 + \frac{1}{\eta}\right) + \zeta$</td>
<td>$\sqrt{\lambda^2 \Gamma\left(1 + \frac{2}{\eta}\right) - \lambda^2 \Gamma\left(1 + \frac{1}{\eta}\right)^2}$</td>
<td>$X \geq \zeta$ $\lambda &gt; 0$ $\eta &gt; 0$</td>
</tr>
</tbody>
</table>

Table C3.2: Mean, standard deviation and restrictions for the distributions available in PARASODE.
<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
<th>ζ</th>
<th>λ</th>
<th>η</th>
<th>RESTRICTIONS</th>
<th>SUBROUTINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>μ</td>
<td>σ</td>
<td>------</td>
<td>λ &gt; 0</td>
<td>PNormal</td>
</tr>
<tr>
<td>LOG-NORMAL</td>
<td>$\ln(\mu) \cdot \frac{\lambda^2}{2}$</td>
<td>$\sqrt{\ln\left(1 + \frac{\sigma^2}{\mu^2}\right)}$</td>
<td>------</td>
<td>λ &gt; 0</td>
<td>PLogNormal</td>
</tr>
<tr>
<td>MAXIMA TYPE I (GUMBEL)</td>
<td>$\frac{\pi}{\sqrt{6}\sigma}$</td>
<td>$\mu - \frac{0.57722}{\zeta}$</td>
<td>------</td>
<td>ζ &gt; 0</td>
<td>PGumbel</td>
</tr>
<tr>
<td>RECTANGULAR (UNIFORM)</td>
<td>$\mu - \sqrt{3}\sigma$</td>
<td>$\mu + \sqrt{3}\sigma$</td>
<td>------</td>
<td>ζ ≤ x ≤ λ; ζ &lt; λ</td>
<td>PRectangular</td>
</tr>
<tr>
<td>GAMMA</td>
<td>$\frac{\mu^2}{\sigma^2}$</td>
<td>$\frac{\mu}{\sigma^3}$</td>
<td>------</td>
<td>ζ &gt; 0; λ &gt; 0</td>
<td>PGamma</td>
</tr>
<tr>
<td>BETA</td>
<td>$\frac{(x - \mu_1)}{(x_2 - x_1)} + \frac{(x_2 - \mu)(x - x_1)}{\sigma^2(x_2 - x_1)}$</td>
<td>$\frac{(x_2 - x_1)\zeta}{(\mu - \xi)} - \zeta$</td>
<td>------</td>
<td>x, y, y &gt; 0; ζ &gt; 0</td>
<td>PBeta</td>
</tr>
<tr>
<td>MAXIMA TYPE II (FRÉCHET)</td>
<td>No explicit form</td>
<td>$\frac{\mu}{\Gamma\left(1 - \frac{1}{\zeta}\right)}$</td>
<td>------</td>
<td>ζ &gt; 2; λ &gt; 0</td>
<td>PFrechet, Gamma, GG</td>
</tr>
<tr>
<td>EXPONENTIAL</td>
<td>μ - σ</td>
<td>σ</td>
<td>------</td>
<td>x ≥ ζ; λ &gt; 0</td>
<td>PExponential</td>
</tr>
<tr>
<td>RAYLEIGH</td>
<td>$\mu \sqrt{\frac{2}{\pi}}$</td>
<td>------</td>
<td></td>
<td>x ≥ 0; ζ &gt; 0</td>
<td>PRayleigh</td>
</tr>
<tr>
<td>MINIMA TYPE III (WEIBULL)</td>
<td>Given</td>
<td>$\frac{(\mu - \zeta)}{\Gamma\left(1 + \frac{1}{\eta}\right)}$</td>
<td>No explicit form</td>
<td>x ≥ ζ; λ &gt; 0; η &gt; 0</td>
<td>PWeibull, Gamma, GG</td>
</tr>
</tbody>
</table>

Table C3.3: Parameters and restrictions for the distributions available in PARASODE (the subroutines used for their calculation are also tabulated).
C4.1 Observed Water Levels At Liverpool

The PDF and CDF are based upon 45992 values collected into classes of 0.1m interval by Proudman Oceanographic Laboratory (POL), UK. Results have been smoothed using a nine-point moving average.
The distributions of extreme water levels are based on information provided by Blackman (1997), POL.

Figure C4.2: Probability density function and cumulative distribution function of observed extreme water levels at Liverpool (LBD=Liverpool Bay Datum; OD=Ordnance Datum; OD=LBD-4.93m).
C4.2 Predicted Tide Levels At Liverpool

Figure C4.3: Probability density function and cumulative distribution function of predicted tide levels at Liverpool (LBD=Liverpool Bay Datum; OD=Ordnance Datum; OD=LBD-4.93m).

The PDF and CDF are based upon 47076 values collected into classes of 0.1m interval by POL. Results have been smoothed using a nine-point moving average.
APPENDIX C5 - Details Of Dune Erosion Calculations

In PARASODE, the methods used to calculate the areas of erosion and accretion and to test the required balance between these areas depend on the direction of the sand movements:

- If movements are allowed only seaward (see Figures C5.1 and C5.2), PARASODE:
  1) Calculates area C which lies between the surge level, the nourished profile above surge level and the gradient 1:md. Note that if \((S1,T1)=(S3,T3)\) then area C is zero (see Figures C5.1(a) and C5.2(a)).
  2) Calculates area B which lies between the surge level and the gradient 1:mt. Note that if \((S9,T9)\) is below the nourished profile, area B is zero (see Figure C5.2).
  3) Calculates area E which lies between points \((S9,T9), (S2,T2)\), the surge level and the nourished profile. Note that if \((S9,T9)\) is below the nourished profile, area E is zero (see Figure C5.2).
  4) Calculates the Y-coordinates, \(YPV\), of Vellinga's points, \((XPV,YPV)\), corresponding to the X-values of the points in the nourished profile; the total number of points in Vellinga's profile is \(NPV\).
  5) Determines the intersection points between the two profiles. Where the nourished profile is above Vellinga's profile, it is considered a hump. A hump starts at \((XHStart,YHStart)\) and finishes landward at \((XHEnd,YHEnd)\). If the nourished profile is below Vellinga's profile, it is considered a depression. A depression starts at \((XDStart,YDStart)\) and finishes landward at \((XDEnd,YDEnd)\). The total number of humps is \(NumHump\) and the total number of depressions is \(NumDep\).
  6) Calculates the areas of the humps, \(BH\), and of the depressions, \(BD\). The corresponding cumulative areas, \(SHump\) and \(SDep\), from the seaward to the landward limits of the profiles, are also calculated. These areas include area C and area E-B. Note that if \((S9,T9)\) is below the nourished profile (see Figure C5.2), since movements of sand are allowed only seaward, the first hump and its area have to be neglected, and the number of humps, together with areas BH and SHump have to be adjusted.
  7) Calculates the error, \(Err\), in the balance between erosion and accretion, i.e. determines the difference in the areas of the humps and the areas of the depressions. The idea is that sand in a hump is used to fill the depressions situated seaward of the...
hump until it is depleted. A value for the absolute error of less than 1m$^3$/m is required. If $\text{Err}>1\text{m}^3/\text{m}$, then accretion exceeds erosion and Vellinga's profile has to be moved landward. If $\text{Err}<1\text{m}^3/\text{m}$, then erosion exceeds accretion and Vellinga's profile has to be moved seaward if $S_8 \neq S_1$; if $S_8 = S_1$, no erosion of the nourished profile is expected ($C=0$).

8) If a balance has not yet been achieved, the new $X$-coordinate of the starting point of the parabolic part of the profile, $S_8$, is calculated, along with the new points $(S_9, T_9)$, $(S_2, T_2)$, and $(S_3, T_3)$. Calculations 1) to 7) are then repeated until a balance is achieved.

- If movements are allowed in both directions (see Figures C5.3 and C5.4), PARASODE:
  1) Calculates area $A$ which lies between the surge level and the parabolic part of Vellinga's profile.
  2) Calculates area $C$ which lies between the surge level, the nourished profile above surge level and the gradient 1:md. Note that if $(S_1, T_1) = (S_3, T_3)$ then area $C$ is zero (see Figures C5.3(a) and C5.4(a)).
  3) Calculates area $B$ which lies between the surge level and the gradient 1:mt. Note that if $(S_9, T_9)$ is below the nourished profile, area $B$ is zero (see Figure C5.4).
  4) Calculates area $Q$ which lies between the surge level and the nourished profile below surge level.
  5) Calculates the error, $\text{Err}$, in the balance between erosion and accretion ($\text{Err} = Q - B - A - C$). A value for the absolute error of less than 1m$^3$/m is required. If $\text{Err}>1\text{m}^3/\text{m}$, then accretion exceeds erosion and Vellinga's profile has to be moved landward. If $\text{Err}<1\text{m}^3/\text{m}$, then erosion exceeds accretion and Vellinga's profile has to be moved seaward if $S_8 \neq S_1$; if $S_8 = S_1$, no erosion of the nourished profile is expected ($C=0$).
  6) If a balance has not yet been achieved, the new $X$-coordinate of the starting point of the parabolic part of the profile, $S_8$, is calculated, along with the new points $(S_9, T_9)$, $(S_2, T_2)$, and $(S_3, T_3)$. Calculations 1) to 5) are then repeated until a balance is achieved.

Once the final position of Vellinga's profile and erosion area $C$ are known, a surcharge on $C$, $\text{TSurch}$, is required (see Figure C5.5) as follows:

$$\text{TSurch} = \text{SurchEros} + \text{SurchLongT} \quad (C5.1)$$
Details Of Dune Erosion Calculations

SurchEros is a surcharge on erosion area C which takes account of the effects of the storm surge duration, of the gust bumps, and of the accuracy of the computation (see Section 4.5.2). SurchLongT is a surcharge on erosion area C which accounts for the effect of a gradient in the longshore transport rate (according to eq.(4.9)). These surcharges are expressed as an additional movement, SurD, of the dune face. Note that TSurch can be positive (see Figure C5.5(a)) or negative (see Figure C5.5(b)) since the effects of the surge duration and of the accuracy of the computation can take positive or negative values.

The surcharge distance, SurD, is determined such that the area D between the two 1:md gradients, the surge level and the nourished profile, is equal to TSurch. An absolute error, Err1, of 1m³/m in the difference between D and the absolute value of TSurch is allowed. If Err1>1m³/m, then the surcharge distance is too big, and a new calculation of D is required for a smaller value of SurD. If Err1<-1m³/m, then the surcharge distance is too small, and a new calculation of D is required for a bigger value of SurD.

The X-coordinate of the point of intersection between the surge level and the gradient 1:md of the surcharge is S10. The intersection point between the nourished profile and the surcharge gradient 1:md has coordinates (S4,T4).
Figure C5.1: Notation used for sand balance when movements of sand are allowed only seaward - example 1.
Figure C5.2: Notation used for sand balance when movements of sand are allowed only seaward - example 2.
Figure C5.3: Notation used for sand balance when movements of sand are allowed seaward and landward - example 1.
Figure C5.4: Notation used for sand balance when movements of sand are allowed seaward and landward - example 2.
Figure C5.5: Surcharge on erosion area C.
APPENDIX C6 - Contents Of Input And Output Files

In all five input data files required by PARASODE, each piece of information has to be typed on a different line, except if otherwise specified. Data provided on the same line should be separated by a few blanks. The decimal places are separated by ".". In all input files, the character "!" is followed by a comment. It describes the input given on that line and the following lines, until the next comment appears. Examples of input files are given in Appendices D1 and D3.

PARASODE contains pre-set constants. The user must be aware of these values when providing input data. They are tabulated below.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum number of FORM calculations</td>
<td>10</td>
</tr>
<tr>
<td>Maximum number of iterations in each FORM calculation</td>
<td>200</td>
</tr>
<tr>
<td>Maximum number of time-varying actions</td>
<td>5</td>
</tr>
<tr>
<td>Maximum number of combinations of actions</td>
<td>16</td>
</tr>
<tr>
<td>Maximum number of random variables describing the failure mode</td>
<td>15</td>
</tr>
<tr>
<td>Maximum number of points defining the initial dune profile</td>
<td>100</td>
</tr>
<tr>
<td>Maximum number of iterations to find the position of Vellinga's profile</td>
<td>999</td>
</tr>
<tr>
<td>Maximum number of iterations to find the position of the dune final face</td>
<td>999</td>
</tr>
</tbody>
</table>

Table C6.1: Pre-set constants in PARASODE.

Note that the program execution may be terminated at any time by pressing simultaneously the keys "Ctrl" and "Pause".
The file *general.dad* should contain the following information in the order shown below.

- Choose the failure mode to be studied by giving a number as follows:
  1 - Overtopping (H&R)
  2 - Overtopping (Owen)
  3 - Dune Erosion (Vellinga)

- Choose how the still-water-level is defined by giving a number as follows:
  1 - Total level
  2 - Tide + Surge

- If "Overtopping (H&R)" is selected, choose the confidence value of the maximum run-up to be considered by giving a number as follows:
  1 - 37%
  2 - 99%

- If "Overtopping (H&R)" or "Overtopping (Owen)" are selected, choose the method of calculation of the first partial derivatives of the failure function by answering “Y” or “N” to the following question:
  “Are the first derivatives of the failure function supplied (Y/N) ?”

- If "Dune Erosion (Vellinga)" is selected, choose the direction of the sand movements occurring during a storm surge by giving a number as follows:
  1 - Movements of sand in both directions
  2 - Movements of sand only seaward

- Choose the purpose of the analysis by giving a number as follows:
  1 - Reliability analysis for a specified design (Mode 1)
  2 - Design for a specified reliability level (Mode 2)

- Input the design life of the structure (in years).

- If Mode 1 has been selected, choose whether combinations of actions are to be considered or not by answering “Y” or “N” to the following question:
  “Would you like to consider combinations of actions (Y/N) ?”
If “Y” is selected:

- Choose the number of combinations of actions to be considered by giving a number as follows:
  1 - The number of time-varying actions (k)
  2 - $2^{k-1}$

- Choose which distributions should be provided for each combination of actions by giving a number as follows:
  1 - The basic distributions
  2 - The modified distributions

- Give the number of time-varying actions (the maximum number allowed by the program is 5).

- If the distributions provided are the basic distributions, give the number of time-varying actions and corresponding repetitions in the design life of the structure. For each time-varying action, these two pieces of data are given on the same line. The number of lines taken is equal to the number of time-varying actions and they are presented in increasing order of the number of repetitions.

- If the program is to be run for Mode 1, give the value of the design parameter for which the failure probability is to be found.

- If the program is to be run for Mode 2, give:
  - A starting value of the design parameter.
  - The target probability of failure.

**form.dad**

The file *form.dad* should contain the specifications required for a FORM calculation:

- If no combinations of actions are considered, choose the starting point for the FORM calculations by giving a number as follows:
  1 - Default values (mean values)
  2 - User specified values

  If "User specified values" are chosen, give the starting values of the random variables (deterministic variables should be ignored).

- If combinations of actions are considered and the distributions provided are the modified distributions, then either "Default values (mean values)" or "User specified values" should be supplied for each combination of actions.
• If no combinations of actions are considered, choose the limiting values for the variables by giving a number as follows:
  1 - Default values (+/- 1E25)
  2 - User specified values
If "User specified values" are chosen, give, in the same line, the minimum value and the maximum value for each random variable (deterministic variables should be ignored).

• If combinations of actions are considered and the distributions specified are the basic ones, give the same information as above. The program assumes, subsequently, that the limiting values are the same for all combinations of actions.

• If combinations of actions are considered and the distributions specified are the modified ones, give the same information as above as many times as the number of combinations of actions considered.

• Give the maximum number of iterations allowed for each FORM calculation (the maximum number allowed by the program is 200).

• Give the number of FORM calculations to be performed (the maximum number allowed by the program is 10).

• Give the target values for the FORM calculations.

• Choose the required relative accuracy of the reliability index by giving a number as follows:
  1 - Default value (1%)
  2 - User specified value
If "User specified value" is chosen, give a value within the range [0,1].

• Choose the required smoothing coefficient for the iteration process by giving a number as follows:
  1 - Default value (0)
  2 - User specified value
If "User specified value" is chosen, give a value within the range [0,1].

• Choose the required accuracy of the failure function by giving a number as follows:
  1 - Default value (1%)
  2 - User specified value
If "User specified value" is chosen, give a value within the range [0,1].
The file *meandev.dad* should contain the characteristics of the random variables. For each variable, the following input is required:

- **The type of distribution - Specify a number as follows:**

  **Pre-Defined Distributions**

<table>
<thead>
<tr>
<th>Number</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Deterministic</td>
</tr>
<tr>
<td>1</td>
<td>Normal</td>
</tr>
<tr>
<td>2</td>
<td>Log-Normal</td>
</tr>
<tr>
<td>3</td>
<td>Maxima Type I (Gumbel)</td>
</tr>
<tr>
<td>4</td>
<td>Rectangular (Uniform)</td>
</tr>
<tr>
<td>5</td>
<td>Gamma</td>
</tr>
<tr>
<td>6</td>
<td>Beta</td>
</tr>
<tr>
<td>7</td>
<td>Maxima Type II (Frechet)</td>
</tr>
<tr>
<td>8</td>
<td>Exponential</td>
</tr>
<tr>
<td>9</td>
<td>Rayleigh</td>
</tr>
<tr>
<td>10</td>
<td>Minima Type III (Weibull)</td>
</tr>
</tbody>
</table>

  **User-Defined Distributions**

<table>
<thead>
<tr>
<th>Number</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Water Levels</td>
</tr>
<tr>
<td>12</td>
<td>Extreme Water Levels</td>
</tr>
<tr>
<td>13</td>
<td>Tide Levels</td>
</tr>
</tbody>
</table>

  The abbreviations used in the program are given above in inverted commas.

- **If the variable is deterministic, give its constant value.**

- **If the variable has a user-defined distribution, the mean and the standard deviation are required in the same line. Note that in this case, an ASCII data file has to be prepared containing the values of the variable, \( X_i \), and the corresponding values of the PDF and the CDF. Intermediate values are obtained by linear interpolation.**

- **If the variable is not deterministic or it does not have a user-defined distribution, choose the type of truncation, by giving a number as follows:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Not truncated</td>
</tr>
<tr>
<td>1</td>
<td>Truncated for ( X ) above ( X_0 )</td>
</tr>
<tr>
<td>2</td>
<td>Truncated for ( X ) below ( X_0 )</td>
</tr>
</tbody>
</table>

- **For the failure mode overtopping, if the variable is the significant wave height, the seawall toe level, \( TL \), has to be given. In this case, if the distribution is truncated on the right side, \( X_0 \) is calculated from eq. (5.4). If the distribution is truncated on the left side, give the value of \( X_0 \).**

- **For other truncated distributions, give the value of \( X_0 \).**
- If the variable is described by a Weibull distribution then the mean, the standard deviation and the lower limit have to be specified in this order, in the same line.
- If the variable is described by a Beta distribution then the mean, the standard deviation, the lower limit and the upper limit have to be specified in this order, in the same line.
- If the variable is not Weibull or Beta distributed, only the mean and the standard deviation are required, in the same line.

If combinations of actions are considered with modified distributions, then the input data for all the variables have to be repeated for each combination of actions. Alternatively, if basic distributions are given, the input data for all the variables are required only once. This is because the characteristics of the variables are the same for all combinations of actions; what differs in each combination is the power to which each distribution is raised (see Section 2.3.3.3).

coeffcor.dad

The file coeffcor.dad should contain the correlation coefficients, $\rho_{ij}$, of the random variables. For example, for $i=1,...,3$ and $j=1,...,3$, they should be listed in the following order:

\begin{align*}
\rho_{11} \\
\rho_{12} \\
\rho_{13} \\
\rho_{21} \\
\rho_{22} \\
\rho_{23} \\
\rho_{31} \\
\rho_{32} \\
\rho_{33}
\end{align*}
The file *perfil.dad* should contain the following dune erosion parameters:

- Coastal curvature in degrees per 1000m.
- Number of points defining the initial profile (the maximum number allowed by the program is 100).
- Beach profile coordinates, (XP,YP). The coordinates of each point should be provided in the same line, with the Y-coordinate following the X-coordinate. The coordinates should be given, starting at the most landward location and moving seaward. Note that the X values are positive seaward and the Y values are positive upward, with respect to the origin of coordinates.
- Number of points to be changed in the initial profile.
- Number of the first point to be changed.
- Gradient of the eroded dune face, 1:md (md should be provided).
- Gradient of the toe of the post-storm profile, 1:mt (mt should be provided).
- Nourishment top level.
- Gradient of the nourished face, 1:mnour (mnour should be provided).
APPENDIX C7 - Program Listing

###############################
PARASODE (Probabilistic Assessment of Risks Associated with
Seawall Overtopping & Dune Erosion)
###############################

PARASODE is a Level II FORTRAN 77 program which uses the First
Order Reliability Method (FORM) for assessing the safety of
coastal structures. In particular, it concentrates on the
potential failure mechanisms associated with wave overtopping of
seawalls & dune erosion. The amount of wave overtopping is
calculated by both the H&H equation & Owen’s formula; dune
erosion is calculated using Vellinga’s model. Although the
program incorporates these two specific failure mechanisms, the
majority of the code is generic & can be adapted to other types
of failure without undue difficulty.

PARASODE incorporates routines for transforming the correlated
variables to a set of non-correlated variables & for mapping
non-Normal distributions to equivalent Normal distributions.
PARASODE operates in two modes: 1) Mode 1, the analysis mode,
in which the failure probability is calculated for a given value
of the design parameter; 2) Mode 2: the design mode, in which
the value of a specific design parameter is calculated for a
target probability of failure. Mode 1 allows for combinations of
time-varying actions using the method of Perry Borges &
Castanheta (1983).

SI units are used within the program, except if otherwise
specified.

The program execution may be terminated at any time by pressing,
simultaneously, the keys "Ctrl" and "Pause".

###############################
INPUT VARIABLES:
Optio - Data source
Opti - End the run of the program or restart calculations

MODELING VARIABLES:
Opt - Failure mode
TL - Seawall toe level
FDer - Method of calculation of the first partial derivatives of
the failure function for overtopping
DSWL - Definition of the SWL
Mode - Purpose of the analysis
Comb - Consideration or not of combination of actions
NCombAc, CombAc - Number of combinations of actions
Q - Maximum number of combinations of actions allowed by the
program
NR - Power to which each distribution is raised for each
combination of actions
Distr - Distributions provided for the combination of actions
N - Number of variables
L - Maximum number of variables allowed by the program
Ext - Abbreviation of the name of the variable
ExtExt - Description of the variable
ParamDesc - Description of the design parameter
Rho - Correlation coefficient
Mux - Mean of X
Sigmax - Standard deviation of X
VarDis - Type of distribution
Abrev - Abbreviation of the name of the distribution
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
XMIn - Minimum value of X
XMax - Maximum value of X
StartPt - Starting value of the variables
MaxIter - Maximum number of iterations
NumCalc - Number of FORM calculations
L2 - Maximum number of FORM calculations allowed by the program
ReqBetaAcc - Required relative accuracy of the reliability index
Smooth - Smoothing coefficient for the iteration process
ReqOBJFAcc - Required accuracy of the failure function
Life - Design life of the structure
**Program Listing**

C StartParam - For Mode=1, it is the prescribed value of the
design parameter; For Mode=2, it is the starting
value of the design parameter (from which the
program iterates to find the required value of the
design parameter)

Pf - Design target failure probability
RelInd - Reliability index which corresponds to Pf
Prob - Probability of failure for a specific FORM calculation
Pro - Probability of failure for a specific FORM calculation &
for a specific combination of actions

j, k0, Ex, Aux - Auxiliary variables
TR, Zeta, Lamda, Eta, xI, x2, NPD, t, NPch, NP01d, MuxN, XP,
YP, md, mt, mnoun, courtlev, ctcurv, XPOld, YPOld, Ac, GB, SD,
It, AuxRstar, C, CI,
T3, OptC, OptD - Variables mentioned in the Common statements
but not used here

**OUTPUT VARIABLES:**

PFTotal - Total probability of failure considering all the
combinations of actions
Reliab - Reliability associated with PFTotal

Implicit None
Integer*4 j,k0,Q,L,L2,N,NPD,Mode,Opt,Optio,FDer,Life,t,NPch,
1 NPD01d,OptB,Comb,CombAc,NCombAc,Distr,MaxIter,
1 NumCalc,Aux,It,AuxRstar,Ex,DSWL
Parameter (L=15)
Parameter (L2=10)
Parameter (Q=16)
Character*1 Opti
Character*3 Abrev(Q,L),Ext(L)
Character*17 ExtExt(L)
Character*19 ParamDesc
Integer*4 VarDis(Q,L),Trunc(Q,L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L),Eta(Q,L),
1 xI(Q,L),x2(Q,L),Rho(L,L),Prob(Q),PFTotal(L2),Reliab(L2),
1 Pro(Q,L2),StartParam,Pf,RelInd,MuxN,XP,YP,md,mt,mnoun,
1 XPOld,YPOld,nourtlev,Ac,GB,SD,C,T3,ctcurv,TR(L2),
1 StartPt(Q,L),Xo(Q,L),XMax(Q,L),XMin(Q,L),ReqBetaAcc,
1 Smooth,ReqOBJFAcc,RR(Q,L),TL,C1
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK2/t,NPch,XPOld(100),YP01d(100),NP01d
Common/BLOCK3/MuxN(15),C,T3
Common/BLOCK4/md,mt,mnoun,ctcurv
Common/BLOCK5/Ac,GB,SD
Common/BLOCK6/Tr
Common/BLOCK7/TR
Common/BLOCK8/Zeta,Lamda,Eta,xI,x2
Common/BLOCK9/k0,It,AuxRstar
Common/BLOCK10/C1
Common/BLOCK11/DSWL,TL

Open(Unit=40, File='summary.dat', Status='unknown')
Open(Unit=50, File='results.dat', Status='unknown')

C Definition of the data source.
C =============================================================================
80 Write(*,21)
21 Format(/ 3X,'WHAT IS THE DATA SOURCE ? ' // 11X,
1 1 'The Screen ...... [ 1 ]' / 11X,
1 2 'A Datafile ...... [ 2 ]'/)
878 Write(*,99)
99 Format(3X,'Select Option: ',G)
Read(*,*) Optio
If (Optio.NE.1.AND.Optio.NE.2) goto 878
Write(40,2387) Optio
Write(50,2387) Optio
2387 Format(3X,'Select Option: ',I1)

C Definition of the input data.
C =============================================================================
If (Optio.EQ.1) then
Call Dadscreen(NCombAc,Distr,Comb,Mode,Opt,N,Mux,Signmax,
1 VarDis,Abrev,Ext,ParamDesc,Trunc,Xo,MaxIter,
1 NumCalc,StartPt,XMax,XMin,ReqBetaAcc,Smooth,
1 ReqOBJFAcc,RR,Rho,Life,CombAc,StartParam,Pf,
1 RelInd,ExtExt,FDer)
else
  Call Dadfile(NCombAc,Distr,Comb,Mode,Opt,N,Mux,Sigmax,VarDis,Abrev,Ext,ParamDesc,Trunc,Xo,MaxIter,NumCalc,StartPt,XMax,XMin,ReqBetaAcc,Smooth,ReqOBJFAcc,
  NR,Rho,Life,CombAc,StartParam,Pf,RelInd,ExtExt,FDer)
Endif

C    =========================================== =====================
C    Level II calculations for Mode 1 & Mode 2.
C    =========================================== =====================
If (Mode.EQ.1) then
  Do 3010 j=1,NCombAc
  If ((NCombAc).GT.1) then
    Write(*,713) j
    Write(40,713) j
    Write(50,713) j
  713      Format(/// 3X,'COMBINATION No.',I2)
  Endif
  Do 5950 k0=1,NumCalc
    Ex=0
    Call D1Point(j,N,Opt,Mux,Sigmax,VarDis,Abrev,Rho,FDer,
    NR,StartParam,Prob,Ext,ExtExt,ParamDesc,Trunc,Xo,MaxIter,StartPt,XMax,XMin,
    ReqBetaAcc,Smooth,ReqOBJFAcc,Comb,Ex)
    If (Ex.EQ.0) then
      Pro(j,k0)=Prob(j)
      PAUSE
  5950    continue
  C         ----------
  C         Calculation of the total probability of failure, PFTotal,
  C         considering all the combinations of actions.
  C         ----------
  If (Ex.EQ.0) then
    Do 6400 k0=1,NumCalc
    PFTotal(k0)=0.
    Do 6868 j=1,NCombAc
       PFTotal(k0)=PFTotal(k0)+Pro(j,k0)
    6868         continue
    Reliab(k0)=1.-PFTotal(k0)
    Aux=1
    Write(*,133)
    Write(40,133)
    Write(50,133)
  133      Format(/)
    Call Allowed(Opt,Aux)
    Write(*,68) Life,(100.*PFTotal(k0)),Reliab(k0)
    Write(40,68) Life,(100.*PFTotal(k0)),Reliab(k0)
    Write(50,68) Life,(100.*PFTotal(k0)),Reliab(k0)
  68      Format(3X,'DESIGN LIFE OF THE STRUCTURE = ',I3
  1                     / 3X,'TOTAL PROBABILITY OF FAILURE (%) = ',F10.6 /
  1                     3X,'RELIABILITY = ',E17.10)
  6400    continue
Endif
else
  Do 5960 k0=1,NumCalc
    j=1
    Call D2Point(j,N,Opt,Mux,Sigmax,VarDis,Abrev,Rho,FDer,
    NR,StartParam,Pf,Ext,ExtExt,ParamDesc,Trunc,
    Xo,RelInd,MaxIter,StartPt,XMax,XMin,Smooth,
    ReqOBJFAcc,Comb)
    PAUSE
  5960    continue
Endif
C    =========================================== =====================
C    End of the main program.
C    =========================================== =====================
1122  Write(*,14)
  14      Format(/ 3X,'WOULD YOU LIKE TO RESTART (Y/N) ? ',$,)
    Read(*,2244) Opti
  2244    Format(A1)
    If (Opti.NE.'Y'.AND.Opti.NE.'y'.AND.
    1      Opti.NE.'N'.AND.Opti.NE.'n') goto 1122
    Write(*,5677)
  5677    Format(/)
    Write(40,256) Opti
    Write(50,256) Opti
  256    Format(/ 3X,'WOULD YOU LIKE TO RESTART (Y/N) ? ',A1 ///)
    If (Opti.EQ.'Y'.OR.Opti.EQ.'y') goto 80
End
Subroutine Dadscreen(NCombAc,Distr,Comb,Mode,Opt,N,Mux,Sigmax,VarDis,Abrev,Ext,ParamDesc,Trunc,Xo,MaxIter,NumCalc,StartPt,XXMax,XXMin,ReqBetaAcc,Smooth,ReqOBJFAcc,NR,Rho,Life,CombAc,StartParam,Pf,RelInd,ExtExt,FDer)

Reads the required input data from the screen.

MODELING VARIABLES:
- OptC - Confidence value of the maximum run-up
- Der - Method of calculation of the first partial derivatives of the failure function for overtopping
- CAcc - Consideration or not of combination of actions
- Q - Maximum number of combinations of actions allowed by the program
- NumTVAc - Number of time-varying actions
- r - Repetitions of each action in the design life
- TVAc - Number of the time-varying actions in increasing order of the number of repetitions
- Carac - Name of the distribution
- Def1 - Definition of the limiting values of X
- Def - Definition of the starting point for the FORM calculations
- Def2 - Definition of the required relative accuracy of the reliability index
- Def3 - Definition of the required smoothing coefficient for the iteration process
- Def4 - Definition of the required accuracy of the failure function
- i, j, k, k0, Aux - Auxiliary variables
- It, AuxRstar - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLES:
- Opt - Failure mode
- DSWL - Definition of the SWL
- C1 - Parameter used in the H&R model to calculate C; it depends on the confidence value assigned to the maximum run-up
- TL - Seawall toe level
- Mode - Purpose of the analysis
- FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
- Comb - Consideration or not of combination of actions
- NCombAc, CombAc - Number of combinations of actions
- NR - Power to which each distribution is raised for each combination of actions
- Distr - Distributions provided for the combination of actions
- N - Number of variables
- Ext - Abbreviation of the name of the variable
- ExtExt - Description of the variable
- ParamDesc - Description of the design parameter
- Rho - Correlation coefficient
- Mux - Mean of X
- Sigmax - Standard deviation of X
- VarDis - Type of distribution
- Abrev - Abbreviation of the name of the distribution
- Trunc - Type of truncation
- Xo - Point of truncation (if the distribution is truncated)
- XXMin - Minimum value of X
- XXMax - Maximum value of X
- Zeta, Lambda, Eta - Parameters of a distribution
- x1 - Lower limit on X for a Beta distribution
- x2 - Upper limit on X for a Beta distribution
- StartPt - Starting value of the variables
- MaxIter - Maximum number of iterations
- NumCalc - Number of FORM calculations
- TR - Target values for each FORM calculation
- ReqBetaAcc - Required relative accuracy of the reliability index
- Smooth - Smoothing coefficient for the iteration process
- ReqOBJFAcc - Required accuracy of the failure function
- Life - Design life of the structure
- StartParam - For Mode=1, it is the prescribed value of the
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C design parameter; For Mode=2, it is the starting
value of the design parameter (from which the
program iterates to find the required value of the
design parameter)
Pf - Design target failure probability
RelInd - Reliability index which corresponds to Pf
tccurv - Coastal curvature in degrees per 100mn
NPD, NPD0ld - Number of points defining the initial profile
(XP,YP), (XP0ld,YP0ld) - Coordinates of the points defining the
initial profile
NPch - Number of points to be changed in the initial profile
t - First point to be changed in the initial profile, point no.t
1:md - Gradient of the eroded dune face
1:mt - Gradient of the toe of the post-storm profile
nourtlev - Nourishment top level
1:mmour - Gradient of the nourished face

Integer*4 i,j,k,k0,L,N,NPD,Mode,Opt,NCombAc,Q,t,NPch,NPD0ld,Aux,
1 MaxIter,NumCalc,Def,Def1,Def2,Def3,Def4,Comb,Distr,L2,
1 Life,CombAc,NumTVAc,It,AuxRstar,OptC,OptD,Der,DSWL
1 Parameter (L=15)
Parameter (L2=10)
Parameter (Q=16)
Character*3 Abrev(Q,L),Ext(L)
Character*1 Der,CAcc
Character*17 ExtExt(L)
Character*30 Carac(Q,L)
Character*19 ParamDesc
Integer*4 VarDis(Q,L),Trunc(Q,L),TVAC(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L),Eta(Q,L),
1 x1(Q,L),x2(Q,L),XP,YP,md,mt,mnour,nourtlev,
1 tccurv,TR(L2),StartPt(Q,L),Xo(Q,L),X Max(Q,L),XMin(Q,L),
1 ReqBetaAcc,Smooth,ReqOBFAcc,RP(Q,L),Rho(L,L),StartParam,
1 Pf,RelInd,TL,r(L),C1
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK2/t,NPch,XP0ld(100),YP0ld(100),NPD0ld
Common/BLOCK4/md,mt,mnour,nourtlev,tccurv
Common/BLOCK5/OptD
Common/BLOCK7/TR
Common/BLOCK8/Lamda,Eta,x1,x2
Common/BLOCK9/K0,It,AuxRstar
Common/BLOCK10/C1
Common/BLOCK11/DSWL,TL
Aux=0

---

Definition of the failure mode to be studied.
---

Write(*,2)
Write(40,2)
Write(50,2)
2 Format(// 3X,'WHAT IS THE FAILURE MODE TO BE STUDIED: ')
1 11X, 'Overtopping (H&R) ................. [ 1 ]' /
1 11X, 'Overtopping (Owen) ............... [ 2 ]'
1 11X, 'Dune Erosion (Vellinga) ........ [ 3 ]'
9778 Write(*,97)
97 Format(3X,'Select Option: ',$)
Read(*,*) Opt
If (Opt.NE.1.AND.Opt.NE.2.AND.Opt.NE.3) goto 9778
Write(40,9779) Opt
Write(50,9779) Opt
9779 Format(3X,'Select Option: ',I2)

---

Definition of the still-water-level.
---

If (Opt.EQ.1.AND.Opt.EQ.2.AND.Opt.EQ.3) then
Write(*,9320)
Write(40,9320)
Write(50,9320)
9320 Format(3X,'HOW IS THE STILL-WATER-LEVEL DEFINED ?'
1 11X, 'Total Level .... [ 1 ]'
1 11X, 'Tide + Surge ... [ 2 ]'
3925 Write(*,9473)
39 Format(3X,'Select Option: ',$)
Read(*,*) DSWL
If (DSWL.NE.1.AND.DSWL.NE.2.AND.DSWL.NE.3) goto 3925
Write(40,1437) DSWL
Write(50,1437) DSWL
1437 Format(3X,'Select Option: ',I2)

---
Endif
--------
C Definition of the confidence value of the maximum run-up to be
considered.
--------
If (Opt.EQ.1) then
Write(*,1324)
Write(40,1324)
Write(50,1324)
1324 Format(''/ 3X,'WHAT IS THE CONFIDENCE VALUE OF THE MAXIMUM',1X,'RUN-UP' / 3X,'THAT YOU WOULD LIKE TO CONSIDER ?'
1 // 11X, '37 % ... [ 1 ]' / 1
1 // 11X, '99 % ... [ 2 ]' /)
3225 Write(*,9373)
9373 Format(3X,'Select Option: ',$)
Read('*,' OptC
If (OptC.NE.1.AND.OptC.NE.2) goto 3225
Write(40,1037) OptC
Write(50,1037) OptC
1037 Format(3X,'Select Option: ',I1)
If (OptC.EQ.1) then
C1=1.52
else
C1=2.15
Endif
Endif
--------
C Definition of the method of calculation of the first partial
derivatives of the failure function for overtopping.
--------
2255 If (Opt.NE.3) then
Write(*,333)
333 Format(''/ 3X,'ARE THE FIRST DERIVATIVES OF THE FAILURE',1X,'FUNCTION SUPPLIED (Y/N) ? ',$)
Read('*,3377) Der
3377 Format(3X) Der
If (Der.NE.'Y'.AND.Der.NE.'y'.AND.Der.NE.'N'.AND.Der.NE.'n')
goto 2255
Write(40,6681) Der
Write(50,6681) Der
6681 Format(''/ 3X,'ARE THE FIRST DERIVATIVES OF THE FAILURE',1X,'FUNCTION SUPPLIED (Y/N) ? ',A1)
If ((Der.EQ.'Y').OR.(Der.EQ.'y')) FDer=1
If ((Der.EQ.'N').OR.(Der.EQ.'n')) FDer=2
else
FDer=2
Endif
--------
C Definition of the direction of the sand movements occurring
during a storm surge.
--------
OptD=0
If (Opt.EQ.3) then
Write(*,1329)
Write(40,1329)
Write(50,1329)
1329 Format(''/ 3X,'DURING A STORM SURGE, WOULD YOU LIKE TO TAKE',1X,'INTO ACCOUNT: '
1 // 11X, 'Movements of Sand in Both Directions ?'
1 // 11X, 'Movements of Sand',
1 // 11X,'only Seaward ? .......... [ 2 ]' /)
3115 Write(*,937)
937 Format(3X,'Select Option: ',$)
Read('*,' OptD
If (OptD.NE.1.AND.OptD.NE.2) goto 3115
Write(40,1937) OptD
Write(50,1937) OptD
1937 Format(3X,'Select Option: ',I1)
Endif
--------
C If the failure mode is dune erosion the following quantities
have to be read:
- coastal curvature in degrees per 1000m, ctcurv
- number of points defining the initial profile, NP
- beach profile coordinates, (XP,YP)
- number of points to be changed in the initial profile, NPch
- first point to be changed, point no. t
- gradient of the eroded dune face, 1:md
- gradient of the toe of the post-storm profile, 1:mt
C    - nourishment top level, nourtlev
C    - gradient of the nourished face, 1:mnour

----------

If (Opt.EQ.3) then
  Write(*,7668)
7668    Format(' // 3X,'DUNE EROSION' // 11X,1X,'Coastal Curvature (Deg/1000m) = ',$/
Read(*,*) ctcurv
If (ctcurv.LT.0.OR.ctcurv.GT.24) then
  Write(*,3278)
3278       Format(' // 3X,'ERROR: The Coastal Curvature is not',1X,1X,'Within [0,24] !')
goto 8667
Endif
Write(40,7669) ctcurv
Write(50,7669) ctcurv
7669    Format(' // 3X,'DUNE EROSION' // 11X,1X,'Coastal Curvature (Deg/1000m) = ',E17.10)
7700    Write(*,7700)
7070    Write('/ 11X,'Number of Points Defining the Initial',1X,1X,'Profile (Max=100) = ',$/
Read(*,*) NPD
If (NPD.GT.100.OR.NPD.LE.0) then
  Write(*,3251)
3251       Format(' // 3X,
1X,'ERROR: The Maximum Number of Points Defining the' //
1X,'Initial Profile is not Within [0,100] !')
goto 7070
Endif
Write(40,7701) NPD
Write(50,7701) NPD
7701    Format('/ 11X,'Number of Points Defining the Initial',1X,1X,'Profile (Max=100) = ',I3)
Write(*,0211)
0211       Format('/ 23X, 'Initial Profile' // 19X,'X',18X,'Y' /)
Do 3120 i=1,NPD
  Write(*,0311) i
0311       Format(26X,'X(',I3,') = ',$)
  Read(*,*) XP(i)
  If (i.GE.2) then
    If (XP(i).LT.XP(i-1)) then
      Write(*,1466) i,(i-1)
1466           Format(' // 3X,'ERROR: XP(',I3,') < XP(',I3,') !')
goto 9090
Endif
Endif
Write(*,0317) i
0317       Format(26X,'Y(',I3,') = ',$/
  Read(*,*) YP(i)
  Write(40,0011) XP(i),YP(i)
  Write(50,0011) XP(i),YP(i)
0011       Format(11X,E17.10,2X,E17.10)
XPOld(i)=XP(i)
YPOld(i)=YP(i)
3120     continue
NPD0ld=NPD
9072    Write(*,5325)
5325    Format('/ 11X,'Number of Points to be Changed in the Initial',1X,1X,'Profile ? ',$/
Read(*,*) NPch
If (NPch.LT.1.OR.NPch.GT.NPD) then
  Write(*,1366) NPD
1366       Format('/ 3X, 'ERROR: The Number of Points to be Changed',1X,'in the',1X,'Initial Profile is not Within [1,',1X,'I3,'] !')
goto 9072
Endif
0635    Write(*,5036)
5036    Format('/ 11X,'First Point to be Changed ? Point No. ',$/
Read(*,*) t
If (t.LT.1.OR.t.GT.NPD) then
  Write(*,1322) NPD
1322       Format('/ 3X, 'ERROR: The First Point to be Changed is not Within [1,',1X,'I3,'] !')
goto 6305
Endif
Write(40,7917) NPch,t
Write(50,7917) NPch,t

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7917 Format(/ 11X,'Number of Points to be Changed in the Initial',
1 1X,'Profile = ',I3 // 11X,'First Point to be Changed',
1 1X,'Point No.',I3)
4546 Write(*,3007)
3007 Format(/ 11X,'Gradient of the Eroded Dune Face = 1:',$)
Read(*,*) md
If (md.LE.0) then
Write(*,1311)
1311 Format(/ 3X,'ERROR: The Gradient of the Eroded Dune',
1 1X,'Face is <= 1:0 !')
goto 4546
Endif
4547 Write(*,4396)
4396 Format(/ 11X,'Gradient of the Toe of the Post-Storm Profile',
1 1X,’= 1:',$)
Read(*,*) mt
If (mt.LE.0) then
Write(*,1151)
1151 Format(/ 3X,'ERROR: The Gradient of the Toe of the',
1 1X,’Post-Storm Profile is <= 1:0 !')
goto 4547
Endif
Write(*,3407)
3407 Format(/ 11X,'Nourishment Top Level = ',$)
Read(*,*) nourtlev
5599 Write(*,4346)
4346 Format(/ 11X,'Gradient of the Nourished Face = 1:',$)
Read(*,*) mnour
If (mnour.LE.0) then
Write(*,7103)
7103 Format(/ 3X,'ERROR: The Gradient of the Nourished',
1 1X,’Face is <= 1:0 !')
goto 5599
Endif
Write(40,7933) md,mt,nourtlev,mnour
Write(50,7933) md,mt,nourtlev,mnour
7933 Format(/ 11X,'Gradient of the Eroded Dune Face = 1:',F4.1
1 // 11X,'Gradient of the Toe of the Post-Storm Profile',
1 1X,’= 1:',F4.1 // 11X,’Nourishment Top Level = ',E17.10
1 // 11X,’Gradient of the Nourished Face = 1:',F4.1)
Endif
C
Definition of the number, N, of variables for each failure mode.
C
If (Opt.EQ.2.OR.Opt.EQ.3) then
N=7
else
N=8
Endif
If (DSWL.EQ.2) then
If (Opt.EQ.2.OR.Opt.EQ.3) N=8
If (Opt.EQ.1) N=9
Endif
C
Description of each variable for the failure mode chosen.
C
Call VarExt(N,Opt,Ext,ExtExt,ParamDesc)
Write(*,1009)
Write(40,1009)
Write(50,1009)
1009 Format(/ 3X,'DESCRIPTION OF THE VARIABLES' /
Do 1603 i=1,N
Write(*,1018) i,Ext(i),ExtExt(i)
Write(40,1018) i,Ext(i),ExtExt(i)
Write(50,1018) i,Ext(i),ExtExt(i)
1018 Format(11X,'X(',I3,') = ',A3,’ = ',A17)
1603 continue
C
Definition of the purpose of the analysis.
C
Write(*,1017)
Write(40,1017)
Write(50,1017)
1017 Format(/ 3X,'WHAT IS THE PURPOSE OF THE ANALYSIS ? ' // 11X,
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1 'Reliability Analysis for a Specified Design ... [ 1 ]'
1 / 11X,
1 'Design for a Specified Reliability Level ...... [ 2 ]' /
7101 Write(*,96)
96 Format(3X,'Select Option: ',$,)
Read(*,*) Mode
If (Mode.NE.1.AND.Mode.NE.2) goto 7101
Write(40,9696) Mode
Write(50,9696) Mode
9696 Format(3X,'Select Option: ',I1)

C
C Reads the design life of the structure, Life.
C
888 Write(*,091)
091 Format(// 3X,'DESIGN LIFE OF THE STRUCTURE = ',$,)
Read(*,*) Life
If (Life.LE.0) then
Write(*,7882)
7882 Format(// 3X,'ERROR: The Design Life of the Structure <= 0 !')
goto 888
Endif
Write(40,791) Life
Write(50,791) Life
791 Format(// 3X,'DESIGN LIFE OF THE STRUCTURE = ',I3)

C
C Definition of the combination of actions for Mode 1.
C
If (Mode.EQ.1) then
4901 Write(*,3334)
3334 Format(// 3X,'WOULD YOU LIKE TO CONSIDER',1X,
1 ' COMBINATION OF ACTIONS (Y/N) ? ',$,)
Read(*,3335) CAcc
3335 Format(A1)
If (CAcc.NE.'Y'.AND.CAcc.NE.'y'.AND.
1 CAcc.NE.'N'.AND.CAcc.NE.'n') goto 4901
Write(40,6622) CAcc
Write(50,6622) CAcc
6622 Format(/ 3X,'WOULD YOU LIKE TO CONSIDER',1X,
1 ' COMBINATION OF ACTIONS (Y/N) ? ',A1)
If ((CAcc.EQ.'Y').OR.(CAcc.EQ.'y')) Comb=1
If ((CAcc.EQ.'N').OR.(CAcc.EQ.'n')) Comb=2
else
Comb=2
Endif
C
C Definition of the number of combination of actions, NCombAc.
C
If (Comb.EQ.2) then
2388 continue
else
Write(*,1320)
Write(40,1320)
Write(50,1320)
1320 Format(/ 3X,'HOW MANY COMBINATIONS WOULD YOU LIKE TO',
1 ' CONSIDER ? ' / 11X,
1 ' The Number of Time-Varying Actions (k) ... [ 1 ]' /
1 ' 2^(k-1) ............................. [ 2 ]' //
8832 Write(*,9370)
9370 Format(3X,'Select Option: ',$,)
Read(*,*) CombAc
If (CombAc.NE.1.AND.CombAc.NE.2) goto 8832
Write(40,1007) CombAc
Write(50,1007) CombAc
1007 Format(3X,'Select Option: ',I1)

C
C Definition of the distributions provided for each combination
C of actions.
C
Write(*,1550)
Write(40,1550)
Write(50,1550)
1550 Format(/ 3X,'WHICH DISTRIBUTIONS WOULD YOU LIKE TO',
1 ' PROVIDE ? ' / 11X,
1 ' The Basic Distributions ...... [ 1 ]' / 11X,
1 ' The Modified Distributions ... [ 2 ]' //
551 Write(*,9550)
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9550 Format(3X,'Select Option: ',$)
Read(*,*), Distr
If (Distr.NE.1.AND.Distr.NE.2) goto 551
Write(40,1557) Distr
Write(50,1557) Distr
Endif

If (Comb.EQ.1) then
----------
C
C  Reads the number of time-varying actions, NumTVAc (maximum=5)
& calculates the number of combinations of actions, NCombAc.
C
----------
290 Write(*,092)
092 Format(/ 3X,'NUMBER OF TIME-VARYING ACTIONS (Max=5) = ',$)
Read(*,*), NumTVAc
If (NumTVAc.GT.5.OR.NumTVAc.LE.0) then
Write(*,7887)
7887 Format(// 3X,'ERROR: The Maximum Number of Time Varying Actions is not Within [0,5] !')
goto 290
Endif
Write(40,7991) NumTVAc
Write(50,7991) NumTVAc
7991 Format(// 3X,'NUMBER OF TIME-VARYING ACTIONS (Max=5) = ',I1)
If (CombAc.EQ.1) then
NCombAc=NumTVAc
else
NCombAc=2**(NumTVAc-1)
Endif
If (Distr.EQ.1) then
----------
C
C  Reads the number of the time-varying actions, TVAC, in increasing order of the number of repetitions, r. Reads the number of repetitions of each action in the design life of the structure.
C
----------
7833 Format(// 3X,'NUMBER OF THE TIME-VARYING ACTIONS IN INCREASING ORDER OF THE NUMBER OF REPETITIONS' /)
Do 140 i=1,NumTVAc
2377 Write(*,7832) i
7832 Format(11X,'Action ('I2,') = ',$)
Read(*,*), TVAc(i)
If (TVAc(i).GT.5.OR.TVAc(i).LE.0) then
Write(*,7697) N
7697 Format(//3X,'ERROR: The Value of the Number of the Time Varying Action is not Within ]0,5] !')
goto 2377
Endif
140 continue
Do 4000 i=1,NumTVAc
Write(40,788) Ext(TVAc(i)),r(TVAc(i))
Write(50,788) Ext(TVAc(i)),r(TVAc(i))
788 Format(11X,'r(',A3,') = ',F8.0)
4000 continue
Call Combination(N,NumTVAc,CombAc,NCombAc,r,TVAc,NR)
Do 8711 i=1,NCombAc
  Do 8712 j=1,N
    Write(50,*) 'NR(i,j)=',i,j,NR(i,j)
  8712 continue
8711 continue
else
  Do 88 i=1,NCombAc
    Do 88 j=1,N
      NR(i,j)=1
    88 continue
  88 continue
Endif
Endif

If (Mode.EQ.1) then
  C =============================================================
  C Reads the value of the design parameter, StartParam, for which
  C the failure probability (or reliability) is to be found.
  C ==============================================================
2009 Write(*,9002) ParamDesc
9002 Format(// 3X,'PRESCRIBED VALUE OF THE DESIGN PARAMETER' //
1 11X,A19,' = ',$,)
  Read(*,*) StartParam
  If (StartParam.LT.0) then
    Write(*,6554)
  6554 Format(// 3X,'ERROR: The Prescribed Value of the',
1 1X,'Design Parameter < 0 !')
  goto 2009
Endif
Write(40,7891) ParamDesc,StartParam
Write(50,7891) ParamDesc,StartParam
7891 Format(// 3X,'PRESCRIBED VALUE OF THE DESIGN PARAMETER' //
1 11X,A19,' = ',E17.10 /)
else
  C ==============================================================
  C Reads the design target failure probability, Pf, & starting
  C value of the design parameter, StartParam, from which the
  C program iterates to find the required value of the design
  C parameter for the given failure probability (or reliability).
  C ==============================================================
2101 Write(*,1044)
1044 Format(// 3X, 'DESIGN TARGET FAILURE PROBABILITY [0,1] = ',$)
  Read(*,*) Pf
  If (Pf.LT.0.OR.Pf.GT.1) then
    Write(*,6234)
  6234 Format(//3X,'ERROR: The Value of the Design Target Failure',
1 3X,'Probability is not Within [0,1] !')
  goto 2101
Endif
Write(40,7791) (100.*Pf),ParamDesc,StartParam
Write(50,7791) (100.*Pf),ParamDesc,StartParam
7791 Format(// 3X,'DESIGN TARGET FAILURE PROBABILITY (%) = '
1 3X,'STARTING VALUE OF THE DESIGN PARAMETER - ',A19 / 3X,'
1 'Parameter should not be set to zero !' / 11X,
1 'So, the program assumes a starting value of 5m.')
  StartParam=5.
else
  Write(40,7751) (100.*Pf),ParamDesc,StartParam
  Write(50,7751) (100.*Pf),ParamDesc,StartParam
7751 Format(// 3X,'DESIGN TARGET FAILURE PROBABILITY (%) = '
1 3X,'STARTING VALUE OF THE DESIGN PARAMETER - ',A19 /
1 / 3X,’(to start iteration) = ’,E17.10) 
Endf

Calculation of the reliability index, RelInd, which
corresponds to the target failure probability, Pf.
Call InvNormal(Pf,RelInd)
Write(40,8792) RelInd
Write(50,8792) RelInd

Endf

Calculation of the reliability index, RelInd, which
corresponds to the target failure probability, Pf.
Call InvNormal(Pf,RelInd)
Write(40,8792) RelInd
Write(50,8792) RelInd

Do 6633 j=1,NCombAc
If ((NCombAC.GT.1).AND.(Distr.EQ.2)) then
Write(*,7073) j
Write(40,7073) j
Write(50,7073) j
Endif
Write(*,8777)
Write(40,8777)
Write(50,8777)

Do 18 i=1,N
Write(*,717) Ext(i)
Endef

Do 18 i=1,N
Write(*,717) Ext(i)
Endef

Write(*,796)
Read(*,*) VarDis(j,i)
If (VarDis(j,i).NE.0.AND.VarDis(j,i).NE.1.AND.
VarDis(j,i).NE.2.AND.VarDis(j,i).NE.3.AND.
VarDis(j,i).NE.4.AND.VarDis(j,i).NE.5.AND.
VarDis(j,i).NE.6.AND.VarDis(j,i).NE.7.AND.
VarDis(j,i).NE.8.AND.VarDis(j,i).NE.9.AND.
VarDis(j,i).NE.10.AND.VarDis(j,i).NE.11.AND.
VarDis(j,i).NE.12.AND.VarDis(j,i).NE.13.goto 697
If (VarDis(j,i).NE.0.AND.VarDis(j,i).LE.10) then
Write(*,275) Ext(i)
Endif

If (Trunc(j,i).NE.0.AND.Trunc(j,i).NE.1.AND.
Trunc(j,i).NE.2) goto 6971
If (Trunc(j,i).NE.0) then

---
For the failure mode of overtopping, if the variable is
the significant wave height, Hs, & if Hs is limited by the
available water depth, then the point of truncation is
Xo=0.6(SWL-TL) or Xo=0.6(Tide+Surge-TL), where TL is the
seawall toe level.

---

If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(i.EQ.2).AND.(Trunc(j,i).EQ.1)) then
    Write(*,7511)
    7511 Format(/ 11X,'Seawall Toe Level (TL) = ',$)
    Read(*,*) TL
    goto 5555
Endif
Write(*,751) i
751 Format(/ 11X,'Value of X(',I2,') at which the distribution is to be truncated = ',$)
Read(*,*) Xo(j,i)
Endif
If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(i.EQ.2)) then
    Write(*,7561)
    7561 Format(/ 11X,'Seawall Toe Level (TL) = ',$)
    Read(*,*) TL
    Write(40,7881) TL
    Write(50,7881) TL
    7881 Format(11X,'Seawall Toe Level (TL) = ',E17.10)
Endif
5555 If (VarDis(j,i).NE.0) then
    Write(*,75) Ext(i)
    75 Format(/ 11X,'Mean Value of ',A3,' = ',$)
    Read(*,*) Mux(j,i)
    Write(*,775) Ext(i)
    775 Format(11X,'Standard Deviation of ',A3,' = ',$)
    Read(*,*) Sigmax(j,i)
else
    Write(*,7533) Ext(i)
    7533 Format(/ 11X,'Value of ',A3,' = ',$)
    Read(*,*) Mux(j,i)
    Sigmax(j,i)=0
Endif
If (VarDis(j,i).EQ.6) then
    1313 Write(*,9952) Ext(i)
    9952 Format(/ 11X,'Insert Limit a (a <= ',A3,' <= b): ',$)
    Read(*,*) x1(j,i)
    Write(*,9953) Ext(i)
    9953 Format(/ 11X,'Insert Limit b (a <= ',A3,' <= b): ',$)
    Read(*,*) x2(j,i)
    If (x1(j,i).GE.x2(j,i)) then
        Write(*,8703)
        8703 Format(/// 11X,'ERROR: a >= b !'/)
        goto 1313
    Endif
Endif
Endif
If (VarDis(j,i).EQ.10) then
    6633 continue
Endif
C Calculation of the distribution's parameters, Zeta, Lambda & Eta, for each variable.
C=================================================================================================
Write(50,4495)
4495 Format('// 6X,'DISTRIBUTION'S PARAMETERS')
Do 6070 i=1,N
    Call Parameters(i,j,Mux,Sigmax,VarDis,Ext)
6070 continue
If (Distr.EQ.1) then
    If (NCombAc.GT.1) then
        Call EqCharac(N,NCombAc,Abrev,Carac,VarDis,Trunc,Xo,Mux,
        Sigmax)
        Write(*,9911)
        Write(40,9911)
        Write(50,9911)
    9911 Format(/// 11X,'THE CHARACTERISTICS OF THE VARIABLES ARE ',$
    1 / 11X,'THE SAME FOR ALL COMBINATIONS OF ACTIONS,','$
    1 //)
    Endif
goto 1314
Endif
6633 continue
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C Reads the correlation coefficients, Rho, of the variables.
C
1314 Write(*,971)
Write(40,971)
Write(50,971)
971 Format('/3X, 'CORRELATION COEFFICIENTS' /)
Do 38 i=1,N
Do 37 j=1,N
Write(*,74) Ext(i),Ext(j)
Format(11X,'(',A3,',',A3,') = ',$)
Read(*,*) Rho(i,j)
Write(40,536) Ext(i),Ext(j),Rho(i,j)
Write(50,536) Ext(i),Ext(j),Rho(i,j)
536 Format(11X,'(',A3,',',A3,') = ',E17.10)
37 continue
38 continue
9988 Do 4005 i=1,N
Do 4050 j=1,N
If ((Rho(i,j) .NE. Rho(j,i)).OR.(ABS(Rho(i,j)).GT.1.)) then
If (Rho(i,j) .NE. Rho(j,i)) then
Write(*,7801) i,j
Format('/// 11X,'ERROR: Rho(',I2,',',I2,') is not equal to Rho(',I2,',',I2,') !' //)
Write(*,7444) i,j
7801 Format(// 11X,'COMBINATION No.',I2)
Write(40,7079) j
Write(50,7079) j
7079 Format(/// 11X,'STARTING POINT FOR THE FORM CALCULATIONS: ' //
1 11X,'Default Values (means) ... [ 1 ]' /)
4455 Format(11X,'(',A3,',',A3,') = ',E17.10)
362 Format('/// 11X,')
Endif
Endif
739 Write(*,9477)
7901 Format('/// 11X,'ERROR: |Rho(',I2,',',I2,')| > 1 !' //)
7455 Write(*,7445) j,i
Write(40,7079) i,j
Write(50,7079) i,j
7079 Format(/// 11X,'COMBINATION No.',I2)
Write(40,7079) j
Write(50,7079) j
7079 Format(/// 3X,'COMBINATION No.',I2)
Endif
4455 Format(11X,'(',A3,',',A3,') = ',E17.10)
11X,')
Endif
4455 Write(*,7445) j,i
Write(40,7079) i,j
Write(50,7079) i,j
7079 Format(/// 3X,')
Endif
4455 Format(11X,'COMBINATION No.',I2)
Write(*,9477)
Endif
4050 continue
4005 continue

C Reads the characteristics of the FORM calculations:
C - starting value of the variables, StartPt
C - minimum value, XMin, & maximum value, XMax, of the variables
C - maximum number of iterations, MaxIter
C - number of FORM calculations, NumCalc
C - target values for each FORM calculation, TR
C - required accuracy of the reliability index, ReqBetaAcc
C - smoothing coefficient for the iteration process, Smooth
C - required accuracy of the failure function, ReqOBJFAcc
C
--------
C Starting value of the variables, StartPt.
C
--------
Do 78 j=1,NCombAC
If (NCombAC.GT.1) then
If (Distr.EQ.2) Write(*,7079) j
Write(40,7079) j
Write(50,7079) j
7079 Format('/3X,'COMBINATION No.',I2)
Endif
4455 If ((Distr.EQ.2).OR.(Comb.EQ.2)) then
Write(*,1328)
Write(40,1328)
Write(50,1328)
1328 Format('/3X,'STARTING POINT FOR THE FORM CALCULATIONS: ' //
1 11X, 'Default Values (mean values) ... [ 1 ]' /)
4455 Write(*,1328)
Write(40,1328)
Write(50,1328)
1328 Format('/3X,'COMBINATION No.',I2)
Endif
739 Write(*,9477)
9477 Format(3X,'Select Option: ',5$)
Read(*,*) Def
If (Def.NE.1.AND.Def.NE.2) goto 739
Write(40,8833) Def
Write(50,8833) Def
8833 Format(3X,'Select Option: ',1I1)
Write(40,5671)
Write(50,5671)
Write(50,5671)
5671 Format(/ 3X,'STARTING POINT' /)
If (Def.EQ.1) then
  Do 2156 i=1,N
    StartPt(j,i)=Mux(j,i)
    Write(*,5670) Ext(i),StartPt(j,i)
    Write(40,5670) Ext(i),StartPt(j,i)
    Write(50,5670) Ext(i),StartPt(j,i)
    5670 Format(11X,A3,' = ',E17.10)
  continue
else
  Do 2155 i=1,N
  If (VarDis(j,i).NE.0) then
    Write(*,1132) Ext(i)
    Format(11X,A3,' = '$)
    Read(*,*) StartPt(j,i)
    Write(40,5680) Ext(i),StartPt(j,i)
    Write(50,5680) Ext(i),StartPt(j,i)
    5680 Format(11X,A3,' = ',E17.10)
  else
    StartPt(j,i)=Mux(j,i)
  endif
  2155 continue
endif
If (Distr.EQ.1) then
  Write(40,5636)
  Write(50,5636)
  5636 Format(/ 3X,'STARTING POINT' /)
  Do 2956 i=1,N
  If (NR(j,i).NE.1.) then
    Call Inverse(j,i,Mux,NR,VarDis,StartPt)
  else
    StartPt(j,i)=Mux(j,i)
  endif
  Write(40,5629) Ext(i),StartPt(j,i)
  Write(50,5629) Ext(i),StartPt(j,i)
  5629 Format(11X,A3,' = ',E17.10)
  2956 continue
endif
If (Aux.EQ.1) goto 115
78 continue

C -------
C Minimum value, XMin, & maximum value, XMax, of the variables.
C -------
115 Do 789 j=1,NCombAc
  If ((NCombAC.GT.1).AND.(Distr.EQ.2)) then
    Write(*,7074) j
    Write(40,7074) j
    Write(50,7074) j
    7074 Format(/ 3X,'COMBINATION No.',I2)
  endif
  Write(*,1327)
  Write(40,1327)
  Write(50,1327)
  1327 Format(/ 3X,'LIMITING VALUES FOR THE VARIABLES: '/
1 11X, 'Default Values (+/- 1E25) ... [ 1 ]' /
1 11X, 'User Specified Values .......... [ 2 ]' /)
7397 Write(*,7937)
7937 Format(3X,'Select Option: ',4$)
Read(*,*) Def1
If (Def1.NE.1.AND.Def1.NE.2) goto 7397
Write(40,9375) Def1
Write(50,9375) Def1
9375 Format(3X,'Select Option: ',1I1)
If (Def1.EQ.1) then
  Do 2176 i=1,N
  If (VarDis(j,i).NE.0) then
    XMin(j,i)=1E25
    XMax(j,i)=-1E25
  end if
  If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(DSWL.EQ.1)) then
    XMin(j,5)=TL
    XMax(j,5)=StartParam
  end if
  If (XMin(j,5).GT.XMax(j,5)) then

Write(*,8653)
Write(50,8653)

8653
  Format(// 11X,'ERROR: The Toe Level is Above the',
  1 1X,'Seawall Crest Level ')
  STOP
Endif
Endif
else
  XMin(j,i)=Mux(j,i)
  XMax(j,i)=Mux(j,i)
Endif
continue
else
  Write(*,7974)
7974
  Format(// 3X,'LIMITING VALUES FOR THE VARIABLES')
Do 2175 i=1,N
  If (VarDis(j,i).NE.0) then
    Write(*,7874) Ext(i)
    Format(/ 11X,'XMin(',A3,') = ',$)
    Read(*,*) XMin(j,i)
    Write(*,7978) Ext(i)
    Format(11X,'XMax(',A3,') = ',$)
    Read(*,*) XMax(j,i)
    If (XMin(j,i).GT.XMax(j,i)) then
      Write(*,8733) i,i
      Format(// 11X,'ERROR: XMin(',I2,') > XMax(',I2,') !')
goto 1310
    Endif
  Endif
  If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(DSWL.EQ.1)) then
    XMin(j,5)=TL
    XMax(j,5)=StartParam
    If (XMin(j,5).GT.XMax(j,5)) then
      Write(*,8153)
      Write(50,8153)
      8153
        Format(// 11X,'ERROR: The Toe Level is Above the',
        1 1X,'Seawall Crest Level ')
        STOP
      Endif
    Endif
  else
    XMin(j,i)=Mux(j,i)
    XMax(j,i)=Mux(j,i)
  Endif
2175
  continue
Endif
Endif
else
  Write(*,7974)
7974
  Format(// 3X,'LIMITING VALUES FOR THE VARIABLES')
Do 2788 i=1,N
  Call MinMax(i,j,Opt,VarDis,Ext,Trunc,Xo,Max,Si
g-Xm,
1
Max)
  If (StartPt(j,i).LT.XMin(j,i)) then
    Write(*,8977) Ext(i),StartPt(j,i),Ext(i),XMin(j,i)
    8977
      Format(// 3X,'ERROR: Starting Value of ',A3,' = ',E17.10
      / 10X,'< Minimum of ',A3,' = ' ,E17.10,' !')
      Aux=1
    If (Distr.EQ.1) then
      StartPt(j,i)=XMin(j,i)
    else
      goto 4455
    Endif
  Endif
  If (StartPt(j,i).GT.XMax(j,i)) then
    Write(*,8477) Ext(i),StartPt(j,i),Ext(i),XMax(j,i)
    8477
      Format(// 3X,'ERROR: Starting Value of ',A3,' = ',E17.10
      / 10X,'> Maximum of ',A3,' = ' ,E17.10,' !')
      Aux=1
    If (Distr.EQ.1) then
      StartPt(j,i)=XMax(j,i)
    else
      goto 4455
    Endif
2788
  continue
Endif
Endif
If (Distr.EQ.1) then
  If (NCombAc.GT.1) then
    Do 8001 k=2,NCombAc
    Do 7001 i=1,N
      XMin(k,i)=XMin(i,i)
      XMax(k,i)=XMax(i,i)
    7001
      continue
    8001
      continue
    Write(40,9111)
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Write(50,9111)
   Format(// 11X,
      1       'THE MINIMUM AND MAXIMUM VALUES OF THE VARIABLES' /
      1       'ARE THE SAME FOR ALL COMBINATIONS OF ACTIONS.'
      1       //)
Endif
goto 1315
Endif
789 continue

C Maximum number of iterations, MaxIter.
C ----------
1315 Write(*,5925)
5925 Format(// 3X,'MAXIMUM NUMBER OF ITERATIONS (Max=200) = ',$) Read(*,*) MaxIter
If (MaxIter.GT.200.OR.MaxIter.LE.0) then
   Write(*,7851)
   7851 Format(// 3X,'ERROR: The Maximum Number of Iterations' /
               3X,'is not Within [0,200] !')
   goto 1315
Endif
Write(40,5905) MaxIter
Write(50,5905) MaxIter
5905 Format(/ 3X,'MAXIMUM NUMBER OF ITERATIONS (Max=200) = ',I3)
C Number of FORM calculations, NumCalc.
C ----------
7979 Write(*,5095)
5095 Format(/ 3X,'NUMBER OF FORM CALCULATIONS (Max=10) = ',$) Read(*,*) NumCalc
If (NumCalc.GT.10.OR.NumCalc.LE.0) then
   Write(*,3279)
   3279 Format(// 3X,'ERROR: The Maximum Number of FORM Calculations' /
               3X,'is not Within [0,10] !')
   goto 7979
Endif
Write(40,5005) NumCalc
Write(50,5005) NumCalc
5005 Format(/ 3X,'NUMBER OF FORM CALCULATIONS (Max=10) = ',I2 /)
C Target values for each FORM calculation, TR.
C ----------
If (Opt.NE.3) then
   If ((Opt.EQ.1).OR.(Opt.EQ.2)) then
      Write(*,5023)
      Write(40,5023)
      Write(50,5023)
      5023 Format(/ 3X,'ALLOWABLE DISCHARGE (m3/s/m)' /)
      Do 2007 k0=1,NumCalc
     8799 Write(*,5021) k0
     5021 Format(11X,'Qa(',I2,') = ','$)
      Read(*,*) TR(k0)
      If (TR(k0).LT.0) then
         Write(*,3271)
         3271 Format(// 3X,'ERROR: The Allowable Discharge is < 0 !')
         goto 8799
      Endif
      Write(40,6023) k0,TR(k0)
      Write(50,6023) k0,TR(k0)
      6023 Format(11X,'Qa(',I2,') = ',E17.10)
   else
      Write(*,5055)
      Write(40,5055)
      Write(50,5055)
      5055 Format(/ 3X,'ALLOWABLE TARGET' /)
      Do 2055 k0=1,NumCalc
     5033 Write(*,5033) k0
     5033 Format(11X,'Target(',I2,') = ','$)
      Read(*,*) TR(k0)
      Write(40,6055) k0,TR(k0)
      Write(50,6055) k0,TR(k0)
      6055 Format(11X,'Target(',I2,') = ',E17.10)
   2055 continue
   else
      Write(*,5022)
      Write(40,5022)
      Write(50,5022)
      5022 Format(/ 3X,'ALLOWABLE EROSION DISTANCE (m)' /)
   Endif
else
   Write(*,5023)
   Write(40,5023)
   Write(50,5023)
   5023 Format(// 3X,'ALLOWABLE DISCHARGE (m3/s/m)' /)
   Do 2007 k0=1,NumCalc
  8799 Write(*,5021) k0
  5021 Format(11X,'Qa(',I2,') = ','$)
   Read(*,*) TR(k0)
   If (TR(k0).LT.0) then
      Write(*,3271)
      3271 Format(// 3X,'ERROR: The Allowable Discharge is < 0 !')
      goto 8799
   Endif
   Write(40,6023) k0,TR(k0)
   Write(50,6023) k0,TR(k0)
  6023 Format(11X,'Qa(',I2,') = ',E17.10)
  2007 continue
else
   Write(*,5055)
   Write(40,5055)
   Write(50,5055)
   5055 Format(/ 3X,'ALLOWABLE TARGET' /)
   Do 2055 k0=1,NumCalc
  5033 Write(*,5033) k0
  5033 Format(11X,'Target(',I2,') = ','$)
   Read(*,*) TR(k0)
   Write(40,6055) k0,TR(k0)
   Write(50,6055) k0,TR(k0)
  6055 Format(11X,'Target(',I2,') = ',E17.10)
  2055 continue
else
   Write(*,5022)
   Write(40,5022)
   Write(50,5022)
  5022 Format(/ 3X,'ALLOWABLE EROSION DISTANCE (m)' /)
end
Do 2006 k0=1,NumCalc
9975       Write(*,6022) k0
6022       Format(11X,'Eda(',I2,') = ',$) 
Read(*,*) TR(k0)
If (TR(k0).LT.0) then
   Write(*,3244)
3244         Format(// 3X,
1               'ERROR: The Allowable Erosion Distance is < 0 !' /)
goto 9975
Endif
Write(40,6062) k0,TR(k0)
Write(50,6062) k0,TR(k0)
2006     continue
Endif

----------
C       Required accuracy of the reliability index, ReqBetaAcc.
----------
Write(*,1997)
Write(40,1997)
Write(50,1997)
1997       Format(// 3X,
1               'REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX: '//'
1               11X, 'Default Value [1] ........... [ 1 ]' /
1               11X, 'User Specified Value ....... [ 2 ]' /)
9000       Write(*,9000)
7977       Format(3X,'Select Option: ',$)
Read(*,*) Def2
If (Def2.NE.1.AND.Def2.NE.2) goto 9000
Write(40,9399) Def2
Write(50,9399) Def2
9399       Format(3X,'Select Option: ',$)
If (Def2.EQ.1) then
   ReqBetaAcc=1.
else
   4788       Write(*,8874)
8874       Format(// 3X,'REQUIRED RELATIVE ACCURACY OF THE RELIABILITY',
1               1X,'INDEX [0,1] = ',$)
Read(*,*) ReqBetaAcc
If ((ReqBetaAcc.LT.0).OR.(ReqBetaAcc.GT.1)) then
   Write(*,5577)
5577       Format(// 3X,'ERROR: The Required Relative Accuracy of the',
1               10X,'Reliability Index is not Within [0,1] ! ')
goto 4788
Endif
Write(40,9971) ReqBetaAcc
Write(50,9971) ReqBetaAcc
9971       Format(// 3X,'Required Relative Accuracy of the',
1               3X,'Reliability Index [0,1] = ',E17.10 /)
Endif

----------
C       Smoothing of the iteration process (0<=Smooth<=1).
C       If Smooth=0 then there is no smoothing; if Smooth=0.5 then there
C       is averaging between the last two calculated values of X.
----------
Write(*,1227)
Write(40,1227)
Write(50,1227)
1227       Format(// 3X,'REQUIRED SMOOTHING COEFFICIENT FOR THE',
1               1X,' ITERATION PROCESS: '//'
1               11X, 'Default Value (0) ........... [ 1 ]' /
1               11X, 'User Specified Value ....... [ 2 ]' /)
2792       Write(*,2972)
2972       Format(3X,'Select Option: ',$)
Read(*,*) Def3
If (Def3.NE.1.AND.Def3.NE.2) goto 2792
Write(40,9229) Def3
Write(50,9229) Def3
9229       Format(3X,'Select Option: ',$)
If (Def3.EQ.1) then
   Smooth=0.
else
   2288       Write(*,8822)
8822       Format(// 3X,'REQUIRED SMOOTHING COEFFICIENT FOR',
1               1X,' THE ITERATION PROCESS [0,1] = ',$)
Read(*,*) Smooth
If ((Smooth.LT.0).OR.(Smooth.GT.1)) then
   Write(*,1177)
1177       Format(//3X,'ERROR: The Required Smoothing Coefficient for',
1               1X,'the' / 10X,
1               'Iteration Process is not Within [0,1] !')

