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# Finding the elastic coefficients of a damaged zone in a concrete dam using material optimization to fit measured modal parameters

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The structural safety control of large dams is based on the continuous observation of its behavior under static and dynamic actions and on the use of mathematical models for the simulation of the real dam behavior. These mathematical models should be calibrated taking into account the real behavior of the system dam-reservoir-foundation (mainly influenced by the reservoir water level). For the safety control of old dams, usually with cracking, the mathematical models should include some features for crack simulation.

In this paper the dynamic behavior of a cracked arch dam - Cabril dam - is studied by means an elastic 3D FE model that was calibrated using an optimization technique for the computation of the main elastic parameters that must be used for the cracked zone in order to fit the experimentally measured natural frequencies. Changes on modal parameters are associated not only to water level variations but could be also correlated with structural changes (due to accidental loads, as strong earthquakes, or deterioration processes along the time - e.g. deterioration associated with the development of concrete swelling). For this reason the identification of these changes may be interesting for the structural safety control. The Cabril dam is the highest concrete dam in Portugal, with 130 meters of height and crest length of 290 meters. It is an interesting case study because a large number of cracks have arisen in a specific part of the dam since the beginning of its operations in 1954 (see figure 1(a)). These cracks have remained even after reparation works done in 1981, see [1].

The free vibrations of the dam are continuously measured by a dynamic monitoring system installed in 2008 (see [2] and [3]). The values of the measured natural frequencies are used in our approach. Since the cracks are mainly disposed in the horizontal direction, a transversely isotropic constitutive law is chosen to model the damaged zone of the dam (in blue in figure 1(b)). The rest of the dam and the adjacent valley (in yellow in figure 1(b)) are modeled using an isotropic constitutive law. The transversely isotropic constitutive law

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_p} & \frac{-\nu_p}{E_p} & \frac{-\nu_{zp}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_p}{E_p} & \frac{1}{E_p} & \frac{-\nu_{zp}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{zp}}{E_z} & \frac{-\nu_{zp}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_p)}{E_p} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{zp}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zp}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} \tag{1}$$

is characterized by five independent elastic constants: $E_p$, $\nu_p$, $E_z$, $\nu_{zp}$ and $G_{zp}$. For $E_p$ and $\nu_p$ the standard values of the concrete can be used, as done for the isotropic constitutive law of the undamaged part of the dam. The other three constants $E_z$, $\nu_{zp}$ and $G_{zp}$ remain unknown.

The goal of the present work is to find an approximation of these three values by solving an inverse problem: given the knowledge of the first natural frequencies of the dam, we will search for elastic constants such that the solutions of the corresponding eigenvalue problem are the closest to the given frequencies.

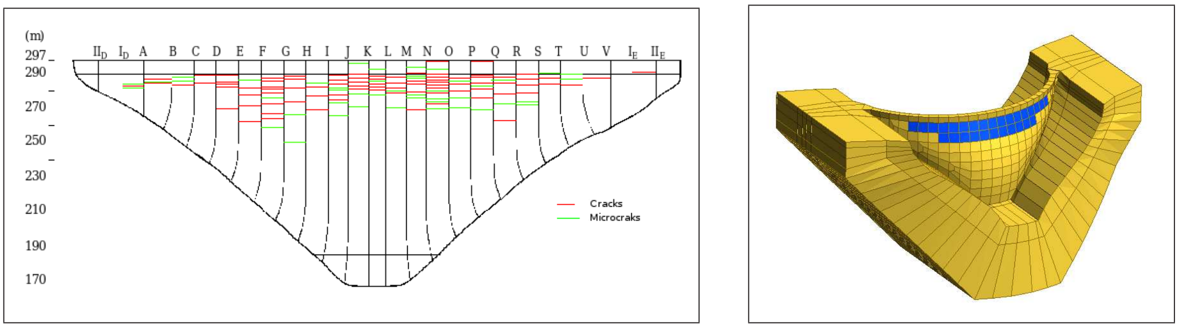
More precisely, the eigenvalue problem below is considered

$$\begin{cases} -div\big(C\varepsilon(u)\big) = \lambda u\,, & \text{in the dam,} \\ u = 0\,, & \text{on the fixed boundary,} \\ C\varepsilon(u)\,.\,n = 0\,, & \text{on the free boundary,} \end{cases} \tag{2}$$

where $C$ is the fourth-order elasticity tensor. Its solutions $(\lambda_k, u_k)_{k\geq 1}$ depend on the five elastic constants characterizing the transversely isotropic law according to (1). We consider the following parametric optimization problem:

$$\min_{E_z,\nu_{zp},G_{zp}} (\lambda_1 - \overline{\lambda}_1)^2 + (\lambda_2 - \overline{\lambda}_2)^2 + (\lambda_3 - \overline{\lambda}_3)^2 \tag{3}$$

where $\overline{\lambda}_1$, $\overline{\lambda}_2$, $\overline{\lambda}_3$ are the first three (physically measured) natural frequencies of the dam. A steepest descent method is employed for solving (3). It makes use of the derivatives of the natural frequencies $\lambda_1$, $\lambda_2$, $\lambda_3$ with respect to the optimization parameters $E_z$, $\nu_{zp}$ and $G_{zp}$. This approach is related to the field of free material optimization, where the elastic moduli are the variables to be optimized to satisfy some functional cost (see section 3 of [4]). The FEM is used for solving numerically problem (2). The whole algorithm is implemented in C++ using the open-source library libMesh (see [5]).




(a) (b)

Figure 1: Cabril dam: outline with the location of cracks (a); mesh with a subdomain relative to the damaged zone (b)

## References

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