----------
C       ----------
C       Required accuracy of the reliability index, ReqBetaAcc.
C       ----------
Write(*,1997)
Write(40,1997)
Write(50,1997)
1997       Format(// 3X,
1               'REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX: '//'
1               11X, 'Default Value [1] ........... [ 1 ]' /
1               11X, 'User Specified Value ....... [ 2 ]' /)
9000       Write(*,9000)
7977       Format(3X,'Select Option: ',$)
Read(*,*) Def2
If (Def2.NE.1.AND.Def2.NE.2) goto 9000
Write(40,9399) Def2
Write(50,9399) Def2
9399       Format(3X,'Select Option: ',$)
If (Def2.EQ.1) then
   ReqBetaAcc=1.
else
   4788       Write(*,8874)
8874       Format(// 3X,'REQUIRED RELATIVE ACCURACY OF THE RELIABILITY',
1               1X,'INDEX [0,1] = ',$)
Read(*,*) ReqBetaAcc
If ((ReqBetaAcc.LT.0).OR.(ReqBetaAcc.GT.1)) then
   Write(*,5577)
5577       Format(// 3X,'ERROR: The Required Relative Accuracy of the',
1               10X,'Reliability Index is not Within [0,1] ! ')
goto 4788
Endif
Write(40,9971) ReqBetaAcc
Write(50,9971) ReqBetaAcc
9971       Format(// 3X,'Required Relative Accuracy of the',
1               3X,'Reliability Index [0,1] = ',E17.10 /)
Endif

----------
C       Smoothing of the iteration process (0<=Smooth<=1).
C       If Smooth=0 then there is no smoothing; if Smooth=0.5 then there
C       is averaging between the last two calculated values of X.
----------
Write(*,1227)
Write(40,1227)
Write(50,1227)
1227       Format(// 3X,'REQUIRED SMOOTHING COEFFICIENT FOR THE',
1               1X,' ITERATION PROCESS: '//'
1               11X, 'Default Value (0) ........... [ 1 ]' /
1               11X, 'User Specified Value ....... [ 2 ]' /)
2792       Write(*,2972)
2972       Format(3X,'Select Option: ',$)
Read(*,*) Def3
If (Def3.NE.1.AND.Def3.NE.2) goto 2792
Write(40,9229) Def3
Write(50,9229) Def3
9229       Format(3X,'Select Option: ',$)
If (Def3.EQ.1) then
   Smooth=0.
else
   2288       Write(*,8822)
8822       Format(// 3X,'REQUIRED SMOOTHING COEFFICIENT FOR',
1               1X,' THE ITERATION PROCESS [0,1] = ',$)
Read(*,*) Smooth
If ((Smooth.LT.0).OR.(Smooth.GT.1)) then
   Write(*,1177)
1177       Format(//3X,'ERROR: The Required Smoothing Coefficient for',
1               1X,'the' / 10X,
1               'Iteration Process is not Within [0,1] !')
Program Listing

goto 2288 
else 
    Write(40,1178) Smooth 
    Write(50,1178) Smooth 
1178    Format(// 3X,'Required Smoothing Coefficient for' / 
1           3X,'the Iteration Process [0,1] = ',E17.10 /)
    Endif
Endif
--------
C Required accuracy of the failure function OBJF, ReqOBJFAcc. 
C If ReqOBJFAcc=1% then ABS(OBJF)<0.01SigmaOBJF, where SigmaOBJF 
C is the standard deviation of the failure function. 
--------
Write(*,3397) 
Write(40,3397) 
Write(50,3397) 
3397   Format(// 3X,'REQUIRED ACCURACY OF THE FAILURE FUNCTION: '// 
1           11X, 'Default Value (1%) ........ [ 1 ]' / 
1           11X, 'User Specified Value ....... [ 2 ]' /)
2744   Write(*,4472) 
4472   Format(3X,'Select Option: ',S) 
Read(*,*) Def4 
If (Def4.NE.1.AND.Def4.NE.2) goto 2744 
Write(40,4499) Def4 
Write(50,4499) Def4 
4499   Format(3X,'Select Option: ',I1) 
If (Def4.EQ.1) then 
    ReqOBJFAcc=1. 
else 
    5588   Write(*,8855) 
8855   Format(// 3X,'REQUIRED ACCURACY OF THE FAILURE',1X, 
1           'FUNCTION [0,1] = ',$) 
Read(*,*) ReqOBJFAcc 
If ((ReqOBJFAcc.LT.(0.)).OR.(ReqOBJFAcc.GT.1.)) then 
    Write(*,5544) 
5544   Format(// 3X,'ERROR: The Required Accuracy of the',1X, 
1           'Function is not Within [0,1] !') 
    goto 5588 
Endif
Write(40,9448) ReqOBJFAcc 
Write(50,9448) ReqOBJFAcc 
9448   Format(// 3X,'Required Accuracy of the Failure' / 
1           3X,'Function [0,1] = ',E17.10 /) 
Endif 
return 

C Subroutine Dadfile(NCombAc,Distr,Comb,Mode,Opt,N,Mux,Sigmax, 
1   VarDis,Abrev,Ext,ParamDesc,Trunc,Xo, 
1   MaxIter,NumCalc,StartPt,XMax,XMin, 
1   L - Maximum number of variables allowed by the program 
1   Der - Method of calculation of the first partial derivatives of 
1   the failure function for overtopping 
1   Carac - Name of the distribution 
1   Def1 - Definition of the limiting values of X 
1   Def2 - Definition of the required relative accuracy of the 

C Reads the required input data from data files. 

C MODELING VARIABLES: 
C OptC - Confidence value of the maximum run-up 
C CAcc - Consideration or not of combination of actions 
C Q - Maximum number of combinations of actions allowed by the program 
C NumTVAc - Number of time-varying actions 
C r - Repetitions of each action in the design life 
C TVAc - Number of the time-varying actions in increasing order of 
C the number of repetitions 
C L - Maximum number of variables allowed by the program 
C Der - Method of calculation of the first partial derivatives of 
C the failure function for overtopping 
C Carac - Name of the distribution 
C Def1 - Definition of the limiting values of X 
C Def2 - Definition of the starting point for the FORM calculations 
C L2 - Maximum number of FORM calculations allowed by the program 
C
OUTPUT VARIABLES:
Opt - Failure mode
DSWL - Definition of the SWL
TL - Seawall toe level
OptD - Direction of the sand movements in dune erosion
C1 - Parameter used in the H&R model to calculate C; it depends on the confidence value assigned to the maximum run-up
Mode - Purpose of the analysis
Comb - Consideration or not of combination of actions
NCombAc, CombAc - Number of combinations of actions
NR - Power to which each distribution is raised for each combination of actions
Distr - Distributions provided for the combination of actions
N - Number of variables
FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
Ext - Abbreviation of the name of the variable
ExtExt - Description of the variable
ParamDesc - Description of the design parameter
Rho - Correlation coefficient
Mux - Mean of X
Sigmax - Standard deviation of X
VarDis - Type of distribution
Abrev - Abbreviation of the name of the distribution
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
XMin - Minimum value of X
XMax - Maximum value of X
Zeta, Lamda, Eta - Parameters of a distribution
x1, x2 - Lower limit on X for a Beta distribution
x2 - Upper limit on X for a Beta distribution
StartPt - Starting value of the variables
MaxIter - Maximum number of iterations
NumCalc - Number of FORM calculations
TR - Target values for each FORM calculation
ReqBetaAcc - Required relative accuracy of the reliability index
Smooth - Smoothing coefficient for the iteration process
ReqOBJFAcc - Required accuracy of the failure function
Life - Design life of the structure
StartParam - For Mode=1, it is the prescribed value of the design parameter; For Mode=2, it is the starting value of the design parameter (from which the program iterates to find the required value of the design parameter)
Pf - Design target failure probability
RelInd - Reliability index which corresponds to Pf
cctcurv - Coastal curvature in degrees per 1000m
NPD, NPDOld - Number of points defining the initial profile (XP,YP), (XP0ld,YP0ld) - Coordinates of the points defining the initial profile
NPch - Number of points to be changed in the initial profile t = First point to be changed in the initial profile, point no.t
1:md - Gradient of the eroded dune face
1:mt - Gradient of the toe of the post-storm profile
nourtlev - Nourishment top level
1:mnour - Gradient of the nourished face

####################################################################
Integer*4 i,j,k,k0,L,L2,N,NPD,Mode,Opt,NCombAc,Q,t,NPch,NP0ld, 1 Life,MaxIter,NumCalc,Def,Def1,Def2,Def3,Def4,Comb, 1 Distr,Aux,CombAc,NumTVAc,It,AuxRstar,OptC,OptD,FDer, 1 DSWL Parameter (L=15)
Parameter (L2=10)
Parameter (Q=16)
Character*1 Der,CAcc
Character*3 Abrev(Q,L),Ext(L)
Character*17 ExtExt(L)
Character*30 Carac(Q,L)
Character*19 ParamDesc
Integer*4 VarDis(Q,L),Trunc(Q,L),TVAC(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L),Eta(Q,L), 1 x1(Q,L),x2(Q,L),XP,YP,md,mc,mnour,XP0ld,YP0ld,nourtlev,
Program Listing

ctcurv, Xo(Q,L), XMax(Q,L), XMin(Q,L), TR(L2), ReqBetaAcc,
Smooth, ReqOBJAcc, NR(Q,L), StartPt(Q, L), Rho(L,L),
StartParam, Pf, RelInd, r(L), TL, C1
Common/BLOCK1/NPD, XP(100), YP(100)
Common/BLOCK2/X, NPch, XPOld(100), YPOld(100), NPDOld
Common/BLOCK4/md, mt, mnour, nourtlev, ctcurv
Common/BLOCK6/OptD
Common/BLOCK7/TR
Common/BLOCK8/eta, Lambda, Eta, x1, x2
Common/BLOCK9/x0, It, AuxRstar
Common/BLOCK10/C1
Common/BLOCK11/DSWL, TL
Open(Unit=15, File='general.dad', Status='Old')
Open(Unit=30, File='meandev.dad', Status='Old')
Open(Unit=35, File='coefcor.dad', Status='Old')
Open(Unit=45, File='perfil.dad', Status='Old')
Open(Unit=65, File='form.dad', Status='Old')

C
C Definition of the failure mode to be studied.
C
C Read(15,*) Opt
If (Opt.NE.1.AND.Opt.NE.2.AND.Opt.NE.3) then
Write(*,3370)
Write(50,3370)
3370 Format(// 3X,'ERROR: Wrong Value for the Failure',1X,
1 'Mode to be Studied !!')
STOP
Endif
Write(40,2)
Write(50,2)
2 Format(// 3X,'WHAT IS THE FAILURE MODE TO BE STUDIED: '/
1 11X, 'Overtopping (H&R) ............ ........ [ 1 ]'/
1 11X, 'Overtopping (Owen) ........... ........ [ 2 ]'/
1 11X, 'Dune Erosion (Vellinga) ......... ....... [ 3 ] '/
Write(40,9779) Opt
Write(50,9779) Opt
9779 Format(3X,'Select Option: ',I2)

C
C Definition of the still-water-level.
C
C If (Opt.EQ.1.OR.Opt.EQ.2.OR.Opt.EQ.3) then
Read(15,*) DSWL
If (DSWL.NE.1.AND.DSWL.NE.2) then
Write(*,3017)
Write(50,3017)
3017 Format(// 3X,'ERROR: Wrong Value for the Definition of' /
1 10X,'the Still-Water-Level !')
STOP
Endif
Write(40,9320)
Write(50,9320)
9320 Format(// 3X,'HOW IS THE STILL-WATER-LEVEL DEFINED ?'/
1 // 11X, 'Total Level .... [ 1 ]'/
1 11X, 'Tide + Surge ... [ 2 ]'/
Write(40,9473) DSWL
Write(50,9473) DSWL
9473 Format(3X,'Select Option: ',I1)
Endif

C
C Definition of the confidence value of the maximum run-up to be
C considered.
C
C If (Opt.EQ.1) then
Read(15,*) OptC
If (OptC.NE.1.AND.OptC.NE.2) then
Write(*,3317)
Write(50,3317)
3317 Format(// 3X,'ERROR: Wrong Value for the Confidence Value' /
1 10X,'of the Maximum Run-Up !')
STOP
Endif
Write(40,1324)
Write(50,1324)
1324 Format(// 3X,'WHAT IS THE CONFIDENCE VALUE OF THE MAXIMUM',
1 11X,'RUN-UP' / 3X,'THAT YOU WOULD LIKE TO CONSIDER ?'/
1 // 11X, '37 % ... [ 1 ]'/
1 11X, '99 % ... [ 2 ]'/
Write(40,1037) OptC
Write(50,1037) OptC
1037     Format(3X,'Select Option: ',I1)
If (OptC.EQ.1) then
    C1=1.52
else
    C1=2.15
Endif
Endif

C       ----------
C       Definition of the method of calculation of the first partial
C       derivatives of the failure function for overtopping.
C       ----------
If (Opt.NE.3) then
    Read(15,3377) Der
3377     Format(A1)
If (Der.NE.'Y'.AND.Der.NE.'y'.AND.Der.NE.'N'.AND.Der.NE.'n')
    then
        Write(*,3373)
        Write(50,3373)
3373     Format(// 3X,
1             'ERROR: Wrong Value for the Method of Calculation' /
1             10X,'of the First Partial Derivatives of the' /
1             10X,'Failure Function for Overtopping !')
STOP
Endif
Write(40,6681) Der
Write(50,6681) Der
6681     Format(// 3X,'ARE THE FIRST DERIVATIVES OF THE FAILURE',1X,
1           'FUNCTION SUPPLIED (Y/N) ? ',A1)
If ((Der.EQ.'Y').OR.(Der.EQ.'y')) FDer=1
If ((Der.EQ.'N').OR.(Der.EQ.'n')) FDer=2
else
    FDer=2
Endif

C       ----------
C       Definition of the direction of the sand movements occurring
C       during a storm surge.
C       ----------
OptD=0
If (Opt.EQ.3) then
    Read(15,*) OptD
If (OptD.NE.1.AND.OptD.NE.2) then
        Write(*,3372)
        Write(50,3372)
3372     Format(// 3X,'ERROR: Wrong Value for the Direction' /
1             10X,'of the Sand Movements !')
STOP
Endif
Write(40,1302)
Write(50,1302)
1302     Format(// 3X,'DURING A STORM SURGE, WOULD YOU LIKE TO TAKE',
1             1X,'INTO ACCOUNT: '//'
1             1X,'Movements of Sand in Both Directions ?,
1             1X,'... [ 1 ]' / 11X,'Movements of Sand',
1             1X,'only Seaward ? ........ [ 2 ]' /)
Write(40,1937) OptD
Write(50,1937) OptD
1937     Format(3X,'Select Option: ',I1)
Endif

--------
C       If the failure mode is dune erosion the following quantities
C       have to be read:
C         - coastal curvature in degrees per 1000m, ctcuv
C         - number of points defining the initial profile, NPD
C         - beach profile coordinates, (XP,YP)
C         - number of points to be changed in the initial profile, NPch
C         - first point to be changed, point no. t
C         - gradient of the eroded dune face, 1:md
C         - gradient of the toe of the post-storm profile, 1:mt
C         - nourishment top level, nourtlev
C         - gradient of the nourished face, 1:mnour
C         --------
If (Opt.EQ.3) then
    Read(45,*) ctcuv
If (ctcuv.LT.0.OR.ctcuv.GT.24) then
        Write(*,3278)
        Write(50,3278)
3278     Format(// 3X,'ERROR: The Coastal Curvature is not',1X,
1             'Within [0,24] !')
STOP
Endif
Write(40,7522) ctcruv
Write(50,7522) ctcruv

7522 Format(// 3X,'DUNE EROSION' // 11X,
1     'Coastal Curvature (Deg/1000m) = ',E17.10 /)

Read(45,*) NPD
If (NPD.GT.100.OR.NPD.LE.0) then
Write(*,3251)
Write(50,3251)
3251 Format(// 3X,
1     'ERROR: The Maximum Number of Points Defining the' // 3X,
1     'Initial Profile is not Within [0,100] !!')
STOP
Endif
Write(40,6011) NPD
Write(50,6011) NPD

6011 Format(11X,'Number of Points Defining the Initial Profile',1X,
1     '(Max=100) = ',I3 // 11X,'Initial Profile ' // 19X,
1     'X', 18X, 'Y')

Do 3120 i=1,NPD
Read(45,*) XP(i),YP(i)
If (i.GE.2) then
If (XP(i).LT.XP(i-1)) then
Write(*,1466) i,(i-1)
Write(50,1466) i,(i-1)
1466 Format(// 3X,'ERROR: XP(',I3,') < XP(',I3,') !!')
STOP
Endif
Endif
Write(40,9011) XP(i),YP(i)
Write(50,9011) XP(i),YP(i)
9011 Format(11X,'Number of Points to be Changed in the Initial Profile = ',I3 // 11X,'First Point to be Changed = Point No.',I3)

XPOld(i)=XP(i)
YPOld(i)=YP(i)
3120 continue

NPDOld=NPD

Read(45,*) NPch
If (NPch.LT.1.OR.NPch.GT.NPD) then
Write(*,1366) NPD
Write(50,1366) NPD
1366 Format(// 3X,'ERROR: The Number of Points to be Changed in the Initial Profile is not Within [1, ',I3,'] !!')
STOP
Endif
Read(45,*) t
If (t.LT.1.OR.t.GT.NPD) then
Write(*,1322) NPD
Write(50,1322) NPD
1322 Format(// 3X,'ERROR: The First Point to be Changed in the Initial Profile is not Within [1, ',I3,'] !!')
STOP
Endif
Write(40,7517) NPch,t
Write(50,7517) NPch,t

7517 Format(// 11X,'Number of Points to be Changed in the Initial Profile = ',I3 // 11X,'First Point to be Changed = ',I3
1     'X,','= Point No.',I3)

Read(45,*) md
If (md.LE.0) then
Write(*,1311)
Write(50,1311)
1311 Format(// 3X,'ERROR: The Gradient of the Eroded Dune is <= 1:0 !')
STOP
Endif
Read(45,*) mt
If (mt.LE.0) then
Write(*,1151)
Write(50,1151)
1151 Format(// 3X,'ERROR: The Gradient of the Toe of the Post-Storm Profile is <= 1:0 !')
STOP
Endif
Read(45,*) mnourlev
Read(45,*) mnour
If (mnour.LE.0) then
Program Listing

3X,'ERROR: The Gradient of the Nourished Face is <= 1:0 !')
STOP
Endif

7103       Format(// 3X,'ERROR: The Gradient of the Eroded Dune Face = 1:',F4.1 //
11X,'Gradient of the Toe of the Post-Storm Profile = 1:',
F4.1 // 11X,'Nourishment Top Level = ',E17.10 // 11X,
1 'Gradient of the Nourished Face = 1:',F4.1)
Endif

C       Definition of the number, N, of variables for each failure mode.
C       =========================================== =====================
If (Opt.EQ.1) then
N=8
else
N=7
Endif
If (DSWL.EQ.2) then
If (Opt.EQ.2.OR.Opt.EQ.3) N=8
If (Opt.EQ.1) N=9
Endif

C       Description of each variable for the failure mode chosen.
C       =========================================== =====================
Call VarExt(N,Opt,Ext,ExtExt,ParamDesc)

C       Definition of the purpose of the analysis.
C       =========================================== =====================
Read(15,*) Mode
If (Mode.NE.1.AND.Mode.NE.2) then
Write(*,3374)
Write(50,3374)
3374     Format(// 3X,'ERROR: Wrong Value for the Definition !')
10X,'of the Purpose of the Analysis !')
STOP
Endif
Write(40,1017)
Write(50,1017)
1017     Format(// 3X,'WHAT IS THE PURPOSE OF THE ANALYSIS ? ' // 11X,
1 'Reliability Analysis for a Specified Design ... [ 1 ]'
1 / 11X,
1 'Design for a Specified Reliability Level ...... [ 2 ]' //
Write(40,9696) Mode
Write(50,9696) Mode
9696     Format(3X,'Select Option: ',I1)

C       Reads the design life of the structure, Life.
C       =========================================== =====================
Read(15,*) Life
If (Life.LE.0) then
Write(*,7882)
Write(50,7882)
7882     Format(// 3X,
1 'ERROR: The Design Life of the Structure is <= 0 !')
STOP
Endif
Write(40,791) Life
Write(50,791) Life
791     Format(// 3X,'DESIGN LIFE OF THE STRUCTURE = ',I3)

C       Definition of the combination of actions for Mode 1.
C       =========================================== =====================
If (Mode.EQ.1) then
Read(15,3335) CAcc

C-24
3335      Format(A1)  
1       If (CAcc.NE.'Y'.AND.CAcc.NE.'y'.AND.
1       CAcc.NE.'N'.AND.CAcc.NE.'n') then
1       Write(*,3357)
1       Write(50,3357)
3357      Format(// 3X,'ERROR: Wrong Value for the Definition of the'/
1       10X,'Combination of Actions for Mode 1 !')
1       STOP
Endif
Write(40,6622) CAcc
Write(50,6622) CAcc
6622      Format(// 3X,'WOULD YOU LIKE TO CONSIDER',1X,
1       'COMBINATION OF ACTIONS (Y/N) ? ',A1)
1       If ((CAcc.EQ.'Y').OR.(CAcc.EQ.'y')) Comb=1
1       If ((CAcc.EQ.'N').OR.(CAcc.EQ.'n')) Comb=2
else
1       Comb=2
Endif
C         ----------
C         Definition of the number of combination of actions, NCombAc.
C         ----------
If (Comb.EQ.2) then
1       NCombAc=1
1       Do 2388 i=1,N
1       NR(1,i)=1
2388     continue
else
1       Read(15,*) CombAc
1       If (CombAc.NE.1.AND.CombAc.NE.2) then
1       Write(*,3367)
1       Write(50,3367)
3367      Format(// 3X,'ERROR: Wrong Value for the Definition of the'/
1       10X,'Number of Combination of Actions !')
1       STOP
Endif
Write(40,1320)
Write(50,1320)
1320     Format(// 3X,'HOW MANY COMBINATIONS WOULD YOU LIKE TO',
1       1X,'CONSIDER ? ' // 11X,
1       'The Number of Time-Varying Actions (k) ... [ 1 ]' /
1       '2'*(k-1) '................................. [ 2 ]' /)
1       Write(40,1007) CombAc
1       Write(50,1007) CombAc
1007     Format(3X,'Select Option: ',I1)
C         ----------
C         Definition of the distributions provided for each combination
C         of actions.
C         ----------
Read(15,*) Distr
1       If (Distr.NE.1.AND.Distr.NE.2) then
1       Write(*,9977)
1       Write(50,9977)
9977      Format(// 3X,'ERROR: Wrong Value for the Definition of the'/
1       10X,'Distributions Provided for Each' /
1       10X,'Combination of Actions !')
1       STOP
Endif
Write(40,1550)
Write(50,1550)
1550     Format(// 3X,'WHICH DISTRIBUTIONS WOULD YOU LIKE TO',
1       1X,'PROVIDE ? ' // 11X,
1       'The Basic Distributions ...... [ 1 ]' / 11X,
1       'The Modified Distributions ... [ 2 ]' /)
1       Write(40,1557) Distr
1       Write(50,1557) Distr
1557     Format(3X,'Select Option: ',I1)
Endif
If (Comb.EQ.1) then
C         ----------
C         Reads the number of time-varying actions, NumTVAc (maximum=5)
C         & calculates the number of combinations of actions, NCombAc.
C         ----------
Read(15,*) NumTVAc
1       If (NumTVAc.GT.5.OR.NumTVAc.LE.0) then
1       Write(*,7887)
1       Write(50,7887)
7887      Format(// 3X,'ERROR: The Maximum Number of Time Varying' /
1       3X,' Actions is not Within [0,5] !')
1       STOP
Endif
Write(40,7911) NumTVAc
Write(50,7911) NumTVAc

7911 Format(// 3X, 'NUMBER OF TIME-VARYING ACTIONS (Max=5) = ',I1)

If (CombAc.EQ.0) then
   NCombAc=NumTVAc
else
   NCombAc=2**(NumTVAc-1)
Endif

If (Distr.EQ.1) then
   ---------------
   Reads the number of the time-varying actions, TVAc, in increasing order of the number of repetitions, r. Reads the number of repetitions of each action in the design life of the structure.
   ---------------
   Write(40,782)
   Write(50,782)

782 Format(//3X, 'REPETITIONS OF EACH ACTION IN THE DESIGN LIFE', /)

1 Do 140 i=1,NumTVAc
   Read(15,*), TVAc(i), r(TVAc(i))
   If (TVAc(i).GT.N.OR.TVAc(i).LE.0) then
      Write(*,4886) N
      Write(50,4886) N
      4886 Format(//3X,'ERROR: The Value of the Number of the Time-Varying Action is not Within [0,]!',
               //3X,' ] !')
      STOP
   Endif
   If (r(TVAc(i)).LE.0) then
      Write(*,6884)
      Write(50,6884)
      6884 Format(// 3X,'ERROR: The Number of Repetitions is',1X,
               ' <= 0 !')
      STOP
   Endif
   If (r(TVAc(i)).LT.r(TVAc(i-1))) then
      Write(*,6334)
      Write(50,6334)
      6334 Format(// 3X, 'ERROR: The Actions are not Listed in Increasing'
               // 3X, ' Order of the Number of Repetitions !')
      STOP
   Endif
   Write(40,788) Ext(TVAc(i)),r(TVAc(i))
   Write(50,788) Ext(TVAc(i)),r(TVAc(i))
1 788 Continue

Do 8711 i=1,NCombAc
   Do 8712 j=1,N
      Write(50,*) 'NR(i,j)=',i,j,NR(i,j)
8712 Continue
8711 Continue
else
   Do 87 i=1,NCombAc
      Do 88 j=1,N
         NR(i,j)=1
     88 Continue
87 Continue
Endif
Endif

If (Mode.EQ.1) then
   --------------------------------------------------------------
   Reads the value of the design parameter, StartParam, for which the failure probability (or reliability) is to be found.
   --------------------------------------------------------------
   Read(15,*) StartParam
   If (StartParam.LT.0) then
      Write(*,6554)
      Write(50,6554)
      6554 Format(// 3X,'ERROR: The Prescribed Value of the',
               // 1X,'Design Parameter is < 0 !')
      STOP
   Endif
   Write(40,7781) ParamDesc,StartParam
   Write(50,7781) ParamDesc,StartParam
87781 Format(// 3X, 'PRESCRIBED VALUE OF THE DESIGN PARAMETER' //
          11X,A19,' = ',E17.10)
else
   --------------------------------------------------------------
C7-26
C         Reads the design target failure probability, Pf, & starting
C         value of the design parameter, StartParam, from which the
C         program iterates to find the required value of the design
C         parameter for the given failure probability (or reliability).
C         ========================================= =====================
Read(15,*) StartParam
If (StartParam.LE.0) then
  Write(*,6224)
  Write(50,6224)
6224       Format(1x,'ERROR: The Prescribed Value of the',1x,'Design Parameter
1        is < 0 !')
  STOP
Endif
Read(15,*) Pf
If (Pf.LT.0.OR.Pf.GT.1) then
  Write(*,6234)
  Write(50,6234)
6234       Format(1x,'ERROR: The Value of the Design Target Failure',
1        Probability is not Within [0,1] !')
  STOP
Endif
If (StartParam.EQ.0.) then
  Write(40,7191) (100.*Pf),ParamDesc,StartParam
  Write(50,7191) (100.*Pf),ParamDesc,StartParam
7191       Format(1x,'DESIGN TARGET FAILURE PROBABILITY (%) = ',
1        F10.6 // 3X,
1        'STARTING VALUE OF THE DESIGN PARAMETER - ',A19/
1        1X, 'to start iteration') = ',E17.10 // 5X,
1        'NOTE: In design for a specified reliability level,'
1        ',/ 3X,'value of the design',/ 3X,
1        'parameter should not be set to zero !' / 11X,
1        'So, the program assumes a starting value of 5m.')
  StartParam=5.
else
  Write(40,7791) (100.*Pf),ParamDesc,StartParam
  Write(50,7791) (100.*Pf),ParamDesc,StartParam
7791       Format(1x,'DESIGN TARGET FAILURE PROBABILITY (%) = ',
1        F10.6 // 3X,
1        'STARTING VALUE OF THE DESIGN PARAMETER - ',A19/
1        1X, 'to start iteration') = ',E17.10)
  Endif
C         ========================================= =====================
C         Calculation of the reliability index, RelInd, which
C         corresponds to the target failure probability, Pf.
C         ========================================= =====================
Call InvNormal(Pf,RelInd)
Write(40,8792) RelInd
Write(50,8792) RelInd
8792     Format(1x,'Reliability Index = ',E17.10 )
Endif
C         ---------------------------------------------------------------
C         Reads the characteristics of the variables:
C         - Type of distribution, VarDist
C         - Type of truncation, Trunc
C         - Point of truncation, Xo (if the distribution is truncated)
C         - Mean, Mux, standard deviation, Sigmax, & lower limit, Zeta
C         ---------------------------------------------------------------
Do 6633 j=1,NCombAc
  If ((NCombAc.GT.1).AND.(Distr.EQ.2)) then
    Write(40,7073) j
    Write(50,7073) j
7073       Format(1x,'COMBINATION No.',I2)
  Endif
  Write(40,8777)
  Write(50,8777)
8777     Format(1x,'CHARACTERISTICS OF THE VARIABLES ')
Enddo
Do 13 i=1,N
  Read(30,*) VarDis(j,i)
C     If (VarDis(j,i).NE.0.AND.VarDis(j,i).NE.1.AND.
1     VarDis(j,i).NE.2.AND.VarDis(j,i).NE.3.AND.
1     VarDis(j,i).NE.4.AND.VarDis(j,i).NE.5.AND.
1     VarDis(j,i).NE.6.AND.VarDis(j,i).NE.7.AND.
1     VarDis(j,i).NE.8.AND.VarDis(j,i).NE.9.AND.
1     VarDis(j,i).NE.10.AND.VarDis(j,i).NE.11.AND.
1     VarDis(j,i).NE.12.AND.VarDis(j,i).NE.13) then
    Write(*,3375) Ext(i)
    Write(50,3375) Ext(i)
3375         Format(1x,'ERROR: Wrong Value for the Type of',1X,1x,'
1        Distribution for ',A3,' !')
  Endif
Enddo
C         ---------------------------------------------------------------
Trunc(j,i)=0
If (VarDis(j,i).NE.0.AND.VarDis(j,i).LE.10)
1      Read(30,*) Trunc(j,i)
1      If (Trunc(j,i).NE.0.AND.Trunc(j,i).NE.1.AND.
1         Trunc(j,i).NE.2) then
1         Write(*,3376) Ext(i)
1         Write(50,3376) Ext(i)
3376     Format('3X,'ERROR: Wrong Value for the Type of',1X,
1         'Truncation for ',A3,' !')
1      STOP
Endif
Xo(j,i)=0.
If (Trunc(j,i).NE.0) then
C             ----------
C             For the failure mode of overtopping, if the variable is
C             the significant wave height, Hs, & if Hs is limited by the
C             available water depth, then the point of truncation is
C             Xo=0.6(SWL-TL) or Xo=0.6(Tide+Surge-T L), where TL is the
C             seawall toe level.
C             ----------
1      If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(i.EQ.2).AND.
1          (Trunc(j,i).EQ.1)) then
1         Read(30,*) TL
goto 5553
Endif
5553     If (VarDis(j,i).NE.0) then
1         If (VarDis(j,i).EQ.6) then
1           Read(30,*) Mux(j,i),Sigmax(j,i),x1(j,i),x2(j,i)
1           If (x1(j,i).GE.x2(j,i)) then
1             Write(*,8703)
1             Write(50,8703)
8703       Format('11X,'ERROR: a >= b !')
1             STOP
1           Endif
1           goto 5555
1         Endif
5555     Endif
1         If (VarDis(j,i).EQ.10) then
1           Read(30,*) Mux(j,i),Sigmax(j,i),Zeta(j,i)
goto 5555
Endif
5555     Endif
else
1         Read(30,*) Mux(j,i)
Endif
5555     Call WhatDist(i,j,VarDis,Abrev,Carac)
1        Call WriCharVar(i,j,VarDis,Trunc,Ext,Carac,Xo,Mux,Sigmax,
1                           Opt)
13       continue
C         ================================================================
C         Calculation of the distribution’s parameters, Zeta, Lambda &
C         Eta, for each variable.
C         ==============================================================
Write(50,4495)
4495    Format('6X,'DISTRIBUTION’S PARAMETERS')
Do 6070 i=1,N
Call Parameters(i,j,Mux,Sigmax,VarDis,Ext)
6070    continue
If (Distr.EQ.1) then
If (NCombAc.GT.1) then
Call EqCharac(N,NCombAc,Abrev,Carac,VarDis,Trunc,Xo,Mux,
1               Sigmax)
Write(40,9911)
Write(50,9911)
9911   Format('11X,'THE CHARACTERISTICS OF THE VARIABLES ARE ',
1        '/11X,'THE SAME FOR ALL COMBINATIONS OF ACTIONS,')
1        (/)
Endif
goto 1314
Endif
6633   continue
C         ================================================================
C         Reads the correlation coefficients, Rho, of the variables.
1314 Write(40,870)
Write(50,870)
870 Format(// 3X, 'CORRELATION COEFFICIENTS ' /)
Do 40 i=1,N
Do 45 j=1,N
   Read(35,*) Rho(i,j)
   Write(40,78) Ext(i),Ext(j),Rho(i,j)
   Write(50,78) Ext(i),Ext(j),Rho(i,j)
78 Format(11X,'(',A3,',',A3,') = ',E17.10)
continue
45 continue
40 continue
Do 4005 i=1,N
Do 4050 j=1,N
   If ((Rho(i,j).NE.Rho(j,i)).OR.(ABS(Rho(i,j)).GT.1)) then
      If (Rho(i,j).NE.Rho(j,i)) then
         Write(*,7801) i,j,j,i
         Write(50,7801) i,j,j,i
         7801 Format(/// 11X,'ERROR: Rho(',I2,',', I2,') is not equal to Rho(',I2, ',',I2,') !')
      else
         Write(*,7901) i,j
         Write(50,7901) i,j
         7901 Format(/// 11X,'ERROR: |Rho(',I2,',' ,I2,')| > 1 !')
      Endif
   Endif
4050 continue
4005 continue
Write(40,*) ' '
Write(50,*) ' '
C ================================================================
C Reads the characteristics of the FORM calculations:
C - starting value of the variables, StartPt
C - minimum value, XMin, & maximum value, XMax, of the variables
C - maximum number of iterations, MaxIter
C - number of FORM calculations, NumCalc
C - target values for each FORM calculation, TR
C - required accuracy of the reliability index, ReqBetaAcc
C - smoothing coefficient for the iteration process, Smooth
C - required accuracy of the failure function, ReqOBJFAcc
C ==============================================================
C ----------
C Starting value of the variables, StartPt.
C ----------
Do 678 j=1,NCombAC
   If (NCombAC.GT.1) then
      Write(40,7079) j
      Write(50,7079) j
      7079 Format(/// 3X,'COMBINATION No.',I2)
   Endif
   If ((Distr.EQ.2).OR.(Comb.EQ.2)) then
      Write(40,132)
      Write(50,132)
      132 Format(/ 3X,'STARTING POINT FOR THE FORM CALCULATIONS: '//' 1
      11X, 'Default Values (mean values) ... | 1 !' /
      11X, 'User Specified Values ........ | 2 !' /)
      Read(65,*) Def
      If (Def.NE.1.AND.Def.NE.2) then
         Write(*,3307)
         Write(50,3307)
         3307 Format(/ 3X,'ERROR: Wrong Value for the Type of',1X,
         1 'Starting Point !')
         STOP
      Endif
      Write(40,937) Def
      Write(50,937) Def
      937 Format(3X,'Select Option: ',I1)
      Write(40,5666)
      Write(50,5666)
      5666 Format(/ 3X,'STARTING POINT' /)
      If (Def.EQ.1) then
         Do 2156 i=1,N
            StartPt(j,i)=Mux(j,i)
            Write(40,5620) Ext(i),StartPt(j,i)
            Write(50,5620) Ext(i),StartPt(j,i)
         2156 continue
         Else
            Do 2155 i=1,N
               If (VarDis(j,i).NE.0) then
                  Read(Dis(j,i),*) StartPt(j,i)
               Endif
         2155 continue
   Endif
Write(40,5620) Ext(i),StartPt(j,i)
Write(50,5620) Ext(i),StartPt(j,i)
5620 Format(11X,A3,' = ',E17.10)
5666 continue
Write(40,5600) Ext(i),StartPt(j,i)
Write(50,5600) Ext(i),StartPt(j,i)
5600
Format(11X,A3,’ ’,’E17.10)
else
StartPt(j,i)=Mux(j,i)
Endif
2155
continue
Endif
Endif
If (Distr.EQ.1) then
Write(40,5636)
Write(50,5636)
5636
Format(/ 3X,’STARTING POINT ’)
Do 2956 i=1,N
If (NR(j,i).NE.1.) then
Call Inverse(j,i,Mux,NR,VarDis,StartPt)
else
StartPt(j,i)=Mux(j,i)
Endif
Write(40,5629) Ext(i),StartPt(j,i)
Write(50,5629) Ext(i),StartPt(j,i)
5629
Format(11X,A3,’ = ’,’E17.10)
2956
continue
Endif
678
continue
C
--------------
C Minimum value, XMin, & maximum value, XMax, of the variables.
C
--------------
Do 789 j=1,NCombAC
If ((NCombAC.GT.1).AND.(Distr.EQ.2)) then
Write(40,7074) j
Write(50,7074) j
7074
Format(/ 3X,’COMBINATION No.’,’I2) 
Endif
Write(40,1327)
Write(50,1327)
1327
Format(/ 3X,’LIMITING VALUES FOR THE VARIABLES: ‘//
1 11X,’Default Values +/- 1E25 ... [ 1 ]’//
1 11X,’User Specified Values ....... [ 2 ]’//)
Read(65,*) Def1
If (Def1.NE.1.AND.Def1.NE.2) then
Write(*,3371)
Write(50,3371)
3371
Format(/ 3X,’ERROR: Wrong Value for the Type of,’//
1 ’Limiting Values for the Variables!’)
STOP
Endif
Write(40,9377) Def1
Write(50,9377) Def1
9377
Format(3X,’Select Option: ’,’I1)
If (Def1.EQ.1) then
Do 2176 i=1,N
If (VarDis(j,i).NE.0) then
XMin(j,i)=-1E25
XMax(j,i)=+1E25
If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(DSWL.EQ.1)) then
XMin(j,5)=TL
XMax(j,5)=StartParam
If (XMin(j,5).GT.XMax(j,5)) then
Write(*,8653)
Write(50,8653)
8653
Format(/ 11X,’ERROR: The Toe Level is Above the’,
1 ’Seawall Crest Level!’)
STOP
Endif
Endif
else
XMin(j,i)=Mux(j,i)
XMax(j,i)=Mux(j,i)
Endif
2176
continue
else
Do 2175 i=1,N
If (VarDis(j,i).NE.0) then
Read(65,*) XMin(j,i),XMax(j,i)
If (XMin(j,i).GT.XMax(j,i)) then
Write(*,8733) i,i
Write(50,8733) i,i
8733
Format(/ 11X,’ERROR: XMin(',I2,') > XMax(',I2,')!’)
STOP
Endif
If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(DSWL.EQ.1)) then
XMin(j,5)=TL
XMax(j,5)=StartParam
If (XMin(j,5).GT.XMax(j,5)) then
Write(*,8153)
Write(50,8153)
8153 Format(// 11X,'ERROR: The Toe Level is Above the',
1 1X,'Seawall Crest Level !')
STOP
Endif
Endif
else
XMin(j,i)=Mux(j,i)
XMax(j,i)=Mux(j,i)
Endif
2175 continue
Endif
Write(40,5678)
Write(50,5678)
5678 Format(/ 3X,'LIMITING VALUES FOR THE VARIABLES' /)
Do 2788 i=1,N
Call MinMax(i,j,Opt,VarDis,Ext,Trunc,Xo,Mux,SigmaMax,XMin,
1 XMax)
If (StartPt(j,i).LT.XMin(j,i)) then
Write(*,8477) Ext(i),StartPt(j,i),Ext(i),XMin(j,i)
Write(50,8477) Ext(i),StartPt(j,i),Ext(i),XMin(j,i)
8477 Format(// 3X,'ERROR: Starting Value of ',A3,' = ',E17.10
1 / 10X,'< Minimum of ',A3,' = ' ,E17.10,' !')
STOP
Endif
If (StartPt(j,i).GT.XMax(j,i)) then
Write(*,8977) Ext(i),StartPt(j,i),Ext(i),XMax(j,i)
Write(50,8977) Ext(i),StartPt(j,i),Ext(i),XMax(j,i)
8977 Format(// 3X,'ERROR: Starting Value of ',A3,' = ',E17.10
1 / 10X,'> Maximum of ',A3,' = ' ,E17.10,' !')
STOP
Endif
2788 continue
If (Distr.EQ.1) then
If (NCombAc.GT.1) then
Do 8001 k=2,NCombAc
Do 7001 i=1,N
XMin(k,i)=XMin(1,i)
XMax(k,i)=XMax(1,i)
7001 continue
8001 continue
Write(40,9111)
Write(50,9111)
9111 Format(// 11X,
1 'THE MINIMUM AND MAXIMUM VALUES OF THE VARIABLES' /
1 11X,'ARE THE SAME FOR ALL COMBINATIONS OF ACTIONS.' /
1 //)
Endif
789 continue
C ---------
C Maximum number of iterations, MaxIter.
C ---------
1315 Read(65,*) MaxIter
If (MaxIter.GT.200.OR.MaxIter.LE.0) then
Write(*,7851)
Write(50,7851)
7851 Format(/ 3X,'ERROR: The Maximum Number of Iterations' /
1 3X,' is not Within [0,200] !')
STOP
Endif
Write(40,5995) MaxIter
Write(50,5995) MaxIter
5995 Format(/ 3X,'MAXIMUM NUMBER OF ITERATIONS (Max=200) = ',I3)
C ---------
C Number of FORM calculations, NumCalc.
C ---------
3279 Read(65,*) NumCalc
If (NumCalc.GT.10.OR.NumCalc.LE.0) then
Write(*,3279)
Write(50,3279)
3279 Format(/ 3X,'ERROR: The Maximum Number of FORM Calculations' /
1 3X,' is not Within [0,10] !')
STOP
Endif
Write(40,5005) NumCalc
Write(50,5005) NumCalc

5005  Format(/ 3X,'NUMBER OF FORM CALCULATIONS (Max=10) = ',I2 /)

C

--------

C  Target values for each FORM calculation, TR.

--------

Aux=2

Do 2007 k0=1,NumCalc

Read(65,*) TR(k0)

If (TR(k0).LT.0) then

Write(*,3244)

Write(50,3244)

STOP

Endif

Call Allowed(Opt,Aux)

2007  continue

--------

C  Required accuracy of the reliability index, ReqBetaAcc.

--------

Write(40,1997)

Write(50,1997)

1997  Format(/ 3X,

1  'REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX: ' //


Read(65,*) Def2

If (Def2.NE.1.AND.Def2.NE.2) then

Write(*,6677)

Write(50,6677)

6677     Format(// 3X,'ERROR: The Required Relative Accuracy of the',1X,

1  "Accuracy of the Reliability Index ! ")

STOP

Endif

Write(40,9399) Def2

Write(50,9399) Def2

9399     Format(3X,'Select Option: ',I1)

If (Def2.EQ.1) then

ReqBetaAcc=1.

else

Read(65,*) ReqBetaAcc

If ((ReqBetaAcc.LT.0)).OR.(ReqBetaAcc.GT.1)) then

Write(*,6677)

Write(50,6677)

Endif

Write(40,9978) ReqBetaAcc

Write(50,9978) ReqBetaAcc

9978     Format(// 3X,'Required Relative Accuracy of the',1X,

1  'Reliability Index [0,1] = ',E17.10 /)

Endif

C

--------

C  Smoothing of the iteration process (0<=Smooth<=1).

C  If Smooth=0 then there is no smoothing; if Smooth=0.5 then there

C  is averaging between the last two calculated values of X.

--------

Write(40,1197)

Write(50,1197)

1197  Format(/ 3X,'REQUIRED SMOOTHING COEFFICIENT FOR THE',1X,

1  'ITERATION PROCESS: ' //

1  11X, 'Default Value (0) ........... [ 1 ] ' /


Read(65,*) Def3

If (Def3.NE.1.AND.Def3.NE.2) then

Write(*,3462)

Write(50,3462)

3462     Format(/ 3X,'ERROR: Wrong Value for the Type of',1X,

1  'Smoothing Coefficient' / 10X,

1  'for the Iteration Process ! ')

STOP

Endif

Write(40,9799) Def3

Write(50,9799) Def3

9799     Format(3X,'Select Option: ',I1)

If (Def3.EQ.1) then

Smooth=0.

else

Read(65,*) Smooth
If ((Smooth.LT.(0.)).OR.(Smooth.GT.1.)) then
  Write(*,1177)
  Write(50,1177)
  1177   Format(// 3X,'ERROR: The Required Smoothing',1X,
    1 'Coefficient for the' / 10X,
    1 'Iteration Process is not Within [0,1] !')
  STOP
Endif
Write(40,1178) Smooth
Write(50,1178) Smooth

If ((Smooth.LT.(0.)).OR.(Smooth.GT.1.)) then
  Write(*,1177)
  Write(50,1177)
  1178   Format(// 3X,'Required Smoothing Coefficient for',1X,
    1 'the Iteration Process [0,1] = ',E17.10 /)
Endif

C       Required accuracy of the failure function OBJF, ReqOBJFAcc.
C       If ReqOBJFAcc=1% then ABS(OBJF)<0.01SigmaOBJF, where SigmaOBJF
C       is the standard deviation of the failure function.
C
Write(40,3397)
Write(50,3397)

3397   Format(// 3X,'REQUIRED ACCURACY OF THE FAILURE FUNCTION: '//
  1 11X, 'Default Value (1%) ........ [ 1 ]' /
  1 11X, 'User Specified Value ........ [ 2 ]' /)
Read(65,*) Def4
If (Def4.NE.1.AND.Def4.NE.2) then
  Write(*,3463)
  Write(50,3463)
Endif

Write(40,9448) ReqOBJFAcc
Write(50,9448) ReqOBJFAcc

4499   Format(3X,'Select Option: ',I1)
If (Def4.EQ.1) then
  ReqOBJFAcc=1.
else
  ReqOBJFAcc=4.0
Endif

C       Subroutine D1Point (j,N,Opt,Mux,Sigmax,VarDis,Abrev,Rho,FDer,
C       NR,StartParam,Prob,Ext,ExtExt,ParamDesc,
C       Trunc,Xo,MaxIter,StartPt,XMax,XMin,
C       ReqBetaAcc,Smooth,ReqOBJFAcc,Comb,Ex)

C       Returns the failure probability, Prob, for a given value of the
C       design parameter, StartParam - Mode 1 (analysis mode).
C
C       INPUT VARIABLES:
C       Opt - Failure mode
C       DSWL - Definition of the SWL
C       TL - Seawall toe level
C       Comb - Consideration or not of combination of actions

C       *****************************************************************
C
Subroutine D1Point (j,N,Opt,Mux,Sigmax,VarDis,Abrev,Rho,FDer,
  NR,StartParam,Prob,Ext,ExtExt,ParamDesc,
  Trunc,Xo,MaxIter,StartPt,XMax,XMin,
  ReqBetaAcc,Smooth,ReqOBJFAcc,Comb,Ex)

C       *****************************************************************

C       Close (Unit=35)
Close (Unit=15)
Close (Unit=30)
Close (Unit=45)
Close (Unit=65)
return
Program Listing

j - Number of the combination of actions
NR - Power to which each distribution is raised for each combination of actions
N - Number of variables
Ext - Abbreviation of the name of the variable
ExtExt - Description of the variable
ParamDesc - Description of the design parameter
Rho - Correlation coefficient
Mux - Mean of X
Sigmax - Standard deviation of X
VarDis - Type of distribution
Abrev - Abbreviation of the name of the distribution
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
XMin - Minimum value of X
XMax - Maximum value of X
Zeta - Parameter of a distribution
StartPt - Starting value of the variables
FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
MaxIter - Maximum number of iterations
x0 - Number of the FORM calculation
ReqBetaAcc - Required relative accuracy of the reliability index
Smooth - Smoothing coefficient for the iteration process
ReqOBJFAcc - Required accuracy of the failure function
StartParam - Prescribed value of the design parameter

MODELING VARIABLES:
Q - Maximum number of combinations of actions allowed by the program
L - Maximum number of variables allowed by the program
L1 - Maximum number of iterations allowed by the program
X - Variables of the failure mode
XOld - X in the previous iteration
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X
Y - Non-correlated, Normal transformed variables
YOld - Y in the previous iteration
Muy - Mean of Y
Sigmay - Standard deviation of Y
V - Matrix of eigenvectors
Vt - Transpose of V
OBJF - Failure function
MuOBJF - Mean of OBJF
VarOBJF - Standard deviation of OBJF

OUTPUT VARIABLES:
Prob - Probability of failure which corresponds to RelInd
Ex - Auxiliary variable

*******************************************************************************

Integer*4 i, j, k, x0, L2, N, Opt, L1, It, FDer, NSTATE, Q, MaxIter, Count,
Program Listing

1 Aux, Comb, Aux1, AuxRstar, Ex, DSWL, ii, iii
Parameter (L=15)
Parameter (L2=10)
Parameter (Q=16)
Parameter (L1=200)
Character*3 Abrev(Q,L), Ext(L)
Character*17 ParamDesc
Integer*4 VarDis(Q,L), Trunc(Q,L), Out(L), k1(L), Sin(L1,L),
Real*8 X(L), XOld(L,L), Mux(Q,L), MuOBJF, MuxOBJF, Sigmax(Q,L),
1 Xo(Q,L), SigmaOBJF, Prob(Q), OBJGRD(L), OBJF, VarOBJF,
1 Rho(L,L), Alpha(L), V(L,L), Vt(L,L), Sigmay(L), Muy(L), Y(L, L),
1 Inf(L), OBJGRDPrev(L), Zeta(Q,L), Lamba(Q,L), Pi, SigmaxN(L, L),
1 YOld(L,L), xl(Q,L), x2(Q,L), ExtEnd, RellInd, Startparam,
1 MuxN, C, T3, md, mnour, nourtlev, etcurv, TR(L2), Step(L),
1 StartPt(Q,L), XMax(Q,L), XMin(Q,L), BetaAcc, ReqBetaAcc,
1 RelInd, Smooth, Prob1, Prob2, DiffProb, ReqOBJFAcc, OBJFAcc,
1 NR(Q,L)
Common/BLOCK3/MuxN(15), C, T3
Common/BLOCK4/md, mt, mnour, nourtlev, etcurv
Common/BLOCK7/TR
Common/BLOCK8/Zeta, Lamba, Etal, xl, x2
Common/BLOCK9/It, AuxRstar
Common/BLOCK11/DSWL, TL
Pi=4.*ATAN(1.)
RelInd=0.
Do 3325 i=1,N
Sin(i, i)=0
Step(i)=0.001
Out(i)=0
k1(i)=1
3325 continue
Write(*,25)
Write(40,25)
Write(50,25)
25 Format(/)
Aux1=1
Call Allowed(Opt, Aux1)
Do 7855 i=1,N
X(i)=StartPt(j, i)
7855 continue
C
C Definition of the iteration number, It.
C
C It=1
53 Write(*, 86) It
Write((50, 86) It
86 Format(/ 6X, 'ITERATION No.', I3)
54 It=It+1
C
C For the failure mode of overtopping, if the variable is the
C significant wave height, Hs=X(2), & if Hs is limited by the
C available water depth, then the point of truncation is
C Xo=0.6(SWL-TL) or Xo=0.6(Tide+Surge-TL).
C
C If ((Opt.EQ.1.OR.Opt.EQ.2).AND. (Trunc(j,2).EQ.1)) then
If ((DSWL.EQ.1).AND. (Trunc(j,2).EQ.1)) then
If ((DSWL.EQ.1).AND. (Trunc(j,2).EQ.1)) then
Xo(j,2)=0.6*(X(5)-TL)
else
Xo(j,2)=0.6*(X(5)+X(6)-TL)
Endif
Write(50, 4444) Ext(L), Xo(j,2)
4444 Format(/ 1E16, 'Xo(', A3, ') = ', E17.10)
If (XMax(j,2).NE.Xo(j,2)) then
XMax(j,2)=Xo(j,2)
Write(50, 5971) Ext(L), XMax(j,2)
5971 Format(/ 1E16, 'XMax(', A3, ') = ', E17.10)
If (XMin(j,2).GT.XMax(j,2)) then
Write(*, 8703)
8703 Format(/ 11X, 'ERROR: XMin(2) > XMax(2) !')
STOP
Endif
If (X(2).GT.XMax(j,2)) then
Write(*, 8367) Ext(L), X(2), XMax(j,2)
Write(50, 8367) Ext(L), X(2), XMax(j,2)
X(2)=XMax(j,2)
Write(*,8471) Ext(2),X(2)
Write(50,8471) Ext(2),X(2)
8471 Format(11X,A3,'=',E17.10/)
Endif
Endif

C
=================================================================

C Transformation of non-Normal correlated variables (X with
C mean=Mux & standard deviation=Sigmax) to Normal correlated
C variables (X with mean=MuxN & standard deviation=SigmaxN).
C
Endif
Endif
Endif

C
=================================================================

C
C       Transformation of non-Normal correlated variables (X with
C       mean=Mux & standard deviation=Sigmax) to Normal correlated
C       variables (X with mean=MuxN & standard deviation=SigmaxN).
C       =========================================== =====================

Write(50,4395)
4395 Format(/11X,'EQUIVALENT NORMAL DISTRIBUTION'S PARAMETERS')
Do 7870 i=1,N
Call EqNorDis(i,j,N,X,VarDis,NR,Ext,MuxN,SigmaxN,Trunc,Xo)
7870 continue
If (Opt.EQ.23) SigmaxN(5)=0.11*(X(4)-2.25)
C       =========================================== =====================

C Transformation of correlated Normal variables (X with mean=MuxN
C & standard deviation=SigmaxN) to non-correlated Normal variables
C (Y with mean=Muy & standard deviation=Sigma y).
C
Call Correlated(N,Ext,MuxN,SigmaxN,Rho,Sigmay,Muy,V,Vt)
Write(50,*) '
Do 635 i=1,N
Y(i)=0.
Do 637 k=1,N
Y(i)=Vt(i,k)*X(k)+Y(i)
637 continue
Write(50,1998) Ext(i),Y(i)
1998 Format(11X,'Y(',A3,') = ',E17.10)
635 continue
Write(50,*) '

C
=================================================================

C Calculation of the failure function, OBJF, & its derivatives,
C OBJGRD, at the design point.
C
Endif

If (FDer.EQ.2) then
Call Derivadas(StartParam,N,X,Ext,Opt,FDer,OBJF,OBJGRD)
else
Call OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,Opt,StartParam)
Do 5690 i=1,N
Write(50,9995) Ext(i),OBJGRD(i)
9995 Format(11X,'dZ/d',A3,' = ',E17.10)
5690 continue
Endif

If (AuxRstar.EQ.1) then
If (It.NE.2) X(1)=(X(1)+XOld(1,It-1))/2.
If ((X(1).LT.XMin(j,1)).OR.(X(1).GT.XMax(j,1))) then
If (X(1).LT.XMin(j,1)) then
Write(*,8976) Ext(1),X(1),XMin(j,1)
Write(50,8976) Ext(1),X(1),XMin(j,1)
8976 Format(/11X,'A3,' = ',E17.10, ' ! ' /)
X(1)=1.001*XMin(j,1)
C              X(1)=XOld(1,It-1)+(XMax(j,1)-XOld(1,It-1))/2.
else
Write(*,8461) Ext(1),X(1),XMax(j,1)
Write(50,8461) Ext(1),X(1),XMax(j,1)
8461 Format(/3X,A3,' = ',E17.10, ' ! ' /)
X(1)=1.001*XMax(j,1)
C
Endif
Write(50,8109) Ext(1),X(1)
8109 Format(/11X,'A3,' = ',E17.10/)
goto 53
Endif
Write(50,8277) Ext(1),X(1)
8277 Format(/11X,'A3,' = ',E17.10/)
goto 53
Endif
Write(50,9312) OBJF
9312 Format(/11X,'Z = ',E17.10/)
Count=0
Do 409 i=1,N
OBJGRDPrev(i)=OBJGRD(i)
If (ABS(OBJGRDPrev(i)).LT.(1E-25)) Count=Count+1
409 continue
If (Count.EQ.N) then

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Write(*,99)
Write(50,99)
99    Format(/ 11X,'ERROR: The derivatives are zero !')
STOP
Endif
Do 402 i=1,N
OBJGRD(i)=0.
Do 450 k=1,N
OBJGRD(i)=OBJGRD(k)*V(k,i)+OBJGRD(i)
450    continue
Write(50,810) i,OBJGRD(i)
810    Format(11X, 'dZ/dY(',I2,') = ',E17.10)
402    continue

C Calculation of the mean value, MuOBJF, & the standard deviation, SigmaOBJF, of the failure function. Calculation of the reliability index, RelInd.

MuOBJF=0
VarOBJF=0
Do 84 i=1,N
MuOBJF=(Muy(i)-Y(i))*OBJGRD(i)+MuOBJF
VarOBJF=((OBJGRD(i)*Sigmay(i))**2)+VarOBJF
84    continue
MuOBJF=OBJF+MuOBJF
SigmaOBJF=SQRT(VarOBJF)
RelOld=RelInd
RelInd=MuOBJF/SigmaOBJF
Write(50,33) MuOBJF,SigmaOBJF,RelInd,RelOld
33    Format(/ 11X,'Mean Value of Z = ',E17.10 / 11X,
1           'Standard Deviation of Z = ',E17.10 / 11X,
1           'New Reliability Index = ',E17.10 / 11X,
1           'Old Reliability Index = ',E17.10 / )

C Calculation of the sensitivity factors, Alpha, & the new design point, Y (and consequently X).

Do 9009 i=1,N
Alpha(i)=(Sigmay(i)/SigmaOBJF)*OBJGRD(i)
YOld(i,It-1)=Y(i)
Y(i)=Muy(i)-Alpha(i)*RelInd*Sigmay(i)
9009    continue
Do 1635 i=1,N
XOld(i,It-1)=X(i)
X(i)=0.
Do 1637 k=1,N
X(i)=V(i,k)*Y(k)+X(i)
1637    continue
Write(50,1187) Ext(i),XOld(i,It-1),Ext(i),X(i)
1187    Format(11X,'XOld(',A3,') = ',E17.10,3X,A3,' = ',E17.10)
1635    continue
Write(50,*) 
Do 88 i=1,N
Write(50,87) i,Alpha(i)
87    Format(11X,'Alpha(Y',I2,') = ',E17.10)
88    continue

C If the new calculated design point lies outside [XMin,XMax], the program continues calculations using a new design point between the last computed design point & the boundary which was exceeded.

C

Do 9895 i=1,N
If ((X(i).LT.XMin(j,i)).OR.(X(i).GT.XMax(j,i))) then
If (X(i).LT.XMin(j,i)) then
Write(*,8977) Ext(i),X(i),XMin(j,i)
Write(50,8977) Ext(i),X(i),XMin(j,i)
8977    Format(/ 11X,A3,' = ',E17.10, '< ',E17.10) / 
X(i)=XMin(j,i)+(XOld(i,It-1)-XMin(j,i))/2.
else
Write(*,8467) Ext(i),X(i),XMax(j,i)
Write(50,8467) Ext(i),X(i),XMax(j,i)
8467    Format(/ 3X,A3,' = ',E17.10, ' > ',E17.10) / 
X(i)=XOld(i,It-1)+(XMax(j,i)-XOld(i,It-1))/2.
Endif
Write(50,8166) Ext(i),X(i)
8166    Format(/ 11X,A3,' = ',E17.10 /)
Aux=1
else
Aux=0
Endif
continue
If (Aux.NE.0) goto 53
C
Cambience convergence at the design point.
C
Do 85 i=1,N
If (((ABS(X(i)/X(i)-1).GT.0.0001).AND.(X(i).NE.0.))
 AND (X(i)-X(i)-1).GT.0.00001)) then
Write(*,9049)
Write(40,9049)
Write(50,9049)
9049 Format(11X,'Not converging at the design point !'
 1 6X,'CONVERGENCE NOT FOUND WITHIN THE MAXIMUM',
 1 1X,'NUMBER OF ITERATIONS !')
goto 9999
else
Endif
85 continue
goto 23
C
Smoothing of the iteration process (0<=Smooth<=1).
C
Write(50,* ) ' '
Do 4488 i=1,N
If (Smooth.EQ.0.) then
If (It.GT.3) then
If (((ABS(X(i)/X(i)-1).GT.0.0001).AND.
(X(i).NE.0.)).OR.((X(i).EQ.0.).AND.
(X(i)-X(i)-1).GT.0.00001)).OR.((Out(i).EQ.1)) then
Write(*,*) '*****'
Write(50,*) '*****'
k1(i)=k1(i)+1
If (((Sin(k1(i)-1,i)-Sin(k1(i),i)).NE.0).AND.
(k1(i).GT.2)) Step(i)=Step(i)/10.
X(i)=X(i)+Step(i)*ABS(X(i)-X(i))
Endif
If (X(i).LT.X(i)-1) then
Sin(k1(i),i)=1
If (((Sin(k1(i)-1,i)-Sin(k1(i),i)).NE.0).AND.
(k1(i).GT.2)) Step(i)=Step(i)/10.
X(i)=X(i)+Step(i)*ABS(X(i)-X(i))
Endif
Write(50,2237) Ext(i),X(i)
2237 Format(11X,A3,' = ',E17.10)
Out(i)=1
Endif
Endif
else
X(i)=(1.-Smooth)*X(i)+Smooth*X(i)
Write(50,2287) Ext(i),X(i)
2287 Format(11X,A3,' = ',E17.10)
Endif
Endif
Endif
Endif
Else
X(i)=(1.-Smooth)*X(i)+Smooth*X(i)
Write(50,2287) Ext(i),X(i)
2287 Format(11X,A3,' = ',E17.10)
Endif
Endif
4488 continue
If (Smooth.EQ.0.) then
Do 8509 i=1,N
If (((ABS(X(i)/X(i)-1).GT.0.0001).AND.(X(i).NE.0.))
 AND (X(i)-X(i)-1).GT.0.00001)) then
Write(*,949)
Write(40,949)
Write(50,949)
949 Format(11X,'Not converging at the design point !'
 1 6X,'ITERATION No.',I3)
goto 54
else
Endif
8509 continue
else
Write(*,9491) It
Write(50,9491) It

Program Listing

9491     Format(/ 11X,'Not converging at the design point !' ///
       1 6X,'ITERATION No.',I3)
   goto 54
Endif

23     OBJFAcc=ReqOBJFAcc*SigmaOBJF/100.
If (ABS(OBJF).GT.OBJFAcc) then
   If (It.GT.MaxIter) then
      Write(*,9339)
      Write(40,9339)
      Write(50,9339)
   endif
   goto 9999
Endif
Write(*,3394) It
Write(50,3394) It
3394     Format(/ 11X,'The accuracy of the failure function'
       1 / 11X,'is less than the required value !'
       1 /// 6X,'CONVERGENCE NOT FOUND WITHIN THE MAXIMUM',
       1 1X,'NUMBER OF ITERATIONS !')
   goto 54
Endif

9419     Format(/11X,'The relative accuracy of the reliability index'
       1 / 11X,'is less than the required value !'
       1 /// 6X,'CONVERGENCE NOT FOUND WITHIN THE MAXIMUM',
       1 1X,'NUMBER OF ITERATIONS !')
   goto 9999
Endif
Write(*,2394) It
Write(50,2394) It
2394     Format(/11X,'The relative accuracy of the reliability index'
       1 / 11X,'is less than the required value !'
       1 /// 6X,'ITERATION No.',I3)
   goto 54
Endif

8860     Format(/ 3X,'FINAL RESULTS' ///
       1 Write(40,8860)
Write(50,8860)
8860     Format(// 3X,'FINAL RESULTS' ///
       1 Write(40,100) It,OBJF,MuOBJF,SigmaOBJF,RelInd,Beta Acc,
       1 (100.*Prob(j)),DifProb
Write(50,100) (100.*Prob(j)),DifProb
100     Format(/11X,'Total Number of Iterations = ', I3 / 11X,
       1 'Failure Function Z (X) = ',E17.10 / 11X,
       1 'Mean Value of Z = ', E17.10 / 11X,
       1 'Standard Deviation of Z = ', E17.10 / 11X,
       1 'Reliability Index = ', E17.10 / 11X,
       1 'Relative Accuracy of the Reliability Index (%) = ',
       1 E17.10 / 11X,'Probability of Failure (%) = ', F10.6 /
11X,'Difference in Pf Between the Last 2 Iterations = ',
E17.10)
Write(40,922)
Write(50,922)
922 Format//(11X, 'DESIGN POINT COORDINATES' //)
Do 539 i=1,N
Write(40,726) Ext(i),X(i)
Write(50,726) Ext(i),X(i)
726 Format(19X,A3,' = ',E17.10)
539 continue
Write(40,1677)
Write(50,1677)
1677 Format(/)
iii=0
Do 444 i=1,N
Inf(i)=((Alpha(i))**2)*100
Do 3359 ii=1,N
If (i.NE.ii) then
  If (Rho(i,ii).NE.0.) iii=1
Endif
3359 continue
If (iii.EQ.1) then
  Write(40,459) i,Alpha(i),Ext(i),Inf(i)
  Write(50,459) i,Alpha(i),Ext(i),Inf(i)
459 Format(11X,'Alpha(Y',I2,') = ',E17.10 / 11X,
1             'Influence of Y(',A3,') on the Reliability',1X,
1             'Index = ',E14.7 /)
iii=0
else
  Write(40,449) Ext(i),Alpha(i),Ext(i),Inf(i)
  Write(50,449) Ext(i),Alpha(i),Ext(i),Inf(i)
449 Format(11X,'Alpha(',A3,') = ',E17.10 / 11X,
1             'Influence of ',A3,' on the Reliability',1X,
1             'Index = ',E14.7 /)
Endif
444 continue
9999 Ex=1
return
End

C       ################################################################
C
C       Subroutine Allowed(Opt,Aux)
C
C       ########################################### #####################
C       Writes the target value, TR, for each failure mode in the files
C       summary.dat & results.dat .
C       ########################################### #####################
C
C       INPUT VARIABLES:
C       Opt - Failure mode
C       TR - Target values for each FORM calculation
C       k0 - Number of the FORM calculation
C       Aux - Auxiliary variable
C
C       MODELING VARIABLES:
C       L2 - Maximum number of FORM calculations allowed by the program
C       It, AuxRstar - Variables mentioned in the Common statements but
C       not used here
C
C       ################################################################
C
Integer*4 Opt,L2,k0,Aux,It,AuxRstar
Parameter (L2=10)
Real*8 TR(L2)
Common/BLOCK7/TR
Common/BLOCK9/k0,It,AuxRstar
If (Opt.NE.3) then
  If ((Opt.EQ.1).OR.(Opt.EQ.2)) then
    If (Aux.EQ.1) Write(*,5023) k0,TR(K0)
    Write(40,5023) k0,TR(K0)
    Write(50,5023) k0,TR(K0)
  5023 Format(3X,'ALLOWABLE DISCHARGE - m3/s/m (',I2,') = ',
1             E10.3)
  else
    If (Aux.EQ.1) Write(*,5077) k0,TR(K0)
    Write(40,5077) k0,TR(K0)
    Write(50,5077) k0,TR(K0)
  5077 Format(3X,'ALLOWABLE TARGET (',I2,') = ',E10.3)

C7-40
Endif
else
  If (Aux.EQ.1) Write(*,5022) k0,TR(K0)
  Write(40,5022) k0,TR(K0)
  Write(50,5022) k0,TR(K0)
5022  Format(3X,'ALLOWABLE EROSION DISTANCE - m (',I2,') = ',F6.2)
Endif
return
End

********************************************************************************

Subroutine D2Point(j,N,Opt,Mux,Sigmax,VarDis,Abrev,Rho,FDer,  
1         NR,StartParam,Pf,Ext,ExtExt,ParamDesc,Trunc,  
1         Xo,RelInd,MaxIter,StartPt,XMax,XMin,Smooth,  
1         ReqOBJFAcc,Comb)

Calculates the value of the design parameter, Param, for a  
target value of the failure probability, Pf - Mode 2  
(design mode).

********************************************************************************

INPUT VARIABLES:
Opt - Failure mode
DSWL - Definition of the SWL
TL - Seawall toe level
Comb - Consideration or not of combination of actions
j - Number of the combination of actions
NR - Power to which each distribution is raised for each  
combination of actions
N - Number of variables
Ext - Abbreviation of the name of the variable
ExtExt - Description of the variable
ParamDesc - Description of the design parameter
Rho - Correlation coefficient
Mux - Mean of X
Sigmax - Standard deviation of X
VarDis - Type of distribution
Abrev - Abbreviation of the name of the distribution
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
XMin - Minimum value of X
XMax - Maximum value of X
Zeta - Parameter of a distribution
StartPt - Starting value of the variables
FDer - Method of calculation of the first partial derivatives of  
the failure function for overtopping
MaxIter - Maximum number of iterations
TR - Target values for each FORM calculation
k0 - Number of the FORM calculation
Smooth - Smoothing coefficient for the iteration process
ReqOBJFAcc - Required accuracy of the failure function
Pf - Design target failure probability
RelInd - Reliability index which corresponds to Pf
StartParam - Starting value of the design parameter (to start  
itération)

MODELING VARIABLES:
Q - Maximum number of combinations of actions allowed by the  
program
L - Maximum number of variables allowed by the program
Li - Maximum number of iterations allowed by the program
X - Variables of the failure mode
XOld - X in the previous iteration
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal  
distribution of X
Y - Non-correlated, Normal transformed variables
YOld - Y in the previous iteration
Muy - Mean of Y
Sigmay - Standard deviation of Y
V - Matrix of eigenvectors
Vt - Transpose of V
OBJF, OBJF0Id - Failure function
MuOBJF - Mean of OBJF
SigmaOBJF - Standard deviation of OBJF
VarOBJF - Variance of OBJF
OBJGRD - First partial derivatives of OBJF
Program Listing

C       NSTATE - Variable used by subroutine EO4XAF (for more details see NAG, 1993)
C       ItInt - Total number of iterations
C       It - Iteration number for each value of Param
C       L2 - Maximum number of FORM calculations allowed by the program
C       Inf - Influence of each variable on the reliability index
C       Alpha - Sensitivity factors
C       Step - Change in X if Smooth=0 & if the iteration process is in a loop
C       k1 - Number of times that X is changed if Smooth=0 & if the iteration process is in a loop
C       OBJFAcc - Accuracy of the failure function
C       Count - Number of variables for which OBJGRD=0
C       Param - Value of the design parameter which corresponds to Pf
C       dparamOBJF - Inverse of the first partial derivative of OBJF with respect to Param
C       nourtlev - Nourishment top level
C       Pi - 3.14159...
C       i, ii, iii, k, Aux, Aux1, Out, OBJGRDPrev, Mu1OBJF, Sin, AuxRstar - Auxiliary variables
c_itcurv, md, mt, C, T3, Lamda,
C       Eta, x1, x2 - Variables mentioned in the Common statements but not used here

# Definitions of variables and parameters

Integer*4 i,j,k,l0,l1,l2,l,N,Opt,It,FDer,NSTATE,Q,MaxIter,ItInt,
                   Count,Aux,Comb,Aux1,AuxRstar,DSWL,ii,iii
Parameter (L=15)
Parameter (L2=10)
Parameter (L1=200)
Parameter (Q=16)
Character*3 Ext (L),Abrev (Q,L)
Character*17 ExtExt (L)
Character*19 ParamDesc
Integer*4 VarDis (Q,L),Trunc (Q,L),Sin (L,L),Out (L),k1 (L)
Real*8 X (L),Yold (L,L),MuX (Q,L),MuOBJF,MuOBJF,Max (Q,L),
   Vt (L,L),Sigma (L,L),Muy (L,L),Yold (L,L),OBJGRD (L),
   OBJGRDPrev (L),Beta (Q,L),Lamda (Q,L),Eta (Q,L),X (Q,L),
   x2 (Q,L),Pi,OBJF,Param,Pf,StartParam,RelInd,MuxX,
   SigmaMax (L),dparamOBJF,OBJF0old,TL,Xo (Q,L),TR (L2),
   StartPt (Q,L),XMax (Q,L),XMin (Q,L),Smooth,ReqOBJFAcc,
   OBJFAcc,Step (L),NR (Q,L),md,mt,mnour,nourtlev,C,T3
Common/BLOCK3/MuxN (15),C,T3
Common/BLOCK4/md,mt,mnour,nourtlev,ctcurv
Common/BLOCK7/TR
Common/BLOCK8/Beta,Lamda,Eta,x1,x2
Common/BLOCK9/X0,It,AuxRstar
Common/Block11/DSWL,TL
Pi=4.*ATAN (1.)
ItInt=0
Do 3325 i=1,N
   Sin (1,i)=0
   X (1)=StartPt (i,1)
3325   continue

Program = StartParam

Write(*,8877)
Write(40,8877)
Write(50,8877)
8877   Format (/)
Aux1=1
Call Allowed (Opt,Aux1)

Do 7855 i=1,N
   X (i)=StartPt (j,i)
7855   continue

Definition of the iteration number, It.
It=1
5399 Write (*,860) It
Write (50,860) It
860   Format (/,'ITERATION No.',I3)
5499 It=It+1
XMax (j,5)=Param
If (Opt.EQ.1.OR.Opt.EQ.2) then
If (VarDis(j,5).EQ.11) XMax(j,5)=5.77
If (VarDis(j,5).EQ.12) XMax(j,5)=6.995
If (VarDis(j,5).EQ.13) XMax(j,5)=5.57

Endif

If (Opt.EQ.3) then
  If (VarDis(j,1).EQ.11) XMax(j,1)=5.77
  If (VarDis(j,1).EQ.12) XMax(j,1)=6.995
  If (VarDis(j,1).EQ.13) XMax(j,1)=5.57
Endif

C       ================================================================
C       For the failure mode of overtopping, if the variable is the
C       significant wave height, Hs=X(2), & if Hs is limited by the
C       available water depth, then the point of truncation is
C       Xo=0.6(SWL-TL) or Xo=0.6(Tide+Surge-TL).
C       ================================================================
If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(Trunc(j,2).EQ.1)) then
  If (DSWL.EQ.1) then
    Xo(j,2)=0.6*(X(5)-TL)
  else
    Xo(j,2)=0.6*(X(5)+X(6)-TL)
  Endif
  Write(50,4444) Ext(2),Xo(j,2)
  4444     Format(16X,'Xo(',A3,') = ',E17.10)
  If (XMax(j,2).NE.Xo(j,2)) then
    XMax(j,2)=Xo(j,2)
    Write(50,5971) Ext(2),XMax(j,2)
    5971       Format(16X,'XMax(',A3,') = ',E17.10)
  Endif
Endif

C       ================================================================
C       Transformation of non-Normal correlated variables (X with
C       mean=Mux & standard deviation=Sigmax) to Normal correlated
C       variables (X with mean=MuxN & standard deviation=SigmaxN).
C       Transformation of correlated Normal variables (X with mean=MuxN
C       & standard deviation=SigmaxN) to non-correlated Normal variables
C       (Y with mean=Muy & standard deviation=Sigmay).
C       ================================================================
Write(50,4395)
  4395  Format(// 11X, 'EQUIVALENT NORMAL DISTRIBUTION`S PARAMETERS')
Do 7870 i=1,N
  Call EqNorDis(i,j,N,X,VarDis,NR,Ext,MuxN,SigmaxN,Trunc,Xo)
  7870  continue
Call Correlated(N,Ext,MuxN,SigmaxN,Rho,Sigmay,Muy,V,Vt)
Do 6351 i=1,N
  Y(i)=0.
  Do 6371 k=1,N
    Y(i)=Vt(i,k)*X(k)+Y(i)
  6371  continue
Write(50,2998) Ext(i),Y(i)
  2998 Format(11X, 'Y(',A3,') = ',E17.10)
  6351  continue

C       Calculation of the failure function, OBJF, & its derivatives,
C       OBJGRD, at the design point.
C       ================================================================
If (FDer.EQ.2) then
  Call Derivadas(Param,N,X,Ext,Opt,FDer,OBJF,OBJGRD)
else
  Call OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,Opt,Param)
Do 5691 i=1,N
  Write(50,9905) Ext(i),OBJGRD(i)
  5691  continue
  9905 Format(11X,'OBJGRD(',A3,') = ',E17.10)
Endif
Endif

C       ================================================================
C       Transformation of non-Normal correlated variables (X with
C       mean=Mux & standard deviation=Sigmax) to Normal correlated
C       variables (X with mean=MuxN & standard deviation=SigmaxN).
C       ================================================================
Write(50,4395)
  4395  Format(// 11X, 'EQUIVALENT NORMAL DISTRIBUTION`S PARAMETERS')
Do 7870 i=1,N
  Call EqNorDis(i,j,N,X,VarDis,NR,Ext,MuxN,SigmaxN,Trunc,Xo)
  7870  continue
Call Correlated(N,Ext,MuxN,SigmaxN,Rho,Sigmay,Muy,V,Vt)
Do 6351 i=1,N
  Y(i)=0.
  Do 6371 k=1,N
    Y(i)=Vt(i,k)*X(k)+Y(i)
  6371  continue
Write(50,2998) Ext(i),Y(i)
  2998 Format(11X, 'Y(',A3,') = ',E17.10)
  6351  continue

C       Calculation of the failure function, OBJF, & its derivatives,
C       OBJGRD, at the design point.
C       ================================================================
If (FDer.EQ.2) then
  Call Derivadas(Param,N,X,Ext,Opt,FDer,OBJF,OBJGRD)
else
  Call OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,Opt,Param)
Do 5691 i=1,N
  Write(50,9905) Ext(i),OBJGRD(i)
  5691  continue
  9905 Format(11X,'OBJGRD(',A3,') = ',E17.10)
9905 Format(11X,'dZ/d\',A3,' = ',E17.10) 
5691 continue 
Endif 

If (AuxRstar.EQ.1) then 
If (It.NE.2) X(1)=(X(1)+XOld(1,It-1))/2. 
If ((X(1).LT.XMin(j,1)) .OR. (X(1).GT.XMax(j,1)) ) then 
If (X(1).LT.XMin(j,1)) then 
Write(*,1076) Ext(1),X(1),XMin(j,1) 
Write(50,1076) Ext(1),X(1),XMin(j,1) 
1076 Format(// 11X,A3,' = ',E17.10, ' ! ' /) 
X(1)=XMin(j,1)+(XOld(1,It-1)-XMin(j,1))/2. 
else 
Write(*,8401) Ext(1),X(1),XMax(j,1) 
Write(50,8401) Ext(1),X(1),XMax(j,1) 
8401 Format(// 3X,A3,' = ',E17.10, ' ! ' /) 
X(1)=XOld(1,It-1)+(XMax(j,1)-XOld(1,It-1))/2. 
Endif 
Write(50,8100) Ext(1),X(1) 
8100 Format(// 11X,A3,' = ',E17.10 /) 
goto 5399 
Endif 
Write(50,8244) Ext(1),X(1) 
8244 Format(// 11X,A3,' = ',E17.10 /) 
goto 5399 
Endif 
Write(50,9312) OBJF 
9312 Format(// 11X,'Z=',E17.10 /) 
OBJFOld=OBJF 
C Write(50,8822) OBJFOld 
C 8822 Format(// 11X,'ZOld=Z=',E17.10) 
Count=0 
Do 1409 i=1,N 
OBJGRODPrev(i)=OBJGRD(i) 
If (ABS(OBJGRODPrev(i)).LT.(1E-25)) Count=Count+1 
1409 continue 
If (Count.EQ.N) then 
Write(*,99) 
Write(50,99) 
99 Format(// 11X,'ERROR: The derivatives are zero !') 
STOP 
Endif 
Do 4012 i=1,N 
OBJGRD(i)=0. 
Do 4501 k=1,N 
OBJGRD(i)=OBJGRODPrev(k)*V(k,i)+OBJGRD(i) 
4501 continue 
Write(50,8108) i,OBJGRD(i) 
8108 Format(11X,'dZ/dY(',I2,') = ',E17.10) 
4012 continue 
Write(50,*) ' ' 
C ================================================================ 
C Calculation of the mean value, MuOBJF, & the standard deviation, 
C SigmaOBJF, of the failure function. 
C ============================================================== 
MuOBJF=0 
VarOBJF=0 
Do 844 i=1,N 
MuOBJF=(Muy(i)-Y(i))*OBJGRD(i)+MuOBJF 
VarOBJF=((OBJGRD(i)*Sigmay(i))**2)+VarOBJF 
844 continue 
MuOBJF=OBJFOld+MuOBJF 
SigmaOBJF=SQRT(VarOBJF) 
C ============================================================== 
C Calculation of the sensitivity factors, Alpha, & the new design 
C point, Y, ( & consequently X). 
C ============================================================== 
Do 9019 i=1,N 
Alpha(i)=(Sigmay(i)/SigmaOBJF)*OBJGRD(i) 
YOld(i,It-1)=Y(i) 
Y(i)=Muy(i)-Alpha(i)*RelInd*Sigmay(i) 
9019 continue 
Do 2635 i=1,N 
XOld(i,It-1)=X(i) 
X(i)=0. 
Do 1657 k=1,N 
X(i)=V(i,k)*Y(k)+X(i) 
1657 continue 
Write(50,1987) Ext(i),XOld(i,It-1),Ext(i),X(i)
1987     Format(11X,'XOld(',A3,') = ',E17.10,3X,A3 ,' = ',E17.10)
2635   continue
879    Format(11X,'Alpha(Y',I2,') = ',E17.10)
889    continue

C       ================================================================
C       If the new calculated design point lies outside [XMin,XMax], the
C       program continues calculations using a new design point between
C       the last computed design point & the boundary which was
C       exceeded.
C       ================================================================

Do 9895 i=1,N
   If ((X(i).LT.XMin(j,i)).OR.(X(i).GT.XMax(j,i))) then
      If (X(i).LT.XMin(j,i)) then
         Write(*,8977) Ext(i),X(i),XMin(j,i)
         Write(50,8977) Ext(i),X(i),XMin(j,i)
         X(i)=XMin(j,i)+(XOld(i,It-1)-XMin(j,i))/2.
         Write(50,8177) Ext(i),X(i)
      else
         Write(*,8477) Ext(i),X(i),XMax(j,i)
         Write(50,8477) Ext(i),X(i),XMax(j,i)
         X(i)=XOld(i,It-1)+(XMax(j,i)-XOld(i,It-1))/2.
         Write(50,8166) Ext(i),X(i)
      endif
   endif
   Aux=1
   else
      Aux=0
   endif
9895   continue
   If (Aux.NE.0) goto 5399

C       ================================================================
C       Check convergence at the design point.
C       ================================================================

Do 859 i=1,N
   If (((ABS(XOld(i,It-1)/X(i)-1).GT.0.0001).AND.(X(i).NE.0.))
1       .OR.((X(i).EQ.0.).AND.(X(i)-XOld(i,It-1).GT.0.00001))) then
      If (It.GT.MaxIter) then
         Write(*,9049)
         Write(40,9049)
         Write(50,9049)
         9049         Format(/ 11X,'Not converging at the design point !'///
1               6X,'CONVERGENCE NOT FOUND WITH IN THE MAXIMUM',
1               1X,'NUMBER OF ITERATIONS !')
         goto 9999
      else
         goto 13
      endif
   endif
859    continue
   goto 23

C       ================================================================
C       Smoothing of the iteration process (0<=Smooth<=1).
C       ================================================================

13    Write(50,*) ' '

Do 4488 i=1,N
   If (Smooth.EQ.0.) then
      If (It.GT.3) then
         If (((ABS(XOld(i,It-1)/X(i)-1).GT.0.0001).AND.
1             (X(i).NE.0.)).OR.((X(i).EQ.0.).AND.
1             (X(i)-XOld(i,It-1).LE.0.00001)).OR.(Out(i).EQ.1)) then
            If (((ABS(XOld(i,It-2)/X(i)-1).LE.0.0001)
1                 .AND.(X(i).NE.0.)).OR.((X(i).EQ.0.)
1                 .AND.(X(i)-XOld(i,It-2).LE.0.00001)).OR.
1                 (((ABS(XOld(i,It-3)/X(i)-1).LE.0.0001)
1                     .AND.(X(i).NE.0.)).OR.((X(i).EQ.0.)
1                     .AND.(X(i)-XOld(i,It-3).LE.0.00001)).OR.(Out(i).EQ.1)) then
                Write(*,'****
1                   ')
                Write(50,*),'****
1                   'k1(i)=k1(i)+1
                If (X(i).GT.XOld(i,It-1)) then
                   Sin(k1(i),i)=4
                Write('*','****
1                   ')
               endif
         endif
      else
         If (((Sin(k1(i)-1,i)-Sin(k1(i),i)).NE.0.).AND.
1             (k1(i).GT.3)) Step(i)=Step(i)/10.
         X(i)=XOld(i,It-1)+Step(i)*ABS(XOld(i,It-1)-X(i))
Endif
If (X(i).LT.XOld(i,It-1)) then
  Sin(k1(i),i)=1
  If (((Sin(k1(i)-1,i)-Sin(k1(i),i)).NE.0).AND. 
   (k1(i).GT.3)) Step(i)=Step(i)/10.
  X(i)=X(i)+Step(i)*ABS(XOld(i,It-1)-X(i))
Endif
Write(50,2237) Ext(i),X(i)
2237  Format(11X,A3,' = ',E17.10)
Out(i)=1
Endif
Endif
else
  X(i)=(1.-Smooth)*X(i)+Smooth*XOld(i,It-1)
Write(50,2287) Ext(i),X(i)
2287       Format(11X,A3,' = ',E17.10)
Endif
4488   continue
If (Smooth.EQ.0.) then
  Do 8509 i=1,N
    If (((ABS(XOld(i,It-1)/X(i)-1).GT.0.0001).AND.(X(i).NE.0.)) 
     .OR.((X(i).EQ.0.).AND.(X(i)-XOld(i,It-1).GT.0.00001))) then
      Write(*,949) It
      Write(50,949) It
      949          Format(/ 11X,'Not converging at the design point !' /// 
                   6X, 'ITERATION No.',I3)
      goto 5499
    Endif
  Enddo 8509
  It=1
  Do 3295 i=1,N
    Step(i)=0.001
    Out(i)=0
    k1(i)=1
  Enddo 3295
Write(*,2394) It
Write(50,2394) It
2394       Format(/ 11X,'The accuracy of the failure function' 
            1 / 11X,'is less than the required value !' 
            1 /// 6X, 'CONVERGENCE NOT FOUND WITHIN THE MAXIMUM', 
            1 1X, 'NUMBER OF ITERATIONS !')
      goto 9999
Endif
23     ItInt=ItInt+(It-1)
OBJFAcc=ReqOBJFAcc*SigmaOBJF/100.
If (ABS(OBJFOld)>OBJFAcc) then
  If (It.GT.MaxIter) then
    Write(*,9419)
    Write(40,9419)
    Write(50,9419)
    9419       Format(/ 11X,'The accuracy of the failure function' /// 
                        6X, 'ITERATION No.',I3)
    goto 9999
  Endif
  It=1
  Do 3295 i=1,N
    Step(i)=0.001
    Out(i)=0
    k1(i)=1
  Enddo 3295
  continue
Endif
2384   continue
Write(*,2394) It
Write(50,2394) It
2394       Format(/ 11X,'The accuracy of the failure function' 
            1 / 11X,'is less than the required value !' 
            1 /// 6X, 'ITERATION No.',I3)
      goto 9999
Endif
---
Call OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,Opt,Param)
Do 4690 i=1,N
Write(50,9095) Ext(i),OBJGRD(i)
4690       Format(11X,'dZ/d',A3, ' = ',E17.10)
Endif
If (AuxRstar.EQ.1) then
   If (It.NE.2) X(1)=(X(1)+XOld(1,It-1))/2.
   If ((X(1).LT.XMin(j,1)).OR.(X(1).GT.XMax(j,1))) then
      If (X(1).LT.XMin(j,1)) then
         Write(*,5975) Ext(1),X(1),XMin(j,1)
         Write(50,5975) Ext(1),X(1),XMin(j,1)
      5975         Format(/ 11X,A3,' = ',E17.10, '< ',E17.10, ' ! ' /)
      else
         Write(*,8551) Ext(1),X(1),XMax(j,1)
         Write(50,8551) Ext(1),X(1),XMax(j,1)
      8551         Format(/ 3X,A3,' = ',E17.10, ' > ',E17.10, ' ! ' /)
      X(1)=XMin(j,1)+(XOld(1,It-1)-XMin(j,1))/2.
   Endif
   Write(50,8116) Ext(1),X(1)
   8116         Format(/ 11X,A3,' = ',E17.10 /)
   goto 5399
   Endif
   Write(50,8233) Ext(1),X(1)
   8233         Format(/ 11X,A3,' = ',E17.10 /)
   goto 5399
   dparamdOBJF=1./OBJGRD(N)
   Write(50,9772) OBJFOld
   9772     Format(/ 11X,'Z=',E17.10)
   Write(50,9395) dparamdOBJF
   9395     Format(11X,'dparamdOBJF = ',E17.10)
   C         ----------
   C         New estimate of the value of the design parameter, param.
   C         ----------
   Param=Param-OBJFOld*dparamdOBJF
   If (Param.LT.0) then
      If (Opt.EQ.3) then
         Write(*,4007)
         Write(50,4007)
      4007         Format(// 11X,'ERROR: Nourishment Width < 0 !')
      else
         Write(*,4037)
         Write(50,4037)
      4037         Format(// 11X,'ERROR: Crest Level < 0  !')
      STOP
   Endif
   Endif
   Write(50,5599) Param
   5599     Format(/ 11X,'New Value of the Design Parameter = ',E17.10 /)
   N=N-1
   goto 5499
Endif
C       =========================================== =====================
C       Print the final results.
C       =========================================== =====================
It=ItInt
Write(40,8760)
Write(50,8760)
8760   Format(// 3X,'FINAL RESULTS' //)
Write(40,100) It,OBJFOLD,MuOBJF,SigmaOBJF,RelInd,(100.*Pf),Param
Write(50,100) It,OBJFOLD,MuOBJF,SigmaOBJF,RelInd,(100.*Pf),Param
100   Format(11X,'Total Number of Iterations = ',I3 / 11X,
     1 'Failure Function Z (X) = ',E17.10 / 11X,
     1 'Mean Value of Z = ',E17.10 / 11X,
     1 'Standard Deviation of Z = ',E17.10 / 11X,
     1 'Reliability Index = ',E17.10 / 11X,
     1 'Target Probability of Failure (%) = ',F10.6 / 11X,
     1 'Design Parameter = ',E17.10)
Write(40,922)
Write(50,922)
922   Format(// 11X, 'DESIGN POINT COORDINATES' //)
   Do 539 i=1,N
      Write(40,726) Ext(i),X(i)
      Write(50,726) Ext(i),X(i)
   726   continue
   Write(40,1677)
   Write(50,1677)
   1677   Format(/)
   iii=0
Do 444 i=1,N
Inf(i)=((Alpha(i))**2)*100
Do 3359 ii=1,N
If (i.NE.ii) then
  If ((Rho(i,ii)).NE.0.) iii=1
Endif
3359     continue
If (iii.EQ.1) then
  Write(40,459) i,Alpha(i),Ext(i),Inf(i)
  Write(50,459) i,Alpha(i),Ext(i),Inf(i)
459        Format(11X,'Alpha(',A3,') = ',E17.10 / 11X,
1             'Influence of ',A3,' on the Reliability',1X,
1             'Index = ',E14.7 /)
  iii=0
else
  Write(40,449) Ext(i),Alpha(i),Ext(i),Inf(i)
  Write(50,449) Ext(i),Alpha(i),Ext(i),Inf(i)
449        Format(11X,'Alpha(',A3,') = ',E17.10 / 11X,
1             'Influence of ',A3,' on the Reliability',1X,
1             'Index = ',E14.7 /)
Endif
444    continue
9999   return
End

*********************************************************************

Subroutine VarExt(N,Opt,Ext,ExtExt,ParamDesc)

*********************************************************************

Defines the variables specific to each failure mode, ExtExt, their abbreviation, Ext, & the design parameter, ParamDesc.

*********************************************************************

INPUT VARIABLES:
Opt - Failure mode
N - Number of variables
DSWL - Definition of the SWL

MODELING VARIABLE:
L - Maximum number of variables allowed by the program
TL - Variable mentioned in the Common statements but not used here

OUTPUT VARIABLES:
Ext - Abbreviation of the name of the variable
ExtExt - Description of the variable
ParamDesc - Description of the design parameter

*********************************************************************

Integer*4 L,N,Opt,DSWL
Character*3 Ext(L)
Character*17 ExtExt(L)
Character*19 ParamDesc
Parameter (L=15)
Real*8 TL
Common/BLOCK11/DSWL,TL

If (Opt.EQ.1) then
  Ext(1)='Tp'  
  Ext(2)='Hs'  
  Ext(3)='A'   
  Ext(4)='B'   
  Ext(5)='SWL' 
  Ext(6)='TAl' 
  Ext(7)='r '  
  Ext(8)='eB' 
  ExtExt(1)='Peak Wave Period' 
  ExtExt(2)='Wave Height'    
  ExtExt(3)='H&R Parameter'  
  ExtExt(4)='H&R Parameter'  
  ExtExt(5)='Still-Water-Level' 
  ExtExt(6)='Seawall Slope'  
  ExtExt(7)='Roughness'      
  ExtExt(8)='Model Parameter' 
If (DSWL.EQ.2) then
  Ext(5)='Tid'    
  Ext(6)='Sur'
Ext(7)="TAl"
Ext(8)="r"
Ext(9)="eB"
ExtExt(5)="Tide Level"
ExtExt(6)="Surge"
ExtExt(7)="Seawall Slope"
ExtExt(8)="Roughness"
ExtExt(9)="Model Parameter"
Endif
ParamDesc='Seawall Crest Level'
elseif (Opt.EQ.2) then
Ext(1)="Tm"
Ext(2)="Hs"
Ext(3)="A"
Ext(4)="B"
Ext(5)="SWL"
Ext(6)="r"
Ext(7)="eB"
ExtExt(1)="Mean Wave Period"
ExtExt(2)="Wave Height"
ExtExt(3)="Owen Parameter"
ExtExt(4)="Owen Parameter"
ExtExt(5)="Still-Water-Level"
ExtExt(6)="Roughness"
ExtExt(7)="Model Parameter"
If (DSWL.EQ.2) then
Ext(5)="Tid"
Ext(6)="Sur"
Ext(7)="r"
Ext(8)="eB"
ExtExt(5)="Tide Level"
ExtExt(6)="Surge"
ExtExt(7)="Roughness"
ExtExt(8)="Model Parameter"
Endif
ParamDesc='Seawall Crest Level'
elseif (Opt.EQ.3) then
Ext(1)="Hs"
Ext(2)="D50"
Ext(3)="DP"
Ext(4)="SD"
Ext(5)="GB"
Ext(6)="Ac"
Ext(7)="h"
ExtExt(1)="Wave Height"
ExtExt(2)="Particle Size"
ExtExt(3)="Initial Profile"
ExtExt(4)="Surge Duration"
ExtExt(5)="Gust Bumps"
ExtExt(6)="Accuracy Comput."
ExtExt(7)="Surge Level"
If (DSWL.EQ.2) then
  Ext(1)="Tid"
  Ext(2)="Sur"  
  Ext(3)="Hs"
  Ext(4)="D50"
  Ext(5)="DP"
  Ext(6)="SD"
  Ext(7)="GB"
  Ext(8)="Ac"
  ExtExt(1)="Tide Level"
  ExtExt(2)="Surge"
  ExtExt(3)="Wave Height"
  ExtExt(4)="Particle Size"
  ExtExt(5)="Initial Profile"
  ExtExt(6)="Surge Duration"
  ExtExt(7)="Gust Bumps"
  ExtExt(8)="Accuracy Comput."
ExtExt(7)="Tid"
ExtExt(8)="Sur"
Endif
ParamDesc='Nourishment Width'
else
ParamDesc='Nothing'
If (N.GE.1) then
Ext(1)="X1"
ExtExt(1)="X(1)"
If (N.GE.2) then
Ext(2)="X2"
ExtExt(2)="X(2)"
Endif
If (N.GE.3) then
Ext(3)='X3 '
ExtExt(3)='X(3)'
If (N.GE.4) then
Ext(4)='X4 '
ExtExt(4)='X(4)'
If (N.GE.5) then
Ext(5)='X5 '
ExtExt(5)='X(5)'
If (N.GE.6) then
Ext(6)='X6 '
ExtExt(6)='X(6)'
If (N.GE.7) then
Ext(7)='X7 '
ExtExt(7)='X(7)'
If (N.GE.8) then
Ext(8)='X8 '
ExtExt(8)='X(8)'
If (N.GE.9) then
Ext(9)='X9 '
ExtExt(9)='X(9)'
If (N.GE.10) then
Ext(10)='X10'
ExtExt(10)='X(10)'
If (N.GE.11) then
Ext(11)='X11'
ExtExt(11)='X(11)'
If (N.GE.12) then
Ext(12)='X12'
ExtExt(12)='X(12)'
If (N.GE.13) then
Ext(13)='X13'
ExtExt(13)='X(13)'
If (N.GE.14) then
Ext(14)='X14'
ExtExt(14)='X(14)'
If (N.GE.15) then
Ext(15)='X15'
ExtExt(15)='X(15)'
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Endif
Return
End

Subroutine Combination(N,NumTVAc,CombAc,NCombAc,r,TVAc, NR)

Returns, for each combination of actions, the power, NR, to which each distribution is raised.

INPUT VARIABLES:
N - Number of variables
NCombAc, CombAc - Number of combinations of actions
NumTVAc - Number of time-varying actions
c - Repetitions of each action in the design life
TVAc - Number of the time-varying actions in increasing order of the number of repetitions

MODELING VARIABLES:
Q - Maximum number of combinations of actions allowed by the program
L - Maximum number of variables allowed by the program
Program Listing

C       i, j, Aux - Auxiliary variables
C
C       OUTPUT VARIABLE:
C       NR - Power to which each distribution is raised for each
C            combination of actions
C
C       ##################################################################

Integer*4 j,i,L,Q,Aux,NumTVAc,NCombAc,CombAc,N
Parameter (L=15)
Parameter (Q=16)
Integer*4 TVAc(L)
Real*8 NR(Q,L),r(L)
Aux=-1
If (CombAc.EQ.2) then
  Do 144 i=1,N
      Do 199 j=1,NCombAc
          NR(j,i)=1
      continue
 199     continue
 144     continue
  Do 14 i=1,NumTVAc
      Do 13 j=1,NCombAc
          If (i.EQ.1) then
              Aux=Aux*(-1)
          If (Aux.LT.0) then
              NR(j,TVAc(1))=1
          else
              NR(j,TVAc(1))=r(TVAc(1))
          Endif
          elseif (i.EQ.2) then
              If (j.EQ.1.OR.j.EQ.5.OR.j.EQ.9.OR.j.EQ.13) then
                  NR(j,TVAc(2))=r(TVAc(2))/r(TVAc(1))
              elseif (j.EQ.2.OR.j.EQ.6.OR.j.EQ.10.OR.j.EQ.14) then
                  NR(j,TVAc(2))=r(TVAc(2))
              else
                  NR(j,TVAc(2))=1
              Endif
          elseif (i.EQ.3) then
              If (j.EQ.1.OR.j.EQ.2.OR.j.EQ.3.OR.j.EQ.4) then
                  NR(j,TVAc(3))=r(TVAc(3))/r(TVAc(2))
              elseif (j.EQ.5.OR.j.EQ.6) then
                  NR(j,TVAc(3))=r(TVAc(3))/r(TVAc(1))
              elseif (j.EQ.7) then
                  NR(j,TVAc(3))=r(TVAc(3))
              else
                  NR(j,TVAc(3))=1
              Endif
          elseif (i.EQ.4) then
              If (j.EQ.9.OR.j.EQ.10.OR.j.EQ.11.OR.j.EQ.12) then
                  NR(j,TVAc(5))=r(TVAc(5))/r(TVAc(3))
              elseif (j.EQ.13.OR.j.EQ.14) then
                  NR(j,TVAc(5))=r(TVAc(5))/r(TVAc(1))
              elseif (j.EQ.15) then
                  NR(j,TVAc(5))=r(TVAc(5))
              else
                  NR(j,TVAc(5))=r(TVAc(5))/r(TVAc(4))
              Endif
          endif
      Endif
  13     continue
  Do 14 i=1,NumTVAc
      Do 17 j=1,N
          NR(i,j)=1
      continue
  Do 16 j=1,NumTVAc
      If (i.EQ.j) then
          NR(i,TVAc(j))=r(TVAc(j))
      else
          If (i.GT.j) then
              If (j.EQ.1.OR.j.EQ.2.OR.j.EQ.3.OR.j.EQ.4) then
                  NR(j,TVAc(4))=r(TVAc(4))/r(TVAc(3))
              elseif (j.EQ.5.OR.j.EQ.6) then
                  NR(j,TVAc(4))=r(TVAc(4))/r(TVAc(2))
              elseif (j.EQ.7) then
                  NR(j,TVAc(4))=r(TVAc(4))
              else
                  NR(j,TVAc(4))=1
              Endif
          endif
      Endif
  17     continue
C    C7-51
NR(i,TVAc(j))=1
else
   NR(i,TVAc(j))=(r(TVAc(j)))/(r(TVAc(j-1)))
Endif
16 continue
15 continue
Endif
return
End

************************************************************************
Subroutine InvNormal(C,Rel)
************************************************************************
Returned the reliability level, Rel, for a specific value, C, of
the exceedance cumulative distribution function of the standard
Normal distribution (tabulated in statistical books, e.g.
Abramowitz & Stegun, 1964). It uses the data file distnorm.dad
which contains the tabulated values.
************************************************************************
INPUT VARIABLE:
C - Value of the exceedance cumulative distribution function of
the standard Normal distribution
MODELING VARIABLES:
M - Number of points of the standard Normal distribution
   tabulated in file distnorm.dad
Phi - Value tabulated in file distnorm.dad of the exceedance
   cumulative distribution function of the standard Normal
   distribution
Beta - Standard Normal variable, tabulated in file distnorm.dad,
   which corresponds to Phi
k - Auxiliary variable
OUTPUT VARIABLE:
Rel - Standard Normal variable which corresponds to C
************************************************************************
Integer*4 k,M
Parameter (M=453)
Real*8 C,Rel,Beta(M),Phi(M)
Open(Unit=20, File='distnorm.dad', Status='Old')
Do 910 k=1,M
  Read(20,*) Beta(k), Phi(k)
910 continue
Do 1132 k=1,M
  If (C.LE.0.5) then
     If (Phi(k).LE.C) then
        If (Phi(k).LT.C) then
           Rel=((C-Phi(k))*((Beta(k+1))-Beta(k)))/
                ((Phi(k+1))-Phi(k))+Beta(k)
(goto 1150)
        else
           Rel=Beta(k)
(goto 1150)
     Endif
  Endif
  else
     If (Phi(k).LE.(1.-C)) then
        If (Phi(k).LT.(1.-C)) then
           Rel=-(((1.-C)-Phi(k))*((Beta(k+1))-Beta(k)))/
                ((Phi(k+1))-Phi(k))+Beta(k)
(goto 1150)
        else
           Rel=-Beta(k)
(goto 1150)
     Endif
  Endif
1132 continue
1150 Close (Unit=20)
return
End
Subroutine WhatDist(i,j,VarDis,Abrev,Carac)

Returns the name, Carac, & the abbreviation, Abrev, of the chosen statistical distribution, VarDis.

INPUT VARIABLES:
i - Number of the variable
j - Number of the combination of actions
VarDis - Type of distribution

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Carac - Name of the distribution
Abrev - Abbreviation of the name of the distribution

Integer*4 L,Q,i,j
Parameter (L=15)
Parameter (Q=16)
Character*30 Carac(Q,L)
Character*3 Abrev(Q,L)
Integer*4 VarDis(Q,L)

If (VarDis(j,i).EQ.0) then
  Carac(j,i)='Deterministic'  
  Abrev(j,i)='Det'
elseif (VarDis(j,i).EQ.1) then
  Carac(j,i)='Normal (Gaussian)'  
  Abrev(j,i)='Nor'
elseif (VarDis(j,i).EQ.2) then
  Carac(j,i)='Log-Normal'  
  Abrev(j,i)='LgN'
elseif (VarDis(j,i).EQ.3) then
  Carac(j,i)='Maxima Type I (Gumbel)'  
  Abrev(j,i)='Gum'
elseif (VarDis(j,i).EQ.4) then
  Carac(j,i)='Rectangular (Uniform)'  
  Abrev(j,i)='Uni'
elseif (VarDis(j,i).EQ.5) then
  Carac(j,i)='Gamma'  
  Abrev(j,i)='Gam'
elseif (VarDis(j,i).EQ.6) then
  Carac(j,i)='Beta'  
  Abrev(j,i)='Bet'
elseif (VarDis(j,i).EQ.7) then
  Carac(j,i)='Maxima Type II (Frechet)'  
  Abrev(j,i)='Fre'
elseif (VarDis(j,i).EQ.8) then
  Carac(j,i)='Exponential'  
  Abrev(j,i)='Exp'
elseif (VarDis(j,i).EQ.9) then
  Carac(j,i)='Rayleigh'  
  Abrev(j,i)='Ray'
elseif (VarDis(j,i).EQ.10) then
  Carac(j,i)='Minima Type III (Weibull)'  
  Abrev(j,i)='Wei'
elseif (VarDis(j,i).EQ.11) then
  Carac(j,i)='User-Defined Distribution'  
  Abrev(j,i)='UD1'
elseif (VarDis(j,i).EQ.12) then
  Carac(j,i)='User-Defined Distribution'  
  Abrev(j,i)='UD2'
else
  Carac(j,i)='User-Defined Distribution'  
  Abrev(j,i)='UD3'
Endif
return
End
Subroutine WriCharVar(i,j,VarDis,Trunc,Ext,Carac,Xo,Mux,Sigmax,Opt)

# Writes the following characteristics of the variables in the files summary.dat & results.dat:
# - Type of distribution
# - Type of truncation & point of truncation, Xo (if the distribution is truncated)
# - Mean, Mux, standard deviation, Sigmax, & lower limit, Zeta

# INPUT VARIABLES:
# Opt - Failure mode
# TL - Seawall toe level
# DSWL - Definition of the SWL
# i - Number of the variable
# j - Number of the combination of actions
# Ext - Abbreviation of the name of the variable
# Mux - Mean of X
# Sigmax - Standard deviation of X
# VarDis - Type of distribution
# Carac - Name of the distribution
# Trunc - Type of truncation
# Xo - Point of truncation (if the distribution is truncated)
# Zeta - Parameter of a distribution
# x1 - Lower limit on X for a Beta distribution
# x2 - Upper limit on X for a Beta distribution

# MODELING VARIABLES:
# L - Maximum number of variables allowed by the program
# Q - Maximum number of combinations of actions allowed by the program
# Lamda, Eta - Variables mentioned in the Common statement but not used here

#**************************************************************************************

Integer*4 i,j,L,Q,Opt,DSWL
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Character*30 Carac(Q,L)
Integer*4 VarDis(Q,L),Trunc(Q,L)
Real*8 Xo(Q,L),Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L),
Real*8 x1(Q,L),x2(Q,L),TL
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
Common/BLOCK11/DSWL,TL
Write(40,8020) Ext(i),Carac(j,i)
Write(50,8020) Ext(i),Carac(j,i)
8020 Format(11X,'Probability Distribution of ',A3,' = ',A30)
If (Trunc(j,i).NE.0) then
If (Trunc(j,i).EQ.1) then
If ((Opt.EQ.1.OR.Opt.EQ.2).AND.(i.EQ.2)) then

Write(40,802) Ext(i),TL
Write(50,802) Ext(i),TL
802            Format(11X,'The Distribution of ',A3,
1                 ' is truncated above Xo = 0.6(SWL-TL)' / 11X,
1                 'Seawall Toe Level (TL) = ',E17.10)
else
Write(40,829) Ext(i),TL
Write(50,829) Ext(i),TL
8029           Format(11X,'The Distribution of ',A3,
1             ' is truncated below Xo = ',E17.10)
Endif
else
Write(40,82) Ext(i),Xo(j,i)
Write(50,82) Ext(i),Xo(j,i)
82           Format(11X,'The Distribution of ',A3,
1             ' is truncated above Xo = ',E17.10)
Endif
else
Write(40,829) Ext(i),Xo(j,i)
Write(50,829) Ext(i),Xo(j,i)
829          Format(11X,'The Distribution of ',A3,
1             ' is truncated below Xo = ',E17.10)
Endif
Subroutine Parameters(i,j,Mux,Sigmax,VarDis,Ext)

Calls the subroutines required to calculate the distribution's parameters, Zeta, Lambda & Eta, for each variable.

INPUT VARIABLES:
- i - Number of the variable
- j - Number of the combination of actions
- Ext - Abbreviation of the name of the variable
- Mux - Mean of X
- Sigmax - Standard deviation of X
- VarDis - Type of distribution
- x1 - Lower limit on X for a Beta distribution
- x2 - Upper limit on X for a Beta distribution

MODELING VARIABLES:
- L - Maximum number of variables allowed by the program
- Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
- Zeta, Lambda, Eta - Parameters of a distribution

Subroutine Parameters(i,j,Mux,Sigmax,VarDis,Ext)

Calls the subroutines required to calculate the distribution's parameters, Zeta, Lambda & Eta, for each variable.

INPUT VARIABLES:
- i - Number of the variable
- j - Number of the combination of actions
- Ext - Abbreviation of the name of the variable
- Mux - Mean of X
- Sigmax - Standard deviation of X
- VarDis - Type of distribution
- x1 - Lower limit on X for a Beta distribution
- x2 - Upper limit on X for a Beta distribution

MODELING VARIABLES:
- L - Maximum number of variables allowed by the program
- Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
- Zeta, Lambda, Eta - Parameters of a distribution

If (VarDis(j,i).EQ.0.OR.VarDis(j,i).GT.10) then
  Zeta(j,i)=Mux(j,i)
  Lambda(j,i)=Sigmax(j,i)
  Write(30,2245) Ext(i),Zeta(j,i),Ext(i),Lambda(j,i)
  2245   Format(16X,'Mean (',A3,') = ',E17.10 / 16X,'Standard Deviation (',A3,') = ',E17.10)
elseif (VarDis(j,i).EQ.1) then
  Call PNormal(j,i,Ext,Mux,Sigmax,Zeta,Lambda)
elseif (VarDis(j,i).EQ.2) then
  Call PLogNormal(j,i,Ext,Mux,Sigmax,Zeta,Lambda)
elseif (VarDis(j,i).EQ.3) then
  Call PGumbel(j,i,Ext,Mux,Sigmax,Zeta,Lambda)
elseif (VarDis(j,i).EQ.4) then
  Call PRectangular(j,i,Ext,Mux,Sigmax,Zeta,Lambda)
else
  Write(30,2245) Ext(i),Zeta(j,i),Ext(i),Lambda(j,i)
  2245   Format(16X,'Mean (',A3,') = ',E17.10 / 16X,'Standard Deviation (',A3,') = ',E17.10)
end
Subroutine EqCharac(N,NCombAc,Abrev,Carac,VarDis,Trunc,Xo,Mux,Sigmax)

If the distributions provided for the variables are the basic distributions, then the characteristics of the variables are the same for all the combinations considered. What differs in each combination is the power to which each distribution is raised. This subroutine gives to the variables in all combinations of actions the same characteristics which the variables have in combination 1.

INPUT VARIABLES:
N - Number of variables
NCombAc - Number of combinations of actions

INPUT/OUTPUT VARIABLES:
Mux - Mean of X
Sigmax - Standard deviation of X
VarDis - Type of distribution
Carac - Name of the distribution
Abrev - Abbreviation of the name of the distribution
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
Zeta, Lamda, Eta - Parameters of a distribution
x1 - Lower limit on X for a Beta distribution
x2 - Upper limit on X for a Beta distribution

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
i, k - Auxiliary variables

Do 8000 k=2,NCombAc
   Do 7000 i=1,N
      Abrev(k,i)=Abrev(1,i)
      Carac(k,i)=Carac(1,i)
      VarDis(k,i)=VarDis(1,i)
      Trunc(k,i)=Trunc(1,i)
      Xo(k,i)=Xo(1,i)
      Mux(k,i)=Mux(1,i)
      Sigmax(k,i)=Sigmax(1,i)
      Zeta(k,i)=Zeta(1,i)
      If (VarDis(1,i).EQ.9) Lamda(k,i)=Lamda(1,i)
      If (VarDis(1,i).EQ.10) Eta(k,i)=Eta(1,i)
      If (VarDis(1,i).EQ.6) then
         x1(k,i)=x1(1,i)
   Enddo
Enddo

Subroutine Inverse(j,i,Mux,NR,VarDis,StartPt)

Returns the argument, StartPt, corresponding to a value of 0.5 of a chosen cumulative distribution function, i.e. CDF(StartPt)=0.5.

INPUT VARIABLES:
i - Number of the variable
j - Number of the combination of actions
NR - Power to which each distribution is raised for each combination of actions
Mux - Mean of X
VarDis - Type of distribution
Zeta, Lambda, Eta - Parameters of a distribution

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
CDF - Value of the cumulative distribution function
C - Value of the exceedance cumulative distribution function
Rel - Standard Normal variable which corresponds to C
x1, x2 - Variables mentioned in the Common statement but not used here

OUTPUT VARIABLE:
StartPt - Starting value of the variables

### Subroutine Inverse(j,i,Mux,NR,VarDis,StartPt)

```fortran
C       ################################################################
C
Subroutine Inverse(j,i,Mux,NR,VarDis,StartPt)
C
C       ################################################################
C
C       Returns the argument, StartPt, corresponding to a value of 0.5 of a chosen cumulative distribution function, i.e. CDF(StartPt)=0.5 .
C
C       ################################################################
C
C       INPUT VARIABLES:
C       i - Number of the variable
J - Number of the combination of actions
NR - Power to which each distribution is raised for each combination of actions
Mux - Mean of X
VarDis - Type of distribution
Zeta, Lambda, Eta - Parameters of a distribution
C
C       MODELING VARIABLES:
C       L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
CDF - Value of the cumulative distribution function
C - Value of the exceedance cumulative distribution function
Rel - Standard Normal variable which corresponds to C
x1, x2 - Variables mentioned in the Common statement but not used here
C
C       OUTPUT VARIABLE:
C       StartPt - Starting value of the variables
C
C       ################################################################
C
Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Integer*4 VarDis(Q,L)
Real*8 Mux(Q,L),Lambda(Q,L),Zeta(Q,L),x1(Q,L),x2(Q,L),
      Rel,CDF,C,StartPt(Q,L),NR(Q,L)
Common/BLOCK8/Zeta,Lambda,Eta,x1,x2

CDF=0.5**(1./NR(j,i))
If (VarDis(j,i).EQ.0) then
   StartPt(j,i)=Mux(j,i)
elseif (VarDis(j,i).EQ.1) then
   C=1.-CDF
   Call InvNormal(C,Rel)
   StartPt(j,i)=Rel*Lambda(j,i)+Zeta(j,i)
elseif (VarDis(j,i).EQ.2) then
   C=1.-CDF
   Call InvNormal(C,Rel)
   StartPt(j,i)=Rel*Lambda(j,i)+Zeta(j,i)
elseif (VarDis(j,i).EQ.3) then
   StartPt(j,i)=Lambda(j,i)-LOG(-LOG(CDF))/(Zeta(j,i))
elseif (VarDis(j,i).EQ.4) then
   StartPt(j,i)=Zeta(j,i)+(Lambda(j,i)-Zeta(j,i))*CDF
elseif (VarDis(j,i).EQ.5) then
   Call InvGamma(j,i,CDF,StartPt)
elseif (VarDis(j,i).EQ.6) then
   Call InvBeta(j,i,CDF,StartPt)
elseif (VarDis(j,i).EQ.7) then
   StartPt(j,i)=Lambda(j,i)*(-LOG(CDF))**(-1./Zeta(j,i))
elseif (VarDis(j,i).EQ.8) then
   StartPt(j,i)=Zeta(j,i)-Lambda(j,i)*LOG(1.-CDF)
elseif (VarDis(j,i).EQ.9) then
   StartPt(j,i)=Zeta(j,i)*SQRT(-2.*LOG(1.-CDF))
elseif (VarDis(j,i).EQ.10) then
   StartPt(j,i)=Zeta(j,i)*LOG(1.-CDF)**(1./Zeta(j,i))
elseif (VarDis(j,i).EQ.11) then
   Call InvUser1(j,i,CDF,StartPt)
1
C6-57
elseif (VarDis(j,i).EQ.12) then
  Call InvUser2(j,i,CDF,StartPt)
else
  Call InvUser3(j,i,CDF,StartPt)
Endif
return
End

Subroutine MinMax(i,j,Opt,VarDis,Ext,Trunc,Xo,Mux,Sigmax,XMin, XMax)

INPUT VARIABLES:
Opt = Failure mode
i = Number of the variable
j = Number of the combination of actions
Ext = Abbreviation of the name of the variable
Mux = Mean of X
Sigmax = Standard deviation of X
VarDis = Type of distribution
Trunc = Type of truncation
Xo = Point of truncation (if the distribution is truncated)
Zeta, Lamda = Parameters of a distribution
x1 = Lower limit on X for a Beta distribution
x2 = Upper limit on X for a Beta distribution

INPUT/OUTPUT VARIABLES:
XMin = Minimum value of X
XMax = Maximum value of X

MODELING VARIABLES:
L = Maximum number of variables allowed by the program
Q = Maximum number of combinations of actions allowed by the program
Eta = Variable mentioned in the Common statement but not used here

...
If (VarDis(j,i).EQ.11) then
   If (XMin(j,i).LT.(-5.53)) XMin(j,i)=-5.53
   If (XMax(j,i).GT.(5.77)) XMax(j,i)=5.77
Endif
If (VarDis(j,i).EQ.12) then
   If (XMin(j,i).LT.(5.45)) XMin(j,i)=5.45
   If (XMax(j,i).GT.(6.995)) XMax(j,i)=6.995
Endif
If (VarDis(j,i).EQ.13) then
   If (XMin(j,i).LT.(-5.33)) XMin(j,i)=-5.33
   If (XMax(j,i).GT.(5.57)) XMax(j,i)=5.57
Endif
Write(40,5971) Ext(i),XMin(j,i),Ext(i),XMax(j,i)
Write(50,5971) Ext(i),XMin(j,i),Ext(i),XMax(j,i)
5971 Format(11X,'XMin(',A3,') = ',E17.10,3X,'XMax(',A3,') = ',E17.10)
return
End

***********************************************************************

Subroutine EqNorDis(i,j,N,X,VarDis,NR,Ext,MuxN,SigmaxN,Trunc,Xo)

***********************************************************************

Calls the subroutines required to calculate the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution for each variable.

***********************************************************************

INPUT VARIABLES:
  j - Number of the combination of actions
  NR - Power to which each distribution is raised for each combination of actions
  N - Number of variables
  i - Number of the variable
  Ext - Abbreviation of the name of the variable
  X - Variables of the failure mode
  VarDis - Type of distribution
  Trunc - Type of truncation
  Xo - Point of truncation (if the distribution is truncated)
  Zeta, Lamda, Eta - Parameters of a distribution
  x1 - Lower limit on X for a Beta distribution
  x2 - Upper limit on X for a Beta distribution

MODELING VARIABLES:
  Q - Maximum number of combinations of actions allowed by the program
  L - Maximum number of variables allowed by the program

OUTPUT VARIABLES:
  MuxN - Mean of the equivalent Normal distribution of X
  SigmaxN - Standard deviation of the equivalent Normal distribution of X

***********************************************************************

Integer*4 i,j,L,N,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 X(L),MuxN(L),SigmaxN(L),Xo(Q,L),x1(Q,L),x2(Q,L),
        Zeta(Q,L),Lamda(Q,L),Eta(Q,L),NR(Q,L)
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2

If (VarDis(j,i).EQ.0) then
   MuxN(i)=Zeta(j,i)
   SigmaxN(i)=Lamda(j,i)
   Call NorWrite(i,Ext,MuxN,SigmaxN)
elseif (VarDis(j,i).EQ.1) then
   Call NormalD(i,j,N,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,
               NR)
elseif (VarDis(j,i).EQ.2) then
   Call LogNormal(i,j,N,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,
                NR)
elseif (VarDis(j,i).EQ.3) then
   Call Rectangular(i,j,N,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,
                   NR)
else
1
elseif (VarDis(j,i).EQ.5) then
  Call Gamma(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.6) then
  Call BetaDis(i,j,Ext,X,Zeta,Lamda,x1,x2,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.7) then
  Call Frechet(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.8) then
  Call Exponential(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.9) then
  Call Rayleigh(i,j,Ext,X,Zeta,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.10) then
  Call Weibull(i,j,Ext,X,Zeta,Lamda,Eta,MuxN,SigmaxN,Trunc,Xo,NR)
elseif (VarDis(j,i).EQ.11) then
  Call User1(i,j,Ext,X,MuxN,SigmaxN,NR)
elseif (VarDis(j,i).EQ.12) then
  Call User2(i,j,Ext,X,MuxN,SigmaxN,NR)
else
  Call User3(i,j,Ext,X,MuxN,SigmaxN,NR)
Endif
return
End

***********************************************************************

Subroutine Correlated(N,Ext,MuxN,SigmaxN,Rho,Sigmay,Muy,V,Vt)

***********************************************************************

Returns the means, Muy, & the standard deviations, Sigmay, of the non-correlated Normal variables, Y. It also returns the matrix V & its transpose, Vt, required to transform the variables X into Y & vice versa.

***********************************************************************

INPUT VARIABLES:
N - Number of variables
Ext - Abbreviation of the name of the variable
Rho - Correlation coefficient
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Covx - Covariance matrix
D - Vector of eigenvalues
i, k - Auxiliary variables

OUTPUT VARIABLES:
Muy - Mean of Y (non-correlated, Normal transformed variables)
Sigmay - Standard deviation of Y
V - Matrix of eigenvectors
Vt - Transpose of V

***********************************************************************

Integer*4 i,k,L,N
Parameter (L=15)
Character*3 Ext(L)
Real*8 MuxN(L),SigmaxN(L),Rho(L,L),Covx(L,L),D(L),V(L,L),
1         Sigmay(L),Vt(L,L),Muy(L)
Do 701 i=1,N
  Do 702 k=1,N
    Covx(i,k)=Rho(i,k)*SigmaxN(i)*SigmaxN(k)
  continue
702    continue
Call Algebra(N,Covx,D,V)
Write(50,400)
400 Format(/ 11X, 'EIGENVALUES' /)
Do 93 i=1,N
  Write(50,35) Ext(i),D(i)
35 Format(11X,'D(',A3,') = ',E17.10)
93     continue
Write(50,500)
500 Format(/ 11X, 'EIGENVECTORS' /)

Write(50,*) ' ' 
Do 138 k=1,N  
   Do 139 i=1,N  
      Write(50,392) Ext(i),Ext(k),V(i,k)  
      392 Format(11X,'V(',A3,',',A3,') = ',E17.10)  
   139   continue  
138   continue  
Write(50,*) ' ' 
Do 492 i=1,N  
   Do 551 k=1,N  
      Vt(i,k)=V(k,i)  
      Write(50,9891) Ext(i),Ext(k),Vt(i,k)  
      9891 Format(11X,'Vt(',A3,',',A3,') = ',E17.10)  
   551   continue  
492   continue  
Do 235 i=1,N  
   Muy(i)=0.  
   Do 237 k=1,N  
      Muy(i)=Vt(i,k)*MuxN(k)+Muy(i)  
   237   continue  
235   continue  
Write(50,*) ' ' 
Do 333 i=1,N  
   Sigmay(i)=SQRT(D(i))  
   Write(50,992) Ext(i),Muy(i),Ext(i),Sigmay(i)  
   992 Format(11X,'Muy(',A3,') = ',E17.10,3X,'Si gmay(',A3,') = ',E17.10)  
333   continue  
return 
End

Subroutine Derivadas(Par,N,X,Ext,Opt,FDer,OBJF,OBJGRD)

Returns the value of the failure function, OBJF, & the values of the first partial derivatives, OBJGRD. If the expressions for the first partial derivatives are given (FDer=1), they are used to calculate OBJGRD; subroutine OBJFUN is called for this purpose. Otherwise (FDer=2), the derivatives have to be calculated using the subroutine E04XAF. E04XAF is a NAG Fortran subroutine.

For more details see NAG (1993).

INPUT VARIABLES:
Opt - Failure mode
Ext - Abbreviation of the name of the variable
N - Number of variables
X - Variables of the failure mode
FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
Par - For Mode=1 it is the prescribed value of the design parameter (Param); for Mode=2 it is the starting value of the design parameter (StartParam)

MODELING VARIABLES:
IUSER - Failure mode
L - Maximum number of variables allowed by the program
LHES, NSTATE, IFAIL, IWARN, MSGLVL, INFO, EPSRF, HCNTRL, HESIAN, HFORM, WORK - Variables used by subroutine E04XAF (for more details see NAG, 1993)
E04XAF, OBJFUN - External subroutines
USER - For Mode=1 it is the prescribed value of the design parameter (Param); for Mode=2 it is the starting value of the design parameter (StartParam)
i - Auxiliary variable

OUTPUT VARIABLES:
OBJF - Failure function
OBJGRD - First partial derivatives of OBJF

C       ########################################### ####################
C
C       Subroutine Derivadas(Par,N,X,Ext,Opt,FDer,OBJF,OBJGRD)
C
C       Returns the value of the failure function, OBJF, & the values of the first partial derivatives, OBJGRD. If the expressions for the first partial derivatives are given (FDer=1), they are used to calculate OBJGRD; subroutine OBJFUN is called for this purpose. Otherwise (FDer=2), the derivatives have to be calculated using the subroutine E04XAF. E04XAF is a NAG Fortran subroutine.
C
C       For more details see NAG (1993).
C
C       ################################################################
C
C       INPUT VARIABLES:
C       Opt - Failure mode
C       Ext - Abbreviation of the name of the variable
C       N - Number of variables
C       X - Variables of the failure mode
C       FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
C       Par - For Mode=1 it is the prescribed value of the design parameter (Param); for Mode=2 it is the starting value of the design parameter (StartParam)
C
C       MODELING VARIABLES:
C       IUSER - Failure mode
C       L - Maximum number of variables allowed by the program
C       LHES, NSTATE, IFAIL, IWARN, MSGLVL, INFO, EPSRF, HCNTRL, HESIAN, HFORM, WORK - Variables used by subroutine E04XAF (for more details see NAG, 1993)
C       E04XAF, OBJFUN - External subroutines
C       USER - For Mode=1 it is the prescribed value of the design parameter (Param); for Mode=2 it is the starting value of the design parameter (StartParam)
C       i - Auxiliary variable
C
C       OUTPUT VARIABLES:
C       OBJF - Failure function
C       OBJGRD - First partial derivatives of OBJF
C
C       ################################################################

Integer*4 N,L,i
Parameter (L=15)
Character*3 Ext(L)
Program Listing

Integer*4 LHES, IFAIL, IWARN, FDer, MSGVLVL, NSTATE,
   INFO(L), IUSER(1), Opt
Real*8 EPSRF, OBJF, HCNTRL(L), HESIAN(L, L), HFORW(L), OBJGRD(L),
   USER(1), WORK(L*L+L), X(L), Par
External E04XAF, OBJFUN

LHES=N
MSGVLVL=1
EPSRF=-1.
Do 20 i=1,N
   HFORW(i)=-1.
20     continue
IFAIL=1
IUSER(1)=Opt
USER(1)=Par
If (FDer.EQ.2) then
   Call E04XAF(MSGVLVL,N, EPSRF, X, FDer, OBJFUN, LHES, HFORW, OBJF,
     OBJGRD, HESIAN, IWARN, WORK, IUSER,
     USER, INFO, IFAIL)
   If ((IFAIL.EQ.0).OR.(IFAIL.EQ.2)) then
     else
       Write(*,9997) IFAIL
       Write(50,9997) IFAIL
   9997       Format(/ 11X,'On exit from E04XAF IFAIL = ',I2)
   PAUSE
   Endif
  else
   Call OBJFUN(FDer, N, OBJF, OBJGRD, NSTATE, IUSER, USER)
   Do 5691 i=1,N
      Write(50,9095) Ext(i), OBJGRD(i)
   9095       Format(11X,'dZ/d',A3,' = ',E17.10)
   5691     continue
   Endif
return
End

Subroutine OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER)

 Calls the subroutines which contain the failure function. For the failure mode of overtopping, they also contain the first partial derivatives of the failure function.

INPUT VARIABLES:
   IUSER - Failure mode
   OptD - Direction of the sand movements in dune erosion
   N - Number of variables
   X - Variables of the failure mode
   FDer - Method of calculation of the first partial derivatives of the failure function for overtopping
   USER - For Mode=1 it is the value of the prescribed design parameter (Param); for Mode=2 it is the starting value of the design parameter (StartParam)

MODELING VARIABLES:
   Opt = Failure mode
   NSTATE - Variable used by subroutine E04XAF (for more details see NAG, 1993)

OUTPUT VARIABLES:
   OBJF - Failure function
   OBJGRD - First partial derivatives of OBJF

Subroutine OBJFUN(FDer,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER)
If (OptD.EQ.2) then
  Call SDunes(N,X,OBJF,USER)
else
  Call LDunes(N,X,OBJF,USER)
Endif
Endif
return
End

Subroutine NormalDist(j,RelInd,Prob)

Returns the value of the exceedance cumulative distribution function of the standard Normal distribution, Prob (tabulated in statistical books, e.g. Abramowitz & Stegun, 1964), for a given RelInd value. It uses the data file distnorm.dad which contains the tabulated values.

INPUT VARIABLES:
RelInd - Standard Normal variable which corresponds to Prob
j - Number of the combination of actions

MODELING VARIABLES:
M - Number of points of the standard Normal distribution tabulated in file distnorm.dad
Phi - Value tabulated in file distnorm.dad of the exceedance cumulative distribution function of the standard Normal distribution
Beta - Standard Normal variable, tabulated in file distnorm.dad, which corresponds to Phi
Q - Maximum number of combinations of actions allowed by the program
Pi - 3.14159...
i - Auxiliary variable

OUTPUT VARIABLES:
Prob - Value of the exceedance cumulative distribution function of the standard Normal distribution

Integer*4 j,i,M,Q
Parameter (M=453)
Parameter (Q=16)
Real*8 RelInd,Prob(Q),Beta(M),Phi(M),Pi
Open(Unit=20, File='distnorm.dad', Status='Old')
Pi=4.*ATAN(1.)
Do 10 i=1,M
Read(20,*) Beta(i), Phi(i)
10 continue
Do 32 i=1,M
  If (Beta(i).GE.ABS(RelInd)) then
    If (Beta(i).GT.ABS(RelInd)) then
      If (RelInd.GE.0.) then
        Prob(j)=
          ((Beta(i)-RelInd)/(Beta(i)-Beta(i-1)))**Phi(i-1)+
          ((RelInd-Beta(i-1))/(Beta(i)-Beta(i-1)))**Phi(i)
      endif
      Prob(j)=1-(((Beta(i)-ABS(RelInd))/(Beta(i)-Beta(i-1)))
      *Phi(i-1)+((ABS(RelInd)-Beta(i-1))/
      (Beta(i)-Beta(i-1)))**Phi(i)
    endif
  endif
  if (RelInd.GE.0.) then
    Prob(j)=Phi(i)
  else
    Prob(j)=1.-Phi(i)
  endif
  goto 50
Endif
Endif
Endif
Endif
continue

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50   Close (Unit=20)
    return
End

******************************************************************************
Subroutine PNormal(j,i,Ext,Mux,Sigmax,Zeta,Lamda)
******************************************************************************
Returns the parameters, Zeta & Lamda, of a Normal distribution.
******************************************************************************

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Normal distribution
******************************************************************************

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L)
Zeta(j,i)=Mux(j,i)
Lamda(j,i)=Sigmax(j,i)
If (Lamda(j,i).GT.0.) then
  Write(50,2245) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
2245     Format(/ 16X,'Zeta(',A3,') = ',E17.10 /
          1 16X,'Lamda(',A3,') = ',E17.10)
else
  Write(*,5777) Ext(i)
  Write(50,5777) Ext(i)
5777     Format(/ 16X,'ERROR - Normal distribution'/ 19X,
          1 'Lamda(',A3,') <= 0 !'/)
STOP
Endif
End

******************************************************************************
Subroutine PLogNormal(j,i,Ext,Mux,Sigmax,Zeta,Lamda)
******************************************************************************
Returns the parameters, Zeta & Lamda, of a Log-Normal distribution.
******************************************************************************

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Log-Normal distribution
******************************************************************************

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L)
Zeta(j,i)=Mux(j,i)
Lamda(j,i)=Sigmax(j,i)
If (Lamda(j,i).GT.0.) then
  Write(50,2245) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
2245     Format(/ 16X,'Zeta(',A3,') = ',E17.10 /
          1 16X,'Lamda(',A3,') = ',E17.10)
else
  Write(*,5777) Ext(i)
  Write(50,5777) Ext(i)
5777     Format(/ 16X,'ERROR - Log-Normal distribution'/ 19X,
          1 'Lamda(',A3,') <= 0 !'/)
STOP
Endif
End
Subroutine PGumbel(j,i,Ext,Mux,Sigmax,Zeta,Lamda)

Returns the parameters, Zeta & Lamda, of a Gumbel distribution.

### INPUT VARIABLES:
- Ext - Abbreviation of the name of the variable
- Mux - Mean of X
- Sigmax - Standard deviation of X
- i - Number of the variable
- j - Number of the combination of actions

#### MODELING VARIABLES:
- L - Maximum number of variables allowed by the program
- Q - Maximum number of combinations of actions allowed by the program
- Pi = 3.14159...

#### OUTPUT VARIABLES:
- Zeta, Lamda - Parameters of the Gumbel distribution

### Implementation:

```c
Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Pi,Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L)

Zeta(j,i)=Pi/((SQRT(6.))*Sigmax(j,i))
Lamda(j,i)=Mux(j,i)-0.57722/(Zeta(j,i))

If (Zeta(j,i).GT.0.) then
  Write(50,6890) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
else
  Write(*,5202) Ext(i)
Endif
Write(50,5202) Ext(i)
```

STOP

Endif
End

Subroutine PRectangular(j,i,Ext,Mux,Sigmax,Zeta,Lamda)

Returns the parameters, Zeta & Lamda, of a Rectangular
distribution.

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Rectangular distribution

Program Listing

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L)
Zeta(j,i)=Mux(j,i)-SQRT(3.)*Sigmax(j,i)
Lamda(j,i)=Mux(j,i)+SQRT(3.)*Sigmax(j,i)
If (Zeta(j,i).LT.Lamda(j,i)) then
Write(50,6871) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
6871     Format(/ 16X,'Zeta(',A3,') = ',E17.10 /)
else
Write(*,5202) Ext(i),Ext(i)
Write(50,5202) Ext(i),Ext(i)
5202     Format(// 16X,'ERROR - Rectangular distribution'/ 19X,
1           'Zeta(',A3,') >= Lamda(',A3,') !' /)
STOP
Endif
return
End

Subroutine PGamma(j,i,Ext,Mux,Sigmax,Zeta,Lamda)

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Gamma distribution

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),Zeta(Q,L),Lamda(Q,L)
Zeta(j,i)=(Mux(j,i)/Sigmax(j,i))**2
Lamda(j,i)=Mux(j,i)/(Sigmax(j,i)**2)
If ((Zeta(j,i).GT.0.0).AND.(Lamda(j,i).GT.0.00)) then
Write(50,6820) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
6820     Format(/ 16X,'Zeta(',A3,') = ',E17.10 /)
else
Write(*,5202) Ext(i),Ext(i)
Write(50,5202) Ext(i),Ext(i)
5202     Format(// 16X,'ERROR - Gamma distribution'/ 19X,
1           'Zeta(',A3,') >= Lamda(',A3,') !' /)
STOP
Endif
return
End
 Program Listing

If (Zeta(j,i).EQ.0.0) then
    Write(*,5670) Ext(i)
Endif
5670   Format(/ 16X,'ERROR - Gamma distribution'/ 19X,
     'Zeta(',A3,') = 0 !'/)

If (Lamda(j,i).LE.0.0) then
    Write(*,5689) Ext(i)
Endif
5689   Format(/ 16X,'ERROR - Gamma distribution'/ 19X,
     'Lamda(',A3,') <= 0 !'/)

STOP
Endif
return
End

*************************************************************************

Subroutine PBeta(j,i,Ext,Mux,Sigmax,Zeta,Lamda,x1,x2)
*************************************************************************

Returns the parameters, Zeta & Lamda, of a Beta distribution.

*************************************************************************

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
x1 - Lower limit on X
x2 - Upper limit on X
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Beta distribution

*************************************************************************

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),x1(Q,L),x2(Q,L),Zeta(Q,L),Lamda(Q,L)
Zeta(j,i)=((x2(j,i)-x1(j,i))/((x2(j,i)-x1(j,i))**2) /((Sigmax(j,i)**2)*
1   (x2(j,i)-x1(j,i)))*Zeta(j,i))/(Mux(j,i)-x1(j,i))-
1   Lamda(j,i)=((x2(j,i)-x1(j,i))**2)/((Sigmax(j,i)**2)*
1   (x2(j,i)-x1(j,i)))*Zeta(j,i)/(Mux(j,i)-x1(j,i))-
1   Zeta(j,i)
If ((Zeta(j,i).GT.0.0).AND.(Lamda(j,i).GT.0.0)) then
    Write(50,6861) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
6861   Format(/ 16X,'Zeta(',A3,') = ',E17.10/
     1   16X,'Lamda(',A3,') = ',E17.10)
else
    If (Zeta(j,i).LE.0.0) then
        Write(*,5673) Ext(i)
    Endif
5673   Format(/ 16X,'ERROR - Beta distribution'/ 19X,
     'Zeta(',A3,') <= 0 !'/)
    If (Lamda(j,i).LE.0.0) then
        Write(*,5682) Ext(i)
    Endif
5682   Format(/ 16X,'ERROR - Beta distribution'/ 19X,
     'Lamda(',A3,') <= 0 !'/)
    STOP
Endif
return
End

*************************************************************************
Subroutine PFrechet(j,i,Ext,Mux,Sigmax)

Returns the parameters, Zeta & Lamda, of a Frechet distribution.
Since there is no explicit form for calculating Zeta, it uses subroutine Zbrent to evaluate it.

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
VarDis - Type of distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Zbrent, Gammln - External functions
a1 - Lower limit on Zbrent
a2 - Upper limit on Zbrent
tol - Accuracy of Zbrent
Eta, x1, x2 - Auxiliary variables not used here

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Frechet distribution

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),a1,a2,Tol,Zeta(Q,L),Lamda(Q,L),
1 Eta(Q,L),x1(Q,L),x2(Q,L),VarDis(Q,L),i,Aux1,Aux2,Zbrent,
1 Gammln
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Zbrent
External Gammln
Aux1=Mux(j,i)
Aux2=Sigmax(j,i)
a1=2.05
a2=99999999.0
Tol=0.00000001
VarDis(j,i)=7
Zeta(j,i)=Zbrent(j,i,Aux1,Aux2,a1,a2,Tol,VarDis)
L=1./Zeta(j,i)
Lamda(j,i)=Mux(j,i)/EXP(Gammln(L))
If ((Zeta(j,i).GT.2.0).AND.(Lamda(j,i).GT.0.0)) then
Write(50,8889) Ext(i),Zeta(j,i),Ext(i),Lamda(j,i)
8889 Format(/ 16X,'Zeta(',A3,') = ',E17.10 /
1 16X,'Lamda(',A3,') = ',E17.10)
else
If (Zeta(j,i).LE.2.0) then
Write(*,5699) Ext(i)
5699 Format(/ 16X,'ERROR - Frechet distribution'/ 19X,
1 'Zeta(',A3,') <= 2 !' )
Endif
If (Lamda(j,i).LE.0.0) then
Write(*,9989) Ext(i)
9989 Format(/ 16X,'ERROR - Frechet distribution'/ 19X,
1 'Lamda(',A3,') <= 0 !')
Endif
STOP
Endif
return

Subroutine PExponential(j,i,Ext,Mux,Sigmax,Zeta,Lamda)
Returns the parameters, Zeta & Lamda, of an Exponential distribution.

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
Zeta, Lamda - Parameters of the Exponential distribution

Subroutine PRayleigh(j,i,Ext,Mux,Sigmax,Zeta)

Returns the parameter, Zeta, of a Rayleigh distribution.

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
Mux - Mean of X
Sigmax - Standard deviation of X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

Pi - 3.14159...

OUTPUT VARIABLES:
Zeta - Parameter of the Rayleigh distribution

Subroutine PRayleigh(j,i,Ext,Mux,Sigmax,Zeta)
Subroutine PWeibull(j,i,Ext,Mux,Sigmax)

C       ########################################### #####################
C
C       Returns the parameters, Lamda & Eta, of a Weibull distribution
C       for a given Zeta parameter. Since there is no explicit form
C       for calculating Eta, it uses subroutine Zbrent to evaluate it.
C       ########################################### #####################

C       INPUT VARIABLES:
C       Ext - Abbreviation of the name of the variable
C       Mux - Mean of X
C       Sigmax - Standard deviation of X
C       Zeta - Parameter of the Weibull distribution
C       i - Number of the variable
C       j - Number of the combination of actions

C       MODELING VARIABLES:
C       VarDis - Type of distribution
C       L - Maximum number of variables allowed by the program
C       Q - Maximum number of combinations of actions allowed by the
C           program
C       Zbrent, Gammln - External functions
C       a1 - Lower limit on Zbrent
C       a2 - Upper limit on Zbrent
C       Tol - Accuracy of Zbrent
C       T, Aux1, Aux2 - Auxiliary variables
C       x1, x2 - Variables mentioned in the Common statement but not
C                used here

C       OUTPUT VARIABLES:
C       Lamda, Eta - Parameters of the Weibull distribution

C       ########################################### #####################

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Mux(Q,L),Sigmax(Q,L),a1,a2,Tol,Zeta(Q,L),Lamda(Q,L),
1         Eta(Q,L),x1(Q,L),x2(Q,L),VarDis(Q,L) ,T,Aux1,Aux2,Zbrent,
1         Gammln
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Zbrent
External Gammln
Aux1=Mux(j,i)
Aux2=Sigmax(j,i)
a1=0.012
a2=9999.0
Tol=0.00000001
VarDis(j,i)=10
Eta(j,i)=Zbrent(j,i,Aux1,Aux2,a1,a2,Tol,VarDis)
T=1.+1./Eta(j,i)
Lamda(j,i)=(Mux(j,i)-Zeta(j,i))/EXP(Gammln( T))
Subroutine InvGamma(j,i,CDF,StartPt)

Returns StartPt for a specific value CDF of the cumulative distribution function of a Gamma distribution. Since there is no explicit form for the inverse function of a Gamma distribution, it uses subroutine Zbrent to evaluate it.

INPUT VARIABLES:
Lambda - Parameter of the Gamma distribution
CDF - Value of the cumulative distribution function of a Gamma distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
VarDis - Type of distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Zbrent - External function
a1 - Lower limit on Zbrent
a2 - Upper limit on Zbrent
Tol - Accuracy of Zbrent
Aux1, Aux2 - Auxiliary variables
Zeta, Eta, x1, x2 - Variables mentioned in the Common statement but not used here

OUTPUT VARIABLE:
StartPt - Value of the inverse function of a Gamma distribution

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Subroutine InvBeta(j,i,CDF,StartPt)

########################################################################

Returns StartPt for a specific value CDF of the cumulative
distribution function of a Beta distribution. Since there is no
explicit form for the inverse function of a Beta distribution,
it uses subroutine Zbrent to evaluate it.

########################################################################

INPUT VARIABLES:
x1 - Lower limit on X
x2 - Upper limit on X
CDF - Value of the cumulative distribution function of a Beta
distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
VarDis - Type of distribution
Zbrent - External function
a1 - Lower limit on Zbrent
a2 - Upper limit on Zbrent
Tol - Accuracy of Zbrent
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the
    program
Aux1, Aux2 - Auxiliary variables
Zeta, Lamda, Eta - Variables mentioned in the Common statement
    but not used here

OUTPUT VARIABLE:
StartPt - Value of the inverse function of a Beta distribution

########################################################################

Integer*4 j,i,L,Q
Parameter (L=15)
Parameter (Q=16)
Real*8 a1,a2,Tol,Zeta(Q,L),Lamda(Q,L),Eta(Q,L),x1(Q,L),x2(Q,L),
1         StartPt(Q,L),VarDis(Q,L),CDF,Zbrent,Aux1,Aux2
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Zbrent
Aux1=CDF
Aux2=0.
a1=0
a2=1
Tol=0.00000001
VarDis(j,i)=6
StartPt(j,i)=x1(j,i)+(x2(j,i)-x1(j,i))*
1         Zbrent(j,i,Aux1,Aux2,a1,a2,Tol,VarDis)
return
End

########################################################################

Subroutine InvUser1(j,i,CDF1,StartPt)

########################################################################

Returns the value of the water level, StartPt, for a specific
value, CDF1, of the user-defined distribution of water levels.
It uses the data file wldata.dad which contains the tabulated
values.

########################################################################

INPUT VARIABLE:
i - Number of the variable
j - Number of the combination of actions
CDF1 - Value of the cumulative distribution function of the
    user-defined distribution of water levels

MODELING VARIABLES:
M - Number of points of the user-defined distribution tabulated
    in file wldata.dad
WL - Value tabulated in file wldata.dad of the water level of
    the user-defined distribution
PDF - Value tabulated in file wldata.dad of the probability
density function of the user-defined distribution
CDF - Value tabulated in file wldata.dad of the cumulative distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
k - Auxiliary variable

OUTPUT VARIABLE:
StartPt - Value of the inverse function of the user-defined distribution of water levels which corresponds to CDF1

*******************************************************************************

Integer*4 i,j,k,M,Q,L
Parameter (M=126)
Parameter (Q=16)
Real*8 WL(M),PDF(M),CDF(M),CDF1,StartPt(Q,L)
Open(Unit=70, File='wldata.dad', Status='Old')
Do 10 k=1,M
  Read(70,*) WL(k),PDF(k),CDF(k)
10     continue
Do 32 k=1,M
  If (CDF(k).LE.CDF1) then
    If (CDF(k).LT.CDF1) then
      StartPt(j,i)=((CDF1-CDF(k))*((WL(k+1))-WL(k)))/
      1             ((CDF(k+1))-CDF(k))+WL(k)
    goto 1150
  else
    StartPt(j,i)=WL(k)
    goto 1150
  Endif
32     continue
1150   Close (Unit=70)
return
End

*******************************************************************************

Subroutine InvUser2(j,i,CDF1,StartPt)
*******************************************************************************

Returns the value of the extreme water level, StartPt, for a specific value, CDF1, of the user-defined distribution of extreme water levels. It uses the data file extwldat.dad which contains the tabulated values.

*******************************************************************************

INPUT VARIABLE:
i - Number of the variable
j - Number of the combination of actions
CDF1 - Value of the cumulative distribution function of the user-defined distribution of extreme water levels

MODELING VARIABLES:
M - Number of points of the user-defined distribution tabulated in file extwldat.dad
ExtWL - Value tabulated in file extwldat.dad of the extreme water level of the user-defined distribution
PDF - Value tabulated in file extwldat.dad of the probability density function of the user-defined distribution
CDF - Value tabulated in file extwldat.dad of the cumulative distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
k - Auxiliary variable

OUTPUT VARIABLE:
StartPt - Value of the inverse function of the user-defined distribution of extreme water levels which corresponds to CDF1

*******************************************************************************

Integer*4 i,j,k,M,Q,L
Parameter (M=17)
Parameter (Q=16)
Parameter (L=15)
Real*8 ExtWL(M),PDF(M),CDF(M),CDF1,StartPt(Q,L)
Open(Unit=75, File='extwldat.dad', Status='Old')
Do 10 k=1,M
Read(75,*) ExtWL(k),PDF(k),CDF(k)
10 continue
Do 32 k=1,M
If (CDF(k).LE.CDF1) then
If (CDF(k).LT.CDF1) then
StartPt(j,i)=((CDF1-CDF(k))*((ExtWL(k+1))-ExtWL(k)))/
1             ((CDF(k+1))-CDF(k))+ExtWL(k)
goto 1150
else
StartPt(j,i)=ExtWL(k)
goto 1150
Endif
32 continue
1150 Close (Unit=75)
return
End

*******************************************************************************
Subroutine InvUser3(j,i,CDF1,StartPt)
*******************************************************************************
Returns the value of the tide level, StartPt, for a specific value, CDF1, of the user-defined distribution of tide levels. It uses the data file tide.dat which contains the tabulated values.
*******************************************************************************
INPUT VARIABLE:
i - Number of the variable
j - Number of the combination of actions
CDF1 - Value of the cumulative distribution function of the user-defined distribution of tide levels
MODELING VARIABLES:
M - Number of points of the user-defined distribution tabulated in file tide.dat
Tide - Value tabulated in file tide.dat of the tide level of the user-defined distribution
PDF - Value tabulated in file tide.dat of the probability density function of the user-defined distribution
CDF - Value tabulated in file tide.dat of the cumulative distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
k - Auxiliary variable
OUTPUT VARIABLE:
StartPt - Value of the inverse function of the user-defined distribution of tide levels which corresponds to CDF1
*******************************************************************************
Integer*4 i, j, k, M, Q, L
Parameter (M=122)
Parameter (Q=16)
Parameter (L=15)
Real*8 Tide(M),PDF(M),CDF(M),CDF1,StartPt(Q,L)
Open(Unit=80, File='tide.dat', Status='Old')
Do 10 k=1,M
Read(80,*) Tide(k),PDF(k),CDF(k)
10 continue
Do 32 k=1,M
If (CDF(k).LE.CDF1) then
If (CDF(k).LT.CDF1) then
StartPt(j,i)=((CDF1-CDF(k))*((Tide(k+1))-Tide(k)))/
1             ((CDF(k+1))-CDF(k))+Tide(k)
goto 1150
else
StartPt(j,i)=Tide(k)
goto 1150
Endif
32 continue
1150 Close (Unit=80)
return
End

Subroutine InvUser3(j,i,CDF1,StartPt)
Program Listing

32     continue
1150   Close (Unit=80)
return
End

Subroutine NorWrite(i,Ext,MuxN,SigmaxN)

Wrote the mean, MuxN, & the standard deviation, SigmaxN, of the
equivalent Normal distribution of X in the file results.dat .

INPUT VARIABLES:
Ext - Abbreviation of the name of the variable
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
distribution of X
i - Number of the variable

MODELING VARIABLE:
L - Maximum number of variables allowed by the program

Integer*4 i,L
Parameter (L=15)
Character*3 Ext(L)
Real*8 MuxN(L),SigmaxN(L)

Write(50,8709) Ext(i),MuxN(i),Ext(i),SigmaxN(i)
8709   Format(/ 16X,'Mean Value N(',A3,') = ',E17.10 /
1           16X,'Standard Deviation N(',A3,') = ',E17.10)
return
End

Subroutine NormalD(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,
1                     Xo,NR)

Calculates the value of the probability density function, PDFx,
at X & the value of the cumulative distribution function, CDFx,
at X of a Normal distribution (CDFx is calculated using
subroutine NormalDist). If the Normal distribution is truncated
at X=Xo, it also calculates the value of the cumulative
distribution function at X=X0, CDFX0. These values are used to
return the mean, MuxN, & the standard deviation, SigmaxN, of the
equivalent Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each
combination of actions
Zeta, Lamda - Parameters of the Normal distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Normal
distribution at X
CDFx - Value of the cumulative distribution function of a
Normal distribution at X
CDFX0 - Value of the cumulative distribution function of a
Normal distribution at X=X0
SNX - Standard Normal variable
Prob - Value of the exceedance cumulative distribution function
of the standard Normal distribution which corresponds to
SNX

C7-75
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Pi = 3.14159...

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

____________________________________________________________________________________

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 X(L),MuxN(L),SigmaxN(L),Zeta(Q,L),Lamda(Q,L),CDFx(L),
1 CDFx0(L),PDFx(L),Xo(Q,L),Pi,Prob(Q),SNX,NR(Q,L)

Pi=4.*ATAN(1.)
SNX=(X(i)-Zeta(j,i))/Lamda(j,i)
Call NormalDist(j,SNX,Prob)
CDFx(i)=1.-Prob(j)
1 PDFx(i)=(EXP((-(((X(i)-Zeta(j,i))**2))/
1 (2.*(Lamda(j,i)**2)))/
1 (Lamda(j,i)**2.*SQRT(2.*Pi))
C Write(50,9824) SNX,Prob(j),CDFx(i)
C 9824 Format(16X,'Normd X=',E17.10,3X,'Prob=',E17.10 /
1 16X,'CDFX=',E17.10,3X,'PDFX=',E17.10 /)
If (Trunc(j,i).NE.0) then
SNX=(Xo(j,i)-Zeta(j,i))/Lamda(j,i)
Call NormalDist(j,SNX,Prob)
CDFx0(i)=1.-Prob(j)
C Write(50,9724) SNX,Prob(j),CDFx0(i)
C 9724 Format(16X,'Normd X=',E17.10,3X,'Prob=',E17.10 /
1 16X,'CDFX0=',E17.10,3X,'PDFX=',E17.10 /)
Endif
Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx,NR)
If ((Trunc(j,i).EQ.0).AND. (NR(j,i).EQ.1)) then
MuxN(i)=Zeta(j,i)
SigmaxN(i)=Lamda(j,i)
Call NorWrite(i,Ext,MuxN,SigmaxN)
Endif
return
End

Subroutine LogNormal(i,j,Ext,X,Zeta,Lamda,MuxN,
1 SigmaxN,Trunc,Xo,NR)
Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at X of a Log-Normal distribution (CDFx is calculated using subroutine NormalDist). If the Log-Normal distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0, CDFx0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each combination of actions
Zeta, Lamda - Parameters of the Log-Normal distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Log-Normal distribution at X
C67-77

**Program Listing**

**CDFx** - Value of the cumulative distribution function of a Log-Normal distribution at X

**CDFx0** - Value of the cumulative distribution function of a Log-Normal distribution at X=X0

**SNX** - Standard Normal Variable

**Prob** - Value of the exceedance cumulative distribution function of the standard Normal distribution which corresponds to SNX

**L** - Maximum number of variables allowed by the program

**Q** - Maximum number of combinations of actions allowed by the program

**Pi** - 3.14159...

**OUTPUT VARIABLES:**

**MuxN** - Mean of the equivalent Normal distribution of X

**SigmaxN** - Standard deviation of the equivalent Normal distribution of X

---

```
Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 X(L),MuxN(L),SigmaxN(L),Zeta(Q,L),Lamda(Q,L),CDFx(L),

1 CDFx0(L),PDFx(L),Prob(Q),Pi,SNX,Xo(Q,L),NR(Q,L)

Pi=4.*ATAN(1.)

If (X(i).GT.0.) then

SNX=(LOG(X(i))-Zeta(j,i))/(Lamda(j,i))

Call NormalDist(j,SNX,Prob)

CDFx(i)=1.-Prob(j)

PDFx(i)=(1./(X(i)*Lamda(j,i)))*(EXP(-(SNX**2)/2.))/SQRT(2.*Pi)

If (Trunc(j,i).NE.0) then

SNX=(LOG(Xo(j,i))-Zeta(j,i))/(Lamda(j,i))

Call NormalDist(j,SNX,Prob)

CDFx0(i)=1.-Prob(j)

Endif

Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,PDFx,NR)

If ((Trunc(j,i).EQ.0).AND.(NR(j,i).EQ.1)) then

MuxN(i)=X(i)*(1.-LOG(X(i))+Zeta(j,i))

SigmaxN(i)=X(i)*Lamda(j,i)

Call NorWrite(i,Ext,MuxN,SigmaxN)

Endif

else

Write(*,5209) Ext(i)

Write(50,5209) Ext(i)

5209 Format(/'16X,'ERROR - Log-Normal distribution'/19X,

1 'A3,' <= 0 !' /

STOP

Endif

return

End
```

---

**Subroutine Gumbel(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,NR)**

Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at a Gumbel distribution. If the Gumbel distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0, CDFx0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

---

**INPUT VARIABLES:**

**X** - Variable

**Ext** - Abbreviation of the name of the variable

**Trunc** - Type of truncation

**Xo** - Point of truncation (if the distribution is truncated)

**NR** - Power to which each distribution is raised for each combination of actions

**Zeta, Lamda** - Parameters of the Gumbel distribution

**i** - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Gumbel distribution at X
CDFx - Value of the cumulative distribution function of a Gumbel distribution at X
CDFx0 - Value of the cumulative distribution function of a Gumbel distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 X(L),MuxN(L),SigmaxN(L),Zeta(Q,L),Lamda(Q,L),
1 CDFx(L),CDFx0(L),PDFx(L),Xo(Q,L),NR(Q,L)
CDFx(i)=EXP(-EXP(-Zeta(j,i)*(X(i)-Lamda(j,i))))
PDFx(i)=Zeta(j,i)*EXP(-Zeta(j,i)*(X(i)-Lamda(j,i)))*CDFx(i)
If (Trunc(j,i).NE.0) CDFx0(i)=EXP(-EXP(-Zeta(j,i)*
1 (Xo(j,i)-Lamda(j,i))))
1 Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx,NR)
End

Subroutine Rectangular(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,
Xo,NR)
Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at X of a Rectangular distribution. If the Rectangular distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each combination of actions
Zeta, Lamda - Parameters of the Rectangular distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Rectangular distribution at X
CDFx - Value of the cumulative distribution function of a Rectangular distribution at X
CDFx0 - Value of the cumulative distribution function of a Rectangular distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X
Program Listing

--------------------------------------------

        Integer*4 i,j,L,Q
       Parameter (L=15)
       Parameter (Q=16)

Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 X(L),X0N(L),SigmaxN(L),Zeta(j,i),Lamda(j,i),
1 CDFx(L),CDFx0(L),PDFx(L),Xo(Q,L),NR(Q,L)

IF ((X(i).LE.Lamda(j,i)).AND.(X(i).GE.Zeta(j,i)).AND.
1 (Zeta(j,i).LT.Lamda(j,i))) then
  CDFx(i)=(X(i)-Zeta(j,i))/(Lamda(j,i)-Zeta(j,i))
  PDFx(i)=1./(Lamda(j,i)-Zeta(j,i))
IF (Trunc(j,i).NE.0) CDFx0(i)=(Xo(j,i)-Zeta(j,i))/
1 (Lamda(j,i)-Zeta(j,i))

Call Truncation(i,j,Ext,X,MuxN, SigmaxN,Trunc,Xo,CDFx,CDFx0, PDFx,NR)
else
Write(*,5679) Ext(i),Zeta(j,i),Lamda(j,i)
Write(50,5679) Ext(i),Zeta(j,i),Lamda(j,i)
5679 Format(// 16X,'ERROR - Rectangular distribution'/ 19X,
1 A3,' is not in the required range ['E17.10,','E17.10'] !')
STOP
Endif
return
End

--------------------------------------------

Subroutine Gamma(i,j,Ext,X,Zeta,Lamda,MuxN, SigmaxN,Trunc,Xo,NR)

Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at X of a Gamma distribution (it uses the functions Gammln & Gammp to calculate, respectively, the Gamma function & the incomplete Gamma function). If the Gamma distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0, CDFx0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each combination of actions
Zeta, Lamda - Parameters of the Gamma distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Gamma distribution at X
CDFx - Value of the cumulative distribution function of a Gamma distribution at X
CDFx0 - Value of the cumulative distribution function of a Gamma distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Gam - Gamma function
Gammln, Gammp - External functions
XX, Y, T1 - Auxiliary variables

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

--------------------------------------------

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 Zeta(Q,L),X(L),Lamda(Q,L),T,T1,Gam,MuxN(L),SigmaxN(L),
CDFx(L),CDFx0(L),PDFx(L),Xo(Q,L),Gammln,Gampmp,NR(Q,L),XX
External Gammln
External Gampmp
If (X(i).GE.0.0) then
XX=Zeta(j,i)
Gam=EXP(Gammln(XX))
T=Lamda(j,i)*X(i)
Write(50,9230) Zeta(j,i),Gam
9230 Format(16X,'Gamma(',E17.10,') = ',E17.10)
PDFx(i)=(Lamda(j,i)*(T**(Zeta(j,i)-1.1)))*(EXP(-T))/Gam
CDFx(i)=Gampmp(XX,T)
If (Trunc(j,i).NE.0) then
T1=Lamda(j,i)*Xo(j,i)
CDFx0(i)=Gampmp(XX,T1)
Endif
Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx,NR)
else
Write(*,5679) Ext(i)
Write(50,5679) Ext(i)
5679 Format(// 16X,'ERROR - Gamma distribution'/ 19X,A3,' < 0 !' /)
STOP
Endif
return
End

Subroutine BetaDis(i,j,Ext,X,Zeta,Lamda,x1, x2,MuxN,SigmaxN,
1 Trunc,Xo,NR)
Calculates the value of the probability density function, PDFx,
at X & the value of the cumulative distribution function, CDFx,
at X of a Beta distribution (it uses the functions Beta & Betai
to calculate, respectively, the Beta function & the incomplete
Beta function). If the Beta distribution is truncated at X=Xo,
it also calculates the value of the cumulative distribution
function at X=X0, CDFx0. It uses these values to return the
mean, MuxN, & the standard deviation, SigmaxN, of the equivalent
Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each
combination of actions
Zeta, Lamda - Parameters of the Beta distribution
i - Number of the variable
j - Number of the combination of actions
x1 - Lower limit on X
x2 - Upper limit on X
MODELING VARIABLES:
PDFx - Value of the probability density function of a Beta
distribution at X
CDFx - Value of the cumulative distribution function of a Beta
distribution at X
CDFx0 - Value of the cumulative distribution function of a Beta
distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the
program
Beta, Betai - External functions
XX, YY, T, BetaAux - Auxiliary variables
OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
distribution of X

******************************************************************************
Program Listing

C
Integer*4 i, j, L, Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 Zeta(Q,L), X(L), Lamda(Q,L), x1(Q,L), x2(Q,L), BetaAux(Q,L), T,
1 MuxN(L), SigmaxN(L), CDFx(L), CDFx0(L), PDFx(L), Xo(Q,L),
1 Betai,Beta, NR(Q,L), XX, YY
External Beta
External Betai
If ((X(i).LE.x2(j,i)).AND.(X(i).GE.x1(j,i)).AND.
1 (x1(j,i).LT.x2(j,i))) then
XX=Zeta(j,i)
YY=Lamda(j,i)
BetaAux(j,i)=Beta(XX,YY)
PDFx(i)=(X(i)-x1(j,i))**((Zeta(j,i)-1.))*(x2(j,i)-X(i))**
1 (Lamda(j,i)-1.)/(BetaAux(j,i)*(x2(j,i)-x1(j,i)))
1 T=(X(i)-x1(j,i))/(x2(j,i)-x1(j,i))
CDFx(i)=Betai(XX,YY,T)
If (Trunc(j,i).NE.0) then
T=(Xo(j,i)-x1(j,i))/(x2(j,i)-x1(j,i))
CDFx0(i)=Betai(XX,YY,T)
Endif
Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx, NR)
else
Write(*,-5579) Ext(i),x1(j,i),x2(j,i)
Write(50,-5579) Ext(i),x1(j,i),x2(j,i)
5579     Format(// 16X,'ERROR - Beta distribution'/ 19X,
1      ',A3,',' is not in the specified range '/, 19X,
1      '(',E17.10,',',E17.10,') !' )
1 STOP
Endif
return
End

Subroutine Frechet(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,Xo,
1 NR)
Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at X of a Frechet distribution. If the Frechet distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0, CDFx0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each combination of actions
Zeta, Lamda - Parameters of the Frechet distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Frechet distribution at X
CDFx - Value of the cumulative distribution function of a Frechet distribution at X
CDFx0 - Value of the cumulative distribution function of a Frechet distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
Program Listing

C                 distribution of X

#-----------------------------------------

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 Zeta(Q,L),X(L),Lamda(Q,L),MuxN(L),SigmaxN(L),CDFx(L),
1       CDFx0(L),PDFx(L),Xo(Q,L),NR(Q,L)
1
1 If (X(i).GT.0.0) then
1 CDFx(i)=EXP(-((Lamda(j,i)/X(i))**Zeta(j,i)))
1 PDFx(i)=(Zeta(j,i)/Lamda(j,i))**((Lamda(j,i)/X(i))**
1            (Zeta(j,i)+1.))*CDFx(i)
1 If (Trunc(j,i).NE.0) CDFx0(i)=EXP(-((Lamda(j,i)/
1                                  Xo(j,i))**Zeta(j,i)))
1 Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1                    PDFx,NR)
else
1 Write(*,9979) Ext(i)
1 Write(50,9979) Ext(i)
9979   Format(// 16X,'ERROR - Frechet distribution'/ 19X,
1              A3,' <= 0 !' /)
1 STOP
1 Endif
1 return
1 End

#-----------------------------------------

Subroutine Exponential(i,j,Ext,X,Zeta,Lamda,MuxN,SigmaxN,Trunc,
1                         Xo,NR)
1
1 Calculates the value of the probability density function, PDFx,
1 at X & the value of the cumulative distribution function, CDFx,
1 at X of an Exponential distribution. If the Exponential
1 distribution is truncated at X=Xo, it also calculates the value
1 of the cumulative distribution function at X=X0, CDFx0. It uses
1 these values to return the mean, MuxN, & the standard deviation,
1 SigmaxN, of the equivalent Normal distribution through
1 subroutine Truncation.

#-----------------------------------------

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each
combination of actions
Zeta, Lamda - Parameters of the Exponential distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of an
Exponential distribution at X
CDFx - Value of the cumulative distribution function of an
Exponential distribution at X
CDFx0 - Value of the cumulative distribution function of an
Exponential distribution at X=Xo
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the
program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
distribution of X

#-----------------------------------------

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Program Listing

Real*8 X(L),Zeta(Q,L),MuxN(L),SigmaxN(L),CDFx(L),CDFx0(L),
1 PDFx(L),Xo(Q,L),Lamda(Q,L),NR(Q,L)

If (X(i).GE.Zeta(j,i)) then
  CDFx(i)=1.-EXP(-((X(i)-Zeta(j,i))/Lamda(j,i))
  PDFx(i)=(1./Lamda(j,i))*(1.-CDFx(i))
If (Trunc(j,i).NE.0) CDFx0(i)=1.-EXP(-((Xo(j,i)-Zeta(j,i))/
1 Lamda(j,i)))
Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx,NR)
else
  Write(*,8889) Ext(i),Zeta(j,i)
  Write(50,8889) Ext(i),Zeta(j,i)
8889 Format(// 16X,'ERROR - Exponential distribution'/ 19X,
1 A3,' < ',E17.10,' !' /)
STOP
Endif
return
End

******************************************************************************
Subroutine Rayleigh(i,j,Ext,X,Zeta,MuxN,SigmaxN,Trunc,Xo,NR)
******************************************************************************

Calculates the value of the probability density function, PDFx,
1 at X & the value of the cumulative distribution function, CDFx,
1 at X of a Rayleigh distribution. If the Rayleigh distribution is
2 truncated at X=Xo, it also calculates the value of the
cumulative distribution function at X=X0, CDFx0. It uses these
2 values to return the mean, MuxN, & the standard deviation,
2 SigmaxN, of the equivalent Normal distribution through
2 subroutine Truncation.

******************************************************************************

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each
1 combination of actions
Zeta - Parameter of the Rayleigh distribution
1 i - Number of the variable
2 j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Rayleigh
distribution at X
CDFx - Value of the cumulative distribution function of a
Rayleigh distribution at X
CDFx0 - Value of the cumulative distribution function of a
Rayleigh distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
distribution of X

******************************************************************************

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 X(L),Zeta(Q,L),MuxN(L),SigmaxN(L),CDFx(L),CDFx0(L),
1 PDFx(L),Xo(Q,L),NR(Q,L)
If (X(i).GE.0.0) then
  CDFx(i)=1.-EXP(-((X(i)**2)/(2.*(Zeta(j,i)**2)))
  PDFx(i)=(X(i)/(Zeta(j,i)**2))*EXP(-((X(i)**2)/
1 (2.*(Zeta(j,i)**2)))
If (Trunc(j,i).NE.0) CDFx0(i)=1.-EXP(-((Xo(j,i)**2)/
1 (2.*(Zeta(j,i)**2)))
Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,CDFx0,
1 PDFx,NR)
else
  Write(*,8889) Ext(i),Zeta(j,i)
  Write(50,8889) Ext(i),Zeta(j,i)
8889 Format(// 16X,'ERROR - Rayleigh distribution'/ 19X,
1 A3,' < ',E17.10,' !' /)
STOP
End
Program Listing

Write(*,8779) Ext(i)
Write(50,8779) Ext(i)
8779 Format(/16X,'ERROR - Rayleigh distribution'/,19X,1,A3,'< 0 ! '/) STOP
Endif
return
End

########################################################################

c Subroutine Weibull(i,j,Ext,X,Zeta,Lamda,Eta,MuxN,SigmaxN,Trunc,Xo,NR)
########################################################################

Calculates the value of the probability density function, PDFx, at X & the value of the cumulative distribution function, CDFx, at X of a Weibull distribution. If the Weibull distribution is truncated at X=Xo, it also calculates the value of the cumulative distribution function at X=X0, CDFx0. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine Truncation.

########################################################################

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
Trunc - Type of truncation
Xo - Point of truncation (if the distribution is truncated)
NR - Power to which each distribution is raised for each combination of actions
Zeta, Lamda, Eta - Parameters of the Weibull distribution
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
PDFx - Value of the probability density function of a Weibull distribution at X
CDFx - Value of the cumulative distribution function of a Weibull distribution at X
CDFx0 - Value of the cumulative distribution function of a Weibull distribution at X=X0
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

########################################################################

Integer*4 i,j,L,Q
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Integer*4 Trunc(Q,L)
Real*8 Zeta(Q,L),X(L),Lamda(Q,L),Eta(Q,L),MuxN(L),SigmaxN(L),
1 cumCDFx(L),CDFx0(L),PDFx(L),Xo(Q,L),NR(Q,L)
If (X(i).GE.Zeta(j,i)) then
  CDFx(i)=1.-EXP(-(((X(i)-Zeta(j,i))/Lamda(j,i))**Eta(j,i))
  PDFx(i)=(Eta(j,i)/Lamda(j,i))**((1.-CDFx(i))*
1 - (((X(i)-Zeta(j,i))/Lamda(j,i))**Eta(j,i)-1)
  If (Trunc(j,i).NE.0) CDFx0(i)=1.-EXP(-(((Xo(j,i)-Zeta(j,i))
1 /Lamda(j,i))**Eta(j,i))
  Call Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,
1 CDFx0,PDFx,NR)
else Write(*,9979) Ext(i)
Write(50,9979) Ext(i)
9979 Format(/16X,'ERROR - Weibull distribution'/,19X,1,A3,'< Zeta ! /)
STOP
Endif
return
End
Subroutine User1(i,j,Ext,X,MuxN,SigmaxN,NR)

Returns the value of the probability density function, PDFx, & the value of the cumulative distribution function, CDFx, of the user-defined distribution of water levels, for a given value of the water level, X. It uses the data file wldata.dad which contains the tabulated values. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine PCDF.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
NR - Power to which each distribution is raised for each combination of actions
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
M - Number of points of the user-defined distribution tabulated in file wldata.dad
WL - Value tabulated in file wldata.dad of the water level of the user-defined distribution
PDF - Value tabulated in file wldata.dad of the probability density function of the user-defined distribution
CDF - Value tabulated in file wldata.dad of the cumulative distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
PDFx - Value of the probability density function of the user-defined distribution at X
CDFx - Value of the cumulative distribution function of the user-defined distribution at X
k - Auxiliary variable

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

Integer*4 i,j,k,M,Q,L
Parameter (M=126)
Parameter (Q=16)
Parameter (L=15)
Character*3 Ext(L)
Real*8 WL(M),PDF(M),CDF(M),X(L),PDFx(L),CDFx(L),MuxN(L),
       SigmaxN(L),NR(Q,L)
Open(Unit=70, File='wldata.dad', Status='Old')
Do 10 k=1,M
   Read(70,*) WL(k),PDF(k),CDF(k)
10    continue
Do 32 k=1,M
   If (WL(k).GE.X(i)) then
      If (WL(k).GT.X(i)) then
         PDFx(i)=((WL(k)-X(i))/(WL(k)-WL(k-1)))*PDF(k-1)+
            ((X(i)-WL(k-1))/(WL(k)-WL(k-1)))*PDF(k)
         CDFx(i)=((WL(k)-X(i))/(WL(k)-WL(k-1)))*CDF(k-1)+
            ((X(i)-WL(k-1))/(WL(k)-WL(k-1)))*CDF(k)
         goto 50
      else
         PDFx(i)=PDF(k)
         CDFx(i)=CDF(k)
      endif
   endif
32   continue
50   Call PCDF(i,j,Ext,X,MuxN,SigmaxN,PDFx,CDFx,NR)
Close (Unit=70)
return
End
Subroutine User2(i,j,Ext,X,MuxN,SigmaxN,NR)

Returns the value of the probability density function, PDFx, & the value of the cumulative distribution function, CDFx, of the user-defined distribution of extreme water levels, for a given value of the extreme water level, X. It uses the data file extwldat.dat which contains the tabulated values. It uses these values to return the mean, MuxN, & the standard deviation, SigmaxN, of the equivalent Normal distribution through subroutine PCDF.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
NR - Power to which each distribution is raised for each combination of actions

MODELING VARIABLES:
M - Number of points of the user-defined distribution tabulated in file extwldat.dat
ExtWL - Value tabulated in file extwldat.dat of the extreme water level of the user-defined distribution
PDF - Value tabulated in file extwldat.dat of the probability density function of the user-defined distribution
CDF - Value tabulated in file extwldat.dat of the cumulative distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program

PDFx - Value of the probability density function of the user-defined distribution at X
CDFx - Value of the cumulative distribution function of the user-defined distribution at X

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

Integer*4 i,j,k,M,Q,L
Parameter (M=17)
Parameter (Q=16)
Parameter (L=15)
Character*3 Ext(L)
Real*8 ExtWL(M),PDF(M),CDF(M),X(L),PDFx(L),CDFx(L),MuxN(L),SigmaxN(L),NR(Q,L)
Open(Unit=75, File='extwldat.dat', Status='Old')
Do 10 k=1,M
Read(75,**) ExtWL(k),PDF(k),CDF(k)
10 continue
Do 32 k=1,M
If (ExtWL(k).GE.X(i)) then
  If (ExtWL(k).GT.X(i)) then
    PDFx(i)=((ExtWL(k)-X(i))/(ExtWL(k)-ExtWL(k-1)))*PDF(k-1)+
            ((X(i)-ExtWL(k-1))/(ExtWL(k)-ExtWL(k-1)))*PDF(k)
    CDFx(i)=((ExtWL(k)-X(i))/(ExtWL(k)-ExtWL(k-1)))*CDF(k-1)+
            ((X(i)-ExtWL(k-1))/(ExtWL(k)-ExtWL(k-1)))*CDF(k)
    goto 50
  else
    PDFx(i)=PDF(k)
    CDFx(i)=CDF(k)
  endif
Endif
32 continue
50 Call PCDF(i,j,Ext,X,MuxN,SigmaxN,PDFx,CDFx,NR)
Close (Unit=75)
return
End
Subroutine User3(i,j,Ext,X,MuxN,SigmaxN,QR)

Returns the value of the probability density function, PDFx, & the value of the cumulative distribution function, CDFx, of
the user-defined distribution of tide levels, for a given value
of the tide level, X. It uses the data file tide.dad which
contains the tabulated values. It uses these values to return
the mean, MuxN, & the standard deviation, SigmaxN, of the
equivalent Normal distribution through subroutine PCDF.

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
QR - Power to which each distribution is raised for each
combination of actions
i - Number of the variable
j - Number of the combination of actions

MODEL VARIABLES:
M - Number of points of the user-defined distribution tabulated
in file tide.dad
Tide - Value tabulated in file tide.dad of the tide level of
the user-defined distribution
PDF - Value tabulated in file tide.dad of the probability
density function of the user-defined distribution
CDF - Value tabulated in file tide.dad of the cumulative
distribution function of the user-defined distribution
L - Maximum number of variables allowed by the program
QR - Maximum number of combinations of actions allowed by the
program
PDFx - Value of the probability density function of the
user-defined distribution at X
CDFx - Value of the cumulative distribution function of the
user-defined distribution at X
k - Auxiliary variable

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal
distribution of X

###

```fortran
Integer*4 i,j,k,M,QR,L
Parameter (M=122)
Parameter (QR=16)
Parameter (L=15)
Character*3 Ext(L)
Real*8 Tide(M),PDF(M),CDF(M),X(L),PDFx(L),CDFx(L),MuxN(L),
1 SigmaxN(L),QR(Q,R)
Open(Unit=80, File='tide.dad', Status='Old')
Do 10 k=1,M
  Read(80,*) Tide(k),PDF(k),CDF(k)
  continue
Do 32 k=1,M
  If (Tide(k).GE.X(i)) then
    If (Tide(k).GT.X(i)) then
      PDFx(i)=((Tide(k)-X(i))/(Tide(k)-Tide(k-1)))*PDF(k-1)+
((X(i)-Tide(k-1))/(Tide(k)-Tide(k-1)))*PDF(k)
      CDFx(i)=((Tide(k)-X(i))/(Tide(k)-Tide(k-1)))*CDF(k-1)+
((X(i)-Tide(k-1))/(Tide(k)-Tide(k-1)))*CDF(k)
    goto 50
  else
    PDFx(i)=PDF(k)
    CDFx(i)=CDF(k)
    goto 50
  Endif
Endif
32 continue
50 Call PCDF(i,j,Ext,X,MuxN,SigmaxN,PDFx,CDFx,QR)
Close (Unit=80)
return
End
```
Subroutine Algebra(N,Covx,D,V)

Returns the eigenvalues, D, & the eigenvectors, V, of the matrix Covx. It uses subroutines Tred2 & TQLI to calculate these values.

INPUT VARIABLES:
Covx - Covariance matrix
N - Number of variables

MODELING VARIABLES:
E - Off-diagonal elements of the orthogonal matrix, Q, obtained from Covx
i, k, NP - Auxiliary variables

OUTPUT VARIABLES:
D - Eigenvalues
V - Eigenvectors

Function Zbrent(j,i,Aux1,Aux2,a1,a2,Tol,VarDis)

Using Brent's method, it returns the root of a function (e.g. GG, TT, MM, BB) known to lie between a1 & a2. The root, returned as Zbrent, is refined until its accuracy is within Tol.

For more details, see Press et al (1992), pp.354.
C = B
FC = FB
Do 11 Iter = 1, Itmax
   If (((FB.GT.0.).AND.(FC.GT.0.)).OR.((FB.LT.0.).AND.
   (FC.LT.0.))) then
      C = A
      FC = FA
      D = B - A
      E = D
   Endif
   If (((ABS(FC)).LT.(ABS(FB))) then
      A = B
      B = C
      C = A
      FA = FB
      FB = FC
      FC = FA
   Endif
   Tol1 = 2.*Eps*ABS(B)+0.5*Tol
   XM = 0.5*(C - B)
   If (((ABS(XM)).LE.(ABS(Tol1)).OR.(FB.EQ.0.)) then
      Zbrent = B
      return
   Endif
   If (((ABS(E)).GE.(Tol1).AND.((ABS(FA)).GT. (ABS(FB)))) then
      S = FB/FA
      If (A.EQ.C) then
         P = 2.*XM*S
         T = 1.-S
      else
         T = FA/FC
         R = FB/FC
         P = (2.*XM*T*(T-R) - (B-A)*(R-1.))
         T = (T-1.)*(R-1.)*(S-1.)
      Endif
      If (P.GT.0.) T = -T
      P = ABS(P)
      if (P.*(T).LT.((DMIN1((3.*XM*T-ABS(Tol1*T)),(ABS(E*T))))))
         E = D
         D = P/T
      else
         D = XM
         E = D
      Endif
   else
      D = XM
      E = D
   Endif
   A = B
   FA = FB
   If (((ABS(D)).GT.(Tol1)) then
      B = B + D
   else
      B = B + SIGN(Tol1, XM)
   Endif
   If (VarDis(j,i).EQ.7) then
      FB = GG(Aux1, Aux2, B)
   elseif (VarDis(j,i).EQ.10) then
      FB = TT(i,j, Aux1, Aux2, B)
   elseif (VarDis(j,i).EQ.5) then
      FB = MM(i,j, Aux1, B)
   elseif (VarDis(j,i).EQ.6) then
      FB = BB(i,j, Aux1, B)
   Endif
11 continue
PAUSE 'Zbrent exceeding maximum iterations.'
Zbrent = B
return
End

C C C
C
C Function Gammln(XX)
C
C Returns the value ln(Gamma(XX)) for XX>0. Full accuracy is
C obtained for XX>1. For 0<XX<1, the reflection formula can be
C used first.
C
--------
For more details, see Press et al (1992), pp.207.

For more details, see Press et al (1992), pp.207.

C       ################################################################
C
C       Subroutine Truncation(i,j,Ext,X,MuxN,SigmaxN,Trunc,Xo,CDFx,
C       CDFx0,PDFx,NR)
C
C       ########################################################################
C
C       Returns the value of the probability density function, PDFx, at
C       X & the value of the cumulative distribution function, CDFx, at
C       X of a distribution truncated at X=Xo & with a CDFx=CDFx0 at
C       X=X0. It also returns the mean, MuxN, & the standard deviation,
C       SigmaxN, of the equivalent Normal distribution through
C       subroutine PCDF.
C
C       ########################################################################
C
C       INPUT VARIABLES:
C       X - Variable
C       Ext - Abbreviation of the name of the variable
C       Trunc - Type of truncation
C       Xo - Point of truncation (if the distribution is truncated)
C       NR - Power to which each distribution is raised for each
C             combination of actions
C       CDFx0 - Value of the cumulative distribution function at X=X0
C       i - Number of the variable
C       j - Number of the combination of actions
C
C       INPUT/OUTPUT VARIABLES:
C       PDFx - Value of the probability density function at X
C       CDFx - Value of the cumulative distribution function at X
C
C       MODELING VARIABLES:
C       L - Maximum number of variables allowed by the program
C       Q - Maximum number of combinations of actions allowed by the
C            program
C
C       OUTPUT VARIABLES:
C       MuxN - Mean of the equivalent Normal distribution of X
C       SigmaxN - Standard deviation of the equivalent Normal
C                 distribution of X
C
C       ########################################################################
C
C       Integer*4 i,j,L,Q
C       Parameter (L=15)
C       Parameter (Q=16)
C       Character*3 Ext(L)
C       Integer*4 Trunc(Q,L)
C       Real*8 X(L),MuxN(L),SigmaxN(L),CDFx(L),CDFx0(L),PDFx(L),Xo(Q,L),
C       NR(Q,L)
C
C       If (Trunc(j,i).NE.0) then
C       If (Trunc(j,i).EQ.1) then
C       If (X(i).GT.Xo(j,i)) then
C       PDFx(i)=0.
C       CDFx(i)=1.
C       else
C       PDFx(i)=PDFx(i)/CDFx0(i)
C     End
CDFx(i)=CDFx(i)/CDFx0(i)
Endif
else
  If (X(i).LT.Xo(j,i)) then
    PDFx(i)=0.
    CDFx(i)=0.
  else
    PDFx(i)=PDFx(i)/(1.-CDFx0(i))
    CDFx(i)=(CDFx(i)-CDFx0(i))/(1.-CDFx0(i))
  Endif
Endif
Endif
Call PCDF(i,j,Ext,X,MuxN,SigmaxN,PDFx,CDFx, NR)
return
End

*************************************************************************
Function Gammp(XX,X)
*************************************************************************

Returns the incomplete Gamma function, gamma(XX,X)/Gamma(XX).

--------
For more details, see Press et al (1992), pp.211.

*************************************************************************
Real*8 XX,X,Gamser,Gln,Gammcf,Gammp
If ((X.LT.0.).OR.(XX.LE.0.)) PAUSE 'Bad arguments in Gammp'
If (X.LT.(XX+1.)) then
  Call GSER(Gamser,XX,X,Gln)
  Gammp=Gamser
else
  Call GCF(Gammcf,XX,X,Gln)
  Gammp=1.-Gammcf
Endif
return
End

*************************************************************************
Function Beta(XX,YY)
*************************************************************************

Returns the value of the Beta function, B(XX,YY).

--------
For more details, see Press et al (1992), pp.209.

*************************************************************************
Real*8 XX,YY,XY,Beta,Gammln
External Gammln
XY=XX+YY
Beta=EXP(Gammln(XX)+Gammln(YY)-Gammln(XY))
return
End

*************************************************************************
Function Betai(XX,YY,U)
*************************************************************************

Returns the incomplete Beta function, Bu(XX,YY)/B(XX,YY).

--------
For more details, see Press et al (1992), pp.220.

*************************************************************************
Real*8 XX,YY,XY,U,BT,Gammln,Betaacf,Betai
External Gammln
External Betaacf
If ((U.LT.0.).OR.(U.GT.1.)) PAUSE 'Bad argument U in Betai'
If ((U.EQ.0.).OR.(U.EQ.1.)) then
  BT=0.
else
  Call GCf(Betaacf,XX,YY,U,Gammln)
  Betai=1.-Betaacf
Endif
return
End
else
    XY=XX+YY
    BT=EXP(Gammln(XY)-Gammln(XX)-Gammln(YY)+XX
        *LOG(U)+YY*LOG(1.-U))
Endif
If (U.LT.((XX+1.)/(XX+YY+2.))) then
    Betai=BT*Betacf(XX,YY,U)/XX
    return
else
    XY=1.-U
    Betai=1.-BT*Betacf(YY,XX,XY)/YY
    return
Endif
End

Subroutine Tred2(Covx,N,NP,D,E)

Householder reduction of a real symmetric N by N matrix Covx,
stored in an NP by NP physical array. On output, Covx is
replaced by the orthogonal matrix Q effecting the
transformation. D returns the diagonal elements of the
tridiagonal matrix, & E the off-diagonal elements, with
E(1)=0.

For more details, see Press et al (1992), pp.467.

integer*4 i,t,K,L,N,NP
real*8  H,Scale,F,G,HH,Covx(NP,NP),D(NP),E(NP)
do 18 i=N,2,-1
    L=i-1
    H=0.
    Scale=0.
    if (L.GT.1) then
        do 11 K=1,L
            Scale=Scale+DABS(Covx(i,K))
        continue
        endif
        do 11 K=1,L
            Scale=Scale+DABS(Covx(i,K))
        continue
        endif
    else
        do 12 K=1,L
            Covx(i,K)=Covx(i,K)/Scale
            H=H+(Covx(i,K)**2)
        continue
    endif
    F=Covx(i,L)
    G=-DSIGN(DSQRT(H),F)
    E(i)=Scale*G
    H=H-F*G
    Covx(i,L)=F-G
    F=0.
    do 15 t=1,L
        Covx(t,i)=Covx(i,t)/H
        G=0.
        do 13 K=1,t
            G=G+Covx(t,K)*Covx(i,K)
        continue
        do 13 K=t+1,L
            G=G+Covx(K,t)*Covx(i,K)
        continue
        E(t)=G/H
        F=F+E(t)*Covx(i,t)
    continue
    HH=F/(H+H)
    do 17 t=1,L
        F=Covx(i,t)
        G=E(t)-HH*F
        E(t)=G
        do 16 K=1,t
            Covx(t,K)=Covx(t,K)-F*E(K)-G*Covx(i,K)
        continue
    continue
    continue
endif
else
    E(i)=Covx(i,L)
endif
D(i)=H
Program Listing

[284x790]18 continue
D(1)=0.
E(1)=0.
Do 24 i=1,N
L=i-1
If (D(i).NE.0.) then
Do 22 t=1,L
G=0.
Do 19 K=1,L
G=G+Covx(i,K)*Covx(K,t)
19 continue
Do 21 K=1,L
Covx(K,t)=Covx(K,t)-G*Covx(K,i)
21 continue
22 continue
Endif
D(i)=Covx(i,i)
Covx(i,i)=1.
Do 23 t=1,L
Covx(i,t)=0.
Covx(t,i)=0.
23 continue
24 continue
return
End

Subroutine TQLI(D,E,N,NP,V)

QL algorithm with implicit shifts, to determine the eigenvalues & eigenvectors of a real, symmetric, tridiagonal matrix, or of a real, symmetric matrix previously reduced by TRED2. D is a vector of length NP. On input, its first N elements are the diagonal elements of the tridiagonal matrix. On output, it returns the eigenvalues. The vector E inputs the subdiagonal elements of the tridiagonal matrix, with E(1) arbitrary. On output E is destroyed. The matrix V(N by N matrix stored in an NP by NP array) is input as the identity matrix. If the eigenvectors of a matrix that has been reduced by TRED2 are required, then V is input as the matrix output by TRED2. In either case, the Kth column of V returns the normalized eigenvector corresponding to D(K).

----------
For more details, see Press et al (1992), pp.473.

Integer*4 i,K,L,M,N,NP,Iter
External Pythag
Open(Unit=100, File='liko.dat', Status='unknown')
Do 11 i=2,N
E(i)=E(i-1)
11 continue
E(N)=0.
Do 15 L=1,N
Iter=0
1 Do 12 M=L,N-1
DD=DABS(D(M))+DABS(D(M+1))
Write(100,*)
If (DABS(E(M))+DD.EQ.DD) goto 2
12 continue
M=N
2 If (M.NE.L) then
If (Iter.EQ.30) Pause 'Too many iterations in TQLI'
Iter=Iter+1
G=(D(L+1)-D(L))/(2.*E(L))
Write(100,*)
Minha=1.0
R=Pythag(G,Minha)
G=D(N)-D(L)+E(L)/(G+DSIGN(R,G))
S=1.
C=1.
P=0.
Do 14 i=M-1,L,-1
F=S*E(i)
P=F*C
B=C*E(i)
R=Pythag(F,G)
14 continue
E(i+1)=R
If (R.EQ.0.) then
  D(i+1)=D(i+1)-P
  E(M)=0.
goto 1
Endif
S=F/R
C=G/R
G=D(i+1)-P
R=(D(i)-G)*S+2.*C*B
P=G*R
D(i+1)=G+P
G=C*R-B
Do 13 K=1,N
  F=V(K,i+1)
  V(K,i+1)=S*V(K,i+1)+C*F
  V(K,i)=C*V(K,i)-S*F
13    continue
14    continue
D(L)=D(L)-P
E(L)=G
E(M)=0.
goto 1
Endif
continue
return
End

---------------------------------------------------------------------
Function GG(Aux1,Aux2,AB)
---------------------------------------------------------------------
GG is a function for which the root is to be calculated using subroutine Zbrent. The root of GG gives the value of the parameter Zeta of a Frechet distribution.
---------------------------------------------------------------------
INPUT VARIABLES:
Aux1, Aux2, AB - Auxiliary variables
MODELING VARIABLES:
Gammln - External function
AB1, AB2 - Auxiliary variables
---------------------------------------------------------------------
Real*8 Aux1,Aux2,AB,AB1,AB2,Gammln,GG
External Gammln
AB1=1.-1./AB
AB2=1.-2./AB
GG=(Aux2**2)+(Aux1**2)-((Aux1**2)*
1     (EXP(Gammln(AB2))))/((EXP(Gammln(AB1)))**2)
return
End

---------------------------------------------------------------------
Function TT(i,j,Aux1,Aux2,AB)
---------------------------------------------------------------------
TT is a function for which the root is to be calculated using subroutine Zbrent. The root of TT gives the value of the parameter Eta of a Weibull distribution.
---------------------------------------------------------------------
INPUT VARIABLES:
Zeta - Parameter of a Weibull distribution
i - Number of the variable
j - Number of the combination of actions
Aux1, Aux2, AB - Auxiliary variables
MODELING VARIABLES:
Gammln - External function
L - Maximum number of variables allowed by the program
Program Listing

Q - Maximum number of combinations of actions allowed by the program
AB1, AB2 - Auxiliary variables
Lamda, Eta, x1, x2 - Variables mentioned in the Common statement but not used here

#############################################################

Integer*4 i,j,Q,L
Parameter (Q=16)
Parameter (L=15)
Real*8 Aux1,Aux2,Zeta(Q,L),Lamda(Q,L),Eta(Q,L),x1(Q,L),x2(Q,L),
Gammln,TT,AB,AB1,AB2
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Gammln
AB1=1.+1./AB
AB2=1.+2./AB
TT=Aux2-((Aux1-Zeta(j,i))/EXP(Gammln(AB1)))*
1     SQRT((EXP(Gammln(AB2)))-((EXP(Gammln(AB1 )))**2))
return
End

#############################################################

Function MM(i,j,Aux1,AB)

#############################################################

MM is a function for which the root is to be calculated using subroutine Zbrent. The root of MM gives a value used by subroutine InvGamma to calculate the inverse function of a Gamma distribution.

#############################################################

INPUT VARIABLES:
Zeta - Parameter of a Gamma distribution
i - Number of the variable
j - Number of the combination of actions
Aux1, AB - Auxiliary variables

MODELING VARIABLES:
Gammp - External function
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Aux2 - Auxiliary variable
Lamda, Eta, x1, x2 - Variables mentioned in the Common statement but not used here

#############################################################

Integer*4 j,i,L,Q
Parameter (Q=16)
Parameter (L=15)
Real*8 Zeta(Q,L),Lamda(Q,L),Eta(Q,L),x1(Q,L),x2(Q,L),
AB,Aux1,Aux2,Gammp,MM
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Gammp
Aux2=Zeta(j,i)
MM=Aux1-Gammp(Aux2,AB)
return
End

#############################################################

Function BB(i,j,Aux1,AB)

#############################################################

BB is a function for which the root is to be calculated using subroutine Zbrent. The root of BB gives a value used by subroutine InvBeta to calculate the inverse function of a Beta distribution.

#############################################################

INPUT VARIABLES:
Zeta, Lamda - Parameters of a Beta distribution
i - Number of the variable
j - Number of the combination of actions
Aux1, AB - Auxiliary variables

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Betai - External function
Aux2, Aux3 - Auxiliary variables
Eta, x1, x2 - Variables mentioned in the Common statement but not used here

########################################################################
Integer*4 j,i,L,Q
Parameter (Q=16)
Parameter (L=15)
Real*8 Zeta(Q,L),Lamda(Q,L),Eta(Q,L),x1(Q,L),x2(Q,L),
AB,Aux1,Betai,BB,Aux2,Aux3
Common/BLOCK8/Zeta,Lamda,Eta,x1,x2
External Betai
Aux2=Zeta(j,i)
Aux3=Lamda(j,i)
BB=Aux1-Betai(Aux2,Aux3,AB)
return
End

########################################################################
Subroutine PCDF(i,j,Ext,X,MuxN,SigmaxN,PDFx,CDFx,NR)

########################################################################

INPUT VARIABLES:
X - Variable
Ext - Abbreviation of the name of the variable
NR - Power to which each distribution is raised for each combination of actions
PDFx - Value of the probability density function at X
CDFx - Value of the cumulative distribution function at X
i - Number of the variable
j - Number of the combination of actions

MODELING VARIABLES:
SNX - Standard Normal variable
C - Value of the exceedance cumulative distribution function of the standard Normal distribution which corresponds to SNX
C1, C2 - Value of the cumulative distribution function at X
L - Maximum number of variables allowed by the program
Q - Maximum number of combinations of actions allowed by the program
Pi - 3.14159...

OUTPUT VARIABLES:
MuxN - Mean of the equivalent Normal distribution of X
SigmaxN - Standard deviation of the equivalent Normal distribution of X

########################################################################
Integer*4 i,j,Q,L
Parameter (L=15)
Parameter (Q=16)
Character*3 Ext(L)
Real*8 Pi,X(L),CDFx(L),PDFx(L),MuxN(L),SigmaxN(L),C,C1,C2,SNX,
NR(Q,L)
Pi=4.*ATAN(1.)
C1=CDFx(i)**NR(j,i)
C2=CDFx(i)**(NR(j,i)-1)
C=1.-C1
Call InvNormal(C,SNX)
SigmaxN(i)=(EXP(-(SNX**2)/2.))/(NR(j,i)*C2*PDFx(i)*SQRT(2.*Pi))
Program Listing

MuxN(i)=X(i)-SigmaxN(i)*SNX
Call NorWrite(i,Ext,MuxN,SigmaxN)
return
End

C  ################################################################
C
Subroutine GSER(Gamser,XX,X,Gln)
  ################################################################

Returns the incomplete Gamma function, gamma (XX,X)/Gamma (XX),
evaluated by its series representation as Gamser. Also returns
ln (Gamma (XX)) as Gln.

---

For more details, see Press et al (1992), pp.212.

Integer*4 Itmax,N
Real*8 Eps
Parameter (Itmax=100,Eps=3.0E-7)
Real*8 XX,X,Gamser,Gln,AP,SUM,DEL,Gammln
External Gammln
Gln=Gammln(XX)
If (X.LE.0.) then
  If (X.LT.0.) Pause 'x < 0 in GSER'
  Gamser=0.
  return
Endif
AP=XX
SUM=1./XX
DEL=SUM
Do 11 N=1,Itmax
  AP=AP+1.
  DEL=DEL*X/AP
  SUM=SUM+DEL
  If ((ABS(DEL)).LT.(ABS(SUM)*Eps)) goto 1
11     continue
PAUSE 'XX too large; Itmax too small in GSER'
Gamser=SUM*EXP(-X+XX*LOG(X)-Gln)
return
End

C  ################################################################
C
Subroutine GCF(Gammcf,XX,X,Gln)
  ################################################################

Returns the incomplete Gamma function, gamma (XX,X)/Gamma (XX),
evaluated by its continued fraction representation as Gammcf.
Also returns ln (Gamma (XX)) as Gln.

---

For more details, see Press et al (1992), pp.212.

Integer*4 Itmax,i
Real*8 Eps,Fpmin
Parameter (Itmax=100,Eps=3.0E-7,Fpmin=1.0E-30)
Real*8 XX,X,Gammcf,Gln,a,b,c,d,del,h,Gammln
External Gammln
Gln=Gammln(XX)
b=X+1.-XX
c=1./Fpmin
d=1./b
h=d
Do 11 i=1,Itmax
  an=-i*(i-XX)
  b=b+2.
  d=an*d+b
  If (ABS(d).LT.Fpmin) d=Fpmin
  c=b+an/c
  If (ABS(c).LT.Fpmin) c=Fpmin
  d=1./d
  del=d*c
  h=h*del
  If (ABS(del-1.).LT.Eps) goto 1
Program Listing

11     continue
PAUSE 'XX too large; Itmax too small in GCF'
Gammar=EXP(-X+XX*LOG(X)-Gln)*h
return
End

Function Betacf(ZZ,WW,VV)

Returns continued fraction for the incomplete Beta function by modified Lentz's method.
----------
For more details, see Press et al (1992), pp.221.

Integer*4 Maxit,M,M2
Real*8 Eps,Fpmin
Parameter (Maxit=100,Eps=3.0E-7,Fpmin=1.0E-30)
Real*8 ZZ,WW,VV,aa,c,d,del,h,qab,qam,qap,Betacf
qab=ZZ+WW
qap=ZZ+1.
qam=ZZ-1.
c=1.
d=1.-qab*VV/qap
If (ABS(d).LT.Fpmin) d=Fpmin
d=1./d
h=d
Do 11 M=1,Maxit
M2=2*M
aa=M*(WW-M)*VV/((qam+M2)*(ZZ+M2))
d=1.+aa/c
If (ABS(d).LT.Fpmin) c=Fpmin
d=1./d
h=h*d*c
aa=-(ZZ+M)*(qab+M)*VV/((ZZ+M2)*(qap+M2))
d=1.+aa/d
If (ABS(d).LT.Fpmin) d=Fpmin
c=1.+aa/c
If (ABS(c).LT.Fpmin) c=Fpmin
d=1./d
del=d*c
h=h*del
If ((ABS(del-1.)).LT.Eps) goto 11
PAUSE 'ZZ or WW too big, or Maxit too small in Betacf'
Betacf=h
return
End

Function Pythag(a,b)

Computes SQRT(a*a+b*b) without destructive underflow or overflow.
----------
For more details, see Press et al (1992), pp.62.

Real*8 a,b,aa,ab,Pythag
aa=DBAS(a)
ab=DBAS(b)
If (aa.GT.ab) then
Pythag=aa*DSQRT(1.+(ab/aa)**2)
else
If (ab.EQ.0.) then
Pythag=0.
else
Pythag=ab*DSQRT(1.+(aa/ab)**2)
Endif
Subroutine HandR(FDer,N,X,OBJF,OBJGRD,Par)

Returns the failure function, OBJF, & its first partial derivatives, OBJGRD, for the failure mode of overtopping using the H&R model.

**INPUT VARIABLES:**
- It - Iteration number
- FDer - Method of calculation of the first partial derivatives of the failure function overtopping
- N - Number of variables
- DSWL - Definition of the SWL
- X - Variables of the failure mode
- Par - For Mode=1, it is the prescribed seawall crest level; for Mode=2, it is the starting seawall crest level (from which the program iterates to find the required value of the seawall crest level)
- C1 - Parameter used in the H&R model to calculate C; it depends on the confidence value assigned to the maximum run-up
- TR - Allowable discharge for each FORM calculation
- k0 - Number of the FORM calculation

**MODELING VARIABLES:**
- L - Maximum number of variables allowed by the program
- Tp - Wave period of peak spectral density
- Hs - Significant height of the incident waves
- A - Model coefficient
- B - Model coefficient
- SWL - Still-water-level
- Tide - Tide level
- Surge - Surge
- TangAlpha - Tangent of the seawall slope
- r - Roughness of the seawall's front slope
- eB - Model parameter
- Ep - Surf similarity parameter
- C - Parameter in the H&R model
- CL - Seawall crest level
- TL - Seawall toe level
- Rc - Seawall freeboard
- Rstar - Dimensionless freeboard
- Q - Mean overtopping discharge over unit length of seawall
- Pi - 3.14159...
- AuxRstar - Variable mentioned in the Common statements but not used here

**OUTPUT VARIABLES:**
- OBJF - Failure function
- OBJGRD - First partial derivatives of OBJF

**Program Listing**

```fortran
program listing

C       ########################################### ####################
C
C       Subroutine HandR(FDer,N,X,OBJF,OBJGRD,Par)
C
C       Returns the failure function, OBJF, & its first partial
derivatives, OBJGRD, for the failure mode of overtopping using
the H&R model.

C       INPUT VARIABLES:
C       It - Iteration number
C       FDer - Method of calculation of the first partial derivatives of
the failure function overtopping
C       N - Number of variables
C       DSWL - Definition of the SWL
C       X - Variables of the failure mode
C       Par - For Mode=1, it is the prescribed seawall crest level; for
    Mode=2, it is the starting seawall crest level (from which
    the program iterates to find the required value of the
    seawall crest level)
C       C1 - Parameter used in the H&R model to calculate C; it depends
    on the confidence value assigned to the maximum run-up
C       TR - Allowable discharge for each FORM calculation
C       k0 - Number of the FORM calculation
C
C       MODELING VARIABLES:
C       L - Maximum number of variables allowed by the program
C       Tp - Wave period of peak spectral density
C       Hs - Significant height of the incident waves
C       A - Model coefficient
C       B - Model coefficient
C       SWL - Still-water-level
C       Tide - Tide level
C       Surge - Surge
C       TangAlpha - Tangent of the seawall slope
C       r - Roughness of the seawall's front slope
C       eB - Model parameter
C       Ep - Surf similarity parameter
C       C - Parameter in the H&R model
C       CL - Seawall crest level
C       TL - Seawall toe level
C       Rc - Seawall freeboard
C       Rstar - Dimensionless freeboard
C       Q - Mean overtopping discharge over unit length of seawall
C       Pi - 3.14159...
C       AuxRstar - Variable mentioned in the Common statements but
    not used here
C
C       OUTPUT VARIABLES:
C       OBJF - Failure function
C       OBJGRD - First partial derivatives of OBJF

C       ################################################################
C       Integer*4 N,k0,L,FDer,It,AuxRstar,DSWL
C       Parameter (L=15)
C       Real*8  X(N),Par,OBJF,OBJGRD(N),TR(L)
C       Real*8  Tp,Hs,A,B,C,CL,SWL,Tide,Surge,TangAlpha,r,eB,Q,CL,Rc,
C                  Rstar,Ep,Pi,TL
C       Common/BLOCK7/TR
C       Common/BLOCK9/k0,It,AuxRstar
C       Common/BLOCK10/Cl
C       Common/BLOCK11/DSWL,TL
C       Pi=4.*ATAN(1.)
C       Tp=X(1)
C       Hs=X(2)
C       A=X(3)
C       B=X(4)
C       If (DSWL.EQ.1) then
C         SWL=X(5)
C         TangAlpha=X(6)
C         r=X(7)
C         eB=X(8)

Endif
return
End

C       ########################################### ####################
C
```
else
Tide=X(5)
Surge=X(6)
SWL=Tide+Surge
If (SWL.LT.TL) then
Write(*,789) SWL,TL
Write(50,789) SWL,TL
789        Format('! 3X,'ERROR: SWL Below Toe Level !'
1 / 10X,'SWL=',E17.10,3X,'TL=',E17.10 )
STOP
Endif
TangAlpha=X(7)
r=X(8)
eB=X(9)
Endif

C       Definition of the seawall crest level, CL. CL has the value of
C       the design parameter (in Mode 1: if DSWL=1, X=8; if DSWL=2, X=9)
C       or it is the new variable (in Mode 2: if DSWL=1, X=9; if DSWL=2,
C       X=10).
C       =========================================== =====================
If ((N.EQ.9.AND.DSWL.EQ.1).OR.(N.EQ.10.AND.DSWL.EQ.2)) then
CL=X(N)
else
CL=Par
Endif
If (CL.LT.SWL) then
Write(*,78) SWL,CL
Write(50,78) SWL,CL
78       Format('! 3X,'ERROR: SWL Above Crest Level !'
1           / 10X,'SWL=',E17.10,3X,'CL=',E17.1 0 )
STOP
Endif

C       Calculation of C.
C       =========================================== =====================
C       ----------
C       Calculation of the surf similarity parameter, Ep.
C       ----------
Ep=TangAlpha/SQRT((2.*Pi*Hs)/(9.81*(Tp**2))
If (Ep.LE.2.) then
C=C1*1.35*Ep
else
C=C1*(3.-0.15*Ep)
Endif
C       ================================================================
C       Calculation of the dimensionless freeboard, Rstar.
C       =========================================== =====================
Rc=CL-SWL
Rstar=Rc/(r*C*Hs)
C       =========================================== =====================
C       Calculation, for Mode=2, of the first partial derivative of the
C       failure function in relation to CL=X(N), OBJGRD(N).
C       =========================================== =====================
If ((N.EQ.9.AND.DSWL.EQ.1).OR.(N.EQ.10.AND.DSWL.EQ.2)) then
If (FDer.EQ.1) then
If ((Rstar.LT.1.).AND.(Rstar.GE.0.)) th en
If (Ep.LE.2.) then
OBJGRD(N)=(A*eb*B*SQRT(CL*1.35*Tp*TangAlpha)
  *(9.81**(3./4.))*(Hs**(1./4.)))/(r*(2.*Pi)**(1./4.))**(1.-Rstar)**(eB*1)
else
OBJGRD(N)=(A*eb*B*SQRT(9.81*Hs*C)/r)**(1.-Rstar)**(eB*1)
Endif
Endif
Endif
If (Rstar.GE.1.) then
OBJGRD(N)=0.
goto 15
Endif
Write(*,8700)
Write(50,8700)
8700       Format('! 11X,'ERROR: Mode 2 - R* < 0 !'
STOP
Endif
Program Listing

C
C Calculation of the mean overtopping discharge over unit length
of seawall, Q.
C
Endif

If (Rstar.GE.1.) then
Q=0.
Endif

If ((Rstar.LT.1.).AND.(Rstar.GE.0.)) then
Q=A*SQRT(9.81*((C*Hs)**3))*((1.-Rstar)**(eB*B))
Endif

If (Rstar.LT.0.) then
If (Rc.LT.0.) then
Write(*,8730)
Write(50,8730)
8730 Format(/ 11X,'ERROR: Rc < 0 - R* < 0 !' /)
Endif
If (C.LT.0.) then
Write(*,8720)
Write(50,8720)
8720 Format(// 11X,'ERROR: C < 0 - R* < 0 !' )
STOP
Endif

C
C Calculation of the failure function, OBJF, & of its first
partial derivatives, OBJGRD (if required).
C
OBJF=TR(k0)-Q

If (FDer.EQ.1) then
If ((Rstar.LT.1.).AND.(Rstar.GE.0.)) then
If (Ep.LE.2) then
OBJGRD(1)=-(((1.5*A*SQRT(Tp*(9.81**(5./2.))*(Hs**(3./2.))*
(C1**3)*(1.35**3)*(TangAlpha**3)*((2.*Pi)**
(-3./2.))))*((1.-Rstar)**(eB*B))+((A*eB*B*
((1.-Rstar)**(eB*B-1))))*
((Re*SQRT(C1*1.35*TangAlpha))/(r*SQRT(Tp)*((2.*Pi)**
1*(1./4.)))))
OBJGRD(2)=(((3./4.)*A*Hs**((-1./4.)))*SQRT((9.81**5./*2.)*
((C*Hs)**3)*((1.35**3)*((1.-Rstar)**(eB*B-1)))*
((2.*Pi)**(-3./2.))))*((1.-Rstar)**(eB*B))
OBJGRD(3)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(4)=-A*eB*B*Hs**((1./4.))*(9.81**(3./4.))
OBJGRD(5)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(6)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(7)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(8)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(9)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.))))*LOG(1.-Rstar)

else
OBJGRD(5)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(6)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(7)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(8)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(9)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.))))*LOG(1.-Rstar)
end else

If (DSWL.EQ.1) then
OBJGRD(5)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(6)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(7)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(8)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(9)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.))))*LOG(1.-Rstar)
end else

end if

C
C Calculation of the failure function, OBJF, & of its first
partial derivatives, OBJGRD (if required).
C
OBJF=TR(k0)-Q

If (FDer.EQ.1) then
If ((Rstar.LT.1.).AND.(Rstar.GE.0.)) then
If (Ep.LE.2) then
OBJGRD(1)=-(((1.5*A*SQRT(Tp*(9.81**(5./2.))*(Hs**(3./2.))*
(C1**3)*(1.35**3)*(TangAlpha**3)*((2.*Pi)**
(-3./2.))))*((1.-Rstar)**(eB*B))+((A*eB*B*
((1.-Rstar)**(eB*B-1))))*
((Re*SQRT(C1*1.35*TangAlpha))/(r*SQRT(Tp)*((2.*Pi)**
1*(1./4.)))))
OBJGRD(2)=(((3./4.)*A*Hs**((-1./4.)))*SQRT((9.81**5./*2.)*
((C*Hs)**3)*((1.35**3)*((1.-Rstar)**(eB*B-1)))*
((2.*Pi)**(-3./2.))))*((1.-Rstar)**(eB*B))
OBJGRD(3)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(4)=-A*eB*B*Hs**((1./4.))*(9.81**(3./4.))
OBJGRD(5)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(6)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(7)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(8)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(9)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.))))*LOG(1.-Rstar)

else
OBJGRD(5)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(6)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(7)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(8)=-(A*eB*B*Hs**((1./4.))*(9.81**(3./4.)))*
OBJGRD(9)=-((A*eB*B*Hs**((1./4.))*(9.81**(3./4.))))*LOG(1.-Rstar)
end else

end if

C
C Calculation of the mean overtopping discharge over unit length
of seawall, Q.
C
Endif

Endif

STOP
OBJGRD(8) = -A*eB*B*((1.-Rstar)**(eB*B-1))*(Rc*SQRT(C1)*TangAlpha)*(9.81**((3./4.)))*((Hs**(1./4.)))/(r**2)*((2.*Pi)**((1.-Rstar)**(eB*B-1)))*LOG(1.-Rstar)*((eB*B-1))

Endif
else
OBJGRD(1) = ((3.*A*9.81*C1*0.15*Hs*SQRT(C)*TangAlpha)/((2.*SQRT(2.*Pi)))*((1.-Rstar)**(eB*B-1))*(r*SQRT(r*SQRT(2.*Pi)*C))

OBJGRD(2) = -(3.*A*(C**(3./2.))*SQRT(9.81*Hs))/2.)*((1.-Rstar)**(eB*B-1))*LOG(1.-Rstar)

OBJGRD(3) = -SQRT(9.81*(Hs**3)*(C**3))*((1.-Rstar)**(eB*B-1))*(eB*B)*LOG(1.-Rstar)

OBJGRD(4) = -A*eB*B*SQRT(9.81*(Hs**3)*(C**3))*((1.-Rstar)**(eB*B-1))*(2.*SQRT(2.*Pi)*Hs))

OBJGRD(5) = -A*eB*B*SQRT(9.81*Hs*C)*( (1.-Rstar)**(eB*B-1))/r

OBJGRD(6) = ((3.*A*Hs*9.81*0.15*Tp*C1*SQRT(C))/2.)*SQRT(2.*Pi)))*((1.-Rstar)**(eB*B-1))*LOG(1.-Rstar)*((eB*B-1))

OBJGRD(7) = -(3.*A*9.81*0.15*Tp*C1*SQRT(9.81*Hs))/2.)*((1.-Rstar)**(eB*B-1))*LOG(1.-Rstar)*((eB*B-1))

OBJGRD(8) = -A*B*SQRT(9.81*(Hs**3)*(C**3))*((1.-Rstar)**(eB*B-1))*LOG(1.-Rstar)

Endif
endif

If (Rstar.GE.1.) then
OBJGRD(i) = 0.
2342 continue
Endif
If (Rstar.LT.0.) then
Write(*,8701)
Write(50,8701)
8701 Format(// 11X,'ERROR: R* < 0 !')
STOP
Endif
End

C Subroutine Owen(FDer,N,X,OBJF,OBJGRD,Par)

C Returns the failure function, OBJF, & its first partial derivatives, OBJGRD, for the failure mode of overtopping using Owen's model.

C INPUT VARIABLES:
FDer - Method of calculation of the first partial derivatives of the failure function of overtopping
N - Number of variables

C
Program Listing

DSWL - Definition of the SWL
X - Variables of the failure mode
Par - For Mode=1, it is the prescribed seawall crest level; for
Model 2, it is the starting seawall crest level (from which
the program iterates to find the required value of the
seawall crest level)
TR - Allowable discharge for each FORM calculation
k0 - Number of the FORM calculation

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
Tm - Mean zero-crossing wave period
Hs - Significant height of the incident waves
S - Incident wave steepness
A - Model coefficient
B - Model coefficient
SWL - Still-water-level
Tide - Tide level
Surge - Surge
eB - Model parameter
CL - Seawall crest level
TL - Seawall toe level
Q - Mean overtopping discharge over unit length of seawall
Pi = 3.14159...
It, AuxRstar - Variables mentioned in the Common statements but
not used here

OUTPUT VARIABLES:
OBJF - Failure function
OBJGRD - First partial derivatives of OBJF

# Definition of the seawall crest level, CL. CL has the value of
# the design parameter (in Mode 1: if DSWL=1, X=7; if DSWL=2, X=8)
# or it is the new variable (in Mode 2: if DSWL=1, X=8; if DSWL=2,
# X=9). For Mode=1, the first partial derivative of the failure
# function in relation to CL=X(N), OBJGRD(N), has to be calculated.

Integer*4 N,L,k0,FDer,It,AuxRstar,DSWL
Parameter (L=15)
Real*8 X(N),Par,OBJF,OBJGRD(N),TR(L)
Real*8 Pi,Tm,Hs,A,B,SWL,Tide,Surge,r,eB,S,CL,TL
Common/BLOCK7/TR
Common/BLOCK9/k0,It,AuxRstar
Common/BLOCK11/DSWL,TL
Pi=4.*ATAN(1.)
Tm=X(1)
Hs=X(2)
A=X(3)
B=X(4)
If (DSWL.EQ.1) then
   SWL=X(5)
   r=X(6)
eB=X(7)
else
   Tide=X(5)
   Surge=X(6)
   SWL=Tide+Surge
If (SWL.LT.TL) then
   Write(*,789) SWL,TL
   Write(50,789) SWL,TL
789    Format(//'ERROR: SWL Below Toe Level !'
          / 10X,'SWL=',E17.10,3X,'TL=',E17.10 /)
   STOP
endif
r=X(7)
eB=X(8)
endif
S=(2.*Pi*Hs)/(9.81*(Tm**2))

Definition of the seawall crest level, CL. CL has the value of
the design parameter (in Mode 1: if DSWL=1, X=7; if DSWL=2, X=8)
or it is the new variable (in Mode 2: if DSWL=1, X=8; if DSWL=2,
X=9). For Mode=2, the first partial derivative of the failure
function in relation to CL=X(N), OBJGRD(N), has to be calculated.

If ((N.EQ.8.AND.DSWL.EQ.1).OR.(N.EQ.9.AND.DSWL.EQ.2)) then
   CL=X(N)
   If (FDer.EQ.1) then
      OBJGRD(N)=(A*eB*B*SQR(9.81*Hs)/(r))*
1      EXP((-eB*B*SQR(S)*(CL-SWL))/(r*Hs*SQR(2.*Pi)))
      goto 15
   Endif
else
   ...
   ...
endif

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CL = Par
Endif
If (CL.LT.SWL) then
Write(*,78) SWL,CL
Write(50,78) SWL,CL
78 Format(// 3X,'ERROR: SWL Above Crest Level !' / 1 10X,'SWL=',E17.10,3X,'CL=',E17.10)
STOP
Endif

Calculation of the mean overtopping discharge over unit length of seawall, Q.

Q = A*SQRT((9.81*(Hs**3))/(S/(2.*Pi)))*EXP((-eB*B*SQRT(S)*(CL-SWL))/(r*Hs*SQRT(2.*Pi)))*

Calculation of the failure function, OBJF, & of its first partial derivatives, OBJGRD (if required).

OBJF=TR(k0)-Q
If (FDer.EQ.1) then
OBJGRD(1)=(A*SQRT((9.81*2.*Pi*(Hs**3))/(S))/(2.*S))*
OBJGRD(2)=-A*(9.81*TM)*EXP((-eB*B*(CL-SWL)))/(r*TM*SQRT(9.81*Hs))
OBJGRD(3)=-SQR((9.81*(Hs**3))/(S/(2.*Pi)))*
OBJGRD(4)=(A*B*(CL-SWL)*SQRT(9.81*Hs)/r)*
END

Subroutine SDunes(N,X,OBJF,Par)

Returns the failure function, OBJF, for the failure mode of dune erosion using Vellinga's method & allowing for movements of sand only seaward during the storm surge.

INPUT VARIABLES:
N - Number of variables
X - Variables of the failure mode
DSWL - Definition of the SWL
Par - For Mode=1, it is the prescribed nourishment width; for Mode=2, it is the starting nourishment width (from which the program iterates to find the required value of the nourishment width)
TR - Allowable erosion distance for each FORM calculation
k0 - Number of the FORM calculation
ctcurv - Coastal curvature in degrees per 1000m

(0<=ctcurv<=24Deg/1000m)
NPD – Initially, the number of points defining the initial profile; then, the number of points defining the nourished profile
NPDOld – Number of points defining the initial profile
(XP,YP) – Initially, the coordinates of the points defining the initial profile; then, the coordinates of the points defining the changed profile; finally, the coordinates of the points defining the nourished profile
(XPOld,YPOld) – Coordinates of the points defining the initial profile
NPch – Number of points to be changed in the initial profile
t – First point to be changed in the initial profile, point no.t

MODELING VARIABLES:
L – Maximum number of variables allowed by the program
ka – Maximum number of iterations allowed by the program to find the final position of Vellinga’s profile & the final location of the surcharge face
je – Number of the iteration to find the final position of Vellinga’s profile
j – Number of the iteration to find the final location of the surcharge face
Er – Maximum number of points (XP,YP) allowed by the program
h – Storm surge level
Tide – Tide level
Surge – Surge
W – Fall velocity of dune sand in seawater
Hs – Offshore significant wave height
DP – Change in the initial profile
D50 – Particle size: D50 of the dune sand
Ac – Accuracy of the computation
GB – Gust bumps
SD – Storm surge duration
MuHs – Mean of Hs expressed as a function of h
(W –) – Fall velocity of dune sand in seawater
Depth – Depth of Vellinga’s parabolic post-storm profile
nourwidt – Nourishment width at top level
(S1,T1) – Intersection point between the nourished profile & the surge level
(S9,T9) – Point where the parabolic part of Vellinga’s profile finishes
YPT9 – Y-coordinate of the point in the nourished profile which has X=S9
(S2,T2) – Intersection point between the nourished profile & the gradient 1:mt of Vellinga’s profile
S8 – X-coordinate of the starting point of the parabolic part of Vellinga’s post-storm profile
(S3,T3) – Intersection point between the nourished profile & the gradient 1:md of Vellinga’s profile
NumHump – Number of humps
NumDep – Number of depressions
BH – Area of a hump
SHump – Cumulative area of the humps starting from the seaward end of the profiles
BD – Area of a depression
SDep – Cumulative area of the depressions starting from the seaward end of the profiles
A – Area of erosion between the surge level, the nourished profile above surge & the gradient 1:md
B – Area between the surge level & the gradient 1:mt of Vellinga’s profile
E – Area between points (S9,T9) & (S2,T2), between the surge level & the nourished profile
(S4,T4) – Intersection point between the nourished profile & the surcharge gradient, 1:md
S10 – X-coordinate of the point of intersection between the surge level & the gradient 1:md of the surcharge
SurchEros – Surcharge on erosion area C above surge level to take into account the effects of the accuracy of the computation, of the storm surge duration & of the gust bumps
SurchLongT – Surcharge on erosion area C above surge level to take into account the effect of a gradient in the longshore transport
TSurch – Total surcharge on erosion area C which is the sum of the surcharges SurchEros plus SurchLongT
SurD – Surcharge distance
Err – Error in the balance between erosion & accretion (assuming movements of sand only seaward during the storm surge)
Mov – Value by which S8 is changed in each iteration performed to find the final position of Vellinga’s parabolic profile
i, M1, Q1, Q3, LD, BE, AL, Aux1, Sin, Fim – Auxiliary variables
It, AuxRstar, MuxN,
md, mt, nourtlev, mnour - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLE:
OBJF - Failure function

Integer*4 N,L,NPD,t,k0,ka,je,j,i,MI,Q1,NPch,NP0old1d,Aux1,Fim,
NumDep,NumHump,Er,It,AuxRstar,DSWL

Parameter (L=15)
Parameter (Er=100)
Parameter (ka=999)

Real*8 X(L),DP,h,Tide,Surge,Hs,D50,XP,YP,LD,W,BE,AL,Depth,Le,S1,
T1,S9,T9,B,E,S2,T2,S8(0:Ka),C,S3,T3,SurchEros,OBJF,NumN,
Par,md,mt,XP0ld1d,YP0ld1d,Surch0d(0:ka),mnour,nourtlev,SD,Ac,GB,
S4,T4,nourwidt,Err(0:ka),S10(0:ka),Mov,Sin(0:ka),YPT9,
BD(Er),BH(Er),SDep(0:Er),SHump(0:Er),ctcuvr,SurchLongT,
1
1
1
1

Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK2/t,NPch,XP0ld1d(100),YP0ld1d(100),NPD0ld1d
Common/BLOCK3/NumN(15),C,T3
Common/BLOCK4/md,mt,mnour,nourtlev,ctcuv
Common/BLOCK5/Ac,GB,SD
Common/BLOCK7/TR
Common/BLOCK9/k0,It,AuxRstar
Common/BLOCK11/DSWL,L

Q1=0
Auxl=0
Mov=10.
S8(0)=0.
Err(0)=1
Sin(0)=-1.
je=1
j=1
MuHs=0.

If (DSWL.EQ.1) then
  If (X(7).GE.3.0.AND.X(7).LT.7.0)
    1    MuHs=0.6*X(7)+4.82-0.0063*((7.-X(7))**3.13)
    If (X(7).GT.7.0) MuHs=0.6*X(7)+4.82
    Hs=X(1)+MuHs
    D50=X(2)
    DP=X(3)
    SD=X(4)
    GB=X(5)
    Ac=X(6)
    h=X(7)
  else
    If ((X(7)+X(8)).GE.3.0.AND.(X(7)+X(8)).LT.7.0)
      1    MuHs=0.6*(X(7)+X(8))+4.82-0.0063*((7.-X(7)+X(8))**3.13)
      If ((X(7)+X(8)).GT.7.0) MuHs=0.6*(X(7)+X(8))+4.82
      Hs=X(1)+MuHs
      D50=X(2)
      DP=X(3)
      SD=X(4)
      GB=X(5)
      Ac=X(6)
      h=Tide+Surge
      Tide=X(7)
      Surge=X(8)
      h=Tide+Surge
  Endif
Endif

NPD=NPD0ld
Do 3610 i=1,NPD
  XP(i)=XP0ld1d(i)
  YP(i)=YP0ld1d(i)
  3610 continue

Call Profile(t,NPch,DP,h,X,S1,T1,N,Par,nourwidt)

C Calculation of the shape of Vellinga's parabolic post-storm profile & of the position of the offshore point where the parabolic profile terminates: from the starting point of the profile, the length is Le & the depth is Depth.

LD=LOG10(D50)/LOG10(10.)
W=1./(10**((0.476*(LD**2)+2.18*LD+3.226))
BE=Hs/7.6
AL=(Hs/7.6)**1.28*(0.0268/W)**0.56
Le=250.*AL

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Depth=5.717*BE

Definition of the position of Vellinga's profile relative to the nourished profile in order to obtain a balance between the area of erosion & the area of accretion.

Do 2005 i=NPD,1,-1
If (YP(i).GE.T1) then
  M1=i
goto 3069
Endif
2005 continue
Write(*,9929)
Write(50,9929)
9929 Format('ERROR: Surge Level Above the Dune Height !')
STOP
3069 If (YP(M1).NE.T1) M1=M1+1

Definition of S8 which is the X-coordinate of the starting point of the parabolic part of Vellinga's profile. As a first approximation, S8=S1.

S8(je)=S1

Calculation of the coordinates, (S9,T9), of the point where the parabolic profile finishes.

S9=S8(je)+Le
T9=T1-Depth
Call SeaProf(S9,T9,T1,B,E,S2,T2,YPT9)
Call HumpDep(je,BE,AL,S1,T1,S2,T2,S3,T3,S8,S9,T9,B,C,E,md,YPT9,
1  NumDep,NumHump,BD,BH,SDep,SHump)

Calculation of the balance of erosion & accretion.

If (NumDep.GE.1) then
  If (NumDep.GT.NumHump) then
    If (NumHump.EQ.0) then
      Err(je)=BD(NumDep)
      Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
    else
      Call Error(Q1,BH,BD,NumDep,NumHump,SDep,SHump,Fim,je,
1        Err,T2,T3,S8,Mov,Sin,Aux1)
  Endif
Endif
If (NumDep.EQ.NumHump) then
  Call Error(Q1,BH,BD,NumDep,NumHump,SDep,SHump,Fim,je,Err,
1        T2,T3,S8,Mov,Sin,Aux1)
Endif
If (Fim.EQ.1) goto 7557
If (Fim.EQ.2) goto 999
Endif
If ((NumDep.EQ.0).AND.(NumHump.EQ.0)) then
  S4=0.
  SurchEros=0.
  SurchLongT=0.
  TSurch=0.
  SurD(j)=0.
goto 3355
else
  If (Fim.EQ.1) goto 7557
If (Fim.EQ.2) goto 999
Endif
Endif

Call SurchC(ctcurv,BE,W,S8,T1,S3,S4,T4,S10,SurD,SurchEros,
1  SurchLongT,TSurch)

S8(1)=S8(je)

Program Listing
Calculation of the failure function, OBJF.
==========================================
OBJF=TR(k0)+S4
return
End

Subroutine LDunes(N,X,OBJF,Par)

Returns the failure function, OBJF, for the failure mode of dune erosion using Vellinga's method & allowing for movements of sand in both directions during the storm surge.

INPUT VARIABLES:
N - Number of variables
X - Variables of the failure mode
DSWL - Definition of the SWL
Par - For Mode=1, it is the prescribed nourishment width; for Mode=2, it is the starting nourishment width (from which the program iterates to find the required value of the nourishment width)
TR - Allowable erosion distance for each FORM calculation
k0 - Number of the FORM calculation
tcurv - Coastal curvature in degrees per 1000m (0<tcurv<24Deg/1000m)
NPD - Initialy, the number of points defining the initial profile; then, the number of points defining the nourished profile
NPD0ld - Number of points defining the initial profile
(XP,YP) - Initially, the coordinates of the points defining the initial profile; then, the coordinates of the points defining the changed profile; finally, the coordinates of the points defining the nourished profile
(XP0ld,YP0ld) - Coordinates of the points defining the initial profile
NPch - Number of points to be changed in the initial profile
t - First point to be changed in the initial profile, point no.1

MODELING VARIABLES:
L - Maximum number of variables allowed by the program
ka - Maximum number of iterations allowed by the program to find the final position of Vellinga's profile & the final location of the surcharge face
je - Number of the iteration to find the final position of Vellinga's profile
j - Number of the iteration to find the final location of the surcharge face
h - Storm surge level
Tide - Tide level
Surge - Surge
Hs - Offshore significant wave height
DP - Change in the initial profile
D50 - Particle size: D50 of the dune sand
Ac - Accuracy of the computation
GB - Gust bumps
SD - Storm surge duration
MuHs - Mean of Hs expressed as a function of h
W - Fall velocity of dune sand in seawater
Depth - Depth of Vellinga's parabolic post-storm profile
Le - Length of Vellinga's parabolic post-storm profile
nourwidth - Nourishment width at top level
(S1,T1) - Intersection point between the nourished profile & the surge level
(S9,T9) - Point where the parabolic part of Vellinga's profile finishes
(S2,T2) - Intersection point between the nourished profile & the gradient 1:mt of Vellinga's profile
S8 - X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile
(S3,T3) - Intersection point between the nourished profile & the gradient 1:md of Vellinga's profile
A - Area between the surge level & the parabolic part of Vellinga's profile
C - Area of erosion between the surge level, the nourished profile above surge & the gradient 1:md
B - Area between the surge level & the gradient 1:mt of Vellinga’s profile
Q - Area between the surge level & the nourished profile below surge
(S4,T4) - Intersection point between the nourished profile & the surcharge gradient, 1:md
S10 - X-coordinate of the point of intersection between the surge level & the gradient 1:md of the surcharge
SurchEros - Surcharge on erosion area C above surge level to take into account the effects of the accuracy of the computation, of the storm surge duration & of the gust bumps
SurchLongT - Surcharge on erosion area C above surge level to take into account the effect of a gradient in the longshore transport
TSurch - Total surcharge on erosion area C which is the sum of the surcharges SurchEros plus SurchLongT
SurD - Surcharge distance
Err - Error in the balance between erosion & accretion (assuming movements of sand in both directions during the storm surge)
Mov - Value by which S8 is changed in each iteration performed to find the final position of Vellinga’s parabolic profile
i, M1, Q1, Q3, LD, BE, AL, Aux1,
Sin, Fim, D5 - Auxiliary variables
It, AuxRstar, MuxN,
md, mt, nourtlev, mnour, TL - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLE:
OBJF - Failure function

###############################################################################
Integer*4 N,L,NPD,t,ka,k0,j,i,M1,NPch,NPDOId,Aux1,je,Q1,Fim,It,
1 AuxRstar,DSWL

Parameter (L=15)
Parameter (ka=999)
Real*8 X(L),DP,h,Tide,Surge,Hs,D50,XP,YP,LD,WE,AL,Depth,Le,A,
1 S1,T1,S9,T9,Q,B,S2,T2,S8(0:Ka),DS,C,S3,T3,SurchEros,OBJF,
1 MuxN,Par,md,mt,XP0Id,YF0Id,SurD(0:ka),SD,Ac,GB,mnour,
1 nourtlev,S10(0:ka),S4,T4,nourwidth,Err(0:ka),Mov,
1 Sin(0:ka),ctcurv,SurchLongT,TSurch,TR(L),TL,MuHs
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK2/t,NPch,XP0Id(100),YP0Id(100),NPDOId
Common/BLOCK3/MuxN(15),C,T3
Common/BLOCK4/md,mt,mnour,nourtlev,ctcurv
Common/BLOCK5/AC,GB,SD
Common/BLOCK7/TR
Common/BLOCK9/k0,It,AuxRstar
Common/BLOCK11/DSWL,TL

Fim=0
Q1=0
Aux1=0
Mov=10.
S8(0)=0.
Err(0)=1
Sin(0)=0.
j=1
je=1
MuHs=0.

If (DSWL.EQ.1) then
If (X(7).GE.3.0.AND.X(7).LT.7.0)
1 MuHs=0.6*X(7)+4.82-0.0063*(7.-X(7))**3.13
If (X(7).GT.7.0) MuHs=0.6*X(7)+4.82
Hs=X(1)+MuHs
D50=X(2)
DP=X(3)
SD=X(4)
GB=X(5)
Ac=X(6)
h=X(7)
else
If ((X(7)+X(8)).GE.3.0.AND.(X(7)+X(8)).LT.7.0)
1 MuHs=0.6*(X(7)+X(8))+4.82-0.0063*((7.-X(7)+X(8)))**3.13
If ((X(7)+X(8)).GT.7.0) MuHs=0.6*(X(7)+X(8))+4.82
Hs=X(1)+MuHs
D50=X(2)
DP=X(3)
SD=X(4)
GB=X(5)
Ac=X(6)
Tide=X(7)
Surge=X(8)
h=Tide+Surge

Endif

NP=NP0ld
Do 3610 i=1,NP
 XP(i)=XPOld(i)
 YP(i)=YPOld(i)
3610 continue

Call Profile(t,NPch,DP,h,X,S1,T1,N,Par,nourwidt)

C       =========================================== =====================
C       Calculation of the shape of Vellinga's parabolic post-storm
C       profile & of the position of the offshore point where the
C       parabolic profile terminates: from the starting point of the
C       profile, the length is Le & the depth is Depth. Calculation of
C       the area A between the surge level & the parabolic part of
C       Vellinga's profile.
C       =========================================== =====================
LD=LOG10(D50)/LOG10(10.)
W=1./(10**((0.476*(LD**2)+2.18*LD+3.226))
BE=Hs/7.6
AL=((Hs/7.6)**1.28)*((0.0268/W)**0.56)
Le=250.*AL
Depth=5.717*BE
A=AL*BE*854.8

C       ================================================================
C       Definition of the position of Vellinga's profile relative to the
C       nourished profile in order to obtain a balance between the area
C       of erosion & the area of accretion.
C       =========================================== =====================
Do 2005 i=NPD,1,-1
 If (YP(i).GE.T1) then
  M1=i
  goto 3069
 Endif
2005 continue
Write(*,9929)
Write(50,9929)
9929 Format(// 6X,'ERROR: Surge Level Above the Dune Height !')
STOP
3069 If (YP(M1).NE.T1) M1=M1+1

C       Definition of S8 which is the X-coordinate of the starting
C       point of the parabolic part of Vellinga's profile. As a first
C       approximation, S8=S1.
C       ----------
S8(je)=S1
6556 Call LandProf(je,S8,T1,M1,S1,C,S3,T3)

C       ----------
C       Calculation of the coordinates, (S9,T9), of the point where the
C       parabolic profile finishes.
C       ----------
S9=S8(je)+Le
T9=T1-Depth
Call LSeaProf(S9,T9,S1,M1,T1,B,Q,S2,T2)

C       =========================================== =====================
C       Calculation of Err. Err is the error in the balance between
C       erosion & accretion (assuming movements of sand in both
C       directions during the storm surge).
C       If |Err|>1 then the balance between erosion & accretion is
C       not satisfactory. If Err>1, then accretion exceeds erosion
C       (Vellinga's profile has to be moved landward); if Err<1,
C       then erosion exceeds accretion (Vellinga's profile has to be
C       moved seaward, if S8 not equal to S1; if S8=S1, no erosion
C       is expected). After Vellinga's profile is moved, the new
C       X-coordinate of the starting point of the parabolic part of the
C       profile, S8, is calculated.
C       =========================================== =====================
Err(je)=Q-B-A-C
If ((Err(je).LE.0.).AND.(Q1.EQ.0)) then
  SurchEros=0.
  SurchLongT=0.
  TSurch=0.
  SurD(j)=0.
S4=0.
Fim=3
goto 3355
Endif
If (Err(je).LT.0.) then
Sin(je)=1.
elseresin(je)=-1.
Endif
If (ABS(Err(je)).GE.1.) then
DS=Err(je)/(T3-T2)
je=je+1
If (je.GT.999) then
Write(*,9987)
Write(50,9987)
9987       Format(// 6X, 1,'ERROR: The maximum number of iterations allowed by '/'
1       6X,' the program to find the final position of'/'
1       6X,' Vellinga’s profile has been exceeded !')
STOP
Endif
S8(je)=S8(je-1)-DS
If ((ABS(S8(je)-S8(je-2)).LT.0.0001).OR.(Aux1.EQ.1)) then
If (Sin(je-1).NE.Sin(je-2)) then
Mov=Mov/10.
Endif
S8(je)=S8(je-1)+Sin(je-1)*Mov
Aux1=1
Endif
Q1=1
goto 6556
Endif
S8(1)=S8(je)
Call SurchC(ctcurv,BE,W,S8,T1,S3,S4,T4,S10,SurchD,SurchEros,
1              SurchLongT,TSurch)
C       =========================================== =====================
C       Calculation of the failure function OBJF.
C       =========================================== =====================
3355   OBJF=TR(k0)+S4
return
End

###########################################################################
Subroutine Profile(t,NPch,DP,h,X,S1,T1,N,param,nownidt)

###########################################################################
Returns the coordinates of the nourished profile, which is used for comparison with Vellinga’s profile.

###########################################################################
INPUT VARIABLES:
N - Number of variables
X - Variables of the failure mode
Par - For Mode=1, it is the prescribed nourishment width; for
    Mode=2, it is the starting nourishment width (from which the program iterates to find the required value of the nourishment width)
DSWL - Definition of the SWL
DP - Change in the initial profile
h - Storm surge level
NPch - Number of points to be changed in the initial profile
t - First point to be changed in the initial profile, point no. t
nourlev - Nourishment top level
1:mmour - Gradient of the nourished face

INPUT/OUTPUT VARIABLES:
NPD - Initially, the number of points defining the initial profile; then, the number of points defining the nourished profile
(XP,YP) - Initially, the coordinates of the points defining the initial profile; then, the coordinates of the points defining the changed profile; finally, the coordinates of the points defining the nourished profile
MODELING VARIABLES:
- L - Maximum number of variables allowed by the program
- (XN,YN) - Intersection point between the changed profile & the nourishment top level
- (XM,YM) - The most seaward point at the nourishment top level
- (XQ,YQ) - Intersection point between the nourishment slope 1:mnour & the changed profile
- (XB,YB) - Intersection point between the changed profile & the surge level
- i, k, M, M1, M2, M3, Q3, Zes, G, XPOld, YPOld - Auxiliary variables
- md, mt, ctcurv, TL - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLES:
- nourwidt - Nourishment width at top level
- (S1,T1) - Intersection point between the nourished profile & the surge level

---

Integer*4 M,N,L,NPD,t,i,M1,M2,M3,Q3,NPch,k,DSWL
Parameter (L=15)
Parameter (M=100)
Real*8 X(L),DP,h,XP,YP,S1,T1,Par,XP,YB,md,mt,1:nour,mnour,nourtlev,
1
Q3=0

Do 2291 i=t,(t+NPch-1)
YP(i)=YP(i)+DP
2291   continue

Do 3499 i=1,NPD
XPOld(i)=XP(i)
YPOld(i)=YP(i)
3499   continue

If ((N.EQ.8.AND.DSWL.EQ.1).OR.(N.EQ.9.AND.DSWL.EQ.2)) then
nourwidt=X(N)

---

First, the width of the nourishment will have the value of the design parameter (in Mode 1: if DSWL=1, X=7; if DSWL=2, X=8) or it is the new variable (in Mode 2: if DSWL=1, X=8; if DSWL=2, X=9). For Mode=2, the first partial derivative of the failure function in relation to nourwidt=X(N), OBJGRD(N), has to be calculated.

---

If ((N.EQ.8.AND.DSWL.EQ.1).OR.(N.EQ.9.AND.DSWL.EQ.2)) then
nourwidt=X(N)
else
nourwidt=Par
Endif
If (nourwidt.EQ.0.) then
S1=XB
T1=YB
return
Endif
If (nourwidt.LT.0.) then
Write(*,2155) nourwidt
Write(50,2155) nourwidt
2155 Format(// 11X,'ERROR: Nourishment Width = ',E17.10,' < 0 !')
STOP
Endif

C
---------
C
Second, calculation of the intersection point, (XN,YN), between
C
the changed profile & the nourishment top level.
C
---------
Do 2375 i=NPD,1,-1
If (YP(i).GT.nourtlev) then
M2=i
goto 2386
Endif
If (YP(i).EQ.nourtlev) then
M2=i
Q3=1
goto 2386
Endif
2375 continue
Write(*,1589)
Write(50,1589)
1589 Format(// 6X,'ERROR: Nourishment Top Level Above the', 1 1X,'Dune Height !')
STOP
2386 XN=XP(M2)+(XP(M2+1)-XP(M2))*(nourtlev-YP(M2))/(YP(M2+1)-YP(M2))
YN=nourtlev

C
---------
C
Third, extension of the nourishment seaward by distance nourwidt
C
at the nourishment top level gives point (XM,YM).
C
---------
XM=XN+nourwidt
YM=YN

C
---------
C
Fourth, extension of the nourishment downwards at a slope of
C
1:mnour until it intersects the changed profile gives point
C
(XQ,YQ).
C
---------
Do 4499 i=M2,NPD
If (XP(i).GT.XM) then
M3=i
goto 4398
Endif
4499 continue
Write(*,8225)
Write(50,8225)
8225 Format(// 11X,'Extend the Initial Profile Seaward !')
STOP
4398 Zes=XP(M3)-XM
G=YM-Zes/mnour
If (G.GT.(YP(M3))) then
M3=M3+1
If (M3.GT.NPD) then
Write(*,9225)
Write(50,9225)
9225 Format(// 11X,'The Nourished Face Does Not Intersect the' / 1 11X,'Profile! Extend the Initial Profile Seaward !')
STOP
Endif
goto 4398
Endif
XQ=XP(M3)-Zes*(YP(M3)-G)/(YM-G-(Zes*(YP(M3-1)-YP(M3))/ (XP(M3)-XP(M3-1))))
YQ=YP(M3)+(XP(M3)-XQ)*(YP(M3-1)-YP(M3))/(XP(M3)-XP(M3-1))

C
---------
C
Fifth, if the top level of the nourishment is above the surge
C
level, & the nourished profile intersects the changed profile
C
below surge level, then the new coordinates of the point
C
(XB,YB) in the nourished profile have to be calculated, (S1,T1).

C
---------

C7-113
If ((nourtlev.GE.h).AND.(YQ.LT.YB)) then
    S1=XM+mnour*(YM-YB)
else
    S1=XB
Endif
T1=YB

----------
Finally, definition of the points, (XP,YP), of the whole profile
after the nourishment.
----------
If ((M2-Q3).GE.1) then
    Do 2381 i=1,(M2-Q3)
        XP(i)=XP0old(i)
        YP(i)=YP0old(i)
    2381     continue
Endif
XP(M2-Q3+1)=XN
YP(M2-Q3+1)=YN
XP(M2-Q3+2)=XM
YP(M2-Q3+2)=YM
XP(M2-Q3+3)=XQ
YP(M2-Q3+3)=YQ
NPD=(M2-Q3+3)+(NPD-M3+1)
k=0
    Do 2997 i=(M2-Q3+4),NPD
        XP(i)=XP0old(M3+k)
        YP(i)=YP0old(M3+k)
    k=k+1
2997   continue
return
End

#########################################################################
Subroutine LandProf(je,S8,T1,M1,S1,C,S3,T3)
#########################################################################
Returns the intersection point, (S3,T3), between the nourished
profile & the gradient 1:md of Vellinga's profile. It also
returns the area, C, between the surge level, the nourished
profile above surge & the gradient 1:md.
#########################################################################
INPUT VARIABLES:
je - Number of the iteration to find the final position of
Vellinga's profile
NPD - Number of points defining the nourished profile
1:md - Gradient of the eroded dune face
(XP,YP) - Coordinates of the points defining the nourished
profile
(S1,T1) - Intersection point between the nourished profile & the
surge level
M1 - Number of the point, (XP(M1),YP(M1))<=T1
S8 - X-coordinate of the starting point of the parabolic part of
Vellinga's post-storm profile
MODELING VARIABLES:
ka - Maximum number of iterations allowed by the program to find
the final position of Vellinga's profile & the final
location of the surcharge face
i, M3, U, V - Auxiliary variables
mt, nourtlev, mnour, ctcurv - Variables mentioned in the Common
statements but not used here
OUTPUT VARIABLES:
(S3,T3) - Intersection point between the nourished profile & the
gradient 1:md of Vellinga's profile
C - Area of erosion between the surge leve, the nourished
profile above surge & the gradient 1:md
#########################################################################
Integer*4 NPD,i,je,M1,M3,ka
Parameter (ka=999)
Real*8 T1,XP,YP,S8(0:ka),C,S3,T3,U,V,md,mt,mnour,nourtlev,
Calculation of the intersection point, \((S_3,T_3)\), between the nourished profile and the gradient 1:md of Vellinga’s profile.

\[
\begin{align*}
\text{If } (S_8(je).LT.XP(1)) & \text{ then} \\
& \text{Write(*,8625)} \\
& \text{Write(50,8625)} \\
\text{8625} & \text{Format(//'11X,'Extend the Initial Profile Landward !')} \\
& \text{STOP} \\
& \text{Endif} \\
& \text{Do 0903 } i=1,NPD \\
& \text{If } (XP(i).GT.S8(je)) \text{ then} \\
& \text{M3}=i \\
& \text{goto 4977} \\
0903 & \text{continue} \\
& \text{Endif} \\
& \text{Write(*,8925)} \\
& \text{Write(50,8925)} \\
\text{8925} & \text{Format(//'11X,'Extend the Initial Profile Seaward !')} \\
& \text{STOP} \\
& \text{Endif} \\
& \text{Continue} \\
& \text{Endif} \\
& \text{U}=S8(je)-XP(M3-1) \\
& \text{V}=T1+U/md \\
& \text{If } (V.LT.YP(M3-1)) \text{ then} \\
& \text{M3}=M3-1 \\
& \text{If } (M3.LE.1) \text{ then} \\
& \text{Write(*,8225)} \\
& \text{Write(50,8225)} \\
\text{8225} & \text{Format(//'11X,'Extend the Initial Profile Landward !')} \\
& \text{STOP} \\
& \text{Endif} \\
& \text{goto 4977} \\
& \text{Endif} \\
& \text{S3}=XP(M3-1)+U*(V-YP(M3-1))/(V-T1-(U*(YP(M3-1)-YP(M3)))/} \\
& \text{1} \\
& \text{If } (S3.LT.S8(je)) \text{ then} \\
& \text{T3}=YP(M3-1)-(S3-XP(M3-1))*(YP(M3-1)-YP(M3))/(XP(M3)-XP(M3-1)) \\
& \text{else} \\
& \text{S3}=S8(je) \\
& \text{T3}=T1 \\
& \text{C}=0. \\
& \text{goto 2591} \\
\text{Endif} \\
& \text{End} \\
\text{--------------------------------------------------------------------------------------------------------} \\
\text{Calculation of area, } C, \text{ between the surge level, the nourished profile above surge & the gradient 1:md.} \\
\text{--------------------------------------------------------------------------------------------------------} \\
& \text{If } ((XP(M3).LE.S1).AND.(XP(M1-1).LE.S1)) \text{ then} \\
& \text{C}=0.5*(XP(M3)-S3)*((T3-T1)+(YP(M3)-T1)) + 0.5*(S1-XP(M1-1))*(YP(M1-1)-T1) \\
& \text{else} \\
& \text{C}=0.5*(S1-S3)*(T3-T1) \\
& \text{Endif} \\
& \text{If } ((M1-2).GE.M3) \text{ then} \\
& \text{Do } 4701 \text{ i=M3,(M1-2) } \\
& \text{C=C+0.5*(XP(i+1)-XP(i))*(YP(i+1)-T1)+(YP(i)-T1)} \\
4701 & \text{continue} \\
& \text{Endif} \\
& \text{C=C-0.5*(S8(je)-S3)*(T3-T1)} \\
\text{2591} & \text{return} \\
\text{End} \\
\text{--------------------------------------------------------------------------------------------------------} \\
\text{Subroutine SeaProf(S9,T9,T1,B,E,S2,T2,YPT9)} \\
\text{--------------------------------------------------------------------------------------------------------} \\
\text{Returns the point of intersection, } (S2,T2), \text{ between the nourished profile \& the gradient 1:mt of Vellinga's profile. It also returns the area, } B, \text{ between the surge level \& the gradient 1:mt; \& the area, } E, \text{ between points } (S9,T9) \text{ \& } (S2,T2), \text{ between the surge level \& the nourished profile.} \\
\text{--------------------------------------------------------------------------------------------------------} \\
\text{INPUT VARIABLES:}
NPD - Number of points defining the nourished profile
(XP,YP) - Coordinates of the points defining the nourished profile
1:mt - Gradient of the toe of the post-storm profile
T1 - Y-coordinate of the intersection point between the nourished profile & the surge level
(S9,T9) - Point where the parabolic part of Vellinga's profile finishes

MODELING VARIABLES:

i, M2, R1, R2, Zes, G - Auxiliary variables
md, nourtlev, mnour, ctcurv - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLES:

YPT9 - Y-coordinate of the point in the nourished profile which has X=S9
(S2,T2) - Intersection point between the nourished profile & the gradient 1:mt of Vellinga's profile
B - Area between the surge level & the gradient 1:mt of Vellinga's profile
E - Area between points (S9,T9) & (S2,T2), between the surge level & the nourished profile

Program Listing

```
Integer*4 NPD,i,M2,R1,R2
Real*8 XP,YP,T1,S2,T2,S9,T9,B,E,Zes,G,md,mt,mnour,md,mt
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK4/md,mt,mnour,nourtlev,ctcurv

Calculation of the point of intersection, (S2,T2), between the nourished profile & the gradient 1:mt of Vellinga's profile.  
Calculation of the area, B, between the surge level & the gradient 1:mt.
Calculation of the area, E, between points (S9,T9) & (S2,T2),
```

```
Do 4499 i=1,NPD
   If (XP(i).GT.S9) then
      M2=i
      R1=M2
      YPT9=YP(M2-1)+(YP(M2)-YP(M2-1))*(S9-XP(M2-1))/
      (XP(M2)-XP(M2-1))
      If (YPT9.GE.T9) then
         S2=S9
         T2=YPT9
         B=0.
         E=0.
         M2=M2-1
         return
      else
         goto 4398
   Endif
   Zes=XP(M2)-S9
   G=T9-Zes/mt
   If (G.GT.YP(M2)) then
      M2=M2+1
      If (M2.GT.NPD) then
         Write(*,8225)
         Write(50,8225)
         8225 Format(// 11X,'Extend the Initial Profile Seaward !')
         STOP
      else
         goto 4398
   Endif
   S2=XP(M2)-Zes*(YP(M2)-G)/(T9-G-(Zes*(YP(M2-1)-YP(M2))/
   (XP(M2)-XP(M2-1))))
   T2=YP(M2)+(XP(M2)-S2)*(YP(M2-1)-YP(M2))/(XP(M2)-XP(M2-1))
   M2=M2-1
Endif
```

```
Calculation of the point of intersection, (S2,T2), between the nourished profile & the gradient 1:mt of Vellinga's profile. 
Calculation of the area, B, between the surge level & the gradient 1:mt.
```

```
B=0.5*(S2-S9)*((T1-T2)+(T1-T9))
```
between the surge level & the nourished profile.

Do 101 i=1,NPD
If (XP(i).GT.S2) then
  R2=i-1
  goto 102
Endif
101    continue
Write(*,8925)
Write(50,8925)
8925   Format(// 11X,'Extend the Initial Profile Seaward !')
STOP
102    If ((R2.GT.S9).AND.(R1.LT.S2)) then
  E=0.5*(XP(R1)-S9)*((T1-YP(R1))+(T1-YPT9)) +0.5*(S2-XP(R2))*
     ((T1-T2)+(T1-YP(R2)))
else
  E=0.5*(S2-S9)*((T1-T2)+(T1-YPT9))
Endif
If (R2.GT.R1) then
  Do 3781 i=R1,(R2-1)
    E=E+0.5*(XP(i+1)-XP(i))*((T1-YP(i))+(T1-YP(i+1)))
  3781     continue
Endif
return
End

Subroutine HumpDep(je,BE,AL,S1,T1,S2,T2,S3,T3,S8,S9,T9,B,C,E,
  md,YPT9,NumDep,NumHump,BD,BH,SDep,SHump)
1
Returns the number of humps, Numhump, the number of depressions, NumDep, & the corresponding areas, BH & BD. SHump & SDep are the corresponding cumulatives areas starting from the seaward end of the profiles.

INPUT VARIABLES:
je - Number of the iteration to find the final position of Vellinga's profile
NPD - Number of points defining the nourished profile
(XP,YP) - Coordinates of the points defining the nourished profile
(S1,T1) - Intersection point between the nourished profile & the surge level
(S9,T9) - Point where the parabolic part of Vellinga's profile finishes
YPT9 - Y-coordinate of the point in the nourished profile which has X=S9
(S2,T2) - Intersection point between the nourished profile & the gradient 1:mt of Vellinga's profile
S8 - X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile
(S3,T3) - Intersection point between the nourished profile & the gradient 1:md of Vellinga's profile
1:md - Gradient of the eroded dune face
C - Area of erosion between the surge level, the nourished profile above surge & the gradient 1:md
B - Area between the surge level & the gradient 1:mt of Vellinga's profile
E - Area between points (S9,T9) & (S2,T2), between the surge level & the nourished profile
BE, AL - Auxiliary variables
MODELING VARIABLES:
ka - Maximum number of iterations allowed by the program to find the final position of Vellinga's profile & the final location of the surcharge face
Er - Maximum number of points (XP,YP) allowed by the program
(XPOld1,YPOld1) - Coordinates of the points defining the nourished profile
NPV - Number of points defining Vellinga's profile
(XPV,YPV) - Coordinates of the points defining Vellinga's profile
(XHStart,YHStart) - Coordinates of the starting point of a hump
(XHEnd,YHEnd) - Coordinates of the end point of a hump
BH1 - Area between Vellinga's profile & the surge level at the location of a hump. If it is the last hump & the area C>0,
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```
then BH1 is the area as defined previously plus the area C

(BH1 - Area between the nourished profile & the surge level at
the location of a hump

(XDStart, YDStart) - Coordinates of the starting point of a
  depression

(XYEnd, YYEnd) - Coordinates of the end point of a depression

BD1 - Area between Vellinga's profile & the surge level at the
  location of a depression. If it is the first depression &
  the area B>0, then BD1 is the area as defined previously
  plus the area B

BD2 - Area between the nourished profile & the surge level at
  the location of a depression. If it is the first
  depression & the area B>0, then BD2 is the area as defined
  previously plus the area E

(XFUNC, YFUNC) - Coordinates of intersection point between the
  nourished profile & Vellinga's profile

i, j, k, N1, N2, N3, D0, D1, D2, D3, H0, H1, H2, H3,
AUX3, AUX4 - Auxiliary variables

OUTPUT VARIABLES:

NumHump - Number of humps
NumDep - Number of depressions
BH - Area of a hump
SHump - Cumulative area of the humps starting from the seaward
  end of the profiles
BD - Area of a depression
SDep - Cumulative area of the depressions starting from the
  seaward end of the profiles

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```c
Integer*4 NPD, k, ka, Er, j, je, i, N1, N2, N3, D0, D1, D2, D3, H0, H1, H2, H3,
Aux3, Aux4, NumDep, NumHump, NPV

Parameter (Er=100)
Parameter (ka=999)

Real*8 XP, YP, BE, AL, S1, T1, S2, T2, S3, T3, S8 (0:Ka), S9, T9, B, C, E,
md, YPT9, BD (Er), BD1 (Er), BD2 (Er), BH (Er), BH1 (Er), BH2 (Er),
SDep (0:Er), SHump (0:Er), XPV (Er), YPV (Er), XHStart (Er),
YHStart (Er), YHEnd (Er), XDStart (Er), XDEnd (Er),
YDStart (Er), YDEnd (Er), XFunc, YFunc, XPOld1 (Er), YPOld1 (Er)

Common/BLOCK1/NPD, XP (100), YP (100)

Do 367 k = 1, NPD
XPOld1 (k) = XP (k)
YPOld1 (k) = YP (k)
367 continue

Calculation of the Y-coordinate, YPV, of Vellinga's points,
(XPV, YPV), corresponding to the X-values of the points in the
nourished profile (XP, YP). The total number of points in
Vellinga's profile is NPV.

Do 11 i = 1, NPV
If (XP (i) .GT. S3) then
  N1 = i
  goto 15
Endif
11 continue
Do 12 i = NPV, 1, -1
If (XP (i) .LT. S9) then
  N2 = i
  goto 16
Endif
12 continue
Do 13 i = 1, NPV
If (XP (i) .GT. S8 (je)) then
  N3 = i
  goto 17
Endif
13 continue
If (N2 .GE. N1) then
  NPV = N2 - N1 + 4
else
  NPV = 3
Endif
XPV (i) = S3
YPV (i) = T3
XPV (NPV) = S2
YPV (NPV) = T2
XPV (NPV - 1) = S9
YPV (NPV - 1) = T9
If ((N3 - 1) .GE. N1) then
```
Do 14 i=2,(N3-N1+2)
  XPV(i)=XP(N1+i-2)
  YPV(i)=YPV(i-1)-(XPV(i)-XPV(i-1))/md
14  continue
Endif
If ((N3-N1+2).LE.(NPV-2)) then
  Do 18 i=(N3-N1+2),(NPV-2)
    XPV(i)=XP(N1+i-2)
    YPV(i)=T1-(0.4714*Be*SQRT((AL**(-1))*(XPV(i)-S8(je))+18)-2.*BE)
18  continue
Endif
C       =========================================== =====================
C       Determination of the intersection points be tween the two
C       profiles. Where the nourished profile is above Vellings's
C       profile it is a hump. A hump starts at (XHStart,YHStart) &
C       finishes landward at (XHEnd,YHEnd). If the nourished profile is
C       below Vellings's profile, it is a depression. A depression
C       starts at (XDStart,YDStart) & finishes landward at
C       (XDEnd,YDEnd). The total number of humps is Numhump & the total
C       number of depressions is NumDep.
C       =========================================== =====================
NumDep=0
NumHump=0
If (NPV.EQ.3) then
  If (YPT9.GE.T9) then
    NumHump=1
    XHStart(1)=S9
    YHStart(1)=YPT9
    XHEnd(1)=S3
    YHEnd(1)=T3
    Write(*,*) 'Only 1 hump; Nao sei se isto esta bem !'
    Write(50,*) 'Only 1 hump; Nao sei se isto esta bem !'
  else
    NumDep=1
    XDStart(1)=S9
    YDStart(1)=YPT9
    XDEnd(1)=S3
    YDEnd(1)=T3
    Write(*,*) 'Only 1 depression; Nao sei se isto esta bem !'
    Write(50,*) 'Only 1 depression; Nao sei se isto esta bem !'
  Endif
Endif
goto 2950
Endif
Do 19 i=(NPV-2),2,-1
If (\(\text{YPV}(i).GT.YP(N1+i-2)\)) \&\& (\(\text{YPV}(i).NE.YP(N1+i-2)\)) then
  NumDep=NumDep+1
  If (((i+1).EQ.(NPV-1)).AND.(B.EQ.0.)) then
    XP(N1+i-1)=S9
    YP(N1+i-1)=YPT9
  Endif
Endif
If (((i+1).EQ.(NPV-1)).AND.(B.GT.0.)) then
  XDStart(Nump)=S9
  YDStart(Nump)=YPT9
  goto 20
Endif
Aux3=0
Call XYFunc(je,i,N1,Aux3,Be,AL,T1,S8,XFunc,YFunc)
XDStart(Nump)=XFunc
YDStart(Nump)=YFunc
Do 3673 k=1,NPD
  XP(k)=XPold1(k)
  YP(k)=YPold1(k)
3673 continue
Endif
If ((i-1).EQ.1) then
   XDEnd(NumDep)=S3
   YDEnd(NumDep)=T3
Endif
If (((i+1).EQ.(NPV-1)).AND.(B.EQ.0.)) then
   NumHump=NumHump+1
   XHStart(NumHump)=S2
   YHStart(NumHump)=T2
   XHEnd(NumHump)=XDStart(NumDep)
   YHEnd(NumHump)=YDStart(NumDep)
Endif
Endif
If (   (YPV(i).LT.YP(N1+i-2)).OR.
   (YPV(i).EQ.YP(N1+i-2)).AND.
   (YPV(i+1).LT.YP(N1+i-1)).AND.(i.NE.(NPV-2))).OR.
   ((i+1).EQ.(NPV-1)).AND.(B.EQ.0.).AND.(i.EQ.(NPV-2))
) then
   NumHump=NumHump+1
   XHStart(NumHump)=S2
   YHStart(NumHump)=T2
   goto 21
Endif
If (((i+1).EQ.(NPV-1)).AND.(B.EQ.0.)) then
   YHStart(NumHump)=YP(N1+i-1)=YPT9
Endif
Aux3=0
Call XYFunc(je,i,N1,Aux3,BE,AL,T1,S8,XFunc,YFunc)
XHStart(NumHump)=XFunc
YHStart(NumHump)=YFunc
Do 1367 k=1,NPD
   XP(k)=XPOld1(k)
   YP(k)=YPOld1(k)
1367 continue
Endif
21 If (   (YPV(i-1).GE.YP(N1+i-3)).AND.(i-1).NE.1) then
    Aux3=1
    Call XYFunc(je,i,N1,Aux3,BE,AL,T1,S8,XFunc,YFunc)
    XHEnd(NumHump)=XFunc
    YHEnd(NumHump)=YFunc
Endif
If ((i-1).EQ.1) then
   XHEnd(NumHump)=S3
   YHEnd(NumHump)=T3
Endif
If (((i+1).EQ.(NPV-1)).AND.(B.GT.0.)) then
   NumDep=NumDep+1
   XDStart(NumDep)=S9
   YDStart(NumDep)=YPT9
   XDEnd(NumDep)=XHStart(NumHump)
   YDEnd(NumDep)=YHStart(NumHump)
Endif
Endif
continue

====================================================================
Calculation of the areas of the humps, BH. Shump is the cumulative area of the humps calculated from the seaward end of the profiles.
====================================================================

2950 If (NumHump.GE.1) then
   Shump(0)=0.
   Do 27 j=1,NumHump
      If ((C.GT.0.).AND.(j.EQ.NumHump)) then
         BH1(j)=((2.*0.4714*BE*AL/3.)*
            ((((XHStart(j)-S8(j))/AL+18)**(3./2.))-
            ((B.EQ.0.).AND.(i.EQ.(NPV-2))
            )))
      Endif
      If (   (YPV(i).LT.YP(N1+i-2)).OR.
            (YPV(i).EQ.YP(N1+i-2)).AND.
            (YPV(i+1).LT.YP(N1+i-1)).AND.(i.NE.(NPV-2))).OR.
            ((i+1).EQ.(NPV-1)).AND.(B.EQ.0.).AND.(i.EQ.(NPV-2))
      ) then
         NumHump=NumHump+1
         XHStart(NumHump)=S2
         YHStart(NumHump)=T2
         goto 21
      Endif
      If (((i+1).EQ.(NPV-1)).AND.(B.EQ.0.)) then
         XHStart(NumHump)=S9
         YHStart(NumHump)=YPT9
      Endif
      Aux3=0
      Call XYFunc(je,i,N1,Aux3,BE,AL,T1,S8,XFunc,YFunc)
      XHStart(NumHump)=XFunc
      YHStart(NumHump)=YFunc
      Do 1367 k=1,NPD
         XP(k)=XPOld1(k)
         YP(k)=YPOld1(k)
1367 continue
   Endif
Endif
continue

======================================
Calculation of BH1 & BH2. BH1 is the area between Vellinga's profile & the surge level at the location of a hump. If it is the last hump & the area C>0, then BH1 is the area as defined previously plus the area C. BH2 is the area between the nourished profile & the surge level at the location of a hump. BH is the difference between BH1 & BH2.
======================================
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1               (18**(3./2.))-
1               (2.*BE*(XHStart (j)-S8(je))))+C
Aux4=0
H0=0
Do 32 i=NPD,1,-1
   If (XP(i).LE.XHStart(j)) then
      H1=i
   Else
      If (XP(i).EQ.XHStart(j)) H0=1
      goto 33
   Endif
32     continue
33     H2=0
Do 34 i=1,NPD
   If (XP(i).GE.S1) then
      H3=i
   Else
      If (XP(i).EQ.S1) H2=1
      goto 35
   Endif
34     continue
35     BH2(j)=0.
   If ((H0.NE.1).AND.(XP(XHStart(j)).GE.S1)) then
      BH2(j)=BH2(j)+0.5*(XHStart(j)-XP(H1))*
1    ((T1-YHStart(j))+(T1-YP(H1)))
   Endif
36     Aux4=1
   If ((H2.NE.1).AND.(XP(XHEnd(j)).LE.XHStart(j))) then
      BH2(j)=BH2(j)+0.5*(XP(H3)-S1)*(T1-YP(H3))
      Aux4=1
   Endif
   BH1(j)=(2.*0.4714*BE*AL/3.)*
1    (((XHStart(j)-S8(je))/AL+18)**(3./2.))-
1    (((XHEnd(j)-S8(je))/AL+18)**(3./2.))+
1    2.*BE*(XHStart(j)-XHStart(j))
Aux4=1
H0=0
Do 28 i=NPD,1,-1
   If (XP(i).LE.XHStart(j)) then
      H1=i
   Else
      If (XP(i).EQ.XHStart(j)) H0=1
      goto 29
   Endif
28     continue
29     H2=0
Do 30 i=1,NPD
   If (XP(i).GE.XHEnd(j)) then
      H3=i
   Else
      If (XP(i).EQ.XHEnd(j)) H2=1
      goto 31
   Endif
30     continue
31     BH2(j)=0.
   If ((H0.NE.1).AND.(XHStart(j).GT.H3))
      BH2(j)=BH2(j)+0.5*(XHStart(j)-XP(H1))*
1    ((T1-YHStart(j))+(T1-YP(H1)))
   Endif
32     Aux4=1
   If (H1.GT.H3) then
      Aux4=1
   Endif
   Do 37 i=H3,(H1-1)
      BH2(j)=BH2(j)+0.5*(XP(i+1)-XP(i))*
1    ((T1-YP(i))+(T1-YP(i+1)))
37     continue
   If (Aux4.EQ.0) BH2(j)=0.5*(XHStart(j)-S1)*(T1-YHStart(j))
   If (Aux4.EQ.1.AND.H1.LT.H3) BH2(j)=0.5*(XHStart(j)-
1    XEnd(j))*(T1-YHStart(j))+(T1-YHEnd(j))
   BH(j)=BH1(j)-BH2(j)
   SHump(j)=SHump(j-1)+BH(j)
27     continue
Endif
C
C    ==============================================================
C    Calculation of the areas of the depressions, BD. SDep is the
C    cumulative area of the depressions calculated from the seaward
C    end of the profiles.
C    ==============================================================

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---

Calculation of BD1 & BD2. BD1 is the area between Vellinga's profile & the surge level at the location of a depression. If it is the first depression & the area B>0, then BD1 is the area as defined previously plus the area B. BD2 is the area between the nourished profile & the surge level at the location of a depression. If it is the first depression & the area B>0, then BD2 is the area as defined previously plus the area E. BD is the difference between BD2 & BD1.

---

If (NumDep.GE.1) then
SDep(0)=0.
Do 39 j=1,NumDep
   If (((YPT9.LT.T9).AND.(j.EQ.1)) then
      BD1(j)=((2.*0.4714*BE*AL/3.)*
         1 (((XDStart(j)-S8(je))/AL+18)**(3./2.))-
         1 (((XDEnd(j)-S8(je))/AL+18)**(3./2.)))-
         1 (2.*BE*(XDStart(j)-XDEnd(j))))+B
      Aux4=0
      D0=0
      Do 44 i=NPD,1,-1
         If (XP(i).LE.XDStart(j)) then
            D1=i
            If (XP(i).EQ.XDStart(j)) D0=1
            goto 45
         Endif
      continue
   44 D2=0
   Do 46 i=1,NPD
      If (XP(i).GE.XDEnd(j)) then
         D3=i
         If (XP(i).EQ.XDEnd(j)) D2=1
         goto 47
      Endif
   continue
   46 BD2(j)=0.
      If (((D0.NE.1).AND.(D1.GE.D3)) then
         BD2(j)=BD2(j)+0.5*(XDStart(j)-XP(D1))*
            1 ((T1-YDStart(j))+(T1-YP(D1)))
      Aux4=1
      Endif
      If (((D2.NE.1).AND.(D1.GE.D3)) then
         BD2(j)=BD2(j)+0.5*(XP(D3)-XDEnd(j)) *
            1 ((T1-YP(D3))+(T1-YDEnd(j)))
      Aux4=1
      Endif
      BD2(j)=BD2(j)+E
   else
      BD1(j)=((2.*0.4714*BE*AL/3.)*
         1 (((XDStart(j)-S8(je))/AL+18)**(3./2.))-
         1 (((XDEnd(j)-S8(je))/AL+18)**(3./2.))-
         1 (2.*BE*(XDStart(j)-XDEnd(j))))
      Aux4=1
      D0=0
      Do 40 i=NPD,1,-1
         If (XP(i).LE.XDStart(j)) then
            D1=i
            If (XP(i).EQ.XDStart(j)) D0=1
            goto 41
         Endif
      continue
   40 D2=0
   Do 42 i=1,NPD
      If (XP(i).GE.XDEnd(j)) then
         D3=i
         If (XP(i).EQ.XDEnd(j)) D2=1
         goto 43
      Endif
   continue
   42 BD2(j)=0.
      If ((D0.NE.1) .AND. (D1.GE.D3)) then
         BD2(j)=BD2(j)+0.5*(XDStart(j)-XP(D1)) *
            1 ((T1-YDStart(j))+(T1-YP(D1)))
      Endif
      If ((D2.NE.1) .AND. (D1.GE.D3)) then
         BD2(j)=BD2(j)+0.5*(XP(D3)-XDEnd(j)) *
            1 ((T1-YP(D3))+(T1-YDEnd(j)))
      Endif
      If (D1.GT.D3) then
         Aux4=1
      Do 48 i=D3,(D1-1)
         BD2(j)=BD2(j)+0.5*(XP(i+1)-XP(i)) *
            1 ((T1-YP(i))+(T1-YP(i+1)))
      continue
   48 Endif
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If (Aux4.EQ.0) BD2(j)=BD2(j)+0.5*(XDStart(j)-XDEnd(j))*(T1-YDStart(j)+(T1-YDEnd(j)))

BD(j)=BD2(j)-BD1(j)
SDep(j)=SDep(j-1)+BD(j)
continue
Endif

Since sand movements are only allowed seaward during the storm surge, the first hump & its area have to be neglected for the case B=0 & the number of humps, the areas BH & SHump have to be adjusted.

If (B.EQ.0.) then
NumHump=NumHump-1
If (NumHump.GE.1) then
Do 50 j=1,NumHump
SHump(j)=SHump(j+1)-BH(1)
continue
Do 51 j=1,NumHump
BH(j)=BH(j+1)
50         continue
51         continue
Endif
Endif
return
End

Subroutine Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)

If |Err|>=1 then the balance between erosion & accretion is not satisfactory (assuming movements of sand only seaward during the storm surge).
If Err>1, then accretion exceeds erosion (Vellinga's profile has to be moved landward); if Err<-1, then erosion exceeds accretion (Vellinga's profile has to be moved seaward, if S8 not equal to S1; if S8=S1, no erosion is expected). If Vellinga's profile is moved, the new X-coordinate of the starting point of the parabolic part of the profile, S8, is returned.

INPUT VARIABLES:
T2 - Y-coordinate of the intersection point between the nourished profile & the gradient 1:mt of Vellinga’s profile
T3 - Y-coordinate of the intersection point between the nourished profile & the gradient 1:md of Vellinga’s profile
Err - Error in the balance between erosion & accretion (assuming movements of sand only seaward during the storm surge)

INPUT/OUTPUT VARIABLES:
je - Number of the iteration to find the final position of Vellinga's profile
S8 - X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile
Mov - Value by which S8 is changed in each iteration performed to find the final position of Vellinga's parabolic profile
Q1, Sin, Aux1 - Auxiliary variables

MODELING VARIABLES:
ka - Maximum number of iterations allowed by the program to find the final position of Vellinga's profile & the final location of the surcharge face
DS - Auxiliary variables

OUTPUT VARIABLE:
Fim - Auxiliary variable

Integer*4 je,ka,Aux1,Q1,Fim
Parameter (ka=999)
Real*8 Err(0:ka),Sin(0:ka),T2,T3,DS,S8(0:Ka),Mov
If (Err(je).LT.0.) then
Sin(je)=1.
else
Sin(je)=-1.
Endif
If (ABS(Err(je)).GE.1.) then
DS=Err(je)/(T3-T2)
je=je+1
If (je.GT.999) then
Write(*,9987)
Write(50,9987)
9987 Format(/ 6X,
1 'ERROR: The maximum number of iterations allowed by' /
1 6X,'the program to find the final position of' /
1 6X,'Vellinga`s profile has been exceeded !')
STOP
Endif
S8(je)=S8(je-1)-DS
If ((ABS(S8(je)-S8(je-2)).LT.0.001).OR.(Aux1.EQ.1)) then
If (Sin(je-1).NE.Sin(je-2)) then
Mov=Mov/10.
Endif
S8(je)=S8(je-1)+Sin(je-1)*Mov
Aux1=1
Endif
Q1=1
Fim=1
else
If (Q1.EQ.0) then
Fim=3
else
Fim=2
Endif
Endif
Endif
return
End

******************************************************************************
Subroutine Error(Q1,BH,BD,NumDep,NumHump,SDep,SHump,Fim,je,Err,
T2,T3,S8,Mov,Sin,Aux1)
******************************************************************************

Returns Err. Err is the error in the balance between erosion &
accretion (assuming movements of sand only seaward during the
storm surge). Then, subroutine Balance is called to move
Vellinga's profile depending on the value of Err.

******************************************************************************

INPUT VARIABLES:
T2 - Y-coordinate of the intersection point  between the
nourished profile & the gradient 1:mt of Vellinga's profile
T3 - Y-coordinate of the intersection point  between the
nourished profile & the gradient 1:md of Vellinga's profile

INPUT/OUTPUT VARIABLES:
je - Number of the iteration to find the final position of
Vellinga's profile
S8 - X-coordinate of the starting point of the parabolic part of
Vellinga's post-storm profile
Mov - Value by which S8 is changed in each iteration performed
to find the final position of Vellinga's parabolic profile
NumHump - Number of humps
NumDep - Number of depressions
BH - Area of a hump
BD - Area of a depression
SDep - Cumulative area of the depressions starting from the
seaward end of the profiles
SHump - Cumulative area of the humps starting from the seaward
end of the profiles
Q1, Sin, Aux1 - Auxiliary variables

MODELING VARIABLES:
ka - Maximum number of iterations allowed by the program to find
the final position of Vellinga's profile & the final
location of the surcharge face
Er - Maximum number of points (XP,YP) allowed by the program
SHumpOld - Cumulative area of the humps starting from the
seaward end of the profiles
SDepOld - Cumulative area of the depressions starting from the
seaward end of the profiles
i, j, k - Auxiliary variables
OUTPUT VARIABLES:
Err - Error in the balance between erosion & accretion (assuming
movements of sand only seaward during the storm surge)
Fim - Auxiliary variable

Integer*4 i,j,je,k,ka,Er,Q1,NumDep,NumHump,Fim,Aux1
Parameter (Er=100)
Parameter (ka=999)
Real*8 SHump(0:Er),SDep(0:Er),SHumpOld(Er),SDepOld(Er),BH(Er),
1 BD(Er),Err(0:ka),T2,T3,S8(0:ka),Mov,Sin(0:ka)
Fim=0
i=1
15 If (ABS(BH(i)).GE.ABS(BD(i))) then
If (i.NE.NumDep) then
NumHump=NumHump-1
NumDep=NumDep-1
Do 2399 j=1,NumDep
SDep(j)=SDep(j+1)-BD(1)
2399 continue
Do 2299 j=1,NumDep
BD(j)=BD(j+1)
2299 continue
If (NumHump.GE.1) then
Do 9522 j=1,NumHump
SHump(j)=SHump(j+1)-BH(1)
9522 continue
Do 9922 j=1,NumHump
BH(j)=BH(j+1)
9922 continue
goto 15
else
Err(je)=BD(NumDep)
Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
If (Fim.NE.0) return
Endif
else
Fim=0
If (Q1.EQ.0) then
Fim=3
Endif
If (Q1.GE.1) then
Err(je)=BD(i)-BH(i)
Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
Endif
If (Fim.NE.0) return
Endif
Endif
If (ABS(BH(i)).LT.ABS(BD(i))) then
If (i.EQ.NumDep) then
Fim=0
If (Q1.EQ.0) then
Fim=3
Endif
If (Q1.GE.1) then
Err(je)=BD(i)-BH(i)
Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
Endif
If (Fim.NE.0) return
else
Do 108 j=(i+1),NumDep
If (j.NE.NumDep) then
If (ABS(SHump(j)).GE.ABS(SDep(j))) then
SDepOld(j)=SDep(j)
SHumpOld(j)=SHump(j)
NumHump=NumHump-j
NumDep=NumDep-j
Do 2033 k=1,NumDep
BD(k)=BD(j+k)
SDep(k)=SDep(j+k)-SDepOld(j)
2033 continue
If (NumHump.GE.1) then
Do 3320 k=1,NumHump
BH(k)=BH(j+k)
SHump(k)=SHump(j+k)-SHumpOld(j)
3320 continue
goto 15
else
Err(je)=BD(NumDep)
end
Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
  If (Fim.NE.0) return
Endif
else
  If (NumHump.GT.j) then
    goto 108
  else
    Err(je)=SDep(NumDep)-SHump(NumHump)
    Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
    If (Fim.NE.0) return
Endif
Endif
else
  If (NumHump.EQ.NumDep) then
    Err(je)=SDep(j)-SHump(j)
    Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
    If (Fim.NE.0) return
  else
    Err(je)=SDep(j)-SHump(j-1)
    Call Balance(je,Err,T2,T3,S8,Q1,Fim,Mov,Sin,Aux1)
    If (Fim.NE.0) return
  endif
endif
108        continue
Endif
Endif
return
End

################################################################
C
C Subroutine SurchC(ctcurv,BE,W,S8,T1,S3,S4,T4,S10,SurD,SurchEros,
1                    SurchLongT,TSurch)
C
C       Returns the total surcharge on erosion area C, TSurch, & the
C       corresponding surcharge distance, SurD, & the intersection
C       point, (S4,T4), between the nourished profile & the surcharge
C       gradient, 1:md (through subroutine Surcharge). It also returns
C       the surcharge, SurchEros, on erosion area C above surge level to
C       take into account the effects of the accuracy of the
C       computation, Ac, of the storm surge duration, SD, & of the gust
C       bumps, GB, respectively. It also returns the surcharge,
C       SurchLongT, on erosion area C above surge level to take into
C       account the effect of a gradient in the longshore transport.
C
C       INPUT VARIABLES:
C       MuxN - Mean of the equivalent Normal distribution of X
C       Ac - Accuracy of the computation
C       GB - Gust bumps
C       SD - Storm surge duration
C       W - Fall velocity of dune sand in seawater (m/s)
C       ctcurv - Coastal curvature in degrees per 1000m
C             (0<ctcurv<=24Deg/1000m)
C       T1 - Y-coordinate of the intersection point between the
C            nourished profile & the surge level
C       S8 - X-coordinate of the starting point of the parabolic part of
C           Vellinga's post-storm profile
C       (S3,T3) - Intersection point between the nourished profile & the
C             gradient 1:md of Vellinga's profile
C       C - Area of erosion between the surge level, the nourished
C           profile above surge & the gradient 1:md
C       BE - Auxiliary variable
C
C       MODELING VARIABLES:
C       ka - Maximum number of iterations allowed by the program to find
C            the final position of Vellinga’s profile & the final
C            location of the surcharge face
C       j - Number of the iteration to find the final location of the
C            surcharge face
C       G0 - Coefficient used in the calculation of SurchLongT
C       CTotal - Sum of the erosion area C plus the surcharge SurchEros
C       D - Area between the two 1:md gradients, the surge level & the
C           nourished profile
C       Errl - Error in the balance between the required surcharge,
C           TSurch, & D
C       Jump - Value by which SurD is changed in each iteration
C       performed to find the final location of the surcharge

C
OUTPUT VARIABLES:

(S4,T4) - Intersection point between the nourished profile & the surcharge gradient, 1:md
S10 - X-coordinate of the point of intersection between the surge level & the gradient 1:md of the surcharge
SurchEros - Surcharge on erosion area C above surge level to take into account the effects of the accuracy of the computation, of the storm surge duration, & of the gust bumps
SurchLongT - Surcharge on erosion area C above surge level to take into account the effect of a gradient in the longshore transport
TSurch - Total surcharge on erosion area C which is the sum of the surcharges SurchEros plus SurchLongT
SurD - Surcharge distance

*****************************************************************************

Integer*4 ka,j,Q2,Aux2
Parameter (ka=999)
Real*8 W,BE,T1,S8(0:Ka),C,S3,T3,SurchEros,MuxN,SurD(0:ka),SD,Ac,
GB,S4,T4,D,S10(0:ka),Err1(0:ka),Jump,Sinal(0:ka),ctcurv,
CTotal,SurchLongT,TSurch,G0
Common/BLOCK3/MuxN(15),C,T3
Common/BLOCK5/Ac,GB,SD

*****************************************************************************

Calculation of the surcharge, SurchEros, on erosion area C above surge level to take into account the effects of the accuracy of the computation, Ac, of the storm surge duration, SD, & of the gust bumps, GB, respectively.

SurchEros=(20.+0.1*C)*Ac+0.1*C*SD+0.05*C*GB/MuxN(5)

*****************************************************************************

Calculation of the surcharge, SurchLongT, on erosion area C above surge level to take into account the effect of a gradient in the longshore transport for not too strongly curved coastal sections (the coastal curvature, 0<=ctcurv<=24Deg/1000m).

If ((ctcurv.LE.24.).AND.(ctcurv.GT.6.)) then
  If ((ctcurv.LE.24.).AND.(ctcurv.GT.18.)) G0=100.
  If ((ctcurv.LE.18.).AND.(ctcurv.GT.12.)) G0=75.
  If ((ctcurv.LE.12.).AND.(ctcurv.GT.6.)) G0=50.
CTotal=C+SurchEros
SurchLongT=CTotal*(BE**0.72)*((W/0.0268)**0.56)*G0/300.
else
  SurchLongT=0.
Endif

*****************************************************************************

Calculation of the total surcharge on erosion area C, TSurch, which is the sum of the surcharges SurchEros plus SurchLongT.

TSurch=SurchEros+SurchLongT

*****************************************************************************

Calculation of the surcharge distance, SurD.

Jump=10.
Sinal(0)=0.
Aux2=0.
Q2=1
j=1
S10(0)=S8(1)
SurD(0)=0.
Err1(0)=0.
If (TSurch.LT.0.) Q2=-1

--------

Calculation of a first approximation for SurD. Calculation of the point, X=S10, of intersection between the surge level & the gradient 1:md of the surcharge.

2319
S10(0)=S8(1)‐SurD(0)
If ((S10(0).GT.S8(1)).AND.(TSurch.GT.0.)) then
  S10(0)=(S10(0+1)+S10(0+2))/2.
  SurD(0)=S8(1)‐S10(0)
  Aux2=1
Program Listing

Calculates the surcharge erosion, D, corresponding to SurD.

---

Call Surcharge(j,S8,S10,T1,D,S3,T3,S4,T4,Tsurch)

---

Error1(j) = D - ABS(Tsurch)

If (Error1(j) < 0.) then
    Signal(j) = -1.
else
    Signal(j) = 1.
Endif

If (ABS(Error1(j)).GE.1.) then
    j = j + 1
Endif

---

Subroutine LSeaProf(S9,T9,S1,M1,T1,B,Q,S2,T2)

---

Returns the intersection point, (S2,T2), between the nourished profile & the gradient 1:mt of Vellinga's profile. It also returns the area, B, between the surge level & the gradient 1:mt; & the area, Q, between the surge level & the nourished profile below surge.

---

INPUT VARIABLES:

NPD - Number of points defining the nourished profile
(XP,YP) - Coordinates of the points defining the nourished profile
1:mt - Gradient of the toe of the post-storm profile
(S1,T1) - Intersection point between the nourished profile & the surge level
M1 - Number of the point, (XP(M1),YP(M1)), from seaward for which YP(M1) <= T1
(S9,T9) - Point where the parabolic part of Vellinga's profile finishes

MODELING VARIABLES:

YPT9 - Y-coordinate of the point in the nourished profile which
i, M2, Zes, G - Auxiliary variables
md, nourtlev, mnour, ctcurv - Variables mentioned in the Common statements but not used here

OUTPUT VARIABLES:
(S2, T2) - Intersection point between the nourished profile & the gradient 1:mt of Vellinga's profile
B - Area between the surge level & the gradient 1:mt of Vellinga's profile
Q - Area between the surge level & the nourished profile below surge

***************************************************************************************

Integer*4 NPD,i,M1,M2
Real*8 XP,YP,S1,T1,S2,T2,S9,T9,B,Q,Zes,G,md,mt,mnour,
1 nourtlev,YPT9,ctcurv
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK4/md,mt,mnour,nourtlev,ctcurv

C C C Calculation of the intersection point, (S2, T2), between the nourished profile & the gradient 1:mt of Vellinga's profile.
C C C
Do 4499 i=1,NPD
If (XP(i).GT.S9) then
M2=i
YPT9=YP(M2-1)+(YP(M2)-YP(M2-1))*(S9-XP(M2-1))/
1 (XP(M2)-XP(M2-1))
If (YPT9.GE.T9) then
S2=S9
T2=YPT9
B=0.
M2=M2-1
goto 3902
else
goto 4398
Endif
Endif
4499 continue
Write(*,8275)
Write(50,8275)
8275 Format(// 11X,'Extend the Initial Profile Seaward !')
STOP

4398 Zes=XP(M2)-S9
G=T9-Zes/mt
If (G.GT.YP(M2)) then
M2=M2+1
If (M2.GT.NPD) then
Write(*,8225)
Write(50,8225)
8225 Format(// 11X,'Extend the Initial Profile Seaward !')
STOP
Endif
goto 4398
Endif
S2=XP(M2)-Zes*(YP(M2)-G)/((T9-G-(Zes*(YP(M2-1)-YP(M2)))/
1 (XP(M2)-XP(M2-1))))
T2=YP(M2)+(XP(M2)-S2)*(YP(M2-1)-YP(M2))/(XP(M2)-XP(M2-1))
M2=M2-1

C C C Calculation of the area, B, between the surge level & the gradient 1:mt.
C C C
B=0.5*(S2-S9)*((T1-T9)+(T1-T2))

C C C Calculation of area, Q, between the surge level & the nourished profile below surge.
C C C
3902 If ((XP(M2).GT.T1).AND.(XP(M1).LT.S2)) then
Q=0.5* (XP(M1)-S1)* (T1-YP(M1)) + 0.5* (S2-XP(M2)) *
1 ((T1-T2)+(T1-YP(M2))
else
Q=0.5* (S2-S1) * (T1-T2)
Endif
If ((M2-1).GE.M1) then
Do 3381 i=M1,(M2-1)
Q=Q+0.5* (XP(i+1)-XP(i)) * ((T1-YP(i))+ (T1-YP(i+1)))
3381 continue
Endif

C7-129
Subroutine XYFunc(i,N1,Aux3,XPV,YPV,XFunc,YFunc)
Subroutine XYFunc(je,i,N1,Aux3,BE,AL,T1,S8,XFunc,YFunc)

Returns the intersection point (XFunc,YFunc) between the part of Vellinga's profile defined by the points:
(XPV(i-Aux3),YPV(i-Aux3))
(XPV(i-Aux3+1),YPV(i-Aux3+1))
& the part of the nourished profile defined by the points:
(XP(N1+i-Aux3-1),YP(N1+i-Aux3-1))
(XP(N1+i-Aux3-2),YP(N1+i-Aux3-2))

Returns the intersection point (XFunc,YFunc) between the parabolic part of Vellinga's profile & the part of the nourished profile defined by the points:
(XP(N1+i-Aux3-1),YP(N1+i-Aux3-1))
(XP(N1+i-Aux3-2),YP(N1+i-Aux3-2))

INPUT VARIABLES:
(XP,YP) - Coordinates of the points defining the nourished profile
(XPV,YPV) - Coordinates of the points defining Vellinga's profile
T1 - Y-coordinate of the intersection point between the nourished profile & the surge level
S8 - X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile
i, je, N1, Aux2, Aux3, BE, AL - Auxiliary variables

MODELING VARIABLES:
L - Maximum number of points (XP,YP) & (XPV,YPV) allowed by the program
ka - Maximum number of iterations allowed by the program to find the final position of Vellinga's profile & the final location of the surge face
A, B, C, Aux1, XFunc1, YFunc1, XFunc2, YFunc2 - Auxiliary variables
NPD - Variable mentioned in the Common statement but not used here

OUTPUT VARIABLES:
(XFUNC,YFUNC) - Coordinates of an intersection point between the nourished profile & Vellinga's profile

Integer*4 i,N1,Aux3,NPD,L,ka,je
Parameter (L=100)
Parameter (ka=999)
Real*8 XP,YP,XFunc,YFunc,XFunc1,YFunc1,XFunc2,YFunc2,A,B,C,AL,
BE,S8(0:Ka),T1,Aux1,Aux2
Common/BLOCK1/NPD,XP(100),YP(100)

A=-(YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))**2)/(0.4714*BE*(XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2)))**2
If (A.EQ.0.) then
YFunc=YP(N1+i-Aux3-1)
XFunc=S8(je)+AL*((T1-YFunc+2.*BE)/(0.4714*BE))**2)-18.*AL
return
Endif
B=([AL]**(-1))+(2.*T1*(YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2)))*YP(N1+i-Aux3-2))/
(4.*YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))**2)
C=-(4./(0.4714**2))-((2.*T1*(YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2)))*YP(N1+i-Aux3-2))/
((0.4714**2))*(XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2)))+
(2.*XP(N1+i-Aux3-2)*((YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))**2))/((0.4714**2)*((YP(N1+i-Aux3-1)-
XP(N1+i-Aux3-2))**2))

C7-130
Program Listing

```
1  ((0.4714*BE)**2)-
1  S8(je)*(AL**(-1))+18.-
1  (4.*XP(N1+i-Aux3-2)*(YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2)))/(BE*(0.4714**2)*(XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2)))-
1  ((XP(N1+i-Aux3-2)**2)*((YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))))-
1  2.*XP(N1+i-Aux3-2)*YP(N1+i-Aux3-1)-
1  (XP(N1+i-Aux3-2))**2))/(XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2))-1

If ((B*B-4.*A*C).LT.0.) then
Write(*,9997)
Write(50,9997)
9997       Format(//'ERROR: (B^2-4AC) negative in the quadratic'
1              /'equation AX^2+BX+C=0 ! Roots of the'
1              //equation are complex conjugate! ')
STOP
Endif

Aux2=1.
Aux1=-0.5*(B+DSIGN(Aux2,B)*SQRT(B*B-4.*A*C))
XFunc1=Aux1/A
XFunc2=C/Aux1

If (XFunc1.GT.XP(N1+i-Aux3-2).AND.XFunc1.LT.XP(N1+i-Aux3-1)) then
YFunc1=YP(N1+i-Aux3-2)+(XFunc1-XP(N1+i-Aux3-1))
1             (YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))/
1             (XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2))

If ((YFunc1.GT.YP(N1+i-Aux3-1).AND.YFunc1.LT.YP(N1+i-Aux3-2)).AND.YFunc1.LE.T1).OR.
1             (YFunc1.LT.YP(N1+i-Aux3-1).AND.YFunc1.GT.YP(N1+i-Aux3-2).AND.YFunc1.LE.T1)) then
XFunc=XFunc1
YFunc=YFunc1
Endif
Endif

If (XFunc2.GT.XP(N1+i-Aux3-2).AND.XFunc2.LT.XP(N1+i-Aux3-1).AND.XFunc2.GT.S8(je)) then
YFunc2=YP(N1+i-Aux3-2)+(XFunc2-XP(N1+i-Aux3-2))
1             (YP(N1+i-Aux3-1)-YP(N1+i-Aux3-2))/
1             (XP(N1+i-Aux3-1)-XP(N1+i-Aux3-2))

If ((YFunc2.GT.YP(N1+i-Aux3-1).AND.YFunc2.LT.YP(N1+i-Aux3-2)).AND.YFunc2.LE.T1).OR.
1             (YFunc2.LT.YP(N1+i-Aux3-1).AND.YFunc2.GT.YP(N1+i-Aux3-2).AND.YFunc2.LE.T1)) then
XFunc=XFunc2
YFunc=YFunc2
Endif
Endif
return
End
```

---

```
Subroutine Surcharge(j,S8,S10,T1,D,S3,T3,S4,T4,TSurch)

Returns the intersection point, (S4,T4), between the nourished profile & the surcharge gradient, 1:md. It also returns the area D between the two 1:md gradients, the surge level & the nourished profile.

INPUT VARIABLES:

j - Number of the iteration to find the final location of the surcharge face
NPD - Number of points defining the nourished profile
(XP,YP) - Coordinates of the points defining the nourished profile
T1 - Y-coordinate of the intersection point between the nourished profile & the surge level
1:md - Gradient of the eroded dune face
S8 - X-coordinate of the starting point of the parabolic part of Vellinga's post-storm profile
(S3,T3) - Intersection point between the nourished profile & the gradient 1:md of Vellinga's profile
S10 - X-coordinate of the point of intersection between the surge level & the gradient 1:md of the surcharge
TSurch - Total surcharge on erosion area C which is the sum of

---

C7-131
the surcharges SurchEros plus SurchLongT

MODELING VARIABLES:
  ka - Maximum number of iterations allowed by the program to find
       the final position of Vellinga's profile & the final
       location of the surcharge face
  i, M3, M31, U, V - Auxiliary variables
  mt, nourtlev, mnour, ctcerv - Variables mentioned in the Common
       statements but not used here

OUTPUT VARIABLES:
  (S4,T4) - Intersection point between the nourished profile & the
            surcharge gradient, 1:md
  D - Area between the two 1:md gradients, the surge level & the
      nourished profile

Integer*4 NPD,i,j,M3,M31,ka
Parameter (ka=999)
Real*8 D,XP,YP,T1,S3,T3,S8(0:ka),S4,T4,S10(0:ka),U,V,md,mt,
1      mnour,nourtlev,TSurch,ctcurv
Common/BLOCK1/NPD,XP(100),YP(100)
Common/BLOCK4/md,mt,mnour,nourtlev,ctcurv

Calculation of the intersection point, (S4,T4), between the
nourished profile & the surcharge gradient, 1:md.

Calculation of area D between the two 1:md faces, the surge
level & the nourished profile.
8695     Format(// 11X,'Extend the Initial Profile Seaward !')
STOP

7794     If ((XP(M31).GT.S4).AND.(XP(M3).LT.S3)) then
1     D=0.5*(S8(1)-S3)*(T3-T1)+0.5*(XP(M3)-S4)*
1     ((T4-T1)+(YP(M3)-T1))+
else
1     D=0.5*(S8(1)-S3)*(T3-T1)+0.5*(S3-S4)*((T3-T1)+(T4-T1))
Endif
If (M31.GT.M3) then
Do 4701 i=M3,(M31-1)
1     D=D+0.5*(XP(i+1)-XP(i))*((YP(i+1)-T1)+(YP(i)-T1))
4701   continue
Endif
D=D-0.5*(S10(j)-S4)*(T4-T1)
Endif

C       ---------------------
C       TSurch<0
C       ---------------------
If (TSurch.LT.0.) then
Do 5137 i=1,NPD
1     If (XP(i).GT.S3) then
1     M31=i
1     goto 7549
Endif
5137   continue
Write(*,8629)
Write(50,8629)
8629     Format(// 11X,'Extend the Initial Profile Seaward !')
STOP

7549     If ((XP(M31).LT.S4).AND.(XP(M3-1).GT.S3)) then
1     D=0.5*(S10(j)-S4)*(T4-T1)+0.5*(S4-XP(M3-1))*
1     ((T4-T1)+(YP(M3-1)-T1))+0.5*(XP(M31)-S3)*
else
1     D=0.5*(S10(j)-S4)*(T4-T1)+0.5*(S4-S3)*((T4-T1)+(T3-T1))
Endif
If (M3.GT.M31) then
Do 7410 i=M31,(M3-1)
1     D=D+0.5*(XP(i+1)-XP(i))*((YP(i+1)-T1)+(YP(i)-T1))
7410   continue
Endif
D=D-0.5*(S8(1)-S3)*(T3-T1)
Endif
2591   return
End
APPENDIX D

Input Data And Results From PARASODE
APPENDIX D1 - Examples Of Input And Output Files For Wave Overtopping

Input File *general.dad*: H&R Model

1 ! Failure mode of overtopping (H&R model)
2 ! Tide + Surge
1 ! \( P_{\text{max}} \) 37%
Y ! Derivatives of the failure function supplied
1 ! Mode 1 - Reliability analysis for a specified design
1 ! Design life (in years)
N ! No combinations of actions considered
10 ! Target design parameter - seawall crest level

Input File *form.dad*: H&R Model

2 ! Starting point: user specified values
4.5
2
4
1
0.5
0.95
1.049
2 ! \([X_{\text{Min}}, X_{\text{Max}}]\): user specified values
1 20
0 20
-5.33 5.57
-5 5
0.25 1
0.5 1
-1E25 1E25
200 ! Maximum number of iterations
6 ! Number of FORM calculations
1E-1
1E-2
1E-3
1E-4
1E-5
1E-6
1 ! Accuracy on Beta (%): default value (1%)
2 ! Smoothing of the iteration: user specified value
0.9
1 ! Accuracy on Z0 (%): default value (1%)
**Input File meandev.dad: H&R Model**

```
10                              ! Type of distribution (Weibull)
0                               ! Not truncated
6.4  1.152  4.224               ! Mean; stand. dev.; lower limit
10                              ! Type of distribution (Weibull)
1                               ! Truncated for X above Xo
0                               ! Toe level
1.2  0.7  0.45                  ! Mean; stand. dev.; lower limit
0.00753                         ! Value
0                               ! Type of distribution (Deterministic)
4.17                            ! Value
13                              ! Type of distribution (User-Defined)
0.2749  2.3619                 ! Mean value; standard deviation
3                               ! Type of distribution (Gumbel)
0                               ! Not truncated
0.019483  0.192382475          ! Mean value; standard deviation
1                               ! Type of distribution (Normal)
0                               ! Not truncated
0.5  0.05                       ! Mean value; standard deviation
6                               ! Type of distribution (Beta)
0                               ! Not truncated
0.95  0.01  0.9  1              ! Mean; stand. dev.; lower & upper limits
2                               ! Type of distribution (Log-Normal)
0                               ! Not truncated
1.049  0.241                    ! Mean value; standard deviation
```

**Input File coefcor.dad: H&R Model**

```
1                              ! Rho(1,1)
0.6                            ! Rho(1,2)
0                               ! Rho(1,3)
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0.6                            ! Rho(1,1)
1                               ! Rho(1,2)
0                               ! Rho(1,3)
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
1                               ! Rho(1,1)
0                               ! Rho(1,2)
0                               ! Rho(1,3)
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
1                               ! Rho(1,1)
0                               ! Rho(1,2)
0                               ! Rho(1,3)
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
0                               ! .
```

D1-2
Examples Of Input And Output Files For Wave Overtopping

0
0
0
0
0
0
1
0
0
0
0
0
0
0
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
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Output File *summary.dat*: H&R Model

**WHAT IS THE DATA SOURCE?**

- The Screen ..... [ 1 ]
- A Datafile ..... [ 2 ]

Select Option: 2

**WHAT IS THE FAILURE MODE TO BE STUDIED?**

- Overtopping (H&R) ................. [ 1 ]
- Overtopping (Owen) ................. [ 2 ]
- Dune Erosion (Vellinga) ............ [ 3 ]

Select Option: 1

**HOW IS THE STILL-WATER-LEVEL DEFINED?**

- Total Level .... [ 1 ]
- Tide + Surge ... [ 2 ]

Select Option: 2

**WHAT IS THE CONFIDENCE VALUE OF THE MAXIMUM RUN-UP THAT YOU WOULD LIKE TO CONSIDER?**

- 37 % ... [ 1 ]
- 99 % ... [ 2 ]

Select Option: 1

**ARE THE FIRST DERIVATIVES OF THE FAILURE FUNCTION SUPPLIED (Y/N)?** Y

**DESCRIPTION OF THE VARIABLES**

- X( 1) = Tp = Peak Wave Period
- X( 2) = Hs = Wave Height
- X( 3) = A = H&R Parameter
- X( 4) = B = H&R Parameter
- X( 5) = Tid = Tide Level
- X( 6) = Sur = Surge
- X( 7) = Tal = Seawall Slope
- X( 8) = r = Roughness
- X( 9) = eB = Model Parameter
Examples Of Input And Output Files For Wave Overtopping

WHAT IS THE PURPOSE OF THE ANALYSIS?

Reliability Analysis for a Specified Design ... \[ 1 \]
Design for a Specified Reliability Level ...... \[ 2 \]

Select Option: 1

DESIGN LIFE OF THE STRUCTURE =   1

WOULD YOU LIKE TO CONSIDER COMBINATION OF ACTIONS (Y/N) ? N

PRESENTED VALUE OF THE DESIGN PARAMETER

Seawall Crest Level = 0.1000000000E+02

CHARACTERISTICS OF THE VARIABLES

Probability Distribution of Tp = Minima Type III (Weibull)
Mean Value of Tp = 0.6400000000E+01
Standard Deviation of Tp = 0.1152000000E+01
Lower Limit on Tp = 0.4224000000E+01

Probability Distribution of Hs = Minima Type III (Weibull)
The Distribution of Hs is truncated above Xo = 0.6(Tide+Surge-TL)
Seawall Toe Level (TL) = 0.0000000000E+00
Mean Value of Hs = 0.1200000000E+01
Standard Deviation of Hs = 0.7000000000E+00
Lower Limit on Hs = 0.4500000000E+00

Probability Distribution of A = Deterministic
Mean Value of A = 0.7530000000E-02
Standard Deviation of A = 0.0000000000E+00

Probability Distribution of B = Deterministic
Mean Value of B = 0.4170000000E+01
Standard Deviation of B = 0.0000000000E+00

Probability Distribution of Tid = User-Defined Distribution
Mean Value of Tid = 0.2749000000E+00
Standard Deviation of Tid = 0.2361900000E+01

Probability Distribution of Sur = Maxima Type I (Gumbel)
Mean Value of Sur = 0.1948300000E-01
Standard Deviation of Sur = 0.1923824750E+00

Probability Distribution of TAl = Normal (Gaussian)
Mean Value of TAl = 0.5000000000E+00
Standard Deviation of TAl = 0.5000000000E-01
Probability Distribution of \( r \) = Beta
Mean Value of \( r \) = 0.9500000000E+00
Standard Deviation of \( r \) = 0.1000000000E-01
Limits \( a \) and \( b \) of \( r \) = [0.9000000000E+00, 0.1000000000E+01]

Probability Distribution of \( e_B \) = Log-Normal
Mean Value of \( e_B \) = 0.1049000000E+01
Standard Deviation of \( e_B \) = 0.2410000000E+00

CORRELATION COEFFICIENTS

\[
\begin{align*}
(T_p, T_p) &= 0.1000000000E+01 \\
(T_p, H_s) &= 0.6000000000E+00 \\
(T_p, A) &= 0.0000000000E+00 \\
(T_p, B) &= 0.0000000000E+00 \\
(T_p, Tid) &= 0.0000000000E+00 \\
(T_p, Sur) &= 0.0000000000E+00 \\
(T_p, TAl) &= 0.0000000000E+00 \\
(T_p, r) &= 0.0000000000E+00 \\
(T_p, eB) &= 0.0000000000E+00 \\
(H_s, Tp) &= 0.6000000000E+00 \\
(H_s, Hs) &= 0.1000000000E+01 \\
(H_s, A) &= 0.0000000000E+00 \\
(H_s, B) &= 0.0000000000E+00 \\
(H_s, Tid) &= 0.0000000000E+00 \\
(H_s, Sur) &= 0.0000000000E+00 \\
(H_s, TAl) &= 0.0000000000E+00 \\
(H_s, r) &= 0.0000000000E+00 \\
(H_s, eB) &= 0.0000000000E+00 \\
(A, Tp) &= 0.0000000000E+00 \\
(A, Hs) &= 0.0000000000E+00 \\
(A, A) &= 0.1000000000E+01 \\
(A, B) &= 0.0000000000E+00 \\
(A, Tid) &= 0.0000000000E+00 \\
(A, Sur) &= 0.0000000000E+00 \\
(A, TAl) &= 0.0000000000E+00 \\
(A, r) &= 0.0000000000E+00 \\
(A, eB) &= 0.0000000000E+00 \\
(B, Tp) &= 0.0000000000E+00 \\
(B, Hs) &= 0.0000000000E+00 \\
(B, A) &= 0.0000000000E+00 \\
(B, B) &= 0.1000000000E+01 \\
(B, Tid) &= 0.0000000000E+00 \\
(B, Sur) &= 0.0000000000E+00 \\
(B, TAl) &= 0.0000000000E+00 \\
(B, r) &= 0.0000000000E+00 \\
(B, eB) &= 0.0000000000E+00 \\
(Tid, Tp) &= 0.0000000000E+00 \\
(Tid, Hs) &= 0.0000000000E+00 \\
(Tid, A) &= 0.0000000000E+00 \\
(Tid, B) &= 0.0000000000E+00 \\
(Tid, Tid) &= 0.1000000000E+01 \\
(Tid, Sur) &= 0.0000000000E+00 \\
(Tid, TAl) &= 0.0000000000E+00
\end{align*}
\]
Examples Of Input And Output Files For Wave Overtopping

(Tid, r) = 0.0000000000E+00  
(Tid, eB) = 0.0000000000E+00  
(Sur, Tp) = 0.0000000000E+00  
(Sur, Hs) = 0.0000000000E+00  
(Sur, A) = 0.0000000000E+00  
(Sur, B) = 0.0000000000E+00  
(Sur, Tid) = 0.0000000000E+00  
(Sur, Sur) = 0.1000000000E+01  
(Sur, TAl) = 0.0000000000E+00  
(Sur, r) = 0.0000000000E+00  
(Sur, eB) = 0.0000000000E+00  
(TAl, Tp) = 0.0000000000E+00  
(TAl, Hs) = 0.0000000000E+00  
(TAl, A) = 0.0000000000E+00  
(TAl, B) = 0.0000000000E+00  
(TAl, Tid) = 0.0000000000E+00  
(TAl, Sur) = 0.0000000000E+00  
(TAl, TAl) = 0.1000000000E+01  
(TAl, r) = 0.0000000000E+00  
(TAl, eB) = 0.0000000000E+00  
(r, Tp) = 0.0000000000E+00  
(r, Hs) = 0.0000000000E+00  
(r, A) = 0.0000000000E+00  
(r, B) = 0.0000000000E+00  
(r, Tid) = 0.0000000000E+00  
(r, Sur) = 0.0000000000E+00  
(r, r) = 0.0000000000E+00  
(r, TAl) = 0.0000000000E+00  
(r, eB) = 0.0000000000E+00  
(eB, Tp) = 0.0000000000E+00  
(eB, Hs) = 0.0000000000E+00  
(eB, A) = 0.0000000000E+00  
(eB, B) = 0.0000000000E+00  
(eB, Tid) = 0.0000000000E+00  
(eB, Sur) = 0.0000000000E+00  
(eB, TAl) = 0.0000000000E+00  
(eB, r) = 0.0000000000E+00  
(eB, eB) = 0.1000000000E+00

STARTING POINT FOR THE FORM CALCULATIONS:

Default Values (mean values) ... [ 1 ]
User Specified Values ........... [ 2 ]

Select Option: 2

STARTING POINT

Tp = 0.4500000000E+01
Hs = 0.2000000000E+01
Tid = 0.4000000000E+01
Sur = 0.1000000000E+01
TAl = 0.5000000000E+00
r = 0.9500000000E+00
eB = 0.1049000000E+01
LIMITING VALUES FOR THE VARIABLES:

Default Values (+/- 1E25) ... [1]
User Specified Values ....... [2]

Select Option: 2

LIMITING VALUES FOR THE VARIABLES:

XMin(Tp) = 0.4224000000E+01    XMax(Tp) = 0.2000000000E+02
XMin(Hs) = 0.4500000000E+00    XMax(Hs) = 0.2000000000E+02
XMin(A) = 0.7530000000E-02     XMax(A) = 0.7530000000E-02
XMin(B) = 0.4170000000E+01     XMax(B) = 0.4170000000E+01
XMin(Tid) = -0.5330000000E+01  XMax(Tid) = 0.5570000000E+01
XMin(Sur) = -0.5000000000E+01  XMax(Sur) = 0.5000000000E+01
XMin(TAI) = 0.2500000000E+00   XMax(TAI) = 0.1000000000E+01
XMin(r)  = 0.9000000000E+00    XMax(r)  = 0.1000000000E+01
XMin(eB) = 0.1000000000E-24    XMax(eB) = 0.1000000000E+26

MAXIMUM NUMBER OF ITERATIONS (Max=200) = 200

NUMBER OF FORM CALCULATIONS (Max=10) = 6

ALLOWABLE DISCHARGE - m3/s/m (1) = 0.100E+00
ALLOWABLE DISCHARGE - m3/s/m (2) = 0.100E-01
ALLOWABLE DISCHARGE - m3/s/m (3) = 0.100E-02
ALLOWABLE DISCHARGE - m3/s/m (4) = 0.100E-03
ALLOWABLE DISCHARGE - m3/s/m (5) = 0.100E-04
ALLOWABLE DISCHARGE - m3/s/m (6) = 0.100E-05

REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX:

Default Value (1%) ............ [1]
User Specified Value ........... [2]

Select Option: 1

REQUIRED SMOOTHING COEFFICIENT FOR THE ITERATION PROCESS:

Default Value (0) ............ [1]
User Specified Value ........... [2]

Select Option: 2

Required Smoothing Coefficient for the Iteration Process [0,1] = 0.9000000000E+00

REQUIRED ACCURACY OF THE FAILURE FUNCTION:

Default Value (1%) ............ [1]
User Specified Value ........... [2]

Select Option: 1
ALLOWABLE DISCHARGE - m³/s/m (1) = 0.100E+00

FINAL RESULTS

Total Number of Iterations = 65
Failure Function Z (X) = 0.2620132425E-06
Mean Value of Z = 0.3434493330E+00
Standard Deviation of Z = 0.8345014442E-01
Reliability Index = 0.4115622992E+01
Relative Accuracy of the Reliability Index (%) = 0.2358248898E-05
Probability of Failure (%) = 0.001940
Difference in Pf Between the Last 2 Iterations = 0.7829946879E-11

DESIGN POINT COORDINATES

Tp = 0.7563663838E+01
Hs = 0.2622799378E+01
A = 0.7530000000E-02
B = 0.4170000000E+01
Tid = 0.4460740834E+01
Sur = 0.1081726008E+00
TAi = 0.4805042888E+00
r = 0.9517390888E+00
eB = 0.6076173730E+00

Alpha(Y 1) = -0.2539299171E+00
Influence of Y(Tp ) on the Reliability Index = 0.6448040E+01

Alpha(Y 2) = -0.5427487582E+00
Influence of Y(Hs ) on the Reliability Index = 0.2945762E+02

Alpha(A  ) = 0.0000000000E+00
Influence of A  on the Reliability Index = 0.0000000E+00

Alpha(B  ) = 0.0000000000E+00
Influence of B  on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.5447375721E+00
Influence of Tid on the Reliability Index = 0.2967390E+02

Alpha(Sur) = -0.1509738752E+00
Influence of Sur on the Reliability Index = 0.2279311E+01

Alpha(TAl) = 0.9473934424E-01
Influence of TAl on the Reliability Index = 0.8975543E+00

Alpha(r  ) = -0.4098585879E-01
Influence of r  on the Reliability Index = 0.1679841E+00

Alpha(eB ) = 0.5574548101E+00
Influence of eB  on the Reliability Index = 0.3107559E+02
ALLOWABLE DISCHARGE - m³/s/m (2) = 0.100E-01

FINAL RESULTS

Total Number of Iterations = 66
Failure Function Z (X) = 0.1202995422E-08
Mean Value of Z = 0.7008017973E-01
Standard Deviation of Z = 0.2367117310E-01
Reliability Index = 0.2994367007E+01
Relative Accuracy of the Reliability Index (%) = 0.7225613349E-06
Probability of Failure (%) = 0.137535
Difference in Pf Between the Last 2 Iterations = 0.9736262209E-10

DESIGN POINT COORDINATES

Tp = 0.7509955325E+01
Hs = 0.2256461280E+01
A = 0.7530000000E-02
B = 0.4170000000E+01
Tid = 0.4012849932E+01
Sur = 0.5072785074E-01
TA1 = 0.4886679945E+00
r = 0.9511623469E+00
eB = 0.7826602205E+00

Alpha(Y 1) = -0.3422216243E+00
Influence of Y(Tp ) on the Reliability Index = 0.1171156E+02

Alpha(Y 2) = -0.5978879019E+00
Influence of Y(Hs ) on the Reliability Index = 0.3574699E+02

Alpha(A  ) = 0.0000000000E+00
Influence of A   on the Reliability Index = 0.0000000E+00

Alpha(B  ) = 0.0000000000E+00
Influence of B   on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.5919726309E+00
Influence of Tid on the Reliability Index = 0.3504316E+02

Alpha(Sur) = -0.1142617071E+00
Influence of Sur on the Reliability Index = 0.1305574E+01

Alpha(TA1) = 0.7568882121E-01
Influence of TA1 on the Reliability Index = 0.5728798E+00

Alpha(r  ) = -0.3764135231E-01
Influence of r   on the Reliability Index = 0.1416871E+00

Alpha(eB ) = 0.3934226912E+00
Influence of eB  on the Reliability Index = 0.1547814E+02
ALLOWABLE DISCHARGE - m³/s/m (3) = 0.100E-02

FINAL RESULTS

Total Number of Iterations = 67
Failure Function Z (X) = -0.7517596647E-09
Mean Value of Z = 0.1305319872E-01
Standard Deviation of Z = 0.5122525833E-02
Reliability Index = 0.2548195782E+01
Relative Accuracy of the Reliability Index (%) = 0.2240196757E-06
Probability of Failure (%) = 0.541433
Difference in Pf Between the Last 2 Iterations = 0.8962282045E-10

DESIGN POINT COORDINATES

Tp = 0.7426464897E+01
Hs = 0.2064834403E+01
A = 0.7530000000E-02
B = 0.4170000000E+01
Tid = 0.3774956652E+01
Sur = 0.3390393578E-01
TA1 = 0.4912886033E+00
r = 0.9509318641E+00
eB = 0.8709390940E+00

Alpha(Y 1) = -0.3809074754E+00
Influence of Y(Tp ) on the Reliability Index = 0.1450905E+02

Alpha(Y 2) = -0.6205992250E+00
Influence of Y(Hs ) on the Reliability Index = 0.3851434E+02

Alpha(A  ) = 0.0000000000E+00
Influence of A   on the Reliability Index = 0.0000000E+00

Alpha(B  ) = 0.0000000000E+00
Influence of B   on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.6139291896E+00
Influence of Tid on the Reliability Index = 0.3769090E+02

Alpha(Sur) = -0.9992632726E-01
Influence of Sur on the Reliability Index = 0.9985271E-01

Alpha(TA1) = 0.6837305630E-01
Influence of TA1 on the Reliability Index = 0.4674875E+00

Alpha(r  ) = -0.3545923011E-01
Influence of r on the Reliability Index = 0.1257357E+00

Alpha(eB ) = 0.2773797840E+00
Influence of eB on the Reliability Index = 0.7693954E+01
ALLOWABLE DISCHARGE - m3/s/m (4) = 0.100E-03

FINAL RESULTS

Total Number of Iterations = 50
Failure Function Z (X) = -0.7579302935E-09
Mean Value of Z = 0.2284606485E-02
Standard Deviation of Z = 0.979443295E-03
Reliability Index = 0.2332553690E+01
Relative Accuracy of the Reliability Index (%) = 0.2283641497E-06
Probability of Failure (%) = 0.983635
Difference in Pf Between the Last 2 Iterations = 0.1390272984E-09

DESIGN POINT COORDINATES

Tp    = 0.7369449843E+01
Hs    = 0.1958496021E+01
A     = 0.7530000000E-02
B     = 0.4170000000E+01
Tid   = 0.3639478043E+01
Sur   = 0.2707465900E-01
TAl   = 0.4923807471E+00
r     = 0.9508282991E+00
eB    = 0.9214694518E+00

Alpha(Y 1) = -0.3987835981E+00
Influence of Y(Tp ) on the Reliability Index = 0.1590284E+02

Alpha(Y 2) = -0.6316113956E+00
Influence of Y(Hs ) on the Reliability Index = 0.3989330E+02

Alpha(A ) = 0.0000000000E+00
Influence of A  on the Reliability Index = 0.0000000E+00

Alpha(B ) = 0.0000000000E+00
Influence of B  on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.6238974930E+00
Influence of Tid on the Reliability Index = 0.3892481E+02

Alpha(Sur) = -0.9358862260E-01
Influence of Sur on the Reliability Index = 0.8758830E+00

Alpha(TAl) = 0.6532971068E-01
Influence of TAl on the Reliability Index = 0.4267971E+00

Alpha(r ) = -0.3443007124E-01
Influence of r  on the Reliability Index = 0.1185430E+00

Alpha(eB ) = 0.1964137825E+00
Influence of eB  on the Reliability Index = 0.3857837E+01
ALLOWABLE DISCHARGE - m³/s/m (5) = 0.100E-04

FINAL RESULTS

Total Number of Iterations = 59
Failure Function Z (X) = -0.3298526059E-09
Mean Value of Z = 0.3928047060E-03
Standard Deviation of Z = 0.1771568459E-03
Reliability Index = 0.2217270826E+01
Relative Accuracy of the Reliability Index (%) = 0.4071944254E-06
Probability of Failure (%) = 1.330288
Difference in Pf Between the Last 2 Iterations = 0.3105839509E-09

DESIGN POINT COORDINATES

Tp = 0.7333707910E+01
Hs = 0.1897379159E+01
A = 0.7530000000E-02
B = 0.4170000000E+01
Tid = 0.3562349375E+01
Sur = 0.2354824851E-01
TAl = 0.4929539674E+00
r = 0.9507712973E+00
eB = 0.9541034378E+00

Alpha(Y1) = -0.4077597518E+00
Influence of Y(Tp) on the Reliability Index = 0.1662680E+02

Alpha(Y2) = -0.6362933160E+00
Influence of Y(Hs) on the Reliability Index = 0.4048692E+02

Alpha(A) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.6298563707E+00
Influence of Tid on the Reliability Index = 0.3967190E+02

Alpha(Sur) = -0.8990601037E-01
Influence of Sur on the Reliability Index = 0.8083091E+00

Alpha(TAl) = 0.6355590418E-01
Influence of TAl on the Reliability Index = 0.4039353E+00

Alpha(r) = -0.3372981404E-01
Influence of r on the Reliability Index = 0.1137700E+00

Alpha(eB) = 0.1374176447E+00
Influence of eB on the Reliability Index = 0.1888361E+01
Examples Of Input And Output Files For Wave Overtopping

ALLOWABLE DISCHARGE - m3/s/m (6) = 0.100E-05

FINAL RESULTS

Total Number of Iterations = 73
Failure Function Z (X) = -0.8329562325E-10
Mean Value of Z = 0.6684666168E-04
Standard Deviation of Z = 0.3105163469E-04
Reliability Index = 0.2152758216E+01
Relative Accuracy of the Reliability Index (%) = 0.9605813979E-06
Probability of Failure (%) = 1.566915
Difference in Pf Between the Last 2 Iterations = 0.8085487037E-09

DESIGN POINT COORDINATES

Tp = 0.7311668376E+01
Hs = 0.1861443367E+01
A = 0.7530000000E-02
B = 0.4170000000E+01
Tid = 0.3517012419E+01
Sur = 0.2162300799E-01
TAl = 0.4932691935E+00
r = 0.9507392836E+00
eB = 0.9760302809E+00

Alpha(Y 1) = -0.4124204710E+00
Influence of Y(Tp ) on the Reliability Index = 0.1700906E+02

Alpha(Y 2) = -0.6384045466E+00
Influence of Y(Hs ) on the Reliability Index = 0.4075604E+02

Alpha(A  ) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B  ) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.6329314191E+00
Influence of Tid on the Reliability Index = 0.4006022E+02

Alpha(Sur) = -0.877686672E-01
Influence of Sur on the Reliability Index = 0.7703339E+00

Alpha(TAl) = 0.6253193176E-01
Influence of TAl on the Reliability Index = 0.3910242E+00

Alpha(r  ) = -0.3329704273E-01
Influence of r on the Reliability Index = 0.1108693E+00

Alpha(eB ) = 0.9499754891E-01
Influence of eB on the Reliability Index = 0.9024534E+00

WOULD YOU LIKE TO RESTART (Y/N) ? N
Examples Of Input And Output Files For Wave Overtopping

Input File *general.dad*: Owen's Model

```
2 ! Failure mode of overtopping (Owen)
2 ! Tide + Surge
Y ! Derivatives of the failure function supplied
1 ! Mode 1 - Reliability analysis for a specified design
1 ! Design life (in years)
N ! No combinations of actions considered
10 ! Target design parameter - seawall crest level
```

Input File *form.dad*: Owen's Model

```
2 ! Starting point: user specified values
5
1.2
4
1
0.95
1.027
2 ! [XMin,XMax]: user specified values
1 20
0 20
-5.33 5.57
-5 5
0.5 1
-1E25 1E25
200 ! Maximum number of iterations
6 ! Number of FORM calculations
1E-1
1E-2
1E-3
1E-4
1E-5
1E-6
1 ! Accuracy on Beta (%): default value (1%)
2 ! Smoothing of the iteration: user specified value
0.9
1 ! Accuracy on Z0 (%): default value (1%)
```

Input File *meandev.dad*: Owen's Model

```
10 !Type of distribution (Weibull)
0 !Not truncated
5 0.9 3.3 !Mean; stand. dev.; lower limit
10 !Type of distribution (Weibull)
1 !Truncated for X above Xo
0 !Toe level
1.2 0.7 0.45 !Mean; stand. dev.; lower limit
0 !Type of distribution (Deterministic)
0.0117 !Value
0 !Type of distribution (Deterministic)
```
Examples Of Input And Output Files For Wave Overtopping

21.71                           !Value
13                              !Type of distribution (User-Defined)
0.2749          2.3619          !Mean value; standard deviation
3                              !Type of distribution (Gumbel)
0                              !Not truncated
0.019483        0.192382475     !Mean value; standard deviation
6                              !Type of distribution (Beta)
0                              !Not truncated
0.95    0.01    0.9     1       !Mean; stand. dev.; lower & upper limits
2                              !Type of distribution (Log-Normal)
0                              !Not truncated
1.027           0.15            !Mean value; standard deviation

Input File coefcor.dad: Owen's Model

1                              ! Rho(1,1)
0.6                             ! Rho(1,2)
0                              ! Rho(1,3)
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
1                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
1                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
1                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
1                              !   .
0                              !   .
0                              !   .
0                              !   .
0                              !   .
0
Output File *summary.dat*: Owen’s Model

**WHAT IS THE DATA SOURCE ?**

The Screen ..... [ 1 ]
A Datafile ..... [ 2 ]

Select Option: 2

**WHAT IS THE FAILURE MODE TO BE STUDIED:**

Overtopping (H&R) .................. [ 1 ]
Overtopping (Owen) ................. [ 2 ]
Dune Erosion (Vellinga) .......... [ 3 ]

Select Option: 2

**HOW IS THE STILL-WATER-LEVEL DEFINED ?**

Total Level .... [ 1 ]
Tide + Surge ... [ 2 ]

Select Option: 2

**ARE THE FIRST DERIVATIVES OF THE FAILURE FUNCTION SUPPLIED (Y/N) ? Y**
DESCRIPTION OF THE VARIABLES

X(1) = Tm  = Mean Wave Period
X(2) = Hs  = Wave Height
X(3) = A   = Owen Parameter
X(4) = B   = Owen Parameter
X(5) = Tid = Tide Level
X(6) = Sur = Surge
X(7) = r   = Roughness
X(8) = eB  = Model Parameter

WHAT IS THE PURPOSE OF THE ANALYSIS ?

Reliability Analysis for a Specified Design ... [ 1 ]
Design for a Specified Reliability Level ...... [ 2 ]

Select Option: 1

DESIGN LIFE OF THE STRUCTURE = 1

WOULD YOU LIKE TO CONSIDER COMBINATION OF ACTIONS (Y/N) ? N

PRESCRIBED VALUE OF THE DESIGN PARAMETER

Seawall Crest Level = 0.1000000000E+02

CHARACTERISTICS OF THE VARIABLES

Probability Distribution of Tm  = Minima Type III (Weibull)
Mean Value of Tm  = 0.5000000000E+01
Standard Deviation of Tm  = 0.9000000000E+00
Lower Limit on Tm  = 0.3300000000E+01

Probability Distribution of Hs  = Minima Type III (Weibull)
The Distribution of Hs  is truncated above Xo = 0.6(Tide+Surge-TL)
Seawall Toe Level (TL) = 0.0000000000E+00
Mean Value of Hs  = 0.1200000000E+01
Standard Deviation of Hs  = 0.7000000000E+00
Lower Limit on Hs  = 0.4500000000E+00

Probability Distribution of A   = Deterministic
Mean Value of A   = 0.1170000000E-01
Standard Deviation of A   = 0.0000000000E+00

Probability Distribution of B   = Deterministic
Mean Value of B   = 0.2171000000E+02
Standard Deviation of B   = 0.0000000000E+00
Probability Distribution of $T_{id}$ = User-Defined Distribution
Mean Value of $T_{id}$ = 0.2749000000E+00
Standard Deviation of $T_{id}$ = 0.2361900000E+01

Probability Distribution of $S_{ur}$ = Maxima Type I (Gumbel)
Mean Value of $S_{ur}$ = 0.1948300000E-01
Standard Deviation of $S_{ur}$ = 0.1923824750E+00

Probability Distribution of $r$ = Beta
Mean Value of $r$ = 0.9500000000E+00
Standard Deviation of $r$ = 0.1000000000E-01
Limits $a$ and $b$ of $r$ = [0.9000000000E+00, 0.1000000000E+01]

Probability Distribution of $e_B$ = Log-Normal
Mean Value of $e_B$ = 0.1027000000E+01
Standard Deviation of $e_B$ = 0.1500000000E+00

CORRELATION COEFFICIENTS

$$(T_m, T_m) = 0.1000000000E+01$$
$$(T_m, H_s) = 0.0600000000E+00$$
$$(T_m, A) = 0.0000000000E+00$$
$$(T_m, B) = 0.0000000000E+00$$
$$(T_m, T_{id}) = 0.0000000000E+00$$
$$(T_m, S_{ur}) = 0.0000000000E+00$$
$$(T_m, r) = 0.0000000000E+00$$
$$(T_m, e_B) = 0.0000000000E+00$$
$$(H_s, T_m) = 0.6000000000E+00$$
$$(H_s, H_s) = 0.1000000000E+01$$
$$(H_s, A) = 0.0000000000E+00$$
$$(H_s, B) = 0.0000000000E+00$$
$$(H_s, T_{id}) = 0.0000000000E+00$$
$$(H_s, S_{ur}) = 0.0000000000E+00$$
$$(H_s, r) = 0.0000000000E+00$$
$$(H_s, e_B) = 0.0000000000E+00$$
$$(A, T_m) = 0.0000000000E+00$$
$$(A, H_s) = 0.0000000000E+00$$
$$(A, A) = 0.1000000000E+01$$
$$(A, B) = 0.0000000000E+00$$
$$(A, T_{id}) = 0.0000000000E+00$$
$$(A, S_{ur}) = 0.0000000000E+00$$
$$(A, r) = 0.0000000000E+00$$
$$(A, e_B) = 0.0000000000E+00$$
$$(B, T_m) = 0.0000000000E+00$$
$$(B, H_s) = 0.0000000000E+00$$
$$(B, A) = 0.0000000000E+00$$
$$(B, B) = 0.1000000000E+01$$
$$(B, T_{id}) = 0.0000000000E+00$$
$$(B, S_{ur}) = 0.0000000000E+00$$
$$(B, r) = 0.0000000000E+00$$
$$(B, e_B) = 0.0000000000E+00$$
$$(T_{id}, T_m) = 0.0000000000E+00$$
Examples Of Input And Output Files For Wave Overtopping

(Tid,Hs ) =  0.0000000000E+00
(Tid,A  ) =  0.0000000000E+00
(Tid,B  ) =  0.0000000000E+00
(Tid,Tid) =  0.1000000000E+01
(Tid,Sur) =  0.0000000000E+00
(Tid,r  ) =  0.0000000000E+00
(Tid,eB ) =  0.0000000000E+00
(Sur,Tm ) =  0.0000000000E+00
(Sur,Hs ) =  0.0000000000E+00
(Sur,A  ) =  0.0000000000E+00
(Sur,B  ) =  0.0000000000E+00
(Sur,Tid) =  0.0000000000E+00
(Sur,Sur) =  0.1000000000E+01
(Sur,r  ) =  0.0000000000E+00
(Sur,eB ) =  0.0000000000E+00
(r  ,Tm ) =  0.0000000000E+00
(r  ,Hs ) =  0.0000000000E+00
(r  ,A  ) =  0.0000000000E+00
(r  ,B  ) =  0.0000000000E+00
(r  ,Tid) =  0.0000000000E+00
(r  ,Sur) =  0.0000000000E+00
(r  ,r  ) =  0.1000000000E+01
(r  ,eB ) =  0.0000000000E+00
(eB ,Tm ) =  0.0000000000E+00
(eB ,Hs ) =  0.0000000000E+00
(eB ,A  ) =  0.0000000000E+00
(eB ,B  ) =  0.0000000000E+00
(eB ,Tid) =  0.0000000000E+00
(eB ,Sur) =  0.0000000000E+00
(eB ,r  ) =  0.0000000000E+00
(eB ,eB ) =  0.1000000000E+01

STARTING POINT FOR THE FORM CALCULATIONS:

Default Values (mean values) ... [ 1 ]
User Specified Values ............ [ 2 ]

Select Option: 2

STARTING POINT

Tm  =  0.5000000000E+01
Hs  =  0.1200000000E+01
Tid =  0.4000000000E+01
Sur =  0.1000000000E+01
r   =  0.9500000000E+00
eB  =  0.1027000000E+01

LIMITING VALUES FOR THE VARIABLES:

Default Values (+/- 1E25) ... [ 1 ]
User Specified Values ............ [ 2 ]

Select Option: 2
LIMITING VALUES FOR THE VARIABLES

\[X_{\text{Min}}(Tm) = 0.3300000000E+01 \quad X_{\text{Max}}(Tm) = 0.2000000000E+02\]
\[X_{\text{Min}}(Hs) = 0.4500000000E+00 \quad X_{\text{Max}}(Hs) = 0.2000000000E+02\]
\[X_{\text{Min}}(A) = 0.1170000000E-01 \quad X_{\text{Max}}(A) = 0.1170000000E-01\]
\[X_{\text{Min}}(B) = 0.2171000000E+02 \quad X_{\text{Max}}(B) = 0.2171000000E+02\]
\[X_{\text{Min}}(Tid) = -0.5330000000E+01 \quad X_{\text{Max}}(Tid) = 0.5570000000E+01\]
\[X_{\text{Min}}(Sur) = -0.5000000000E+01 \quad X_{\text{Max}}(Sur) = 0.5000000000E+01\]
\[X_{\text{Min}}(r) = 0.9000000000E+00 \quad X_{\text{Max}}(r) = 0.1000000000E+01\]
\[X_{\text{Min}}(eB) = 0.1000000020E-24 \quad X_{\text{Max}}(eB) = 0.1000000000E+26\]

MAXIMUM NUMBER OF ITERATIONS (Max=200) = 200

NUMBER OF FORM CALCULATIONS (Max=10) = 6

ALLOWABLE DISCHARGE - m3/s/m (1) = 0.100E+00
ALLOWABLE DISCHARGE - m3/s/m (2) = 0.100E-01
ALLOWABLE DISCHARGE - m3/s/m (3) = 0.100E-02
ALLOWABLE DISCHARGE - m3/s/m (4) = 0.100E-03
ALLOWABLE DISCHARGE - m3/s/m (5) = 0.100E-04
ALLOWABLE DISCHARGE - m3/s/m (6) = 0.100E-05

REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX:

Default Value (1%) .......... [ 1 ]
User Specified Value ....... [ 2 ]

Select Option: 1

REQUIRED SMOOTHING COEFFICIENT FOR THE ITERATION PROCESS:

Default Value (0) .......... [ 1 ]
User Specified Value ....... [ 2 ]

Select Option: 2

Required Smoothing Coefficient for the Iteration Process \([0,1] = 0.9000000000E+00\)

REQUIRED ACCURACY OF THE FAILURE FUNCTION:

Default Value (1%) .......... [ 1 ]
User Specified Value ....... [ 2 ]

Select Option: 1
Examples Of Input And Output Files For Wave Overtopping

ALLOWABLE DISCHARGE - m3/s/m (1) = 0.100E+00

FINAL RESULTS

Total Number of Iterations = 64
Failure Function Z (X) = -0.3304572431E-07
Mean Value of Z = 0.3929601531E+00
Standard Deviation of Z = 0.109784458E+00
Reliability Index = 0.3579379120E+01
Relative Accuracy of the Reliability Index (%) = 0.8743530582E-06
Probability of Failure (%) = 0.017243
Difference in Pf Between the Last 2 Iterations = 0.2190748757E-10

DESIGN POINT COORDINATES

Tm = 0.7867682058E+01
Hs = 0.2288306898E+01
A = 0.1170000000E-01
B = 0.2171000000E+02
Tid = 0.3959377043E+01
Sur = 0.4621900285E-01
r = 0.9511036460E+00
eB = 0.8250366186E+00

Alpha(Y 1) = -0.7472518744E+00
Influence of Y(Tm ) on the Reliability Index = 0.5583854E+02

Alpha(Y 2) = -0.2005296001E+00
Influence of Y(Hs ) on the Reliability Index = 0.4021212E+01

Alpha(A ) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B ) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.4816107580E+00
Influence of Tid on the Reliability Index = 0.2319489E+02

Alpha(Sur) = -0.8911581216E-01
Influence of Sur on the Reliability Index = 0.7941628E+00

Alpha(r ) = -0.2990006018E-01
Influence of r on the Reliability Index = 0.8940136E-01

Alpha(eB) = 0.4007716955E+00
Influence of eB on the Reliability Index = 0.1606180E+02
ALLOWABLE DISCHARGE - m3/s/m (Z) = 0.100E-01

FINAL RESULTS

Total Number of Iterations = 54
Failure Function Z (X) = 0.8715043405E-09
Mean Value of Z = 0.4954358667E-01
Standard Deviation of Z = 0.2093853466E-01
Reliability Index = 0.2366143929E+01
Relative Accuracy of the Reliability Index (%) = 0.3568124208E-06
Probability of Failure (%) = 0.898770
Difference in Pf Between the Last 2 Iterations = 0.2051574986E-09

DESIGN POINT COORDINATES

Tm = 0.6714341777E+01
Hs = 0.1768924154E+01
A = 0.1170000000E-01
B = 0.2171000000E+02
Tid = 0.3324831734E+01
Sur = 0.1484800976E-01
r = 0.9506214224E+00
eB = 0.9037968322E+00

Alpha(Y1) = -0.7365302465E+00
Influence of Y(Tm) on the Reliability Index = 0.5424768E+02

Alpha(Y2) = -0.2590956446E+00
Influence of Y(Hs) on the Reliability Index = 0.6713055E+01

Alpha(A) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.5189503568E+00
Influence of Tid on the Reliability Index = 0.2693095E+02

Alpha(Sur) = -0.6424241822E-01
Influence of Sur on the Reliability Index = 0.4127088E+00

Alpha(r) = -0.2546293174E-01
Influence of r on the Reliability Index = 0.6483609E-01

Alpha(eB) = 0.3410391781E+00
Influence of eB on the Reliability Index = 0.1163077E+02
Examples Of Input And Output Files For Wave Overtopping

ALLOWABLE DISCHARGE - m3/s/m (3) = 0.100E-02

FINAL RESULTS

Total Number of Iterations = 64
Failure Function Z(X) = 0.8701465193E-11
Mean Value of Z = 0.5323353420E-02
Standard Deviation of Z = 0.3180215221E-02
Reliability Index = 0.1673897221E+01
Relative Accuracy of the Reliability Index (%) = 0.1547060913E-06
Probability of Failure (%) = 4.707768
Difference in Pf Between the Last 2 Iterations = 0.2540418154E-09

DESIGN POINT COORDINATES

Tm = 0.6104414348E+01
Hs = 0.1383504337E+01
A = 0.1170000000E-01
B = 0.2171000000E+02
Tid = 0.2805428663E+01
Sur = 0.3618371385E-02
r = 0.9504050441E+00
eB = 0.9414699343E+00

Alpha(Y1) = -0.7203268632E+00
Influence of Y(Tm) on the Reliability Index = 0.5188708E+02

Alpha(Y2) = -0.2818109186E+00
Influence of Y(Hs) on the Reliability Index = 0.7941739E+01

Alpha(A) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.5473526962E+00
Influence of Tid on the Reliability Index = 0.2995950E+02

Alpha(Sur) = -0.5360142620E-01
Influence of Sur on the Reliability Index = 0.2873113E+00

Alpha(r) = -0.2346256681E-01
Influence of r on the Reliability Index = 0.5504920E-01

Alpha(eB) = 0.3141547983E+00
Influence of eB on the Reliability Index = 0.9869324E+01
ALLOWABLE DISCHARGE - m/s/m (4) = 0.100E-03

FINAL RESULTS

Total Number of Iterations = 68
Failure Function Z (X) = -0.1654101392E-10
Mean Value of Z = 0.5174503166E-03
Standard Deviation of Z = 0.4304664431E-03
Reliability Index = 0.1202068883E+01
Relative Accuracy of the Reliability Index (%) = 0.1902001380E-06
Probability of Failure (%) = 11.467071
Difference in Pf Between the Last 2 Iterations = 0.4412629845E-09

DESIGN POINT COORDINATES

Tm  = 0.5709854971E+01
Hs  = 0.1112225125E+01
A   = 0.1170000000E-01
B   = 0.2171000000E+02
Tid = 0.2387767135E+01
Sur = -0.2096607008E-02
r   = 0.9502779598E+00
eB  = 0.9643145164E+00

Alpha(Y 1) = -0.6927297329E+00
Influence of Y(Tm ) on the Reliability Index = 0.4798745E+02

Alpha(Y 2) = -0.2879491765E+00
Influence of Y(Hs ) on the Reliability Index = 0.8291473E+01

Alpha(A  ) = 0.0000000000E+00
Influence of A   on the Reliability Index = 0.0000000E+00

Alpha(B  ) = 0.0000000000E+00
Influence of B   on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.5867796063E+00
Influence of Tid on the Reliability Index = 0.3443103E+02

Alpha(Sur) = -0.4781748192E-01
Influence of Sur on the Reliability Index = 0.2286512E+00

Alpha(r  ) = -0.2242283715E-01
Influence of r   on the Reliability Index = 0.5027836E-01

Alpha(eB ) = 0.3001852550E+00
Influence of eB  on the Reliability Index = 0.9011119E+01
ALLOWABLE DISCHARGE - m$^3$/s/m (5) = 0.100E-04

FINAL RESULTS

Total Number of Iterations = 60
Failure Function Z (X) = -0.4879941218E-10
Mean Value of Z = 0.4701425035E-04
Standard Deviation of Z = 0.547744869E-04
Reliability Index = 0.8583244829E+00
Relative Accuracy of the Reliability Index (%) = 0.1489272992E-05
Probability of Failure (%) = 19.535878
Difference in Pf Between the Last 2 Iterations = 0.3538277560E-08

DESIGN POINT COORDINATES

Tm = 0.5432632364E+01
Hs = 0.9176506972E+00
A = 0.1170000000E-01
B = 0.2171000000E+02
Tid = 0.2031448014E+01
Sur = -0.5574932557E-02
r = 0.9501921396E+00
eB = 0.9800548223E+00

Alpha(Y 1) = -0.6511433929E+00
Influence of Y(Tm ) on the Reliability Index = 0.4239877E+02

Alpha(Y 2) = -0.2749863860E+00
Influence of Y(Hs ) on the Reliability Index = 0.7561751E+01

Alpha(A ) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B ) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.6430924749E+00
Influence of Tid on the Reliability Index = 0.4135679E+02

Alpha(Sur) = -0.4388713472E-01
Influence of Sur on the Reliability Index = 0.1926081E+00

Alpha(r ) = -0.2170666113E-01
Influence of r on the Reliability Index = 0.4711791E-01

Alpha(eB ) = 0.2905676829E+00
Influence of eB on the Reliability Index = 0.8442958E+01
ALLOWABLE DISCHARGE - m3/s/m (6) = 0.100E-05

FINAL RESULTS

Total Number of Iterations = 62
Failure Function Z (X) = -0.4468194738E-10
Mean Value of Z = 0.4055884630E-05
Standard Deviation of Z = 0.665864046E-05
Reliability Index = 0.6091230184E+00
Relative Accuracy of the Reliability Index (%) = 0.9391025663E-05
Probability of Failure (%) = 27.122233
Difference in Pf Between the Last 2 Iterations = 0.1900280305E-07

DESIGN POINT COORDINATES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tm</td>
<td>0.5242180513E+01</td>
</tr>
<tr>
<td>Hs</td>
<td>0.7713292909E+00</td>
</tr>
<tr>
<td>A</td>
<td>0.1170000000E-01</td>
</tr>
<tr>
<td>B</td>
<td>0.2171000000E+02</td>
</tr>
<tr>
<td>Tid</td>
<td>0.1698078052E+01</td>
</tr>
<tr>
<td>Sur</td>
<td>-0.7783445552E-02</td>
</tr>
<tr>
<td>r</td>
<td>0.9501336265E+00</td>
</tr>
<tr>
<td>eB</td>
<td>0.9909332316E+00</td>
</tr>
</tbody>
</table>

Alpha(Y1) = -0.5999624020E+00
Influence of Y(Tm) on the Reliability Index = 0.3599549E+02

Alpha(Y2) = -0.2415765075E+00
Influence of Y(Hs) on the Reliability Index = 0.5835921E+01

Alpha(A) = 0.0000000000E+00
Influence of A on the Reliability Index = 0.0000000E+00

Alpha(B) = 0.0000000000E+00
Influence of B on the Reliability Index = 0.0000000E+00

Alpha(Tid) = -0.7060358221E+00
Influence of Tid on the Reliability Index = 0.4984866E+02

Alpha(Sur) = -0.4108099036E-01
Influence of Sur on the Reliability Index = 0.1687648E+00

Alpha(r) = -0.2127042864E-01
Influence of r on the Reliability Index = 0.4524311E-01

Alpha(eB) = 0.2847090553E+00
Influence of eB on the Reliability Index = 0.8105925E+01

WOULD YOU LIKE TO RESTART (Y/N) ? N
<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>10⁻¹</th>
<th>10⁻²</th>
<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Failure Function Z</strong></td>
<td>0.33E-05</td>
<td>0.53E-08</td>
<td>0.53E-09</td>
<td>0.33E-10</td>
<td>0.23E-11</td>
<td>0.23E-12</td>
</tr>
<tr>
<td><strong>Standard Deviation of Z</strong></td>
<td>0.10</td>
<td>0.306E-01</td>
<td>0.662E-02</td>
<td>0.127E-02</td>
<td>0.230E-03</td>
<td>0.404E-04</td>
</tr>
<tr>
<td><strong>Relative Accuracy of Beta (%)</strong></td>
<td>0.21E-05</td>
<td>0.21E-06</td>
<td>0.1E-06</td>
<td>0.4E-07</td>
<td>0.3E-07</td>
<td>0.4E-08</td>
</tr>
<tr>
<td><strong>Probability of Failure Pₙ (%)/Year</strong></td>
<td>0.083640</td>
<td>1.236960</td>
<td>2.998720</td>
<td>4.440561</td>
<td>5.407432</td>
<td>6.010773</td>
</tr>
<tr>
<td><strong>Difference in Pₙ Between Last 2 Iterations</strong></td>
<td>0.2E-09</td>
<td>0.2E-09</td>
<td>0.1E-09</td>
<td>0.6E-10</td>
<td>0.6E-10</td>
<td>0.7E-11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>10⁻¹</th>
<th>10⁻²</th>
<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X₁=TP</strong></td>
<td>7.544</td>
<td>-----</td>
<td>7.305</td>
<td>-----</td>
<td>7.154</td>
<td>-----</td>
</tr>
<tr>
<td><strong>X₂=Hₘ</strong></td>
<td>2.333</td>
<td>-----</td>
<td>1.877</td>
<td>-----</td>
<td>1.661</td>
<td>-----</td>
</tr>
<tr>
<td><strong>Y₁=HₚHₘ</strong></td>
<td>-----</td>
<td>11.29</td>
<td>-----</td>
<td>15.88</td>
<td>-----</td>
<td>17.03</td>
</tr>
<tr>
<td><strong>Y₂=HₚHₘ</strong></td>
<td>-----</td>
<td>33.21</td>
<td>-----</td>
<td>34.66</td>
<td>-----</td>
<td>34.77</td>
</tr>
<tr>
<td><strong>X₃=Y₃=A</strong></td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
</tr>
<tr>
<td><strong>X₄=Y₄=B</strong></td>
<td>4.17</td>
<td>0</td>
<td>4.17</td>
<td>0</td>
<td>4.17</td>
<td>0</td>
</tr>
<tr>
<td><strong>X₅=Y₅=Tide</strong></td>
<td>4.139</td>
<td>36.37</td>
<td>3.600</td>
<td>40.27</td>
<td>3.336</td>
<td>43.15</td>
</tr>
<tr>
<td><strong>X₆=Y₆=Surge</strong></td>
<td>0.064</td>
<td>1.68</td>
<td>0.025</td>
<td>0.96</td>
<td>0.015</td>
<td>0.67</td>
</tr>
<tr>
<td><strong>X₇=Y₇=tanα</strong></td>
<td>0.483</td>
<td>0.51</td>
<td>0.4940</td>
<td>0.28</td>
<td>0.4954</td>
<td>0.23</td>
</tr>
<tr>
<td><strong>X₈=Y₈=r</strong></td>
<td>0.9509</td>
<td>0.07</td>
<td>0.9505</td>
<td>0.08</td>
<td>0.9504</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>X₉=Y₉=e_B</strong></td>
<td>0.7629</td>
<td>16.87</td>
<td>0.8863</td>
<td>7.99</td>
<td>0.9378</td>
<td>4.10</td>
</tr>
</tbody>
</table>

**Table D2.1:** Summary of PARASODE results: normal conditions, H&R model, (Rₘₐₓ)₃7%, slope 1:2, CL=8m OD
(°Note that the correlation coefficient between Tₚ and Hₘ is ρ₁₂ = 0.6).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ ($m^3/dm$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure Function $Z$</td>
<td>0.3E-05</td>
<td>0.1E-05</td>
<td>-0.8E-09</td>
<td>-0.8E-09</td>
<td>-0.3E-09</td>
<td>-0.8E-10</td>
</tr>
<tr>
<td>Standard Deviation of $Z$</td>
<td>0.835E-01</td>
<td>0.237E-01</td>
<td>0.512E-02</td>
<td>0.979E-03</td>
<td>0.177E-03</td>
<td>0.311E-04</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td>0.2E-05</td>
<td>0.7E-05</td>
<td>0.2E-05</td>
<td>0.2E-05</td>
<td>0.4E-05</td>
<td>0.1E-05</td>
</tr>
<tr>
<td>Probability of Failure $P_r$ (%)</td>
<td>0.001940</td>
<td>0.137535</td>
<td>0.541433</td>
<td>0.936385</td>
<td>1.330088</td>
<td>1.596115</td>
</tr>
<tr>
<td>Difference in $P_r$ Between Last 2 Iterations</td>
<td>0.8E-11</td>
<td>0.1E-09</td>
<td>0.9E-10</td>
<td>0.1E-09</td>
<td>0.3E-09</td>
<td>0.8E-09</td>
</tr>
</tbody>
</table>

Table D2.2: Summary of PARASODE results: normal conditions, HSR model, ($R_{max}$,375), slope 1:2, CL=10m OD. (Note that the correlation coefficient between $T_d$ and $H_d$ is $p_{T_dH_d}=0.5$.)
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>$g$ (m/s$^2$)</th>
<th>$DP$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\gamma$</th>
<th>$\epsilon$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
<td>0.2 E-02</td>
</tr>
<tr>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
<td>0.1 E-02</td>
</tr>
<tr>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
<td>0.01 E-02</td>
</tr>
<tr>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
<td>0.001 E-02</td>
</tr>
<tr>
<td>0.0001 E-02</td>
<td>0.0001 E-02</td>
<td>0.0001 E-02</td>
<td>0.0001 E-02</td>
<td>0.0001 E-02</td>
<td>0.0001 E-02</td>
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</tbody>
</table>

#### Table D2.3 Summary of PARASODE results: normal contributions to $\phi$.

Note that the table contains contributions from $W$, $H_T$, and $\phi$. (continued)
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>$\Delta$ (m$^3$/s/m)</th>
<th>10$^{-1}$</th>
<th>10$^{-2}$</th>
<th>10$^{-3}$</th>
<th>10$^{-4}$</th>
<th>10$^{-5}$</th>
<th>10$^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function $\Xi$</td>
<td>0.3E-08</td>
<td>0.1E-08</td>
<td>0.2E-09</td>
<td>-0.1E-09</td>
<td>-0.4E-10</td>
<td>-0.4E-10</td>
</tr>
<tr>
<td>Standard Deviation of $\Xi$</td>
<td>0.74E-01</td>
<td>0.183E-01</td>
<td>0.360E-02</td>
<td>0.644E-03</td>
<td>0.110E-03</td>
<td>0.184E-04</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td>0.6E-05</td>
<td>0.2E-06</td>
<td>0.1E-06</td>
<td>0.1E-06</td>
<td>0.1E-06</td>
<td>0.5E-05</td>
</tr>
<tr>
<td>Probability of Failure $P_i$ (%/Year)</td>
<td>0.000000</td>
<td>0.000115</td>
<td>0.002299</td>
<td>0.008888</td>
<td>0.017747</td>
<td>0.025911</td>
</tr>
<tr>
<td>Difference in $P_i$ Between last 2 Iterations</td>
<td>0.9E-15</td>
<td>0.5E-12</td>
<td>0.5E-11</td>
<td>0.2E-10</td>
<td>0.3E-10</td>
<td>0.1E-10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/s/m)</th>
<th>10$^{-1}$</th>
<th>10$^{-2}$</th>
<th>10$^{-3}$</th>
<th>10$^{-4}$</th>
<th>10$^{-5}$</th>
<th>10$^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_i = T_y$</td>
<td>7.268</td>
<td>7.423</td>
<td>7.478</td>
<td>7.499</td>
<td>7.507</td>
<td>7.510</td>
</tr>
<tr>
<td>$X_i = H_d$</td>
<td>3.026</td>
<td>2.833</td>
<td>2.732</td>
<td>2.670</td>
<td>2.630</td>
<td>2.604</td>
</tr>
<tr>
<td>$Y_{i=T_y,H_d}$</td>
<td>1.71</td>
<td>3.86</td>
<td>5.68</td>
<td>6.97</td>
<td>7.77</td>
<td>8.28</td>
</tr>
<tr>
<td>$Y_{i=T_y,H_d}$</td>
<td>22.08</td>
<td>30.75</td>
<td>36.73</td>
<td>40.53</td>
<td>42.61</td>
<td>44.14</td>
</tr>
<tr>
<td>$X_i = Y_i = A$</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
</tr>
<tr>
<td>$X_i = Y_i = B$</td>
<td>4.17</td>
<td>0</td>
<td>4.17</td>
<td>0</td>
<td>4.17</td>
<td>0</td>
</tr>
<tr>
<td>$X_i = Y_i = T_y$</td>
<td>4.844</td>
<td>19.62</td>
<td>26.22</td>
<td>33.92</td>
<td>4.450</td>
<td>39.72</td>
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<tr>
<td>$X_i = Y_i = S_y$</td>
<td>0.305</td>
<td>5.14</td>
<td>0.179</td>
<td>3.88</td>
<td>0.136</td>
<td>3.34</td>
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<tr>
<td>$X_i = Y_i = S_y$</td>
<td>0.481</td>
<td>1.75</td>
<td>0.46E3</td>
<td>1.69</td>
<td>0.47E3</td>
<td>1.64</td>
</tr>
<tr>
<td>$X_i = Y_i = T_y$</td>
<td>0.95E4</td>
<td>0.95E3</td>
<td>0.63</td>
<td>0.95E3</td>
<td>0.95E3</td>
<td>0.95E3</td>
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<tr>
<td>$X_i = Y_i = e_y$</td>
<td>0.363E2</td>
<td>48.95</td>
<td>0.5631</td>
<td>30.97</td>
<td>0.6905</td>
<td>18.01</td>
</tr>
<tr>
<td>$X_i = Y_i = e_y$</td>
<td>0.7813</td>
<td>10.01</td>
<td>0.8471</td>
<td>5.39</td>
<td>0.8962</td>
<td>2.78</td>
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</table>

**Table D2.4:** Summary of PARASODE results: normal conditions, H&J model, ($R_{max}$,m)$^{1/2}$, slope 1:2, CL=14m OD
(Note that the correlation coefficient between $T_y$ and $H_d$ is $\rho_{2,0.5}$.)
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>Failure Function ζ</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
<th>10^{-7}</th>
<th>10^{-8}</th>
<th>10^{-9}</th>
<th>10^{-10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2E-06</td>
<td>0.1E-06</td>
<td>0.2E-06</td>
<td>0.4E-06</td>
<td>-4E-06</td>
<td>-7E-11</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Standard Deviation of ζ</td>
<td>0.786E-01</td>
<td>0.183E-01</td>
<td>0.34E-02</td>
<td>0.568E-03</td>
<td>0.961E-04</td>
<td>0.198E-04</td>
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</tr>
<tr>
<td>Relative Accuracy of ζ (%)</td>
<td>0.2E-06</td>
<td>0.4E-06</td>
<td>0.6E-06</td>
<td>0.4E-06</td>
<td>0.3E-06</td>
<td>0.2E-06</td>
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<tr>
<td>Probability of Failure P_f (%)/Year</td>
<td>0.000000</td>
<td>0.000001</td>
<td>0.000030</td>
<td>0.000191</td>
<td>0.000508</td>
<td>0.000875</td>
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</tr>
<tr>
<td>Difference In P_f Between Last 2 Iterations</td>
<td>0.8E-18</td>
<td>0.1E-13</td>
<td>0.5E-12</td>
<td>0.2E-11</td>
<td>0.3E-11</td>
<td>0.3E-11</td>
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</table>

<table>
<thead>
<tr>
<th>Q (m^3/s/m)</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
<th>10^{-7}</th>
<th>10^{-8}</th>
<th>10^{-9}</th>
<th>10^{-10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1 = T_p</td>
<td>7.129</td>
<td>7.229</td>
<td>7.276</td>
<td>7.304</td>
<td>7.319</td>
<td>7.328</td>
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<tr>
<td>X_2 = H_3</td>
<td>3.262</td>
<td>3.102</td>
<td>3.028</td>
<td>2.966</td>
<td>2.937</td>
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</tr>
<tr>
<td>Y_1 = Y_1 H_3</td>
<td>0.599</td>
<td>1.89</td>
<td>2.57</td>
<td>3.28</td>
<td>3.69</td>
<td>3.96</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Y_2 = Y_2 H_3</td>
<td>20.46</td>
<td>28.20</td>
<td>34.16</td>
<td>38.36</td>
<td>40.90</td>
<td>42.53</td>
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<td></td>
</tr>
<tr>
<td>X_3 = Y_3 + A</td>
<td>0.00753</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_4 = Y_4 + B</td>
<td>4.17</td>
<td>4.17</td>
<td>4.17</td>
<td>4.17</td>
<td>4.17</td>
<td>4.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_6 = Y_6 + Sid</td>
<td>0.576</td>
<td>9.76</td>
<td>8.88</td>
<td>8.38</td>
<td>8.01</td>
<td>7.62</td>
<td>7.37</td>
<td>7.26</td>
<td>7.21</td>
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</tr>
<tr>
<td>X_7 = Y_7 + Mtl</td>
<td>0.4491</td>
<td>2.06</td>
<td>0.5961</td>
<td>2.40</td>
<td>0.4594</td>
<td>2.85</td>
<td>0.4613</td>
<td>2.89</td>
<td>0.4632</td>
<td>2.94</td>
</tr>
<tr>
<td>X_8 = Y_8 + Thr</td>
<td>0.9968</td>
<td>0.81</td>
<td>0.9561</td>
<td>1.10</td>
<td>0.958</td>
<td>1.27</td>
<td>0.9585</td>
<td>1.40</td>
<td>0.9553</td>
<td>1.43</td>
</tr>
<tr>
<td>X_9 = Y_9 + e_m</td>
<td>0.3313</td>
<td>4.12</td>
<td>0.4870</td>
<td>33.26</td>
<td>0.6089</td>
<td>20.97</td>
<td>0.7080</td>
<td>7.23</td>
<td>0.8412</td>
<td>4.00</td>
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</table>

Table D2.5: Summary of PARASODE results: normal conditions, H&F model, (R_{300})_{1:2}, slope 1:2, CL=16m OD

(Note that the correlation coefficients between T_p and H_3 is \rho_{T_p-H_3} = 0.5).
### Table D2-6: Summary of PARASODE results

Normal conditions, HSR model, \( R_{\text{max}} \) along with slope 1:2, CL-8m OD

( Note that the correlation coefficient between \( T_p \) and \( H_s \) is \( r_{T_p,H_s} = 0.5 \). )

<table>
<thead>
<tr>
<th>( Q (m^3/s/m) )</th>
<th>( 10^{-1} )</th>
<th>( 10^{-2} )</th>
<th>( 10^{-3} )</th>
<th>( 10^{-4} )</th>
<th>( 10^{-5} )</th>
<th>( 10^{-6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>( \epsilon^2 (%) )</td>
<td>DP</td>
<td>( \epsilon^2 (%) )</td>
<td>DP</td>
<td>( \epsilon^2 (%) )</td>
<td>DP</td>
</tr>
<tr>
<td>( X_1 = T_p )</td>
<td>7.599</td>
<td>---</td>
<td>7.303</td>
<td>---</td>
<td>7.113</td>
<td>---</td>
</tr>
<tr>
<td>( X_2 = H_s )</td>
<td>2.372</td>
<td>---</td>
<td>1.868</td>
<td>---</td>
<td>1.695</td>
<td>---</td>
</tr>
<tr>
<td>( \tilde{Y}<em>1 = Y</em>{1,T_pH_s} )</td>
<td>---</td>
<td>11.13</td>
<td>---</td>
<td>15.79</td>
<td>---</td>
<td>16.86</td>
</tr>
<tr>
<td>( \tilde{Y}<em>2 = Y</em>{2,T_pH_s} )</td>
<td>---</td>
<td>33.37</td>
<td>---</td>
<td>35.09</td>
<td>---</td>
<td>35.1</td>
</tr>
<tr>
<td>( X_3 = Y_{1,A} )</td>
<td>0.000542</td>
<td>0</td>
<td>0.000542</td>
<td>0</td>
<td>0.000542</td>
<td>0</td>
</tr>
<tr>
<td>( X_4 = Y_{1,B} )</td>
<td>7.16</td>
<td>0</td>
<td>7.16</td>
<td>0</td>
<td>7.16</td>
<td>0</td>
</tr>
<tr>
<td>( X_5 = Y_{1,Tide} )</td>
<td>4.191</td>
<td>37.19</td>
<td>3.575</td>
<td>39.33</td>
<td>3.226</td>
<td>41.80</td>
</tr>
<tr>
<td>( X_6 = Y_{1,Surge} )</td>
<td>0.069</td>
<td>1.86</td>
<td>0.022</td>
<td>0.81</td>
<td>0.012</td>
<td>0.57</td>
</tr>
<tr>
<td>( X_7 = Y_{1,Tide} )</td>
<td>0.4964</td>
<td>0.53</td>
<td>0.4939</td>
<td>0.30</td>
<td>0.4966</td>
<td>0.24</td>
</tr>
<tr>
<td>( X_8 = Y_{1,Fr} )</td>
<td>0.9509</td>
<td>0.07</td>
<td>0.9905</td>
<td>0.05</td>
<td>0.9503</td>
<td>0.05</td>
</tr>
<tr>
<td>( X_9 = Y_{1,se} )</td>
<td>0.8053</td>
<td>15.85</td>
<td>0.9048</td>
<td>8.63</td>
<td>0.9488</td>
<td>5.54</td>
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</table>

Results From PARASODE For Wave Overtopping
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>$\sigma (m/s)$</th>
<th>$P_f$ (%)</th>
<th>$R_f$ (%)</th>
<th>$P_P$ (%)</th>
<th>$P_{P'}$ (%)</th>
<th>$P_{P''}$ (%)</th>
<th>$P_{P'''}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4E-08</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>0.2E-08</td>
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<tr>
<td>0.1E-08</td>
<td>—</td>
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</tr>
<tr>
<td>0.0E-09</td>
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</tr>
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</table>

#### Summary of PARASODE Results: Normal Conditions, H/S model, $R_{P_{P''}}$

<table>
<thead>
<tr>
<th>$X = Y = H_{0.3}$</th>
<th>$X = Y = H_{0.3}$</th>
<th>$X = Y = H_{0.3}$</th>
<th>$X = Y = H_{0.3}$</th>
<th>$X = Y = H_{0.3}$</th>
<th>$X = Y = H_{0.3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$ (%)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$P_P$ (%)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$P_{P'}$ (%)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$P_{P''}$ (%)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$P_{P'''}$ (%)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: The correlation coefficient between $e_t$ and $H_{0.3}$ is $r = 0.02$. (p = 0.05)
<table>
<thead>
<tr>
<th>$G$ (m$^2$/cm)</th>
<th>DP</th>
<th>$\epsilon$ (%)</th>
<th>DP</th>
<th>$\epsilon$ (%)</th>
<th>DP</th>
<th>$\epsilon$ (%)</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-1}$</td>
<td>0.7E+06</td>
<td>0.8E+06</td>
<td>0.9E+06</td>
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<td>1.1E+06</td>
<td>1.2E+06</td>
<td>1.3E+06</td>
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<td>$10^{-2}$</td>
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<td>0.7E+06</td>
<td>0.8E+06</td>
<td>0.9E+06</td>
</tr>
<tr>
<td>$10^{-3}$</td>
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<td>0.4E+06</td>
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<td>0.6E+06</td>
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</tbody>
</table>

Table 2.8: Summary of PARASODE results: normal conditions, H.I.P. model, $\rho = 0.5$. Note that the correlation coefficient between $T$ and $H$ is $r = 0.6$. 

---

Results From PARASODE For Wave Overtopping

D2-8
<table>
<thead>
<tr>
<th>Failure Rupture</th>
<th>Standard Deviation of Z</th>
<th>Probability of Failure P (year)</th>
<th>Difference in P between 3rd and 2nd simulations</th>
<th>DP</th>
<th>γ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8E-06</td>
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<td>0.1E-06</td>
<td>0.1E-06</td>
<td>0.1E-06</td>
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<tr>
<td>0.13E-06</td>
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<tr>
<td>0.2E-05</td>
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<td>1.0E-00</td>
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<tr>
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<td>2.0E+00</td>
<td>2.0E+00</td>
<td>2.0E+00</td>
<td>2.0E+00</td>
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<tr>
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<td>4.0E+00</td>
<td>4.0E+00</td>
<td>4.0E+00</td>
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<tr>
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<td>8.0E+00</td>
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<td>8.0E+00</td>
</tr>
<tr>
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<td>1.6E+01</td>
<td>1.6E+01</td>
<td>1.6E+01</td>
<td>1.6E+01</td>
</tr>
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<td>3.2E+01</td>
<td>3.2E+01</td>
<td>3.2E+01</td>
<td>3.2E+01</td>
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<tr>
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<td>2.4E+02</td>
<td>2.4E+02</td>
<td>2.4E+02</td>
</tr>
<tr>
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<td>4.8E+02</td>
<td>4.8E+02</td>
<td>4.8E+02</td>
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<tr>
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<td>1.9E+03</td>
<td>1.9E+03</td>
<td>1.9E+03</td>
<td>1.9E+03</td>
</tr>
<tr>
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<td>3.8E+03</td>
<td>3.8E+03</td>
<td>3.8E+03</td>
<td>3.8E+03</td>
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<td>7.6E+03</td>
<td>7.6E+03</td>
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<tr>
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<td>1.5E+04</td>
<td>1.5E+04</td>
</tr>
<tr>
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<td>3.0E+04</td>
<td>3.0E+04</td>
<td>3.0E+04</td>
<td>3.0E+04</td>
</tr>
<tr>
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<td>6.0E+04</td>
<td>6.0E+04</td>
<td>6.0E+04</td>
</tr>
<tr>
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<td>1.2E+05</td>
<td>1.2E+05</td>
<td>1.2E+05</td>
<td>1.2E+05</td>
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<tr>
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<td>9.6E+05</td>
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</table>

Table D2.3: Summary of PARASODE results: normal conditions. H, T, and P model (P) = 0.05 (0.05 = 4).
<table>
<thead>
<tr>
<th>$Q$ (m$^2$/s/m)</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function</td>
<td>$Z$</td>
<td>0.3E-09</td>
<td>0.1E-08</td>
<td>0.4E-09</td>
<td>0.6E-10</td>
<td>-0.3E-09</td>
</tr>
<tr>
<td>Standard Deviation of $Z$</td>
<td></td>
<td>0.62E+01</td>
<td>0.12E+01</td>
<td>0.23E+02</td>
<td>0.34E+03</td>
<td>0.51E+04</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td></td>
<td>0.7E+06</td>
<td>0.4E+06</td>
<td>0.2E+06</td>
<td>0.2E-06</td>
<td>0.5E+05</td>
</tr>
<tr>
<td>Probability of Failure $P_i$ (%/Year)</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
<td>0.000001</td>
</tr>
<tr>
<td>Difference in $P_i$ Between Last 2 Iterations</td>
<td>0.2E-18</td>
<td>0.2E-13</td>
<td>0.1E-11</td>
<td>0.1E-10</td>
<td>0.9E-11</td>
<td>0.4E-10</td>
</tr>
</tbody>
</table>

$Q$ (m$^2$/s/m) | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1=T_p$</td>
<td>7.053</td>
<td>7.281</td>
<td>7.381</td>
<td>7.416</td>
<td>7.436</td>
<td>7.443</td>
</tr>
<tr>
<td>$X_2=H_s$</td>
<td>3.063</td>
<td>2.832</td>
<td>2.694</td>
<td>2.591</td>
<td>2.517</td>
<td>2.461</td>
</tr>
<tr>
<td>$Y_1=1/T_s$</td>
<td>0.75</td>
<td>2.18</td>
<td>3.86</td>
<td>5.30</td>
<td>6.59</td>
<td>7.57</td>
</tr>
<tr>
<td>$Y_1=H_s$</td>
<td>17.87</td>
<td>25.06</td>
<td>31.96</td>
<td>36.74</td>
<td>40.72</td>
<td>43.73</td>
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<tr>
<td>$X_3=Y_1=A$</td>
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<td>0</td>
<td>0.00542</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>7.16</td>
<td>7.16</td>
<td>7.16</td>
<td>7.16</td>
<td>7.16</td>
</tr>
<tr>
<td>$X_4=Y_1=1/T_d$</td>
<td>4.898</td>
<td>13.82</td>
<td>4.660</td>
<td>20.22</td>
<td>4.904</td>
<td>25.29</td>
</tr>
<tr>
<td>$X_4=Y_2=Surge$</td>
<td>0.332</td>
<td>4.04</td>
<td>0.171</td>
<td>2.62</td>
<td>0.117</td>
<td>2.11</td>
</tr>
<tr>
<td>$X_4=Y_1=1/T_d$</td>
<td>0.0468</td>
<td>1.88</td>
<td>0.4638</td>
<td>1.80</td>
<td>0.4638</td>
<td>1.76</td>
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<tr>
<td>$X_4=Y_2=Fr$</td>
<td>0.9589</td>
<td>0.59</td>
<td>0.9546</td>
<td>0.65</td>
<td>0.9538</td>
<td>0.65</td>
</tr>
<tr>
<td>$X_4=Y_1=1/T_d$</td>
<td>0.3396</td>
<td>6.105</td>
<td>0.4970</td>
<td>4.746</td>
<td>0.6196</td>
<td>3.77</td>
</tr>
<tr>
<td>$X_5=Y_1=1/T_d$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table D2.10: Summary of PARASODE results: normal conditions, HSR model, $(P_{max})_{80}$, slope 1:2, CL=16m OD
(Note that the correlation coefficient between $T_p$ and $H_s$ is $\rho_{T_p,H_s}=0.5$).
### Table C.2: Summary of PARASODE results

<table>
<thead>
<tr>
<th>Q (m³/cm²)</th>
<th>DP</th>
<th>DP</th>
<th>DP</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>10⁻¹</td>
<td>0.89E-06</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
</tr>
<tr>
<td>10⁻²</td>
<td>0.69E-05</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
</tr>
<tr>
<td>10⁻³</td>
<td>0.69E-05</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
</tr>
<tr>
<td>10⁻⁴</td>
<td>0.69E-05</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
</tr>
<tr>
<td>10⁻⁵</td>
<td>0.69E-05</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
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<tr>
<td>10⁻⁶</td>
<td>0.69E-05</td>
<td>6.95E-12</td>
<td>1.43E-02</td>
<td>5.39E-09</td>
</tr>
</tbody>
</table>

Note: Bivariate correlation coefficient between \( T \) and \( H \).
<table>
<thead>
<tr>
<th>Q_w (m/s)</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.113</td>
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<tr>
<td>1.000</td>
<td>2.000</td>
<td>1.000</td>
<td>0.500</td>
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<td>0.125</td>
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<tr>
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<td>0.050</td>
<td>0.025</td>
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<td>0.00625</td>
<td>0.003125</td>
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</table>

Table D2.12: Summary of PARASODE results. Normal distribution. Q_w = 0.001, H_L = 1.0m, p = 0.05.
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>G (m³/sec)</th>
<th>D²-13</th>
<th>DP</th>
<th>DP</th>
<th>DP</th>
<th>DP</th>
<th>DP</th>
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</thead>
<tbody>
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<td>0.6E+1</td>
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<tr>
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</tr>
<tr>
<td>100.0</td>
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<td>0.6E+1</td>
<td>0.6E+1</td>
<td>0.6E+1</td>
<td>0.6E+1</td>
<td>0.6E+1</td>
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</table>

Table D2.13: Summary of PARASODE results for normal conditions. Where Y = model, D = data, and H = depth.
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>10⁻¹</th>
<th>10⁻²</th>
<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>0.2E07</td>
<td>-0.4E08</td>
<td>-0.1E08</td>
<td>0.3E-11</td>
<td>0.8E-13</td>
<td>-0.1E-12</td>
</tr>
<tr>
<td>Standard Deviation of Z</td>
<td>0.881E-01</td>
<td>0.16E-01</td>
<td>0.24E-02</td>
<td>0.3E-03</td>
<td>0.44E-04</td>
<td>0.54E-05</td>
</tr>
<tr>
<td>Relative Accuracy of Z (%)</td>
<td>0.1E-05</td>
<td>0.5E-05</td>
<td>0.1E-05</td>
<td>0.2E-05</td>
<td>0.8E-07</td>
<td>0.4E-05</td>
</tr>
<tr>
<td>Probability of Failure P_f (%/Year)</td>
<td>0.000012</td>
<td>0.017938</td>
<td>0.397621</td>
<td>1.97956+</td>
<td>5.286991+</td>
<td>10.0E+09</td>
</tr>
<tr>
<td>Difference in P_f Between Last 2 Iterations</td>
<td>0.5E-14</td>
<td>0.1E-10</td>
<td>0.3E-09</td>
<td>0.2E-09</td>
<td>0.1E-09</td>
<td>0.8E-09</td>
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</table>

### Table D2.14: Summary of PARASODE results: normal conditions, Owen's model, slope 12, CL=14m OD

(Note that the correlation coefficient between T_m and H_s is \( r_{CH} = 0.85 \).)
<table>
<thead>
<tr>
<th>Failure Function</th>
<th>Standard Deviation of E(i)</th>
<th>Probability of Failure P(i)%/Year</th>
<th>Difference in P Between Last 2 Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_i ) (m/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0E-1</td>
<td>2.3E-8</td>
<td>6.8E-3</td>
<td></td>
</tr>
<tr>
<td>1.0E-2</td>
<td>2.5E-8</td>
<td>1.0E-3</td>
<td></td>
</tr>
<tr>
<td>1.0E-3</td>
<td>2.6E-8</td>
<td>1.4E-3</td>
<td></td>
</tr>
<tr>
<td>1.0E-4</td>
<td>2.7E-8</td>
<td>1.8E-3</td>
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</tr>
<tr>
<td>1.0E-5</td>
<td>2.8E-8</td>
<td>2.2E-3</td>
<td></td>
</tr>
<tr>
<td>1.0E-6</td>
<td>2.9E-8</td>
<td>2.6E-3</td>
<td></td>
</tr>
<tr>
<td>1.0E-7</td>
<td>3.0E-8</td>
<td>3.0E-3</td>
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<tr>
<td>1.0E-8</td>
<td>3.1E-8</td>
<td>3.4E-3</td>
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</tr>
<tr>
<td>1.0E-9</td>
<td>3.2E-8</td>
<td>3.8E-3</td>
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**Summary of PARASODE Results**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_i ) (m/s)</td>
<td>1.0E-1</td>
</tr>
<tr>
<td>Standard Deviation of E(i)</td>
<td>2.3E-8</td>
</tr>
<tr>
<td>Probability of Failure P(i)%/Year</td>
<td>6.8E-3</td>
</tr>
<tr>
<td>Difference in P Between Last 2 Iterations</td>
<td>6.8E-3</td>
</tr>
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</table>

Note: The correlation coefficients for the model are: \( T = 0.95 \) and \( H = 0.90 \).
<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>10⁻¹</th>
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<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>0.3E05</td>
<td>0.6E05</td>
<td>-0.1E05</td>
<td>-0.2E05</td>
<td>+0.4E05</td>
<td>-0.6E05</td>
</tr>
<tr>
<td>Standard Deviation of Z</td>
<td>0.193</td>
<td>0.405E01</td>
<td>0.717E02</td>
<td>0.121E02</td>
<td>0.203E03</td>
<td>0.341E04</td>
</tr>
<tr>
<td>Relative Accuracy of Z (%)</td>
<td>0.2E04</td>
<td>0.2E04</td>
<td>0.3E05</td>
<td>0.3E05</td>
<td>0.4E05</td>
<td>0.4E05</td>
</tr>
<tr>
<td>Probability of Failure Pₚ (%/Year)</td>
<td>10.4822E3</td>
<td>33.3049E3</td>
<td>50.0318E7</td>
<td>59.8256E7</td>
<td>65.5018E7</td>
<td>68.7066E3</td>
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<tr>
<td>Difference in Pₚ between last 2 iterations</td>
<td>0.5E-07</td>
<td>0.3E-07</td>
<td>0.1E-08</td>
<td>0.3E-08</td>
<td>0.6E-08</td>
<td>0.8E-08</td>
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<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>10⁻¹</th>
<th>10⁻²</th>
<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xₚ = Tₚ</td>
<td>7.134</td>
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<td>6.549</td>
<td>—</td>
<td>6.262</td>
<td>—</td>
</tr>
<tr>
<td>Xₚ = Hₙ</td>
<td>1.563</td>
<td>—</td>
<td>1.263</td>
<td>—</td>
<td>0.986</td>
<td>—</td>
</tr>
<tr>
<td>Yₚ = Tₚ, Hₙ</td>
<td>—</td>
<td>57.12</td>
<td>—</td>
<td>46.26</td>
<td>—</td>
<td>39.83</td>
</tr>
<tr>
<td>Yₚ = Tₚ, Hₙ</td>
<td>—</td>
<td>38.43</td>
<td>—</td>
<td>49.47</td>
<td>—</td>
<td>56.29</td>
</tr>
<tr>
<td>Xₚ = Yₚ = A</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
</tr>
<tr>
<td>Xₚ = Yₚ = B</td>
<td>0.17</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>Xₚ = Yₚ = 8WL</td>
<td>5.533</td>
<td>0.77</td>
<td>5.618</td>
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<td>5.612</td>
<td>1.12</td>
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<tr>
<td>Xₚ = Yₚ = Tₚ, Hₙ</td>
<td>0.4979</td>
<td>0.11</td>
<td>0.4998</td>
<td>0.12</td>
<td>0.5000</td>
<td>0.15</td>
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<tr>
<td>Xₚ = Yₚ = r</td>
<td>0.9501</td>
<td>0.01</td>
<td>0.9501</td>
<td>0.02</td>
<td>0.9500</td>
<td>0.02</td>
</tr>
<tr>
<td>Xₚ = Yₚ = e</td>
<td>0.9691</td>
<td>3.54</td>
<td>1.0051</td>
<td>3.22</td>
<td>1.0222</td>
<td>2.59</td>
</tr>
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</table>

Table D2.16: Summary of PARASODE results: extreme conditions, H&R model, (Pₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚ波特ₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚ波特ₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚ波特ₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚₚ波特ₚₚₚₚₚₚₚ波特ₚportion of the coefficient between Tₚ and Hₙ is ρₑ = 0.5.}
### Table D2.17: Summary of PARASODE results: extreme conditions, H&R model, (R_{10m})_{20%}, slope 1:2, CL-10m OD

Note: The correlation coefficient between Tp and Hs is ρ_{10} = 0.95.

<table>
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<tr>
<th>Q (m²/s/m)</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>-0.1E-05</td>
<td>-0.1E-06</td>
<td>-0.1E-06</td>
<td>-0.1E-06</td>
<td>-0.1E-06</td>
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<tr>
<td>Standard Deviation of Z</td>
<td>0.1E-01</td>
<td>0.1E-01</td>
<td>0.1E-01</td>
<td>0.1E-01</td>
<td>0.1E-01</td>
<td>0.1E-01</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td>0.2E-04</td>
<td>0.2E-04</td>
<td>0.2E-04</td>
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<tr>
<td>Probability of Failure P_f (%/Year)</td>
<td>2.069E07</td>
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<tr>
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</tr>
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<td>z = 1.0h</td>
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<tr>
<td>Q (m³/year)</td>
<td>DP</td>
<td>Q (m³/year)</td>
<td>DP</td>
<td>Q (m³/year)</td>
<td>DP</td>
<td>Q (m³/year)</td>
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<tr>
<td>Y = f H</td>
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<td>1.0</td>
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</table>

Table D2.18: Summary of PARASODE results: extreme conditions, H.P. model, (Rmax)_ave = \[\text{shape} 1:2, C1=12\text{m}, \text{O.D.}\]
<table>
<thead>
<tr>
<th>(b (m^2/s/m))</th>
<th>(10^{-1})</th>
<th>(10^{-2})</th>
<th>(10^{-3})</th>
<th>(10^{-4})</th>
<th>(10^{-5})</th>
<th>(10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function (Z)</td>
<td>0.2E-05</td>
<td>-0.1E-05</td>
<td>-0.1E-05</td>
<td>-0.1E-07</td>
<td>-0.2E-08</td>
<td>-0.3E-09</td>
</tr>
<tr>
<td>Standard Deviation of (Z)</td>
<td>0.88E-01</td>
<td>0.28E-01</td>
<td>0.73E-02</td>
<td>0.14E-02</td>
<td>0.36E-03</td>
<td>0.47E-04</td>
</tr>
<tr>
<td>Relative Accuracy of (\beta) (%)</td>
<td>0.5E-04</td>
<td>0.1E-04</td>
<td>0.1E-04</td>
<td>0.5E-05</td>
<td>0.2E-06</td>
<td>0.1E-06</td>
</tr>
<tr>
<td>Probability of Failure (P_f) (%)/Year</td>
<td>0.00E+00</td>
<td>0.01E+00</td>
<td>2.24E+00</td>
<td>3.73E+00</td>
<td>4.78E+07</td>
<td>5.46E+11</td>
</tr>
<tr>
<td>Difference in (P_f) Between Last 2 Iterations</td>
<td>0.4E-08</td>
<td>0.5E-08</td>
<td>0.1E-07</td>
<td>0.7E-08</td>
<td>0.4E-08</td>
<td>0.2E-08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(Q (m^2/s/m))</th>
<th>(10^{-1})</th>
<th>(10^{-2})</th>
<th>(10^{-3})</th>
<th>(10^{-4})</th>
<th>(10^{-5})</th>
<th>(10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_1=\hat{X}_p)</td>
<td>7.84E9</td>
<td>—</td>
<td>7.85E3</td>
<td>—</td>
<td>7.66E0</td>
<td>—</td>
</tr>
<tr>
<td>(X_2=H_s)</td>
<td>3.36E3</td>
<td>—</td>
<td>3.04E0</td>
<td>—</td>
<td>2.73E0</td>
<td>—</td>
</tr>
<tr>
<td>(Y_1=\hat{X}_p)</td>
<td>—</td>
<td>10.76E0</td>
<td>—</td>
<td>33.21E0</td>
<td>—</td>
<td>56.21E0</td>
</tr>
<tr>
<td>(Y_2=\hat{X}_p)</td>
<td>—</td>
<td>40.84E0</td>
<td>—</td>
<td>52.25E0</td>
<td>—</td>
<td>41.39E0</td>
</tr>
<tr>
<td>(X_3=\hat{X})</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
<td>0.00753</td>
<td>0</td>
</tr>
<tr>
<td>(X_4=\hat{X}e)</td>
<td>4.17E0</td>
<td>0</td>
<td>4.17E0</td>
<td>0</td>
<td>4.17E0</td>
<td>0</td>
</tr>
<tr>
<td>(X_5=\hat{X}e)</td>
<td>5.84E3</td>
<td>4.91E0</td>
<td>5.66E0</td>
<td>5.63E0</td>
<td>5.62E0</td>
<td>5.62E0</td>
</tr>
<tr>
<td>(X_6=\hat{X}e)</td>
<td>0.47E0</td>
<td>0.45E0</td>
<td>0.45E0</td>
<td>0.45E0</td>
<td>0.45E0</td>
<td>0.45E0</td>
</tr>
<tr>
<td>(X_7=\hat{X}e)</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
</tr>
<tr>
<td>(X_8=\hat{X}e)</td>
<td>0.57E0</td>
<td>0.83E0</td>
<td>0.83E0</td>
<td>0.83E0</td>
<td>0.83E0</td>
<td>0.83E0</td>
</tr>
</tbody>
</table>

Table D2.19: Summary of PARASODE results: extreme conditions, HSR model, \(R=\hat{X}_p\) at 35°, slope 1:2, CL=14m OD
(Note that the correlation coefficient between \(\hat{X}_p\) and \(H_s\) is \(\rho_{X,H} = 0.2\).)
### Results From PARASODE For Wave Overtopping

#### Table 10.20: Summary of PARASODE results: extreme conditions, H-R model, \( P_{\text{main}} = \rho_r = 0.95 \).

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \text{ verbosity} )</th>
<th>( \text{ scour} )</th>
<th>( \text{ DP} )</th>
<th>( \text{ DP} % )</th>
<th>( \text{ DP} )</th>
<th>( \text{ DP} % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X = T = H )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( X = V = A )</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>( X = Y = B )</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>( X = V = H )</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>( X = Y = C )</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Note: Table 10.20's table row indicates the correlation coefficient between \( T \) and \( H \), \( \rho_r = 0.95 \).
<table>
<thead>
<tr>
<th>Q (m²/σm)</th>
<th>10⁻¹</th>
<th>10⁻²</th>
<th>10⁻³</th>
<th>10⁻⁴</th>
<th>10⁻⁵</th>
<th>10⁻⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>-0.6E-05</td>
<td>-0.5E-05</td>
<td>-0.1E-05</td>
<td>-0.1E-07</td>
<td>-0.2E-08</td>
<td>-0.3E-09</td>
</tr>
<tr>
<td>Standard Deviation of Z</td>
<td>0.197</td>
<td>0.373E-01</td>
<td>0.577E-02</td>
<td>0.810E-03</td>
<td>0.108E-03</td>
<td>0.139E-04</td>
</tr>
<tr>
<td>Relative Accuracy of B (%)</td>
<td>0.2E-05</td>
<td>0.2E-05</td>
<td>0.3E-05</td>
<td>0.4E-05</td>
<td>0.4E-05</td>
<td>0.4E-05</td>
</tr>
<tr>
<td>Probability of Failure P (%/Year)</td>
<td>9.9E-06</td>
<td>3.29E-06</td>
<td>5.2E-06</td>
<td>6.3E-06</td>
<td>7.3E-06</td>
<td>7.9E-06</td>
</tr>
<tr>
<td>Difference in P between last 2 iterations</td>
<td>0.5E-08</td>
<td>0.3E-08</td>
<td>0.7E-09</td>
<td>0.5E-09</td>
<td>0.9E-09</td>
<td>0.1E-07</td>
</tr>
<tr>
<td>Q (m²/σm)</td>
<td>10⁻¹</td>
<td>10⁻²</td>
<td>10⁻³</td>
<td>10⁻⁴</td>
<td>10⁻⁵</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>X₀=T₀</td>
<td>7.196</td>
<td>6.567</td>
<td>6.228</td>
<td>6.022</td>
<td>5.882</td>
<td>5.784</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>2.012</td>
<td>1.271</td>
<td>0.956</td>
<td>0.7966</td>
<td>0.7046</td>
<td>0.6475</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>—</td>
<td>38.75</td>
<td>—</td>
<td>34.47</td>
<td>—</td>
<td>31.50</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>—</td>
<td>38.41</td>
<td>—</td>
<td>61.15</td>
<td>—</td>
<td>63.93</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>—</td>
<td>49.44</td>
<td>—</td>
<td>65.93</td>
<td>—</td>
<td>65.95</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>0.00542</td>
<td>0</td>
<td>0.00542</td>
<td>0</td>
<td>0.00542</td>
<td>0</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>7.16</td>
<td>0</td>
<td>7.16</td>
<td>0</td>
<td>7.16</td>
<td>0</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>5.635</td>
<td>0.84</td>
<td>5.618</td>
<td>0.87</td>
<td>5.611</td>
<td>1.05</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>0.4578</td>
<td>0.11</td>
<td>0.4952</td>
<td>0.12</td>
<td>0.5001</td>
<td>0.15</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>0.9502</td>
<td>0.01</td>
<td>0.9501</td>
<td>0.02</td>
<td>0.9500</td>
<td>0.02</td>
</tr>
<tr>
<td>X₀=H₀</td>
<td>0.365</td>
<td>3.07</td>
<td>1.01</td>
<td>2.97</td>
<td>1.0271</td>
<td>2.96</td>
</tr>
</tbody>
</table>

Table D2.21: Summary of PARASODE results: extreme conditions, HSR model, (R_{max})_{50%}, slope 12, CL=8m OD
(Note that the correlation coefficient between T₀ and H₀ is \( \rho_{T,H} = 0.8 \).)
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>Failure Function</th>
<th>$Q$ (m$^3$/min)</th>
<th>$10^1$</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>2.71E4</td>
<td>1.59E4</td>
<td>5.16E3</td>
<td>1.95E3</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>5.69E3</td>
<td>3.85E3</td>
<td>1.41E3</td>
<td>4.93E2</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>7.16E2</td>
<td>2.68E2</td>
<td>8.91E1</td>
<td>3.24E1</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>9.02E1</td>
<td>3.38E1</td>
<td>1.12E1</td>
<td>3.94E0</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>1.00E0</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>1.00E0</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>1.00E0</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>1.00E0</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
</tr>
<tr>
<td>$x_i$ = $H_i$</td>
<td>$H_o$</td>
<td>1.00E0</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
</tr>
</tbody>
</table>

### Table D2.21: Summary of PARASODE results: extreme conditions, H&R model. (Note: data correlation code is between $x$ and $y$).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$Q$ (m$^3$/min)</th>
<th>$10^1$</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>1.0</td>
<td>$2.71E4$</td>
<td>1.59E4</td>
<td>5.16E3</td>
<td>1.95E3</td>
<td>6.59E2</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0</td>
<td>$5.69E3$</td>
<td>3.85E3</td>
<td>1.41E3</td>
<td>4.93E2</td>
<td>1.71E2</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0</td>
<td>$7.16E2$</td>
<td>2.68E2</td>
<td>8.91E1</td>
<td>3.24E1</td>
<td>1.12E1</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0</td>
<td>$9.02E1$</td>
<td>3.38E1</td>
<td>1.12E1</td>
<td>3.94E0</td>
<td>1.39E0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0</td>
<td>$1.00E0$</td>
<td>3.57E-1</td>
<td>1.18E-1</td>
<td>4.07E-2</td>
<td>1.40E-2</td>
</tr>
</tbody>
</table>

D2-22
<table>
<thead>
<tr>
<th>$Q$ ($m^2/s$)</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>DP</td>
<td>$σ^2$ (%)</td>
<td>DP</td>
<td>$σ^2$ (%)</td>
<td>DP</td>
<td>$σ^2$ (%)</td>
</tr>
<tr>
<td>$X_1 = T_0$</td>
<td>7.903</td>
<td>7.509</td>
<td>7.183</td>
<td>6.965</td>
<td>6.857</td>
<td>6.770</td>
</tr>
<tr>
<td>$X_2 = H_3$</td>
<td>3.196</td>
<td>2.516</td>
<td>2.049</td>
<td>1.781</td>
<td>1.618</td>
<td>1.512</td>
</tr>
<tr>
<td>$X_3 = T_{101} H_3$</td>
<td>49.82</td>
<td>39.23</td>
<td>38.35</td>
<td>41.13</td>
<td>43.75</td>
<td>45.80</td>
</tr>
<tr>
<td>$X_4 = Y_{101}$</td>
<td>0.000542</td>
<td>0</td>
<td>0</td>
<td>0.000542</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X_5 = Y_{101}$</td>
<td>0</td>
<td>0</td>
<td>0.00542</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X_6 = Y_{101}$</td>
<td>5.709</td>
<td>5.631</td>
<td>5.622</td>
<td>5.619</td>
<td>5.618</td>
<td>5.617</td>
</tr>
<tr>
<td>$X_7 = Y_{101}$</td>
<td>0.4881</td>
<td>0.63</td>
<td>0.4965</td>
<td>0.4977</td>
<td>0.4983</td>
<td>0.4986</td>
</tr>
<tr>
<td>$X_8 = Y_{101}$</td>
<td>0.9511</td>
<td>0.14</td>
<td>0.9504</td>
<td>0.9502</td>
<td>0.9502</td>
<td>0.9502</td>
</tr>
<tr>
<td>$X_9 = Y_{101}$</td>
<td>0.7682</td>
<td>25.74</td>
<td>9.44</td>
<td>5.38</td>
<td>0.9595</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Table D2.23: Summary of PARASODE results: extreme conditions, HSR model, $(R_{max})_{100}$, slope 1:2, CL-12m OD
(Note that the correlation coefficient between $T_p$ and $H_3$ is $p_{12} = 0.6$).
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>G (m^3/s/m)</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>0.4E-07</td>
<td>-0.6E-03</td>
<td>-0.1E-03</td>
<td>-0.2E-07</td>
<td>-0.4E-08</td>
<td>-0.5E-09</td>
</tr>
<tr>
<td>Standard Deviation of Z</td>
<td>0.79E-01</td>
<td>0.25E-01</td>
<td>0.17E-02</td>
<td>0.98E-03</td>
<td>0.15E-03</td>
<td>0.22E-03</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td>0.6E-04</td>
<td>0.1E-04</td>
<td>0.8E-05</td>
<td>0.3E-05</td>
<td>0.2E-05</td>
<td>0.3E-05</td>
</tr>
<tr>
<td>Probability of Failure P_t (%/Year)</td>
<td>0.003827</td>
<td>0.83008</td>
<td>3.653881</td>
<td>6.691976</td>
<td>9.507379</td>
<td>11.81352</td>
</tr>
<tr>
<td>Difference in P_t Between Last 2 Iterations</td>
<td>0.3E-09</td>
<td>0.8E-08</td>
<td>0.1E-07</td>
<td>0.5E-08</td>
<td>0.5E-08</td>
<td>0.5E-08</td>
</tr>
</tbody>
</table>

### Table D2.24: Summary of PARASODE results: extreme conditions, H&R model, (R_{max})_{0.01}, slope 1:2, CL-14m OD

(Note that the correlation coefficient between T_p and H_3 is ρ_{X,Y} = 0.6).
## Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>Std. Deviation of F(x) (m)</th>
<th>Failure Location (m)</th>
<th>Standard Deviation of F(x) (m)</th>
<th>Probability of Failure, P (m)</th>
<th>Difference in P</th>
<th>Between Last Calculations</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2E-0</td>
<td>T=1.3</td>
<td>0.2E-0</td>
<td>2.7E-0</td>
<td>7.1E-0</td>
<td>2.7E-0</td>
<td>0</td>
</tr>
<tr>
<td>0.2E-0</td>
<td>T=1.3</td>
<td>0.2E-0</td>
<td>3.7E-0</td>
<td>7.1E-0</td>
<td>3.7E-0</td>
<td>0</td>
</tr>
<tr>
<td>0.2E-0</td>
<td>T=1.3</td>
<td>0.2E-0</td>
<td>4.7E-0</td>
<td>7.1E-0</td>
<td>4.7E-0</td>
<td>0</td>
</tr>
<tr>
<td>0.2E-0</td>
<td>T=1.3</td>
<td>0.2E-0</td>
<td>5.7E-0</td>
<td>7.1E-0</td>
<td>5.7E-0</td>
<td>0</td>
</tr>
<tr>
<td>0.2E-0</td>
<td>T=1.3</td>
<td>0.2E-0</td>
<td>6.7E-0</td>
<td>7.1E-0</td>
<td>6.7E-0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table D2.25:** Summary of PARASODE results for extreme conditions. H1P model, R1=0.0, slope 1:2, CI=0.0 OD.

(Note that the correlation coefficient between T and H is p = 0.05.)
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>Q (m³/s/m)</th>
<th>(10^{-1})</th>
<th>(10^{-2})</th>
<th>(10^{-3})</th>
<th>(10^{-4})</th>
<th>(10^{-5})</th>
<th>(10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function Z</td>
<td>0.2E-05</td>
<td>-0.4E-05</td>
<td>-0.1E-05</td>
<td>-0.8E-05</td>
<td>-0.9E-05</td>
<td>-0.1E-05</td>
</tr>
<tr>
<td>Standard Deviation of Z</td>
<td>0.173</td>
<td>0.244E-01</td>
<td>0.260E-02</td>
<td>0.233E-03</td>
<td>0.230E-04</td>
<td>0.231E-05</td>
</tr>
<tr>
<td>Relative Accuracy of Beta (%)</td>
<td>0.2E-04</td>
<td>0.5E-05</td>
<td>0.4E-05</td>
<td>0.6E-05</td>
<td>0.2E-05</td>
<td>0.1E-05</td>
</tr>
<tr>
<td>Probability of Failure P_f (%/Year)</td>
<td>17.252E4</td>
<td>98.131E05</td>
<td>85.366E94</td>
<td>97.651E10</td>
<td>99.862E36</td>
<td>99.957E25</td>
</tr>
<tr>
<td>Difference in P_f Between Last 2 Iterations</td>
<td>0.5E-07</td>
<td>0.3E-08</td>
<td>0.1E-07</td>
<td>0.7E-10</td>
<td>0.2E-10</td>
<td>0.6E-13</td>
</tr>
</tbody>
</table>

### Table D2.26: Summary of PARASODE results: extreme conditions, Owen's model, slope 1:2, CL-8m OD (Note that the correlation coefficient between \(T_m\) and \(H_3\) is \(\rho_{T_m,H_3} = 0.5\)).
## Results From PARASODE For Wave Overtopping

### Table D2.27: Summary of PARASODE results: extreme conditions, Owe's model, slope 1:2, CL=10m OD

<table>
<thead>
<tr>
<th>$Q$ (m$^3$)</th>
<th>10^{-1}</th>
<th>10^{-2}</th>
<th>10^{-3}</th>
<th>10^{-4}</th>
<th>10^{-5}</th>
<th>10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function $Z$</td>
<td>0.5E-05</td>
<td>0.5E-05</td>
<td>-0.4E-07</td>
<td>-0.1E-07</td>
<td>-0.2E-08</td>
<td>-0.1E-09</td>
</tr>
<tr>
<td>Standard Deviation of $Z$</td>
<td>0.153</td>
<td>0.274E-01</td>
<td>0.361E-02</td>
<td>0.417E-03</td>
<td>0.433E-04</td>
<td>0.418E-05</td>
</tr>
<tr>
<td>Relative Accuracy of $Z$ (%)</td>
<td>0.2E-04</td>
<td>0.4E-04</td>
<td>0.6E-05</td>
<td>0.2E-05</td>
<td>0.4E-05</td>
<td>0.7E-05</td>
</tr>
<tr>
<td>Probability of Failure $P_f$ (%/Year)</td>
<td>2.924000</td>
<td>20.359886</td>
<td>45.860913</td>
<td>66.602770</td>
<td>84.06847</td>
<td>94.10410</td>
</tr>
<tr>
<td>Difference in $P_f$ Between Last 2 Iterations</td>
<td>0.2E-07</td>
<td>0.9E-07</td>
<td>0.3E-08</td>
<td>0.4E-08</td>
<td>0.1E-07</td>
<td>0.1E-08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q$ (m$^3$)</th>
<th>DP</th>
<th>$\epsilon^2$ (%)</th>
<th>DP</th>
<th>$\epsilon^2$ (%)</th>
<th>DP</th>
<th>$\epsilon^2$ (%)</th>
<th>DP</th>
<th>$\epsilon^2$ (%)</th>
<th>DP</th>
<th>$\epsilon^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1=T_m$</td>
<td>6.521</td>
<td>5.549</td>
<td>4.970</td>
<td>4.549</td>
<td>4.211</td>
<td>3.937</td>
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<tr>
<td>$X_2=H_3$</td>
<td>2.484</td>
<td>1.536</td>
<td>1.042</td>
<td>0.775</td>
<td>0.525</td>
<td>0.546</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$Y_1=T_m+H_3$</td>
<td>89.98</td>
<td>87.49</td>
<td>79.57</td>
<td>73.25</td>
<td>69.29</td>
<td>65.22</td>
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<tr>
<td>$Y_2=H_3$</td>
<td>2.40</td>
<td>6.01</td>
<td>13.23</td>
<td>17.67</td>
<td>17.78</td>
<td>14.84</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$X_3=Y_1=A$</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_5=Y_3=VWL$</td>
<td>5.647</td>
<td>5.621</td>
<td>5.613</td>
<td>5.606</td>
<td>5.597</td>
<td>5.586</td>
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<tr>
<td>$X_6=Y_4=r$</td>
<td>0.9504</td>
<td>0.9502</td>
<td>0.9500</td>
<td>0.9499</td>
<td>0.9498</td>
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</tr>
<tr>
<td>$X_7=Y_5=b$</td>
<td>0.9450</td>
<td>0.9448</td>
<td>0.9447</td>
<td>0.9446</td>
<td>0.9445</td>
<td>0.9444</td>
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</table>

Note that the correlation coefficient between $T_m$ and $H_3$ is $\rho_{T,H} = 0.05$. 
## Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>$Q$ ($m^3/s/m$)</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_i$</td>
<td>-0.9E-05</td>
<td>0.3E-05</td>
<td>0.1E-05</td>
<td>-0.2E-08</td>
<td>-0.1E-08</td>
<td>-0.2E-08</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>0.133</td>
<td>0.266E01</td>
<td>0.317E02</td>
<td>0.473E03</td>
<td>0.546E04</td>
<td>0.591E05</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4E04</td>
<td>0.2E04</td>
<td>0.2E05</td>
<td>0.4E06</td>
<td>0.2E06</td>
<td>0.3E06</td>
</tr>
<tr>
<td>$P_i$ (%)</td>
<td>0.374653</td>
<td>6.789809</td>
<td>21.909680</td>
<td>40.908400</td>
<td>58.770594</td>
<td>73.418331</td>
</tr>
<tr>
<td>$\Delta P$ (%)</td>
<td>0.1E-07</td>
<td>0.3E-07</td>
<td>0.4E-08</td>
<td>0.4E-08</td>
<td>0.1E-08</td>
<td>0.6E-08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q$ ($m^3/s/m$)</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = T_{eq}$</td>
<td>7.34E5</td>
<td>6.13E5</td>
<td>5.50E3</td>
<td>5.06E3</td>
<td>4.37E3</td>
<td>4.45E3</td>
</tr>
<tr>
<td>$X_2 = H_9$</td>
<td>2.98E2</td>
<td>2.10E2</td>
<td>1.48E1</td>
<td>1.11E1</td>
<td>0.88E0</td>
<td>0.72E0</td>
</tr>
<tr>
<td>$Y_1 = (T_{eq}, H_9)$</td>
<td>-81.70</td>
<td>-89.66</td>
<td>-86.8+</td>
<td>-81.01</td>
<td>75.87</td>
<td>72.12</td>
</tr>
<tr>
<td>$Y_2 = (T_{eq}, H_9)$</td>
<td>-5.12</td>
<td>-2.30</td>
<td>-6.07</td>
<td>-11.37</td>
<td>-15.53</td>
<td>-17.50</td>
</tr>
<tr>
<td>$X_3 = Y_i = A$</td>
<td>0.01E-1</td>
<td>0.01E-1</td>
<td>0.01E-1</td>
<td>0.01E-1</td>
<td>0.01E-1</td>
<td>0.01E-1</td>
</tr>
<tr>
<td>$X_4 = Y_i = B$</td>
<td>2.17E1</td>
<td>2.17E1</td>
<td>2.17E1</td>
<td>2.17E1</td>
<td>2.17E1</td>
<td>2.17E1</td>
</tr>
<tr>
<td>$X_5 = Y_i = 0$</td>
<td>5.65E2</td>
<td>5.62E2</td>
<td>5.61E5</td>
<td>5.61E5</td>
<td>5.61E5</td>
<td>5.61E5</td>
</tr>
<tr>
<td>$X_6 = Y_i = R_0$</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
<td>0.95E0</td>
</tr>
<tr>
<td>$X_7 = Y_i = W$</td>
<td>0.88E5</td>
<td>0.95E5</td>
<td>0.98E5</td>
<td>1.00E5</td>
<td>1.02E5</td>
<td>8.23</td>
</tr>
</tbody>
</table>

Table D2.28: Summary of PARASODE results: extreme conditions, Owen's model, slope 1:2, CL=12m OD (Note that the correlation coefficient between $T_{eq}$ and $H_9$ is $\rho_{T,H} = 0.8$).
### Results From PARASODE For Wave Overtopping

<table>
<thead>
<tr>
<th>$Q (m^3/s/m)$</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Failure Function $Z$</strong></td>
<td>-0.1E-05</td>
<td>-0.2E-05</td>
<td>0.4E-01</td>
<td>0.2E-01</td>
<td>0.5E-09</td>
<td>-0.1E-09</td>
</tr>
<tr>
<td><strong>Standard Deviation of $Z$</strong></td>
<td>0.111</td>
<td>0.243E-01</td>
<td>0.365E-02</td>
<td>0.481E-03</td>
<td>0.581E-04</td>
<td>0.664E-05</td>
</tr>
<tr>
<td><strong>Relative Accuracy of Beta (%)</strong></td>
<td>0.5E-04</td>
<td>0.1E-04</td>
<td>0.2E-04</td>
<td>0.3E-05</td>
<td>0.5E-06</td>
<td>0.2E-06</td>
</tr>
<tr>
<td><strong>Probability of Failure $P_f$ (%)/Year</strong></td>
<td>0.031264</td>
<td>2.063590</td>
<td>9.979423</td>
<td>22.982953</td>
<td>36.001657</td>
<td>52.956676</td>
</tr>
<tr>
<td><strong>Difference in $P_f$ Between Last Two Iterations</strong></td>
<td>0.2E-08</td>
<td>0.1E-07</td>
<td>0.5E-07</td>
<td>0.6E-10</td>
<td>0.5E-09</td>
<td>0.5E-09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q (m^3/s/m)$</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1=T_m$</td>
<td>8.127</td>
<td>6.668</td>
<td>5.848</td>
<td>5.476</td>
<td>5.125</td>
<td>4.845</td>
</tr>
<tr>
<td>$X_2=H_h$</td>
<td>3.320</td>
<td>2.370</td>
<td>1.910</td>
<td>1.499</td>
<td>1.199</td>
<td>0.9536</td>
</tr>
<tr>
<td>$Y_1=\hat{T}^6, H_h$</td>
<td>73.82</td>
<td>87.59</td>
<td>89.03</td>
<td>86.08</td>
<td>81.75</td>
<td>77.55</td>
</tr>
<tr>
<td>$Y_2=\hat{T}^6, H_h$</td>
<td>5.58</td>
<td>2.08</td>
<td>2.76</td>
<td>6.11</td>
<td>10.27</td>
<td>13.89</td>
</tr>
</tbody>
</table>

| $X_3=T_m$ | 0.0117 | 0 | 0.0117 | 0 | 0.0117 | 0 |
| $X_5=Y_2$ | 5.678 | 5.632 | 5.630 | 5.616 | 5.614 | 5.611 |
| $X_6=T_m$ | 0.9512 | 0.11 | 0.9503 | 0.9503 | 0.9503 | 0.9503 |
| $X_7=\hat{T}^6$ | 0.8150 | 19.71 | 0.9253 | 0.9642 | 0.9865 | 1.0042 |

Table D2.29: Summary of PARASODE results: extreme conditions, Owen’s model, slope 1:2, CL=4m OD

(Note that the correlation coefficient between $T_m$ and $H_h$ is $\rho_{T,H} = 0.6$.)
### Table D2.30: Summary of PARASODE results; extreme conditions; Owen's model, slope 1:2, CL-16m OD

Note that the correlation coefficient between $T_e$ and $H_d$ is $r_e = 0.95$.

<table>
<thead>
<tr>
<th>$Q (m^2/s/cm)$</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Function $Z$</td>
<td>$0.4E-06$</td>
<td>$-0.1E-05$</td>
<td>$-0.2E-05$</td>
<td>$-0.1E-07$</td>
<td>$0.2E-06$</td>
<td>$0.2E-09$</td>
</tr>
<tr>
<td>Standard Deviation of $Z$</td>
<td>$0.94E-01$</td>
<td>$0.54E-01$</td>
<td>$0.35E-02$</td>
<td>$0.47E-03$</td>
<td>$0.38E-04$</td>
<td>$0.68E-05$</td>
</tr>
<tr>
<td>Relative Accuracy of $Z$ (%)</td>
<td>$0.4E-05$</td>
<td>$0.2E-04$</td>
<td>$0.4E-05$</td>
<td>$0.2E-05$</td>
<td>$0.3E-05$</td>
<td>$0.6E-05$</td>
</tr>
<tr>
<td>Probability of Failure $P_f$ (%/Year)</td>
<td>$0.00E+02$</td>
<td>$0.54E+00$</td>
<td>$4.31E+02$</td>
<td>$12.43E+02$</td>
<td>$23.65E+02$</td>
<td>$36.05E+02$</td>
</tr>
<tr>
<td>Difference in $P_f$ between last 2 iterations</td>
<td>$0.1E-10$</td>
<td>$0.8E-08$</td>
<td>$0.7E-06$</td>
<td>$0.7E-06$</td>
<td>$0.8E-08$</td>
<td>$0.8E-08$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q (m^2/s/cm)$</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = T_e$</td>
<td>$8.80E+01$</td>
<td>$7.18E+01$</td>
<td>$6.34E+01$</td>
<td>$5.83E+01$</td>
<td>$5.45E+01$</td>
<td>$5.16E+01$</td>
</tr>
<tr>
<td>$X_2 = H_d$</td>
<td>$3.29E+01$</td>
<td>$2.89E+01$</td>
<td>$2.38E+01$</td>
<td>$1.79E+01$</td>
<td>$1.44E+01$</td>
<td>$1.19E+01$</td>
</tr>
<tr>
<td>$Y_1 = T_e - H_d$</td>
<td>$-68.96$</td>
<td>$-62.02$</td>
<td>$-88.57$</td>
<td>$-88.30$</td>
<td>$-85.70$</td>
<td>$-82.20$</td>
</tr>
<tr>
<td>$Y_2 = T_e + H_d$</td>
<td>$-3.74$</td>
<td>$-3.70$</td>
<td>$-1.73$</td>
<td>$-3.20$</td>
<td>$-6.13$</td>
<td>$-9.55$</td>
</tr>
<tr>
<td>$X_3 = Y_1$</td>
<td>$0.01E+01$</td>
<td>$0.01E+01$</td>
<td>$0.01E+01$</td>
<td>$0.01E+01$</td>
<td>$0.01E+01$</td>
<td>$0.01E+01$</td>
</tr>
<tr>
<td>$X_4 = Y_2$</td>
<td>$2.17E+01$</td>
<td>$2.17E+01$</td>
<td>$2.17E+01$</td>
<td>$2.17E+01$</td>
<td>$2.17E+01$</td>
<td>$2.17E+01$</td>
</tr>
<tr>
<td>$X_5 = Y_3$</td>
<td>$5.69E+00$</td>
<td>$5.63E+00$</td>
<td>$5.62E+00$</td>
<td>$5.61E+00$</td>
<td>$5.61E+00$</td>
<td>$5.61E+00$</td>
</tr>
<tr>
<td>$X_6 = Y_4$</td>
<td>$0.95E+01$</td>
<td>$0.95E+01$</td>
<td>$0.95E+01$</td>
<td>$0.95E+01$</td>
<td>$0.95E+01$</td>
<td>$0.95E+01$</td>
</tr>
<tr>
<td>$X_7 = Y_5$</td>
<td>$0.74E+01$</td>
<td>$0.88E+01$</td>
<td>$1.39E+01$</td>
<td>$0.94E+01$</td>
<td>$0.96E+01$</td>
<td>$0.96E+01$</td>
</tr>
</tbody>
</table>

(continued...)
APPENDIX D3 - Examples Of Input And Output Files For Dune Erosion

Input File *general.dad*: Example 1

```
3 ! Failure mode of dune erosion
1 ! Total level
2 ! Movements of sand only seaward
1 ! Mode 1 - Reliability analysis for a specified design
1 ! Design life (in years)
N ! No combinations of actions considered
0 ! Target design parameter - nourishment width
```

Input File *form.dad*: Example 1

```
2 ! Starting point: user specified values
0
2.25E-4
0
-0.3
0
0.4
0
4.2
2 ! [XMin,XMax]: user specified values
0 1E25
0 1E25
-1E25 1E25
-1E25 1E25
-1E25 1E25
-1E25 1E25
3 1E25
200 ! Maximum number of iterations
1 ! Number of FORM calculations
90
1 ! Accuracy on Beta (%): default value (1%)
1 ! Smoothing of the iteration: default value (0)
1 ! Accuracy on Z0 (%): default value (1%)
```

Input File *meandev.dad*: Example 1

```
1 ! Type of distribution (Normal)
0 ! Not truncated
0 0.60 ! Mean value; standard deviation
1 ! Type of distribution (Normal)
0 ! Not truncated
```
2.25E-4 2.25E-5 ! Mean value; standard deviation
1 ! Type of distribution (Normal)
0 ! Not truncated
0. 0.6 ! Mean value; standard deviation
1 ! Type of distribution (Normal)
0 ! Not truncated
0. 1 ! Mean value; standard deviation
1 ! Type of distribution (Normal)
0 ! Not truncated
0.4 0.1 ! Mean value; standard deviation
1 ! Type of distribution (Normal)
0 ! Not truncated
0. 1 ! Mean value; standard deviation
10 ! Type of distribution (Weibull)
0 ! Not truncated
2.52 0.33 2.19 ! Mean; stand. dev.; lower limit

Input File coefcor.dad; Example 1

1 ! Rho(1,1)
0 ! Rho(1,2)
0 ! Rho(1,3)
0 ! .
0 ! .
0 ! .
0
0
1
0
0
0
0
0
0
0
0
0
0
1
0
0
0
0
0
0
0
0
0
0
0
1
0
0
0
0
0
0
0
0
0
0
0
0
Examples Of Input And Output Files For Dune Erosion

Input File *perfil.dat*: Example 1

```
0               !Coastal curvature in degrees per 1000m
21              !Number of points in the initial profile
-200            15      !Coordinates of the initial profile (XP,YP)
-150            13.5
-100            11
-75             13.5
-50             12
-25             15
-20            12.68571429
-10           8.057142857
  0            3.428571429
 10           -1.2
 15           -1.7
 50           -2.082474227
 100          -2.628865979
 150          -3.175257732
 200          -3.721649485
 250          -4.268041237
 300          -4.81443299
 350          -5.360824742
 400          -5.907216495
 450          -6.453608247
 500          -7
2               !Number of points to be changed in the initial profile
10              !First point to be changed, point No.
 1.            !Gradient of the eroded dune face, 1:md
12.5          !Gradient of the toe of the post-storm profile, 1:mt
 15            !Nourishment top level
 1.5          !Gradient of the nourished face, 1:nour
```

Output File *summary.dat*: Example 1

```
WHAT IS THE DATA SOURCE ?

The Screen ..... [ 1 ]
A Datafile ..... [ 2 ]

Select Option: 2

WHAT IS THE FAILURE MODE TO BE STUDIED:

Overtopping (H&R) .................. [ 1 ]
Overtopping (Owen) ................. [ 2 ]
Dune Erosion (Vellinga) .......... [ 3 ]
```
Select Option: 3

HOW IS THE STILL-WATER-LEVEL DEFINED?

Total Level .... [ 1 ]
Tide + Surge ... [ 2 ]

Select Option: 1

DURING A STORM SURGE, WOULD YOU LIKE TO TAKE INTO ACCOUNT:

Movements of Sand in Both Directions ? ... [ 1 ]
Movements of Sand only Seaward ? .......... [ 2 ]

Select Option: 2

DUNE EROSION

Coastal Curvature (Deg/1000m) = 0.0000000000E+00

Number of Points Defining the Initial Profile (Max=100) = 21

Initial Profile

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2000000000E+03</td>
<td>0.1500000000E+02</td>
</tr>
<tr>
<td>-0.1500000000E+03</td>
<td>0.1350000000E+02</td>
</tr>
<tr>
<td>-0.1000000000E+03</td>
<td>0.1100000000E+02</td>
</tr>
<tr>
<td>-0.7500000000E+02</td>
<td>0.1350000000E+02</td>
</tr>
<tr>
<td>-0.5000000000E+02</td>
<td>0.1200000000E+02</td>
</tr>
<tr>
<td>-0.2500000000E+02</td>
<td>0.1500000000E+02</td>
</tr>
<tr>
<td>-0.2000000000E+02</td>
<td>0.1268571429E+02</td>
</tr>
<tr>
<td>-0.1000000000E+02</td>
<td>0.8057142857E+01</td>
</tr>
<tr>
<td>0.0000000000E+00</td>
<td>0.3428571429E+01</td>
</tr>
<tr>
<td>0.1000000000E+02</td>
<td>-0.1200000000E+01</td>
</tr>
<tr>
<td>0.1500000000E+02</td>
<td>-0.1700000000E+01</td>
</tr>
<tr>
<td>0.5000000000E+02</td>
<td>-0.2082474227E+01</td>
</tr>
<tr>
<td>0.1000000000E+03</td>
<td>-0.2628865979E+01</td>
</tr>
<tr>
<td>0.1500000000E+03</td>
<td>-0.3175257732E+01</td>
</tr>
<tr>
<td>0.2000000000E+03</td>
<td>-0.3721649485E+01</td>
</tr>
<tr>
<td>0.2500000000E+03</td>
<td>-0.4268041237E+01</td>
</tr>
<tr>
<td>0.3000000000E+03</td>
<td>-0.4814432990E+01</td>
</tr>
<tr>
<td>0.3500000000E+03</td>
<td>-0.5360824742E+01</td>
</tr>
<tr>
<td>0.4000000000E+03</td>
<td>-0.5907216495E+01</td>
</tr>
<tr>
<td>0.4500000000E+03</td>
<td>-0.6453608247E+01</td>
</tr>
<tr>
<td>0.5000000000E+03</td>
<td>-0.7000000000E+01</td>
</tr>
</tbody>
</table>

Number of Points to be Changed in the Initial Profile = 2

First Point to be Changed = Point No. 10

Gradient of the Eroded Dune Face = 1: 1.0

Gradient of the Toe of the Post-Storm Profile = 1:12.5
Examples Of Input And Output Files For Dune Erosion

Nourishment Top Level = 0.1500000000E+02

Gradient of the Nourished Face = 1: 1.5

DESCRIPTION OF THE VARIABLES

X(1) = Hs = Wave Height
X(2) = D50 = Particle Size
X(3) = DP = Initial Profile
X(4) = SD = Surge Duration
X(5) = GB = Gust Bumps
X(6) = Ac = Accuracy Comput.
X(7) = h = Surge Level

WHAT IS THE PURPOSE OF THE ANALYSIS?

Reliability Analysis for a Specified Design ... [1]
Design for a Specified Reliability Level ...... [2]

Select Option: 1

DESIGN LIFE OF THE STRUCTURE = 1

WOULD YOU LIKE TO CONSIDER COMBINATION OF ACTIONS (Y/N) ? N

PRESCRIBED VALUE OF THE DESIGN PARAMETER

Nourishment Width = 0.0000000000E+00

CHARACTERISTICS OF THE VARIABLES

Probability Distribution of Hs = Normal (Gaussian)
Mean Value of Hs = 0.0000000000E+00
Standard Deviation of Hs = 0.6000000000E+00

Probability Distribution of D50 = Normal (Gaussian)
Mean Value of D50 = 0.2250000000E-03
Standard Deviation of D50 = 0.2250000000E-04

Probability Distribution of DP = Normal (Gaussian)
Mean Value of DP = 0.0000000000E+00
Standard Deviation of DP = 0.6000000000E+00

Probability Distribution of SD = Normal (Gaussian)
Mean Value of SD = 0.0000000000E+00
Standard Deviation of SD = 0.1000000000E+01

Probability Distribution of GB = Normal (Gaussian)
Mean Value of GB = 0.4000000000E+00
Standard Deviation of GB = 0.1000000000E+00

Probability Distribution of Ac = Normal (Gaussian)
Mean Value of Ac = 0.0000000000E+00
Standard Deviation of Ac = 0.1000000000E+01

Probability Distribution of h = Minima Type III (Weibull)
Mean Value of h = 0.2520000000E+01
Standard Deviation of h = 0.3300000000E+00
Lower Limit on h = 0.2190000000E+01

CORRELATION COEFFICIENTS

(Hs ,Hs ) = 0.1000000000E+01
(Hs ,D50) = 0.0000000000E+00
(Hs ,DP ) = 0.0000000000E+00
(Hs ,SD ) = 0.0000000000E+00
(Hs ,GB ) = 0.0000000000E+00
(Hs ,Ac ) = 0.0000000000E+00
(Hs ,h ) = 0.0000000000E+00
(D50,Hs ) = 0.0000000000E+00
(D50,D50) = 0.1000000000E+01
(D50,DP ) = 0.0000000000E+00
(D50,SD ) = 0.0000000000E+00
(D50,GB ) = 0.0000000000E+00
(D50,Ac ) = 0.0000000000E+00
(D50,h ) = 0.0000000000E+00
(DP ,Hs ) = 0.0000000000E+00
(DP ,D50) = 0.0000000000E+00
(DP ,DP ) = 0.1000000000E+01
(DP ,SD ) = 0.0000000000E+00
(DP ,GB ) = 0.0000000000E+00
(DP ,Ac ) = 0.0000000000E+00
(DP ,h ) = 0.0000000000E+00
(SD ,Hs ) = 0.0000000000E+00
(SD ,D50) = 0.0000000000E+00
(SD ,DP ) = 0.0000000000E+00
(SD ,SD ) = 0.1000000000E+01
(SD ,GB ) = 0.0000000000E+00
(SD ,Ac ) = 0.0000000000E+00
(SD ,h ) = 0.0000000000E+00
(GB ,Hs ) = 0.0000000000E+00
(GB ,D50) = 0.0000000000E+00
(GB ,DP ) = 0.0000000000E+00
(GB ,SD ) = 0.0000000000E+00
(GB ,GB ) = 0.1000000000E+01
(GB ,Ac ) = 0.0000000000E+00
(GB ,h ) = 0.0000000000E+00
(Ac ,Hs ) = 0.0000000000E+00
(Ac ,D50) = 0.0000000000E+00
(Ac ,DP ) = 0.0000000000E+00
(Ac ,SD ) = 0.0000000000E+00
(Ac ,GB ) = 0.0000000000E+00
(Ac ,Ac ) = 0.1000000000E+01
(Ac ,h ) = 0.0000000000E+00
Examples Of Input And Output Files For Dune Erosion

\( (h, H_s) = 0.0000000000E+00 \)
\( (h, D_{50}) = 0.0000000000E+00 \)
\( (h, D_P) = 0.0000000000E+00 \)
\( (h, S_D) = 0.0000000000E+00 \)
\( (h, G_B) = 0.0000000000E+00 \)
\( (h, A_c) = 0.0000000000E+00 \)
\( (h, h) = 0.1000000000E+01 \)

STARTING POINT FOR THE FORM CALCULATIONS:

Default Values (mean values) ... [ 1 ]
User Specified Values ............ [ 2 ]

Select Option: 2

STARTING POINT

\( H_s = 0.0000000000E+00 \)
\( D_{50} = 0.2250000000E-03 \)
\( D_P = -0.3000000000E+00 \)
\( S_D = 0.0000000000E+00 \)
\( G_B = 0.4000000000E+00 \)
\( A_c = 0.0000000000E+00 \)
\( h = 0.4200000000E+01 \)

LIMITING VALUES FOR THE VARIABLES:

Default Values (+/- 1E25) ... [ 1 ]
User Specified Values ........ [ 2 ]

Select Option: 2

LIMITING VALUES FOR THE VARIABLES

\( X_{\text{Min}}(H_s) = 0.0000000000E+00 \quad X_{\text{Max}}(H_s) = 0.1000000000E+26 \)
\( X_{\text{Min}}(D_{50}) = 0.0000000000E+00 \quad X_{\text{Max}}(D_{50}) = 0.1000000000E+26 \)
\( X_{\text{Min}}(D_P) = -0.1000000000E+26 \quad X_{\text{Max}}(D_P) = 0.1000000000E+26 \)
\( X_{\text{Min}}(S_D) = -0.1000000000E+26 \quad X_{\text{Max}}(S_D) = 0.1000000000E+26 \)
\( X_{\text{Min}}(G_B) = -0.1000000000E+26 \quad X_{\text{Max}}(G_B) = 0.1000000000E+26 \)
\( X_{\text{Min}}(A_c) = -0.1000000000E+26 \quad X_{\text{Max}}(A_c) = 0.1000000000E+26 \)
\( X_{\text{Min}}(h) = 0.3000000000E+01 \quad X_{\text{Max}}(h) = 0.1000000000E+26 \)

MAXIMUM NUMBER OF ITERATIONS (Max=200) = 200

NUMBER OF FORM CALCULATIONS (Max=10) = 1

ALLOWABLE EROSION DISTANCE - m (1) = 90.00

REQUIRED RELATIVE ACCURACY OF THE RELIABILITY INDEX:

Default Value (1%) ............ [ 1 ]
User Specified Value .......... [ 2 ]

Select Option: 1
REQUIRED SMOOTHING COEFFICIENT FOR THE ITERATION PROCESS:

Default Value (0) ........... [ 1 ]
User Specified Value ........ [ 2 ]

Select Option: 1

REQUIRED ACCURACY OF THE FAILURE FUNCTION:

Default Value (1%) ........... [ 1 ]
User Specified Value ........ [ 2 ]

Select Option: 1

ALLOWABLE EROSION DISTANCE – m ( 1) = 90.00

FINAL RESULTS

Total Number of Iterations = 6
Failure Function Z (X) = 0.5098040390E-07
Mean Value of Z = 0.1053991340E+03
Standard Deviation of Z = 0.3238986303E+02
Reliability Index = 0.3254077794E+01
Relative Accuracy of the Reliability Index (%) = 0.1540871846E-03
Probability of Failure (%) = 0.056884
Difference in Pf Between the Last 2 Iterations = 0.1002823371E-07

DESIGN POINT COORDINATES

Hs = 0.2366829604E+00
D50 = 0.2061481291E-03
DP = -0.7271166310E-01
SD = 0.6350647216E+00
GB = 0.4079383090E+00
Ac = 0.8719123188E+00
h = 0.4302783557E+01

Alpha(Hs ) = -0.1212237770E+00
Influence of Hs on the Reliability Index = 0.1469520E+01

Alpha(D50) = 0.2574803009E+00
Influence of D50 on the Reliability Index = 0.6629611E+01

Alpha(DP ) = 0.3724130548E-01
Influence of DP on the Reliability Index = 0.1386915E+00

Alpha(SD ) = -0.1951596618E-01
Influence of SD on the Reliability Index = 0.3808729E+01

Alpha(GB ) = -0.2439495770E-01
Influence of GB on the Reliability Index = 0.5951140E-01
Alpha(Ac) = -0.2679445219E+00
Influence of Ac on the Reliability Index = 0.7179427E+01

Alpha(h) = -0.8984125452E+00
Influence of h on the Reliability Index = 0.8071451E+02

WOULD YOU LIKE TO RESTART (Y/N) ? N
APPENDIX D4 - Results From PARASODE For Dune Erosion

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.237</td>
<td>1.47</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.206E-03</td>
<td>6.63</td>
</tr>
<tr>
<td>DP</td>
<td>-0.073</td>
<td>0.14</td>
</tr>
<tr>
<td>SD</td>
<td>0.635</td>
<td>3.81</td>
</tr>
<tr>
<td>GB</td>
<td>0.408</td>
<td>0.06</td>
</tr>
<tr>
<td>Ac</td>
<td>0.872</td>
<td>7.18</td>
</tr>
<tr>
<td>h</td>
<td>4.303</td>
<td>80.71</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 6
Failure Function $Z(X) = 0.5E-07$
Mean Value of $Z = 105.399$
Standard Deviation of $Z = 32.390$
Reliability Index = 3.254
Relative Accuracy of the Reliability Index (%) = 0.2E-03

Probability of Failure (%/Year) = 0.056884
Difference in $P_f$ Between the Last 2 Iterations = 0.1E-07

Table D4.1: Summary of PARASODE results for example 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>-0.029</td>
<td>0.01</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.215E-03</td>
<td>0.90</td>
</tr>
<tr>
<td>DP</td>
<td>-0.135</td>
<td>0.23</td>
</tr>
<tr>
<td>SD</td>
<td>0.779</td>
<td>2.82</td>
</tr>
<tr>
<td>GB</td>
<td>0.410</td>
<td>0.04</td>
</tr>
<tr>
<td>Ac</td>
<td>1.178</td>
<td>6.45</td>
</tr>
<tr>
<td>h</td>
<td>6.207</td>
<td>89.55</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 7
Failure Function $Z(X) = 0.5E-03$
Mean Value of $Z = 168.796$
Standard Deviation of $Z = 36.378$
Reliability Index = 4.640
Relative Accuracy of the Reliability Index (%) = 0.2E-03

Probability of Failure (%/Year) = 0.000175
Difference in $P_f$ Between the Last 2 Iterations = 0.7E-10

Table D4.2: Summary of PARASODE results for example 2.
### Results From PARASODE For Dune Erosion

**Table D4.3:** Summary of PARASODE results for example 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2 (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.331</td>
<td>1.62</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.199E-03</td>
<td>7.07</td>
</tr>
<tr>
<td>DP</td>
<td>-0.052</td>
<td>0.04</td>
</tr>
<tr>
<td>SD</td>
<td>0.542</td>
<td>1.57</td>
</tr>
<tr>
<td>GB</td>
<td>0.407</td>
<td>0.02</td>
</tr>
<tr>
<td>Ac</td>
<td>0.780</td>
<td>3.26</td>
</tr>
<tr>
<td>h</td>
<td>5.669</td>
<td>86.42</td>
</tr>
</tbody>
</table>

- Total Number of Iterations = 20
- Failure Function $Z(X) = 0.8E-04$
- Mean Value of $Z = 224.852$
- Standard Deviation of $Z = 51.968$
- Reliability Index = 4.327
- Relative Accuracy of the Reliability Index (%) = 0.3E-04

**Probability of Failure (%)/Year** = 0.000761

Difference in $P_f$ Between the Last 2 Iterations = 0.4E-10

**Table D4.4:** Summary of PARASODE results for example 4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2 (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.298</td>
<td>1.63</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.202E-03</td>
<td>7.06</td>
</tr>
<tr>
<td>DP</td>
<td>-0.058</td>
<td>0.06</td>
</tr>
<tr>
<td>SD</td>
<td>0.584</td>
<td>2.26</td>
</tr>
<tr>
<td>GB</td>
<td>0.407</td>
<td>0.04</td>
</tr>
<tr>
<td>Ac</td>
<td>0.822</td>
<td>4.48</td>
</tr>
<tr>
<td>h</td>
<td>5.058</td>
<td>84.47</td>
</tr>
</tbody>
</table>

- Total Number of Iterations = 7
- Failure Function $Z(X) = -0.3E-06$
- Mean Value of $Z = 165.857$
- Standard Deviation of $Z = 42.682$
- Reliability Index = 3.886
- Relative Accuracy of the Reliability Index (%) = 0.2E-03

**Probability of Failure (%)/Year** = 0.005083

Difference in $P_f$ Between the Last 2 Iterations = 0.2E-08
Results From PARASODE For Dune Erosion

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.337</td>
<td>1.73</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.198E-03</td>
<td>7.97</td>
</tr>
<tr>
<td>DP</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>SD</td>
<td>0.607</td>
<td>2.03</td>
</tr>
<tr>
<td>GB</td>
<td>0.408</td>
<td>0.03</td>
</tr>
<tr>
<td>Ac</td>
<td>0.855</td>
<td>4.02</td>
</tr>
<tr>
<td>$h$</td>
<td>5.512</td>
<td>84.22</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 8
Failure Function $Z(X) = -0.7E-07$
Mean Value of $Z = 204.380$
Standard Deviation of $Z = 47.939$
Reliability Index = 4.263
Relative Accuracy of the Reliability Index (%) = 0.2E-03

**Probability of Failure (%/Year) = 0.001011**
Difference in $P_f$ Between the Last 2 Iterations = 0.4E-09

Table D4.5: Summary of PARASODE results for example 5.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.285</td>
<td>1.17</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.201E-03</td>
<td>5.82</td>
</tr>
<tr>
<td>DP</td>
<td>-0.061</td>
<td>0.05</td>
</tr>
<tr>
<td>SD</td>
<td>0.906</td>
<td>4.26</td>
</tr>
<tr>
<td>GB</td>
<td>0.411</td>
<td>0.07</td>
</tr>
<tr>
<td>Ac</td>
<td>1.177</td>
<td>7.19</td>
</tr>
<tr>
<td>$h$</td>
<td>5.581</td>
<td>81.44</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 14
Failure Function $Z(X) = -0.03$
Mean Value of $Z = 203.719$
Standard Deviation of $Z = 46.421$
Reliability Index = 4.388
Relative Accuracy of the Reliability Index (%) = 0.6E-03

**Probability of Failure (%/Year) = 0.000573**
Difference in $P_f$ Between the Last 2 Iterations = 0.7E-09

Table D4.6: Summary of PARASODE results for example 6.
### Table D4.7: Summary of PARASODE results for example 7.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.273</td>
<td>1.19</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.202E-03</td>
<td>5.80</td>
</tr>
<tr>
<td>DP</td>
<td>-0.063</td>
<td>0.06</td>
</tr>
<tr>
<td>SD</td>
<td>0.873</td>
<td>4.39</td>
</tr>
<tr>
<td>GB</td>
<td>0.411</td>
<td>0.07</td>
</tr>
<tr>
<td>Ac</td>
<td>1.143</td>
<td>7.52</td>
</tr>
<tr>
<td>$h$</td>
<td>5.291</td>
<td>80.97</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 16  
Failure Function $Z(X) = 0.051$  
Mean Value of $Z = 189.313$  
Standard Deviation of $Z = 45.425$  
Reliability Index = 4.168  
Relative Accuracy of the Reliability Index (%) = 0.8E-07  
**Probability of Failure (%/Year) = 0.001547**  
Difference in $P_f$ Between the Last 2 Iterations = 0.2E-12

### Table D4.8: Summary of PARASODE results for example 8.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.264</td>
<td>1.24</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.203E-03</td>
<td>5.90</td>
</tr>
<tr>
<td>DP</td>
<td>-0.065</td>
<td>0.08</td>
</tr>
<tr>
<td>SD</td>
<td>0.805</td>
<td>4.15</td>
</tr>
<tr>
<td>GB</td>
<td>0.410</td>
<td>0.06</td>
</tr>
<tr>
<td>Ac</td>
<td>1.063</td>
<td>7.24</td>
</tr>
<tr>
<td>$h$</td>
<td>5.043</td>
<td>81.33</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 11  
Failure Function $Z(X) = 0.157$  
Mean Value of $Z = 183.466$  
Standard Deviation of $Z = 46.447$  
Reliability Index = 3.950  
Relative Accuracy of the Reliability Index (%) = 0.3E-05  
**Probability of Failure (%/Year) = 0.003900**  
Difference in $P_f$ Between the Last 2 Iterations = 0.2E-10
### Results From PARASODE For Dune Erosion

#### Table D4.9: Summary of PARASODE results for example 9.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.256</td>
<td>1.30</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.204E-03</td>
<td>6.09</td>
</tr>
<tr>
<td>DP</td>
<td>-0.067</td>
<td>0.09</td>
</tr>
<tr>
<td>SD</td>
<td>0.748</td>
<td>4.00</td>
</tr>
<tr>
<td>GB</td>
<td>0.409</td>
<td>0.06</td>
</tr>
<tr>
<td>Ac</td>
<td>0.997</td>
<td>7.11</td>
</tr>
<tr>
<td>$h$</td>
<td>4.811</td>
<td>81.35</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 9
Failure Function $Z(X) = 0.4E-06$
Mean Value of $Z = 176.884$
Standard Deviation of $Z = 47.326$
Reliability Index = 3.738
Relative Accuracy of the Reliability Index (%) = 0.2E-03

**Probability of Failure (%/Year) = 0.009298**
Difference in $P_f$ Between the Last 2 Iterations = 0.3E-08

#### Table D4.10: Summary of PARASODE results for example 10.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.248</td>
<td>1.37</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.205E-03</td>
<td>6.30</td>
</tr>
<tr>
<td>DP</td>
<td>-0.072</td>
<td>0.11</td>
</tr>
<tr>
<td>SD</td>
<td>0.693</td>
<td>3.85</td>
</tr>
<tr>
<td>GB</td>
<td>0.409</td>
<td>0.06</td>
</tr>
<tr>
<td>Ac</td>
<td>0.934</td>
<td>7.01</td>
</tr>
<tr>
<td>$h$</td>
<td>4.589</td>
<td>81.30</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 19
Failure Function $Z(X) = -0.7E-02$
Mean Value of $Z = 168.325$
Standard Deviation of $Z = 47.707$
Reliability Index = 3.528
Relative Accuracy of the Reliability Index (%) = 0.9E-04

**Probability of Failure (%/Year) = 0.020938**
Difference in $P_f$ Between the Last 2 Iterations = 0.3E-08
Results From PARASODE For Dune Erosion

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.219</td>
<td>1.59</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.208E-03</td>
<td>6.96</td>
</tr>
<tr>
<td>DP</td>
<td>-0.076</td>
<td>0.19</td>
</tr>
<tr>
<td>SD</td>
<td>0.588</td>
<td>4.12</td>
</tr>
<tr>
<td>GB</td>
<td>0.407</td>
<td>0.06</td>
</tr>
<tr>
<td>Ac</td>
<td>0.830</td>
<td>8.21</td>
</tr>
<tr>
<td>$h$</td>
<td>3.947</td>
<td>78.87</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 6
Failure Function $Z(X) = -0.2E-07$
Mean Value of $Z = 69.676$
Standard Deviation of $Z = 24.053$
Reliability Index = 2.897
Relative Accuracy of the Reliability Index (%) = 0.8E-04

**Probability of Failure (%/Year) = 0.188552**
Difference in $P_f$ Between the Last 2 Iterations = 0.1E-07

Table D4.11: Summary of PARASODE results for example 11.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
<th>$\alpha^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.195</td>
<td>1.77</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.210E-03</td>
<td>7.45</td>
</tr>
<tr>
<td>DP</td>
<td>-0.079</td>
<td>0.28</td>
</tr>
<tr>
<td>SD</td>
<td>0.533</td>
<td>4.72</td>
</tr>
<tr>
<td>GB</td>
<td>0.407</td>
<td>0.07</td>
</tr>
<tr>
<td>Ac</td>
<td>0.780</td>
<td>10.13</td>
</tr>
<tr>
<td>$h$</td>
<td>3.552</td>
<td>75.58</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 6
Failure Function $Z(X) = -0.2E-07$
Mean Value of $Z = 53.567$
Standard Deviation of $Z = 21.858$
Reliability Index = 2.451
Relative Accuracy of the Reliability Index (%) = 1.0E-04

**Probability of Failure (%/Year) = 0.712880**
Difference in $P_f$ Between the Last 2 Iterations = 0.5E-07

Table D4.12: Summary of PARASODE results for example 12.
### Table D4.13: Summary of PARASODE results for example 13.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.169</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.212E-03</td>
</tr>
<tr>
<td>DP</td>
<td>-0.079</td>
</tr>
<tr>
<td>SD</td>
<td>0.463</td>
</tr>
<tr>
<td>GB</td>
<td>0.406</td>
</tr>
<tr>
<td>Ac</td>
<td>0.707</td>
</tr>
<tr>
<td>$h$</td>
<td>3.178</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 6
Failure Function $Z(X) = -0.4E-08$
Mean Value of $Z = 35.121$
Standard Deviation of $Z = 18.005$
Reliability Index = 1.951
Relative Accuracy of the Reliability Index (%) = 0.9E-04

**Probability of Failure (%/Year) = 2.555196**
Difference in $P_f$ Between the Last 2 Iterations = 1.0E-07

### Table D4.14: Summary of PARASODE results for example 14.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Design Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>0.336</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>0.198E-03</td>
</tr>
<tr>
<td>DP</td>
<td>0.000</td>
</tr>
<tr>
<td>SD</td>
<td>0.608</td>
</tr>
<tr>
<td>GB</td>
<td>0.408</td>
</tr>
<tr>
<td>Ac</td>
<td>0.857</td>
</tr>
<tr>
<td>$h$</td>
<td>5.498</td>
</tr>
</tbody>
</table>

Total Number of Iterations = 18
Failure Function $Z(X) = -0.3E-02$
Mean Value of $Z = 203.088$
Standard Deviation of $Z = 47.734$
Target Probability of Failure (%/Year) = 0.001011
Reliability Index = 4.255

**Design Parameter = 74.87**
APPENDIX E

Results From @Risk
## CONVERGENCE OF Q STATISTICS USING LATIN HYPERCUBE SAMPLING

### NORMAL CONDITIONS: H&R MODEL, \( (R_{\text{max}})_{37\%} \)
SLOPE 1:2, CL=10m OD

<table>
<thead>
<tr>
<th>Number Of Samples</th>
<th>Mean Of Samples</th>
<th>Standard Deviation Of Samples</th>
<th>Coefficient Of Variation Of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2.59E-06</td>
<td>2.57E-05</td>
<td>9.95E+00</td>
</tr>
<tr>
<td>500</td>
<td>8.51E-05</td>
<td>1.90E-03</td>
<td>2.23E+01</td>
</tr>
<tr>
<td>1000</td>
<td>4.48E-05</td>
<td>7.93E-04</td>
<td>1.77E+01</td>
</tr>
<tr>
<td>2000</td>
<td>2.30E-05</td>
<td>5.12E-04</td>
<td>2.23E+01</td>
</tr>
<tr>
<td>3000</td>
<td>4.80E-05</td>
<td>1.65E-03</td>
<td>3.44E+01</td>
</tr>
<tr>
<td>4000</td>
<td>4.65E-05</td>
<td>1.29E-03</td>
<td>2.78E+01</td>
</tr>
<tr>
<td>5000</td>
<td>4.78E-05</td>
<td>1.03E-03</td>
<td>2.16E+01</td>
</tr>
<tr>
<td>6000</td>
<td>4.28E-05</td>
<td>1.17E-03</td>
<td>2.74E+01</td>
</tr>
<tr>
<td>7000</td>
<td>2.23E-05</td>
<td>7.16E-04</td>
<td>3.21E+01</td>
</tr>
<tr>
<td>8000</td>
<td>3.45E-05</td>
<td>8.82E-04</td>
<td>2.56E+01</td>
</tr>
<tr>
<td>9000</td>
<td>3.15E-05</td>
<td>8.28E-04</td>
<td>2.63E+01</td>
</tr>
<tr>
<td>10000</td>
<td>3.10E-05</td>
<td>6.56E-04</td>
<td>2.12E+01</td>
</tr>
<tr>
<td>20000</td>
<td>2.83E-05</td>
<td>6.83E-04</td>
<td>2.41E+01</td>
</tr>
<tr>
<td>30000</td>
<td>2.95E-05</td>
<td>8.00E-04</td>
<td>2.71E+01</td>
</tr>
<tr>
<td>40000</td>
<td>3.88E-05</td>
<td>1.03E-03</td>
<td>2.65E+01</td>
</tr>
<tr>
<td>50000</td>
<td>4.93E-05</td>
<td>1.52E-03</td>
<td>3.08E+01</td>
</tr>
<tr>
<td>60000</td>
<td>3.78E-05</td>
<td>1.10E-03</td>
<td>2.91E+01</td>
</tr>
<tr>
<td>70000</td>
<td>4.01E-05</td>
<td>1.16E-03</td>
<td>2.90E+01</td>
</tr>
<tr>
<td>80000</td>
<td>4.30E-05</td>
<td>1.40E-03</td>
<td>3.25E+01</td>
</tr>
<tr>
<td>90000</td>
<td>4.11E-05</td>
<td>1.14E-03</td>
<td>2.76E+01</td>
</tr>
<tr>
<td>100000</td>
<td>4.28E-05</td>
<td>1.30E-03</td>
<td>3.03E+01</td>
</tr>
</tbody>
</table>

**Table E1:** Example of the convergence of the mean, standard deviation and coefficient of variation of Q using Latin Hypercube Sampling for normal conditions, H&R model, \( (R_{\text{max}})_{37\%} \), slope 1:2, CL=10m OD.
### CONVERGENCE OF THE PROBABILITY OF FAILURE (%/YEAR) USING LATIN HYPERCUBE SAMPLING

**NORMAL CONDITIONS: H&R MODEL, \( (R_{\text{max}})^{37\%} \)**

**SLOPE 1:2, CL=10m OD**

<table>
<thead>
<tr>
<th>Number Of Samples</th>
<th>Allowable Discharge, ( Q (m^3/s/m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10(^{-1})</td>
</tr>
<tr>
<td>10000</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>20000</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>30000</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>40000</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>50000</td>
<td>6.00E-03</td>
</tr>
<tr>
<td>60000</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>70000</td>
<td>1.43E-03</td>
</tr>
<tr>
<td>80000</td>
<td>2.50E-03</td>
</tr>
<tr>
<td>90000</td>
<td>1.11E-03</td>
</tr>
<tr>
<td>100000</td>
<td>3.00E-03</td>
</tr>
</tbody>
</table>

**Table E2:** Example of the convergence of the probability of failure for different allowable discharges using Latin Hypercube Sampling for normal conditions, H&R model, \( (R_{\text{max}})^{37\%} \), slope 1:2, CL=10m OD.
## CONVERGENCE OF Q STATISTICS USING LATIN HYPERCUBE SAMPLING

### NORMAL CONDITIONS: OWEN'S MODEL
**SLOPE 1:2, CL=10m OD**

<table>
<thead>
<tr>
<th>Number Of Samples</th>
<th>Mean Of Samples</th>
<th>Standard Deviation Of Samples</th>
<th>Coefficient Of Variation Of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.57E-04</td>
<td>8.52E-04</td>
<td>5.44E+00</td>
</tr>
<tr>
<td>500</td>
<td>2.62E-04</td>
<td>1.64E-03</td>
<td>6.25E+00</td>
</tr>
<tr>
<td>1000</td>
<td>4.95E-04</td>
<td>7.32E-03</td>
<td>1.48E+01</td>
</tr>
<tr>
<td>2000</td>
<td>2.57E-04</td>
<td>2.12E-03</td>
<td>8.23E+00</td>
</tr>
<tr>
<td>3000</td>
<td>2.76E-04</td>
<td>2.30E-03</td>
<td>8.34E+00</td>
</tr>
<tr>
<td>4000</td>
<td>3.26E-04</td>
<td>3.11E-03</td>
<td>9.54E+00</td>
</tr>
<tr>
<td>5000</td>
<td>2.91E-04</td>
<td>3.09E-03</td>
<td>1.06E+01</td>
</tr>
<tr>
<td>6000</td>
<td>3.77E-04</td>
<td>4.18E-03</td>
<td>1.11E+01</td>
</tr>
<tr>
<td>7000</td>
<td>3.56E-04</td>
<td>4.14E-03</td>
<td>1.16E+01</td>
</tr>
<tr>
<td>8000</td>
<td>3.21E-04</td>
<td>2.84E-03</td>
<td>8.84E+00</td>
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<tr>
<td>9000</td>
<td>3.64E-04</td>
<td>3.91E-03</td>
<td>1.07E+01</td>
</tr>
<tr>
<td>10000</td>
<td>2.68E-04</td>
<td>2.77E-03</td>
<td>1.03E+01</td>
</tr>
<tr>
<td>20000</td>
<td>3.20E-04</td>
<td>3.50E-03</td>
<td>1.09E+01</td>
</tr>
<tr>
<td>30000</td>
<td>2.91E-04</td>
<td>3.15E-03</td>
<td>1.08E+01</td>
</tr>
<tr>
<td>40000</td>
<td>3.14E-04</td>
<td>3.35E-03</td>
<td>1.07E+01</td>
</tr>
<tr>
<td>50000</td>
<td>3.00E-04</td>
<td>3.10E-03</td>
<td>1.03E+01</td>
</tr>
<tr>
<td>75000</td>
<td>3.01E-04</td>
<td>3.22E-03</td>
<td>1.07E+01</td>
</tr>
<tr>
<td>100000</td>
<td>2.99E-04</td>
<td>3.04E-03</td>
<td>1.02E+01</td>
</tr>
</tbody>
</table>

**Table E3:** Example of the convergence of the mean, standard deviation and coefficient of variation of Q using Latin Hypercube Sampling for normal conditions, Owen's model, slope 1:2, CL=10m OD.
CONVERGENCE OF THE PROBABILITY OF FAILURE (%/YEAR) USING LATIN HYPERCUBE SAMPLING

NORMAL CONDITIONS: OWEN’S MODEL
SLOPE 1:2, CL=10m OD

<table>
<thead>
<tr>
<th>Number Of Samples</th>
<th>Allowable Discharge, Q (m$^3$/s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>10000</td>
<td>1.00E-02</td>
</tr>
<tr>
<td>20000</td>
<td>1.50E-02</td>
</tr>
<tr>
<td>30000</td>
<td>1.33E-02</td>
</tr>
<tr>
<td>40000</td>
<td>2.25E-02</td>
</tr>
<tr>
<td>50000</td>
<td>1.60E-02</td>
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<tr>
<td>75000</td>
<td>2.13E-02</td>
</tr>
<tr>
<td>100000</td>
<td>2.10E-02</td>
</tr>
</tbody>
</table>

Table E4: Example of the convergence of the probability of failure for different allowable discharges using Latin Hypercube Sampling for normal conditions, Owen's model, slope 1:2, CL=10m OD.

PROBABILITY OF FAILURE (%/YEAR) FOR NORMAL CONDITIONS
H&R MODEL, (R$_{max}$)37%, SLOPE 1:2

<table>
<thead>
<tr>
<th>CL (m OD)</th>
<th>Allowable Discharge, Q (m$^3$/s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10$^{-1}$</td>
</tr>
<tr>
<td>8</td>
<td>0.05666</td>
</tr>
<tr>
<td>10</td>
<td>0.00000</td>
</tr>
<tr>
<td>12</td>
<td>0.00000</td>
</tr>
<tr>
<td>14</td>
<td>0.00000</td>
</tr>
<tr>
<td>16</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Table E5: Probabilities of failure produced by @Risk, Latin Hypercube Sampling (60000 samples) for normal conditions, H&R model, (R$_{max}$)37%, slope 1:2.
### PROBABILITY OF FAILURE (%/YEAR) FOR NORMAL CONDITIONS

**H&R MODEL, \((R_{\text{max}})_{99\%}, \text{ SLOPE 1:2}\)**

<table>
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<tr>
<th>CL (m OD)</th>
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<th>(10^{-2})</th>
<th>(10^{-3})</th>
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</table>

**Table E6:** Probabilities of failure produced by @Risk, Latin Hypercube Sampling (60000 samples) for normal conditions, H&R model, \((R_{\text{max}})_{99\%}, \text{slope 1:2}\).

### PROBABILITY OF FAILURE (%/YEAR) FOR NORMAL CONDITIONS

**OWEN’S MODEL, SLOPE 1:2**

<table>
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<tr>
<th>CL (m OD)</th>
<th>Allowable Discharge, (Q (m^3/s/m))</th>
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<th>(10^{-2})</th>
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<th>(10^{-4})</th>
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</tr>
</tbody>
</table>

**Table E7:** Probabilities of failure produced by @Risk, Latin Hypercube Sampling (30000 samples) for normal conditions, Owen’s model, slope 1:2.
APPENDIX F

Published Papers
A new regression model for wave overtopping data
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Abstract

Random waves with a significant height $H_s$ produce a maximum run-up of $CH_s$ on the face of a coastal structure, where coefficient $C$ is determined by the wave and wall characteristics. Unless this run-up is greater than $R_c$, the freeboard of the structure, then there is no overtopping (apart from wind-blown spray). That is $1 - R_c / CH_s > 0$ for overtopping to occur. The parameter $1 - R_c / CH_s$ is used in an analysis of overtopping data and a new regression model is evaluated which, unlike existing expressions, satisfies the relevant physical boundary conditions. The new model is inherently suitable for representing the small overtopping discharges associated with normal design conditions.

1 Introduction

An important criterion for the design of a seawall is the allowable degree of wave overtopping which depends upon the activities normally performed in the lee of the structure, the need to prevent erosion of the rear face of the seawall, and the economic consequences of flooding. During 1978 and 1979, Owen (at HRS\textsuperscript{1}, Wallingford) carried out an extensive series of model tests to determine the overtopping discharges for a range of seawall designs subjected to different random wave climates. The modelled seawalls were all of the same general type: a flat-topped embankment fronted in some cases by a flat berm. The tests were aimed at establishing the impact on overtopping discharge of the wave climate, the seawall slope, the crest and berm elevations, and the berm width.

This paper presents a re-analysis of some of Owen’s data: the results for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, subjected to random waves approaching normal to the slope. The purpose has been to construct a new regression model to represent the data which is more
reliable than Owen’s expression. Care has been taken to consider the physical boundary conditions. Regression coefficients determined using the least-absolute-deviations (LAD) method are recommended in preference to those obtained using the least-squares (LS) technique.

2 Dimensional analysis

In general, the mean overtopping discharge per unit length of seawall, $Q$, depends upon the wave motion, the seawall profile, the foreshore characteristics and the water properties. Written in symbolic form:

$$Q = \text{function}(H_s, T_m, \beta, R_c, \alpha, d_s, g, \ldots)$$  \hspace{1cm} (1)

$H_s$ is the significant height of the incident waves; $T_m$ is the mean zero-crossing wave period; $\beta$ is the angle of wave approach measured normal to the seawall; $R_c$ is the seawall’s freeboard (the height of the crest of the structure above the still-water-level); $\alpha$ is the angle of the seawall front slope measured from the horizontal; $d_s$ is the still-water-depth at the toe of the structure; and $g$ is the acceleration due to gravity. Alternatively,

$$\frac{Q}{\sqrt{g H_s^3}} = \text{function}\left(\frac{R_c}{H_s}, \frac{H_s}{g T_m^2}, \frac{d_s}{H_s}, \alpha, \beta, \ldots\right)$$  \hspace{1cm} (2)

$H_s / g T_m^2$ is a measure of the incident wave steepness. Owen combined this dimensionless group both with $Q / \sqrt{g H_s^3}$ and with $R_c / H_s$, to write:

$$\frac{Q}{T_m g H_s} = \text{function}\left(\frac{R_c}{T_m \sqrt{g H_s}}, \frac{H_s}{g T_m}, \frac{d_s}{H_s}, \alpha, \beta, \ldots\right)$$  \hspace{1cm} (3)

However, other arrangements are possible (Hedges & Reis), including use of the wave period of peak spectral density, $T_p$, rather than the zero-crossing period, $T_m$. The dimensionless groups are generally related using one of the two following functions:

$$Q_* = A \exp(-BR_*),$$  \hspace{1cm} (4)

$$Q_* = A (R_*)^{-B}$$  \hspace{1cm} (5)

where $Q_*$ is the dimensionless overtopping discharge, $R_*$ is the dimensionless freeboard, and $A$ and $B$ are best-fit coefficients determined from the experimental data.

Dimensional analysis provides no means for determining which sets of dimensionless groups may be especially informative or helpful in dealing with a particular data set. A possible problem in using certain pairings of groups is the potential for spurious correlation. A spurious correlation may arise when dimensionless groups plotted against one another contain a common variable.
Care must be taken in interpreting such plots. Scatter in the data may be suppressed simply by the presence of this variable.

3 Regression analysis

3.1 Introduction

Once experimental data have been collected, they may be used to confirm the validity of some theory or, where no satisfactory theory exists, they may be used to construct regression models. However, it is always useful to have some theoretical basis for choosing amongst the possible models. Furthermore, there are many techniques available for fitting regression models. In describing a regression model, care should be taken to emphasise the range of conditions over which there are data to support its use.

As a start, let us consider the physical boundary conditions to be met in addressing wave overtopping:

i) when the embankment has a large freeboard (i.e. when its crest elevation is well above the level of wave uprush), the predicted overtopping discharge should be zero (assuming that the effects of wind-blown spray are ignored);

ii) when the embankment has zero freeboard (i.e. when still-water-level is at the crest level of the embankment) then the predicted overtopping discharge may be large but should still remain finite.

Equations (4) and (5) represent two of the more common functions used to predict wave overtopping. However, when $R_*$ is large, both expressions suggest that the discharge will be finite rather than zero (though it is small provided that $A$ is not very large and provided also that $B>1$). When $R_*$ is zero, the first of these expressions gives $Q_* = A$, a finite quantity, whilst the second expression gives $Q_* = \infty$. Thus neither expression satisfies both boundary conditions, with the second of them satisfying neither. Since most seawalls are designed to permit only relatively small overtopping discharges, the first of the two boundary conditions is likely to be the more important to satisfy.

In addition to considering the boundary conditions, we also need to establish the line of “best fit” to the observed data. There are many criteria for defining the best fit. One possibility is to minimise the sum of the squared deviations of the observations from the values predicted from our expression, but real data usually do not completely satisfy the classical assumptions for LS fitting. Reliable inferences may be drawn from regression models fitted by the LS method only if the assumptions are valid. Furthermore, an LS fitting has the disadvantage that the result is not “robust”: it is sensitive to outlying data points. Whilst we could remove “outliers”, such a procedure should only be considered if there is reason to doubt their validity. Such data must not be removed merely because they do not support our regression model: it may be the model which is wrong.
Minimising the sum of the absolute deviations rather than the sum of the squared deviations does not rely upon the Gaussian error assumption and allows us to retain outliers but prevents these points from exerting a disproportionate influence on the values of the regression coefficients. If the errors are assumed to follow a double exponential distribution, which has thicker tails than the Gaussian distribution, then the parameter values are maximum likelihood estimates. In this paper, we have chosen to fit our regression lines using the LAD method. However, we have also compared these results with an LS fitting.

3.2 A new regression model

3.2.1 A simple overtopping theory for regular waves

Let us step back from the complications of random waves to the simpler problem of regular waves of height $H$ approaching normal to a seawall. We will assume that the instantaneous discharge of water over unit length of the seawall, $q$, is given by the weir formula:

$$q = C_d \left( \frac{2}{3} \right) \sqrt{2g} \left( \eta - R_c \right)^{3/2} \quad \text{for} \quad \eta > R_c$$

in which $\eta$ is the water surface elevation above still-water-level at the seawall (a periodic function of time); $C_d$ is a discharge coefficient. Obviously, overtopping occurs only when the water surface is above the structure’s crest. We will also assume that $\eta = kHF(t)$ in which $F(t)$ denotes a function of time, $t$. For simple, sinusoidal, progressive waves, $k=0.5$ and $F(t)=\cos(2\pi t/T)$, where $T$ is the wave period. However, following Kikkawa et al., we will adopt the simpler form for $F(t)$ shown in Figure 1; $k$ remains a coefficient determined by the particular wave and wall details. Then, the mean discharge, $Q$, is determined as follows:

$$Q = C_d \left( \frac{2}{3} \right) \sqrt{2g} \int_{t_1}^{t_2} \left( kHF(t) - R_c \right)^{3/2} dt$$

in which $t_1 < t < t_2$ corresponds to the interval during each wave period for which $kHF(t) > R_c$. Using the form for $F(t)$ given in Figure 1 then yields:

$$\frac{Q}{\sqrt{g}(kH)^{3}} = C_d \left( \frac{2\sqrt{2}}{15} \right) \left\{ \left( \frac{R_c}{kH} \right)^{5/2} \right\} \quad \text{for} \quad 0 < R_c < kH$$

$$= 0 \quad \text{for} \quad R_c \geq kH$$

Note that overtopping occurs only when $R_c < kH$. In other words, $kH$ represents the run-up on the face of the seawall.
3.2.2 The Hedges & Reis (H&R) overtopping model

The above theory suggests a regression equation for the random overtopping data of the following form:

\[ Q^* = A (1 - R^*)^B \]

for \( 0 \leq R^* < 1 \)

\[ = 0 \]

for \( R^* \geq 1 \)  \hspace{1cm} (9)

in which

\[ Q_s = \frac{Q}{\sqrt{gR^3_{\text{max}}}} = \frac{Q}{\sqrt{g(CH_s)^3}} \]

\[ R_s = \frac{R}{R_{\text{max}}} = \frac{R_c}{CH_s} \]  \hspace{1cm} (10)

Coefficient \( k \) in the expression for regular waves has been replaced by \( C \) in this regression model for random waves characterised by \( H_s \). Note that \( CH_s \) represents \( R_{\text{max}} \), the maximum run-up induced by the random waves, not the run-up induced by a wave of height \( H_s \). Consequently, \( C \) will depend upon the duration of the incident wave conditions unless the wave heights in front of the wall are limited by the available water depth. Until the maximum run-up exceeds the freeboard, \( R_c \), there will be no overtopping. It is also clear that coefficient \( B \) is related, in the case of regular waves, to the shape of the function \( F(t) \) which describes the water surface variation on the seaward face of the wall. There will be a similar dependence on the detailed behaviour of the water surface at the face of the wall in the case of random waves. Finally, coefficient \( A \) represents the dimensionless discharge over the seawall when the freeboard is zero. All three coefficients will be influenced by the seaward profile of the structure.

The above model for overtopping has the advantage that \( Q^* = 0 \) when \( R^* \geq 1 \) and that \( Q^* = A \) when \( R^* = 0 \), in accordance with our required boundary conditions. Figure 2 shows the influences of coefficients \( A, B \) and \( C \) in the new overtopping model.
Although $C (=\frac{R_{\text{max}}}{H_s})$ is not normally evaluated during tests involving wave overtopping, its value may be estimated from run-up measurements for random waves acting on slopes for which there is no overtopping. We have adopted this option rather than including $C$ alongside $A$ and $B$ as a regression coefficient.

Owen recorded his overtopping discharges during tests involving sets of five different runs, each of 100 waves, characterised by the same value of $H_s$. Assuming that run-up values may be described by a Rayleigh distribution, then the expected maximum run-up, $R_{\text{max}}$, during each run is related to the significant run-up, $R_s$, by $R_{\text{max}} = \sqrt{\ln(100) / 2} R_s = 1.52 R_s$. The CIRIA/CUR manual gives two equations for evaluating $R_s$ for smooth slopes without overtopping. Rewritten in our notation and allowing for a printing error, the following expressions for $C$ may be derived:

$C = 1.52 R_s / H_s \begin{cases} = 1.52 (1.35 \xi_p) & \text{for } \xi_p < 2 \\ = 1.52 (3.00 - 0.15 \xi_p) & \text{for } \xi_p > 2 \end{cases}$

Here, $\xi_p$ is the surf similarity parameter calculated using $T_p$ ($\xi_p = \tan \alpha / \sqrt{H_s / L_{op}}$; $L_{op} = g T_p^2 / 2 \pi$). $T_p$ was estimated for Owen’s data using the relationships between $H_s$, $T_m$ and $T_p$ provided by Isherwood.

### 4 Results of analysis

Figure 3 shows an example of the overtopping data collected by Owen: the results for a simple seawall with a uniform front slope of 1:2. The data are plotted in the formats required for fitting regression equations using both the H&R and the Owen models. Figure 3(a) shows the best-fit lines established using LS and LAD procedures for the H&R model. Comparison of the regression coefficients shows the relatively stronger influence which outlying data points have on the LS values. For example, the magnitude of $B$ obtained
from LAD is only about 92% of the LS result. Similar comments may be made about the regression lines obtained for Owen’s model. Note, that the values of A and B reported in Figure 3(b) for Owen’s model are not those which Owen himself recommended.

Table 1: Regression coefficients obtained for the H&R model and for Owen’s model. Also included are Owen’s recommended values.

<table>
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</tr>
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<td></td>
<td>B 4.17</td>
<td>4.55</td>
</tr>
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<td>0.00792</td>
</tr>
<tr>
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<td>B 6.27</td>
<td>5.94</td>
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</table>

Table 1 gives the regression coefficients for all three slopes which we have obtained for the H&R model and for Owen’s model, using both LS and
LAD fitting. Also included for reference are Owen’s recommended values. Owen restricted his analysis to a particular set of conditions whilst we have included all available data. These data fell within the following ranges:

\[
0 < \frac{Q}{\sqrt{g(CH_s)^3}} < 0.0056 \quad 0.14 < \frac{R_c}{CH_s} < 0.90 \\
0 < \frac{Q}{T_m gH_s} < 0.0039 \quad 0.053 < \frac{R_c}{T_m \sqrt{gH_s}} < 0.239 \\
0.0053 < \frac{H_s}{gT_m} < 0.0095 \quad 1.65 < \frac{d_s}{H_s} < 5.20
\] (12)

Figure 4: Wave overtopping data for slope 1:2, showing the level of agreement between Q and \(Q_{\text{PRED}}\).

Earlier, we have mentioned briefly the problem of spurious correlation. Along with most other overtopping models, the H&R model employs a dimensionless discharge and a dimensionless freeboard which contain a common variable \((R_{\text{max}} \text{ or } CH_s)\). The presence of this common variable may reduce the apparent scatter in the data. Consequently, in Figure 4 we show
directly the level of agreement between Owen’s measured values of Q (converted by Owen to full-scale discharges for a seawall in 4m water depth) and the predicted values, Q_{PRED}. Under random wave conditions, overtopping will be dominated by the few waves with large run-ups: most waves will contribute no overtopping if the seawall has a substantial freeboard. Thus, particularly for short runs of random waves, as in Owen’s tests, we can expect some variability in the measured values of Q. Indeed, one of the purposes of Owen’s tests was to show this inherent variability.

In Figure 4, most data points lie within a range for Q/Q_{PRED} of 3/4 to 4/3, whichever model is adopted. It is not obvious from the figure which model best fits the data, nor is it obvious from the plots for simple seawalls with 1:1 and 1:4 front slopes. However, it should be noted that full-scale discharges greater than about $0.001 \times 10^{-3} \text{m}^3/\text{s}/\text{m}$ will be unsafe for vehicles at high speed. Conditions become dangerous for pedestrians when the discharge exceeds $0.03 \times 10^{-3} \text{m}^3/\text{s}/\text{m}$. Discharges greater than about $2 \times 10^{-3} \text{m}^3/\text{s}/\text{m}$ may damage embankment seawalls (CIRIA/CUR4). Consequently, we have looked in more detail at the data points for discharges in the ranges of practical interest. The H&R model appears generally better than Owen’s model for discharges of less than $5 \times 10^{-3} \text{m}^3/\text{s}/\text{m}$, owing to its ability to predict zero overtopping at finite values of freeboard. Furthermore, it tends to give lower required crest levels than Owen’s model for small permissible discharges.

5 Concluding remarks

A new regression model has been developed for describing wave overtopping data. The important features of the model are as follows:

i) it satisfies the relevant physical boundary conditions, a feature which is especially important when the model is used near these boundaries;

ii) it explicitly recognises (through its foundations in a simple theoretical model for regular waves) that regression coefficient A depends upon the shape of the structure since the shape, particularly at its crest, affects the discharge coefficient; coefficient A represents the dimensionless discharge when the dimensionless freeboard is zero;

iii) coefficient B depends upon the detailed behaviour of the water surface on the face of the structure; it increases as front slopes become flatter;

iv) coefficient C relates the maximum run-up to the significant height of the incident waves and may be chosen to allow for the influences of the seawall slope, the surface roughness and porosity, and the incident wave steepness; coefficient C can also account for storm duration in influencing $R_{\text{max}}$.

Ideally, any test programme would fix coefficient A by measuring the discharge over a seawall when the freeboard was zero. Likewise, C could be determined from the minimum freeboard giving zero overtopping discharge.
In the absence of this information, C has been estimated from available data on the significant wave run-up on smooth slopes.

For the present test results, the H&R model is little different from Owen’s model in its ability to represent the data, except for small discharges for which the H&R model is better suited. However, there are a number of ways in which the H&R model could be improved. For example, the period of peak spectral density and the maximum run-up could be measured directly, whereas we have had to estimate these values. We would then expect more reliable estimates for coefficient C and, consequently, a closer agreement between our model and the data.

Finally, one of the purposes of Owen’s tests was to show the variability in Q. This property must be considered in design procedures: it is necessary not only to model the expected value of Q, but also the distribution of Q about this value. As a consequence, approaches to coastal engineering design are shifting towards the assessment of the safety of coastal structures using risk analysis rather than using a deterministic procedure. One of the major stages of risk analysis is the formulation of equations to describe the failure mechanisms of a structure. In this connection, the main objective of the present paper has been to improve the mathematical description of wave overtopping of simple seawalls.

Acknowledgements

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References

Random Wave Overtopping of Simple Seawalls: a New Regression Model

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Keywords:
Coastal engineering
Sea defences
Design methods and aids
Abstract

Seawalls are expensive, and fixing a seawall freeboard at too large a value has both a financial penalty and is unnecessarily damaging to the natural environment owing to the increased impact of the structure on its surroundings. On the other hand, if the crest of a seawall is set too low, then there are potential problems with structural safety and flooding from wave overtopping. Hence, it is important to strike the correct balance between satisfying the structural and functional requirements of the project, avoiding unnecessary expense, and having undesirable impacts on the surrounding environment.

The prediction of wave overtopping rates is usually based on empirical equations fitted to laboratory data. These equations do not have any theoretical basis. However, a new model has now been developed which, unlike existing expressions, accounts for the fact that no overtopping (apart from wind-blown spray) occurs if the seawall freeboard exceeds the maximum wave run-up on the face of the structure. This fact is of practical importance because allowable overtopping discharges to ensure the safety of people and property are quite small.

The paper starts with a brief review of existing overtopping equations, then presents the new model, and concludes by giving an example of its practical implications. It is shown that, for some conditions, the new model predicts seawall freeboards which are several metres less than those predicted by the well-known expression given by Owen (1982).

Notation

\( A, B \) regression coefficients;
\( C \) ratio of the maximum run-up to the significant height of the incident waves (=\( R_{\text{max}}/H_s \));
\( C_d \) discharge coefficient;
\( d_s \) still-water-depth at the toe of the seawall;
\( F(t) \) a function of time (in the description of water surface elevation);
\( g \) acceleration due to gravity;
\( H \) wave height;
\( H_s \) significant wave height;
\( k \) coefficient (in the description of water surface elevation);
\( L_m \) Airy wavelength at the toe of the seawall calculated using the mean zero-crossing wave period;
$L_{op}$  Airy wavelength in deep water calculated using the period of peak spectral density ($= gT_p^2 / 2\pi$);

$L_p$  Airy wavelength at the toe of the seawall calculated using the period of peak spectral density;

$L_s$  Airy wavelength at the toe of the seawall calculated using the significant wave period;

$N$  number of run-up values;

$q$  instantaneous discharge of water over unit length of seawall;

$Q$  mean overtopping discharge over unit length of seawall;

$Q_{\text{PRED}}$  predicted mean overtopping discharge over unit length of seawall;

$Q*$  dimensionless overtopping discharge;

$R_c$  seawall freeboard (the height of the crest of the structure above the still-water-level);

$R_{\text{max}}$  maximum run-up ($=CH_\alpha$);

$(R_{\text{max}})_{p\%}$  p% confidence value of the estimated maximum run-up;

$R_s$  significant wave run-up;

$R_{2\%}$  run-up exceeded by only 2% of the incident waves;

$R*$  dimensionless freeboard;

$t$  time;

$T$  wave period;

$T_m$  mean zero-crossing wave period;

$T_p$  wave period corresponding to peak spectral density;

$T_s$  significant wave period;

$\alpha$  angle of seawall front slope measured from horizontal;

$\beta$  angle of wave approach measured from the normal to the seawall;

$\gamma$  reduction factor to account for influences of berms, roughness, shallow water and oblique wave attack on wave run-up and overtopping;

$\eta$  water surface elevation above still-water-level;

$\pi$  3.14159… ;

$\xi_p$  surf similarity parameter calculated using the period of peak spectral density ($= \tan \alpha / \sqrt{H_s / L_{op}}$).
1. Introduction

An important criterion for the design of a seawall is the allowable degree of wave overtopping which depends upon the activities normally performed in the lee of the structure, the need to prevent erosion of the rear face of the seawall, and the economic consequences of flooding. During 1978 and 1979, Owen (Hydraulics Research Station, 1980; Owen, 1982) carried out an extensive series of model tests to determine the overtopping discharges for a range of seawall designs subjected to different random wave climates. The modelled seawalls were all of the same general type: a flat-topped embankment fronted in some cases by a flat berm. The tests were aimed at establishing the impact on overtopping discharge of the wave climate (including the angle of wave attack and the wave steepness), the seawall slope, the crest and berm elevations, and the berm width.

This paper presents a re-analysis of Owen’s data: the results for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, subjected to random waves approaching normal to the slope. The purpose has been to construct a new regression model to represent the data which accounts for the fact that there is no overtopping (apart from wind-blown spray) if the seawall freeboard exceeds the maximum wave run-up on the face of the structure. The practical implications of using the new model are illustrated.

2. Dimensional analysis

In general, the mean overtopping discharge per unit length of seawall, \( Q \), depends upon the wave motion, the seawall profile, the foreshore characteristics and the water properties:

\[
Q = \text{function}(H_s, T_m, \beta, R_c, \alpha, d_s, g, \ldots)
\]

\( H_s \) is the significant height of the incident waves; \( T_m \) is the mean zero-crossing wave period; \( \beta \) is the angle of wave approach measured from the normal to the seawall; \( R_c \) is the seawall’s freeboard (the height of the crest of the structure above the still-water-level); \( \alpha \) is the angle of the seawall front slope measured from the horizontal; \( d_s \) is the still-water-depth at the toe of the structure; and \( g \) is the acceleration due to gravity (see Figure 1).

Equation 1 may be rewritten in the form of dimensionless groups:
\[
\frac{Q}{\sqrt{gH_s^3}} = \text{function}(\frac{R_\epsilon}{H_s}, \frac{H_s}{gT_m^2}, d_s, \alpha, \beta, \ldots) \quad (2)
\]

or
\[
\frac{Q}{g^2T_m^3} = \text{function}(\frac{R_\epsilon}{H_s}, \frac{H_s}{gT_m^2}, d_s, \alpha, \beta, \ldots) \quad (3)
\]

\(H_s / gT_m^2\) is a measure of the incident wave steepness. Owen combined this group both with \(Q / \sqrt{gH_s^3}\) (or \(Q / g^2T_m^3\)) and with \(R_\epsilon / H_s\), to write:

\[
\frac{Q}{T_m gH_s} = \text{function}(\frac{R_\epsilon}{T_m gH_s}, \frac{H_s}{gT_m^2}, d_s, \alpha, \beta, \ldots) \quad (4)
\]

However, other arrangements are possible (Aminti & Franco, 1988; Ahrens & Bender, 1992), including use of the wave period of peak spectral density, \(T_p\), rather than the mean zero-crossing period, \(T_m\). Table 1 summarises some of the options for dimensionless discharge, \(Q^*\), and dimensionless freeboard, \(R^*\). Here, \(H_s\) has been used to denote the significant wave height calculated either as the mean height of the highest one third of the waves in a record or estimated from the zeroth moment of the surface elevation spectrum (IAHR, 1989). \(L_s\) is the Airy wavelength calculated using the water depth at the toe of the structure and the significant wave period, \(T_s\). \(L_m\) and \(L_p\) are the corresponding wavelengths calculated using \(T_m\) and \(T_p\). \(R_{2\%}\) is the run-up exceeded by only 2% of the incident waves. \(\gamma\) is a reduction factor to account for influences of berms, roughness, shallow water and oblique wave attack on wave run-up and overtopping. \(\xi_p\) is the surf similarity parameter calculated using the wave period of peak spectral density \(\left(\xi_p = \tan \alpha / \sqrt{H_s / L_{op}}\right)\) in which \(L_{op} = gT_p^2 / 2\pi\).

Finally, note that \(R^*\) for De Waal & Van der Meer (1992) is not strictly a dimensionless freeboard but the dimensionless excess of the crest level above the 2% run-up level.

The dimensionless groups in Table 1 are generally related using one of the two following functions:

\[
Q^* = A \exp(-BR^*) \quad (5)
\]

\[
Q^* = A (R^*)^B \quad (6)
\]
where A and B are best-fit coefficients determined from the experimental data. Clearly, coefficients A and B must account for all influences on Q* other than R*.

Dimensional analysis provides no means for determining which sets of dimensionless groups may be especially informative or helpful in dealing with a particular data set. A possible problem in using many of the pairings in Table 1 is the potential for spurious correlation. A spurious correlation may arise when dimensionless groups plotted against one another contain a common variable (Massey, 1971). There is nothing wrong with the presence of a common variable, but care must be taken in interpreting such plots. Scatter in the data may be suppressed simply by the presence of this variable.

3. Regression analysis
3.1. Introduction

Once experimental data have been collected, they may be used to confirm the validity of some theory or, where no satisfactory theory exists, they may be used to construct regression models. However, it is always useful to have some theoretical basis for choosing amongst the possible models. Furthermore, there are many techniques available for fitting regression models (Gunst & Mason, 1980). Which ones are appropriate for a particular study depend upon its objectives. For example, it may be possible to develop a model which is good at predicting values of the response variable but which, nevertheless, is incorrectly specified (i.e. the model does not include all relevant predictor variables or it has an incorrect functional form). In describing a regression model, care should be taken to emphasise the range of conditions over which there are data to support its use. Unfortunately, it is sometimes impossible to collect data on the dependent or response variable (in this instance, overtopping) over the entire range of interest of the independent or predictor variables (wave height, structure profile, etc).

As a start, let us consider the physical boundary conditions to be met in addressing wave overtopping:

i) when the embankment has a large freeboard (i.e. when its crest elevation is well above the level of wave uprush), the predicted overtopping discharge should be zero (assuming that the effects of wind-blown spray are ignored);

ii) when the embankment has zero freeboard (i.e. when still-water-level is at the crest level of the embankment) then the predicted overtopping discharge may
be large but should still remain finite.

Equations 5 and 6 represent two of the more common functions used to predict wave overtopping. However, when $R^*$ is large, both expressions suggest that the discharge will be finite rather than zero (though it is small provided that $A$ is not very large and provided also that $B>1$). When $R^*$ is zero, the first of these expressions gives $Q^* = A$, a finite quantity, whilst the second expression gives $Q^* = \infty$. Thus neither expression satisfies both boundary conditions, with the second of them satisfying neither. Since most seawalls are designed to permit only relatively small overtopping discharges, it is especially important to satisfy the first of the two boundary conditions.

In addition to considering the boundary conditions, we also need to establish the line of “best fit” to the observed data. There are many criteria for defining the best fit. One possibility is to minimise the sum of the squared deviations of the observations from the values predicted from our expression. But real data usually do not completely satisfy the classical assumptions for a least-squares (LS) fitting (Rousseeuw & Leroy, 1987). For example, the deviations may not follow a Normal distribution. Reliable inferences may be drawn from regression models fitted by the LS method only if the assumptions are valid (Draper & Smith, 1981; Rousseeuw & Leroy, 1987). Furthermore, an LS fitting has the disadvantage that the result is not “robust”: it is sensitive to outlying data points. Whilst we could remove “outliers”, such a procedure should only be considered if there is reason to doubt their validity. Such data must not be removed merely because they do not support our regression model: it may be the model which is wrong.

Performing a least-absolute-deviations (LAD) fitting, involves minimising the sum of the absolute deviations rather than the sum of the squared deviations. It does not rely upon the Normal assumption and allows us to retain outliers but prevents these points from exerting a disproportionate influence on the values of the regression coefficients. If the deviations are assumed to follow a Double Exponential distribution, which has thicker tails than the Normal distribution, then the parameter values are maximum likelihood estimates. Figure 2 shows an example of the different results obtained from using the two techniques. Figure 2(a) is a plot of five points which lie almost on a straight line. Therefore, the LS fit and the LAD fit are essentially the same. Figure 2(b) displays a situation where, for some reason, point 4 has been wrongly moved from its original position (indicated by the dashed circle). This point is called an outlier in the $y$-direction; it has a rather strong influence on the LS line, which is quite different from the line in Figure 2(a). Figure 2(c) shows the robustness of the LAD fit with
respect to such an outlier; the line remains (approximately) where it was when observation 4 was correct. In this paper, we have chosen to fit our regression lines using the LAD method. However, we have also compared these results with an LS fitting.

3.2. A new regression model

3.2.1. A simple overtopping theory for regular waves

Let us step back from the complications of random waves to the simpler case of regular waves of height $H$ approaching normal to a seawall. We will assume that the instantaneous discharge of water over unit length of the seawall, $q$, is given by the weir formula (Streeter & Wylie, 1979):

$$q = C_d \frac{2}{3} \sqrt{2g} (\eta - R_c)^{3/2} \quad \text{for } \eta > R_c$$

(7)

in which $\eta$ is the water surface elevation above still-water-level at the seawall (a periodic function of time); $C_d$ is a discharge coefficient. Obviously, overtopping occurs only when the water surface is above the structure’s crest.

We will also assume that:

$$\eta = k HF(t)$$

(8)

$F(t)$ denotes a function of time, $t$. For simple, sinusoidal, progressive waves, $k=0.5$ and $F(t)=\cos(2\pi t/T)$, where $T$ is the wave period. However, following Kikkawa et al (1968), we will adopt the simpler form for $F(t)$ shown in Figure 3; $k$ remains a coefficient determined by the particular wave and wall details.

The mean discharge, $Q$, is determined as follows:

$$Q = C_d \frac{2}{3} \sqrt{2g} \int_{t_1}^{t_2} \left\{k HF(t) - R_c\right\}^{3/2} dt$$

(9)

in which $t_1 < t < t_2$ corresponds to the interval during each wave period for which $k HF(t) > R_c$. Using the form for $F(t)$ given in Figure 3 then yields:
Note that overtopping occurs only when \( R_c < kH \). In other words, \( kH \) represents the run-up on the face of the seawall. Since wave run-up is a function of the incident wave height and steepness, and of the seawall slope, the overtopping discharge can be expected also to depend upon these parameters.

### 3.2.2. The Hedges & Reis (H&R) overtopping model

The above theory suggests a regression equation for the random overtopping data of the following form:

\[
Q = A (1 - R_c) R_{\max}^{1 - B} \quad \text{for } 0 \leq R_c < 1
\]

\[
Q = 0 \quad \text{for } R_c \geq 1
\]

in which

\[
Q = \frac{Q}{\sqrt{gR_{\max}^3}} = \frac{Q}{\sqrt{g(CH_s)^3}}
\]

and

\[
R_c = \frac{R_c}{R_{\max}} = \frac{R_c}{CH_s}
\]

Coefficient \( k \) in the expression for regular waves has been replaced by \( C \) in this regression model for random waves characterised by \( H_s \). Note that \( CH_s \) represents \( R_{\max} \), the maximum run-up induced by the random waves, not the run-up induced by a wave of height \( H_s \). Consequently, \( C \) will depend upon the duration of the incident wave conditions unless the wave heights in front of the wall are limited by the available water depth. Unless the maximum run-up, \( R_{\max} \), exceeds the freeboard, \( R_c \), there is no overtopping (apart from wind-blown spray). It is also clear that coefficient \( B \) is related, in the case of regular waves, to the shape of the function \( F(t) \) which describes the water surface variation on the seaward face of the wall. There will be a similar dependence on the detailed behaviour of the water surface at the face of the wall in the case of random waves. Finally, coefficient \( A \) represents the dimensionless discharge over the seawall when the freeboard is zero. All three coefficients will be

\[
\frac{Q}{\sqrt{g(kH)^3}} = C_d 2\sqrt{2} \left\{ 1 - \frac{R_c}{kH} \right\}^{5/2} \quad \text{for } 0 < R_c < kH
\]

\[
= 0 \quad \text{for } R_c \geq kH
\]
influenced by the seaward profile of the structure.

The above model for overtopping has the advantage that \( Q^* = 0 \) when \( R^* \geq 1 \) and that \( Q^* = A \) when \( R^* = 0 \), in accordance with our required boundary conditions. Figure 4 shows the influences of coefficients A, B and C in the new overtopping model.

The value of C (=\( R_{\text{max}}/H_s \)) to be adopted would best be determined from experimental data. Unfortunately, Owen’s data set (and others) do not provide an adequate number of cases involving zero or very small discharges. Consequently, its value has been estimated from run-up measurements for random waves acting on slopes for which there is no overtopping. Although not ideal for our purposes, these additional data on run-up complement Owen’s overtopping results, allowing the new model to be applied outside the range of his experimental data. We have adopted this option rather than including C alongside A and B as a regression coefficient.

A number of equations describing random wave run-up are available (CIRIA/CUR, 1991; Van der Meer & Janssen, 1995). For example, the CIRIA/CUR (1991) manual gives two equations for evaluating the significant wave run-up, \( R_s \), on smooth slopes without overtopping. It notes that the equations, based upon Ahrens' data (Ahrens, 1981), are probably conservative and that data from Allsop et al (1985) give values 20 to 30% lower. Rewritten in our notation and allowing for a printing error, the expressions are:

\[
\frac{R_s}{H_s} = 1.35 \xi_p \quad \text{for} \quad \xi_p < 2
\]

\[
\frac{R_s}{H_s} = 3.00 - 0.15 \xi_p \quad \text{for} \quad \xi_p > 2
\]

Here, \( \xi_p \) is the surf similarity parameter calculated using \( T_p \) which was estimated for Owen’s data using the relationships between \( H_s \), \( T_m \) and \( T_p \) provided by Isherwood (1987).

Assuming that run-up may be described by a Rayleigh distribution, then the \( p\% \) confidence value of maximum run-up (defining a level below which \( p\% \) of the cases should lie) is related to the significant wave run-up by (Hogben, 1990):

\[
(R_{\text{max}})_{p\%} = \left[ \frac{1}{2} \left( \ell \ln N - \ell \ln \left[ - \ell N \left( \frac{p}{100} \right) \right] \right) \right]^{1/2} R_s
\]
N is the number of run-up values, here taken conservatively to be equal to the number of incident waves.

Owen recorded his overtopping discharges during tests involving sets of five different runs, each of 100 waves, characterised by the same significant wave height. The most probable maximum run-up during each run (the value not exceeded in 37% of the cases for a Rayleigh distribution of run-ups) is then:

\[ (R_{\text{max}})_{37\%} = \sqrt{(\frac{n}{100}) / 2 \cdot R_s} = 1.52R_s \]  

(16)

In none of Owen’s cases were there overtopping for freeboards greater than \((R_{\text{max}})_{37\%}\) if \(R_s\) was evaluated using equations 14. In fact, all nine reported cases of zero overtopping were for freeboards of less than this value. Hence, setting \(C = (R_{\text{max}})_{37\%} / H_s\) is conservative in this instance and the following expressions for C then arise from equations 14 and 16:

\[ C = 1.52(1.35 \xi_p) \quad \text{for} \quad \xi_p < 2 \]

\[ C = 1.52(3.00 - 0.15 \xi_p) \quad \text{for} \quad \xi_p > 2 \]  

(17)

The fact that these expressions for C are conservative may be a result either of the conservative nature of equations 14 or of deficiencies in the assumptions relating to the distribution of run-ups. However, setting \(C = (R_{\text{max}})_{37\%} / H_s\) may not always be appropriate. Note that the value of C to be adopted in the regression model depends upon the level of confidence associated with the prediction of \(R_{\text{max}}\). If C is changed then there will be corresponding changes in the values of A and B. The implications for seawall freeboards of adopting different levels of confidence in \(R_{\text{max}}\) are considered later.

4. Results of regression analysis

Figure 5 shows an example of the overtopping data collected by Owen: the results for a simple seawall with a uniform front slope of 1:2. The data are plotted in the formats required for fitting regression equations using both the H&R and the Owen models. Figure 5(a) shows the best-fit lines established using LS and LAD procedures for the H&R model. Comparison of the regression coefficients shows the relatively stronger influence which outlying data points have on the LS values. For example, the
magnitude of B obtained from the LAD fitting is only about 92% of the LS result. Similar comments may be made about the regression lines obtained for Owen’s model. Note, that the values of A and B reported in Figure 5(b) for Owen’s model are not those which Owen himself recommended.

Table 2 gives the regression coefficients for all three slopes which we have obtained for the H&R model and for Owen’s model, using both LS and LAD fitting. Also included for reference are Owen’s recommended values.

Owen restricted his analysis to a particular set of conditions whilst we have included all available data apart from eleven of the 110 results for the 1:4 slope. Of these eleven, nine had Q=0. Figure 4 shows that there will be many values of R* for which Q=0 and data points with Q=0 must be excluded from a regression analysis, otherwise a regression line (if it could be fitted) would pass through these data rather than defining their lower limit. The other two excluded values (for which Q was not zero) were from a set of five runs with the same dimensionless freeboard, three of which had zero overtopping discharge. Including only two of these five data points would have severely biased the positions of the regression lines. Furthermore, the validity of these two data points is doubtful since a full set of five runs at a smaller dimensionless freeboard all had Q recorded as zero. Although removed for the purposes of regression analysis, the eleven points were reinstated for inclusion in Figure 6 (see later). The full data set fell within the following ranges:

\[
0 < \frac{Q}{g(CH_s)^3} < 0.0056 \quad 0.14 < \frac{R_c}{CH_s} < 0.90
\]
\[
0 < \frac{Q}{T_m gH_s} < 0.0039 \quad 0.053 < \frac{R_c}{T_m gH_s} < 0.239
\]
\[
0.0053 < \frac{H_s}{gT_m^2} < 0.0095
\]
\[
1.65 < \frac{d_s}{H_s} < 5.20
\]

Earlier, we have mentioned briefly the problem of spurious correlation. Along with most other overtopping models (see Table 1), the H&R model employs a dimensionless discharge and a dimensionless freeboard which contain a common variable (in our case, it is \( R_{\max} \) or \( CH_s \)). The presence of this common variable may reduce the apparent scatter in the data. Consequently, in Figure 6 we show directly the level of agreement between Owen’s measured values of Q (converted by Owen to
full-scale discharges for a seawall in 4m water depth) and the predicted values, $Q_{\text{PRED}}$. Of course, we could also have attempted to disguise the scatter in the relationship between $Q$ and $Q_{\text{PRED}}$ by plotting against logarithmic scales (Massey, 1971). But such an attempt is both misleading and unnecessary. Under random wave conditions, overtopping will be dominated by the few waves with large run-ups: most waves will contribute no overtopping if the seawall has a substantial freeboard (Jensen & Juhl, 1987). Thus, particularly for short runs of random waves, as in Owen’s tests, we can expect some variability in the measured values of $Q$. Indeed, one of the purposes of Owen’s tests was to show this inherent variability.

In Figure 6, most data points lie within a range for $Q/Q_{\text{PRED}}$ of 3/4 to 4/3, whichever model is adopted. It is not obvious from the figure which model best fits the data, nor is it obvious from the plots for simple seawalls with 1:1 and 1:4 front slopes. Consequently, we have looked in more detail at the data points for discharges in the ranges of practical interest.

Figure 7 shows the critical mean overtopping discharges currently used in the design of seawalls. The main point to note from this figure is that the range of critical discharges runs from as little as $0.001 \times 10^{-3} \text{m}^3/\text{s/m}$ to about $200 \times 10^{-3} \text{m}^3/\text{s/m}$. An overtopping rate greater than about $0.001 \times 10^{-3} \text{m}^3/\text{s/m}$ will be unsafe for vehicles at high speed and may cause minor damage to the fittings of buildings. Conditions become dangerous for pedestrians when the discharge exceeds $0.03 \times 10^{-3} \text{m}^3/\text{s/m}$. Discharges greater than about $2 \times 10^{-3} \text{m}^3/\text{s/m}$ may damage embankment seawalls, whilst $50 \times 10^{-3} \text{m}^3/\text{s/m}$ is approximately the critical discharge for seawalls without back slopes.

Designers will often wish to know the necessary crest height to limit overtopping to the relatively small amounts indicated above. For these purposes, the H&R model appears generally better than Owen’s model owing to its ability to predict zero overtopping at finite values of freeboard. Furthermore, the next section shows that it tends to give lower required crest levels than Owen’s model for small permissible discharges, offering lower environmental impact and potential cost savings.

5. Some implications for seawall freeboards

According to Owen’s model, the freeboard, $R_c$, necessary to limit overtopping to a specified value, $Q$, is given by:
The H&R model gives:

$$R_c = \frac{T_m \sqrt{gH_s}}{B} \ln \left\{ \frac{A T_m gH_s}{Q} \right\}$$  \hspace{0.5cm} (19)

Note that equation 19 incorporates the mean zero-crossing wave period, $T_m$, whilst equation 20 involves coefficient $C$ which has been described in terms of the period of peak spectral density, $T_p$. In order to compare the output from the two expressions, it has been assumed that the incident waves conform to the Pierson-Moskowitz spectrum. In this case (for $H_s$ in metres, with $T_m$ and $T_p$ in seconds):

$$T_m = 3.55 \sqrt{H_s} \quad ; \quad T_p = 5.00 \sqrt{H_s}$$  \hspace{0.5cm} (21)

Figure 8 provides a comparison between the freeboards predicted using equations 19 and 20 for embankments with uniform front slopes of 1:2 subject to random waves with a significant height of 2m. Similar figures could be prepared for embankments with seaward slopes of 1:1 and 1:4, for additional incident significant wave heights and for different values of the confidence level associated with $R_{max}$.

Owen stated (Hydraulics Research Station, 1980) that his empirical coefficients $A$ and $B$ were determined only for particular ranges of the dimensionless groupings given in equation 4. The conditions included the following: $10^{-6} < Q / T_m gH_s < 10^{-2}$ and $0.05 < R_c / T_m \sqrt{gH_s} < 0.30$. Many of the discharges shown in Figure 8 have $Q / T_m gH_s < 10^{-6}$. For the conditions of Figure 8, this limit is approximately equivalent to $Q < 10^{-4} m^3/s/m$. Nevertheless, Owen also suggested that it was possible to use his equation to extrapolate results when the dimensionless freeboard was such that the dimensionless discharge fell below $10^{-6}$. Thus, for a typical seawall in 4m water depth, it is possible to compare the minimum necessary freeboards predicted by the H&R model with those predicted by Owen's expression if overtopping is to be limited to specified values.

Two points are worth noting:
i) There is reasonable agreement between the H&R model and Owen's model only for overtopping discharges in the range of $10^{-2} \text{m}^3/\text{s/m}$ to $2 \times 10^{-1} \text{m}^3/\text{s/m}$. This is irrespective of the confidence level (37% or 99%) assigned in the evaluation of $R_{\text{max}}$. However, it is in the range where there are significant differences that most seawalls are designed (see Figure 7).

ii) As the confidence level in $R_{\text{max}}$ is increased, the freeboards predicted by the H&R model approach those values obtained from Owen's model. Nevertheless, even using $(R_{\text{max}})_{99\%}$ there remain significant differences. This observation has important implications for seawall design. For example, for an expected overtopping discharge of $10^{-4} \text{m}^3/\text{s/m}$, the difference amounts to about 1.9m. It is even greater both for the lower expected overtopping rates associated with functional safety requirements (Figure 7) and for higher $H_s$ values. Owen's model suggests that the freeboard must continue to increase in order to reduce the overtopping rate even when the crest of the seawall is clearly above any possible run-up level induced by the random waves.

6. Concluding remarks

A new regression model has been presented for describing wave overtopping data. Part of our motivation was to improve the methods available to the designers of seawalls by developing a model closely related to the physics of wave overtopping. The important features of the model are as follows:

i) It satisfies the relevant physical boundary conditions, a feature which is especially important when the model is used near these boundaries.

ii) It explicitly recognises (through its foundations in a simple theoretical model for regular waves) that regression coefficient $A$ depends upon the shape of the structure since the shape, particularly at its crest, affects the discharge coefficient; coefficient $A$ represents the dimensionless discharge when the dimensionless freeboard is zero.

iii) Coefficient $B$ depends upon the detailed behaviour of the water surface on the seaward face of the structure; it increases as front slopes become flatter.

iv) Coefficient $C$ relates the maximum run-up to the significant height of the incident waves and may be chosen to allow for the influences of the seawall slope, the surface roughness and porosity, and the incident wave steepness. Coefficient $C$ can also account for storm duration in influencing $R_{\text{max}}$ (though the regression coefficients in the present study have been established only for short
sequences of 100 random waves). Finally, it may be chosen so that there is a specified confidence level associated with $R_{\text{max}}$.

It is suggested that the regression coefficients contained within the model should be established using a robust regression technique. Examples are given of the differences between the LS and the LAD fitting methods in analysing overtopping data collected by Owen (Hydraulics Research Station, 1980; Owen, 1982).

For the present test results, the H&R model is little different from Owen’s model in its ability to represent the data, except for small discharges for which the H&R model is better suited. An example is given of the application of Owen’s model and the H&R model to predict the freeboards necessary to limit overtopping to specified values. The example shows that, for the small allowable discharges associated with normal design conditions, the H&R model predicts seawall crest elevations which may be up to several metres lower than values from Owen's model. These differences may have very significant financial and environmental consequences and are worthy of further investigation.

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<tr>
<th>Authors</th>
<th>Dimensionless Discharge, $Q$</th>
<th>Dimensionless Freeboard, $R$-</th>
<th>Overtopping Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulics Research Station (1980); Owen (1982)</td>
<td>$\frac{Q}{T_m gH_s}$</td>
<td>$\frac{R_c}{T_m \sqrt{gH_s}}$</td>
<td>$Q. = A \exp(-B\cdot R.)$</td>
</tr>
<tr>
<td>Bradbury &amp; Allsop (1988)</td>
<td>$\frac{Q}{T_m gH_s}$</td>
<td>$\frac{R_c^{2}}{T_m \sqrt{gH_s}^{2}}$</td>
<td>$Q. = A (R.)^{-B}$</td>
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<tr>
<td>Ahrens &amp; Heimbaugh (1988)</td>
<td>$\frac{Q}{\sqrt{gH_s^{3}}}$</td>
<td>$\frac{R_c}{(H_s L_p)^{1/3}}$</td>
<td>$Q. = A \exp(-B\cdot R.)$</td>
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<tr>
<td>Sawaragi et al (1988)</td>
<td>$\frac{Q}{\sqrt{gL_s H_s^{2}}}$</td>
<td>$\frac{R_c}{H_s}$</td>
<td>$\text{---------}$</td>
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<tr>
<td>Aminti &amp; Franco (1988)</td>
<td>$\frac{Q}{T_m gH_s}$</td>
<td>$\frac{R_c}{H_s}$</td>
<td>$Q. = A (R.)^{-B}$</td>
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<tr>
<td>Pedersen &amp; Burcharth (1992)</td>
<td>$\frac{QT_m}{L_m^{2}}$</td>
<td>$\frac{R_c}{H_s}$</td>
<td>$Q. = A (R.)^{-B}$</td>
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<td>De Waal &amp; Van der Meer (1992)</td>
<td>$\frac{Q}{\sqrt{gH_s^{3}}}$</td>
<td>$\frac{R_c - R_{2%}}{H_s}$</td>
<td>$Q. = A \exp(-B\cdot R.)$</td>
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<tr>
<td>Van der Meer (1993); Smith et al (1994); Van der Meer &amp; Janssen (1995)</td>
<td>$\frac{Q}{\sqrt{gH_s^{3}}}$ for $\xi_p &lt; 2$</td>
<td>$\frac{R_c}{H_s}$ for $\xi_p &lt; 2$</td>
<td>$Q. = A \exp(-B\cdot R.)$</td>
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<tr>
<td></td>
<td></td>
<td>$\frac{Q}{\sqrt{gH_s^{3}}}$ for $\xi_p &gt; 2$</td>
<td>$\frac{R_c}{H_s}$ for $\xi_p &gt; 2$</td>
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<tr>
<td>Franco et al (1994)</td>
<td>$\frac{Q}{\sqrt{gH_s^{3}}}$</td>
<td>$\frac{R_c}{H_s}$</td>
<td>$Q. = A \exp(-B\cdot R.)$</td>
</tr>
</tbody>
</table>

**Table 1:** Some options for dimensionless discharge, dimensionless freeboard and overtopping model.

<table>
<thead>
<tr>
<th>H&amp;R MODEL (C given by $R_{\text{max}}^{37%}$)</th>
<th>H&amp;R MODEL (C given by $R_{\text{max}}^{99%}$)</th>
<th>OWEN’S MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAD/LD</td>
<td>LS</td>
<td>LAD/LD</td>
</tr>
<tr>
<td>Slope 1:1 A</td>
<td>0.00703</td>
<td>0.00581</td>
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<tr>
<td>B</td>
<td>3.42</td>
<td>3.22</td>
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<td>Slope 1:2 A</td>
<td>0.00753</td>
<td>0.00790</td>
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<tr>
<td>B</td>
<td>4.17</td>
<td>4.55</td>
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<td>Slope 1:4 A</td>
<td>0.0104</td>
<td>0.00792</td>
</tr>
<tr>
<td>B</td>
<td>6.27</td>
<td>5.94</td>
</tr>
</tbody>
</table>

**Table 2:** Regression coefficients for use in the H&R model and Owen’s model. Also included for reference are Owen’s recommended (Rec.) values.